Interference effects between the initial and final state radiation in a QCD medium

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Abstract. Coherence between multiple emitters is one of the most remarkable properties of QCD jet physics in vacuum. In the presence of a QCD medium one expects that the coherence pattern between multiple emitters is affected. In this work we calculate the gluon emission spectrum off an "asymptotic quark" traversing a dilute QCD medium. When interferences between initial and final quark are included, the medium induced gluon distribution gets modified. The coherent, incoherent and soft limit of the medium induced spectrum are studied. In the soft limit we find an elegant and intuitive probabilistic interpretation. We comment on possible phenomenological applications of this setup for studying coherence effects on observables studied in high energy nuclear collisions.

1. Introduction

Jets produced in high energetic hadronic collisions are an important tool to understand the theory of strong interactions, i.e., QCD. One of the most important properties of jets is the coherence pattern of soft gluon emissions which is related with the quantum interferences among different emitters in a parton cascade. One phenomenological consequence of color coherence phenomena in QCD is the suppression of radiation at large angles thanks to the angular ordering of subsequent soft gluon emissions. To illustrate this, let us consider soft gluon radiation in a deep inelastic process for the color singlet configuration (t channel). In this case, the probability to emit a gluon from any of the emitters is [1]

$$\langle dN_{in}\rangle_\phi = \frac{\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\frac{d(\cos\theta)}{1-\cos\theta}\Theta(\cos\theta-\cos\theta_{qq})\,, \tag{1}$$

where θ is the angle of emitted gluon and θ_{qq} is the angle between the incoming and the outcoming parton due to hard scattering with the virtual photon. From last expression we observe the presence of soft and collinear divergences which must be resummed[1]. In addition, Eq.((1)) indicates us that gluon radiation is going to be confined inside the cone defined by the opening angle θ_{qq} along the emitter. Hence, there is no large angle emission.

In the presence of a QCD medium of finite size the efforts so far have been concentrated to understand the medium induced gluon radiation due to one emitter[2]. Nevertheless, not much is known about color coherence effects in the presence of a QCD medium. Recently, this important issue has been started to be studied by considering the radiation pattern of a $q\bar{q}$ antenna immersed in a QCD medium [3]. In this work, we briefly review an extension of color coherence studies inside a QCD medium to a space-like (t-channel) scattering process.

2. Scattering amplitude from Classical Yang Mills equations

A suitable method to describe the radiation of soft gluons is by using semiclassical methods of pQCD. In this approach the radiated gluon A_μ^a is treated as a fluctuation on top of a static background gauge field $A_{\rm med}$ (the medium). In the presence of the current generated by the incoming and outcoming quark, J^μ, the CYM equations read

$$[D_\mu, F^{\mu\nu}] = J^\mu \tag{2}$$

where $D_\mu = \partial_\mu - igA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ and g is the gluon coupling constant. For the case of interest, we have two currents associated to the incoming quark[1] is given by $J_{inc}^{\mu,a} = g\, p^\mu/E\Theta(-t)\delta^{(3)}(\vec{x} - \vec{p}/Et)\Theta(-t)Q_{inc}^a$, where $p \equiv (E, \vec{p})$ is the energy and 3-momentum and Q_{inc}^q the color charge which are found through the continuity equation, $[D_\mu, J^\mu] = 0$ with $J = J_{inc} + J_{out}$. Finally, the amplitude of emitting a gluon with momentum $k \equiv (\omega, \vec{k})$ is given by

$$\mathcal{M}_\lambda^a(\vec{k}) = \lim_{k^2\to 0} -k^2 A_\mu^a(k)\epsilon_\lambda^\mu(\vec{k})\,, \tag{3}$$

where $\epsilon_\lambda^\mu(\vec{k})$ is the gluon polarization vector. In the following, we will work in light-cone gauge $A^+ = 0$. We consider in addition that the medium is created at $t = 0$. The amplitude for soft gluon emission reads[4]

$$\begin{aligned}\mathcal{M}_\lambda^a &= 2ig^2 \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \int_0^{L^+} dx^+ \, [T\cdot A_{\rm med}^-(x^+,\boldsymbol{q})]^{ab} \Bigg\{ Q_{in}^b \, \frac{\boldsymbol{\kappa}-\boldsymbol{q}}{(\boldsymbol{\kappa}-\boldsymbol{q})^2} \\ & -Q_{out}^b \, \left[\frac{\bar{\boldsymbol{\kappa}}-\boldsymbol{q}}{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2} \left[1-\exp\left(i\frac{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2}{2k^+}x^+\right)\right] + \frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2}\exp\left(i\frac{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2}{2k^+}x^+\right)\right]\Bigg\}\,,\end{aligned} \tag{4}$$

where $Q_{in(out)}^b$ is the color charge of the in(out)coming parton, $\boldsymbol{\kappa} = \mathbf{k} - x\mathbf{p}$ is the transverse momentum of the gluon relative to the incoming parton (similar definition for $\bar{\boldsymbol{\kappa}}$) and L^+ is the length of the medium.

3. Gluon Spectrum and its asymptotic limits

After taking the square of the scattering amplitude ((4)), the medium induced gluon spectrum is given by [4]

$$\begin{aligned}\omega\frac{dN^{\rm med}}{d^3\vec{k}} &= \frac{4\alpha_s C_F \hat{q}}{\pi} \int_0^{L^+} dx^+ \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\mathcal{V}^2(\boldsymbol{q}) \Bigg[\frac{1}{(\boldsymbol{\kappa}-\boldsymbol{q})^2} - \frac{1}{\boldsymbol{\kappa}^2} + 2\frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{q}}{\bar{\boldsymbol{\kappa}}^2(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2}\left(1-\cos\left[\frac{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2}{2k^+}x^+\right]\right) \\ &+2\left\{\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\bar{\boldsymbol{\kappa}}^2} - \frac{\bar{\boldsymbol{\kappa}}\cdot(\boldsymbol{\kappa}-\boldsymbol{q})}{\bar{\boldsymbol{\kappa}}^2(\boldsymbol{\kappa}-\boldsymbol{q})^2}\right\} + 2\left\{\frac{\bar{\boldsymbol{\kappa}}\cdot(\boldsymbol{\kappa}-\boldsymbol{q})}{\bar{\boldsymbol{\kappa}}^2(\boldsymbol{\kappa}-\boldsymbol{q})^2} - \frac{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})\cdot(\boldsymbol{\kappa}-\boldsymbol{q})}{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2(\boldsymbol{\kappa}-\boldsymbol{q})^2}\right\}\left(1-\cos\left[\frac{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2}{2k^+}x^+\right]\right)\Bigg]\,,\end{aligned} \tag{5}$$

The gluon spectrum is composed by three parts: (i) the gluon emission off the incoming parton which is the bremstrahlung of an accelerated color charge which undergoes rescattering, (ii)

[1] For the outoming quark one must change the argument of the Heaviside step function $\Theta(-t) \to \Theta(t)$ and the 4-momentum $p^\mu \to \bar{p}^\mu$.

gluon emission off outcoming quark is identified with the medium induced radiation of the N=1 opacity expansion (GLV spectrum) [2] and (iii) the interferences between the incoming and the outcoming parton. In addition, the spectrum contains soft and collinear divergences.

3.1. Incoherent limit

For short formation times $\tau_f << L^+$, the phases cancel and the gluon spectrum ((5)) simplifies to [4]

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\tau_f \ll L^+} = \frac{4\,\alpha_s C_F\,\hat{q}}{\pi} \int\limits_0^{L^+} dx^+ \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \Big\{ \bar{\boldsymbol{L}}^2 + \boldsymbol{C}^2(\boldsymbol{\kappa}-\boldsymbol{q}) - \boldsymbol{C}^2(\boldsymbol{\kappa}) \Big\}\,, \qquad (6)$$

where we use the definition of the Lipatov vertex $\bar{\boldsymbol{L}}$ in the light cone gauge and the transverse emission current $\boldsymbol{C}(\boldsymbol{\kappa})$ respectively[2]

$$\bar{\boldsymbol{L}} = \frac{\bar{\boldsymbol{\kappa}}-\boldsymbol{q}}{(\bar{\boldsymbol{\kappa}}-\boldsymbol{q})^2} - \frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2}\,, \qquad \boldsymbol{C}(\boldsymbol{\kappa}) = \frac{\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2} - \frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2}. \qquad (7)$$

This limit indicates the two main mechanisms of gluon radiation: the first term of Eq. ((6)) is the radiation of an asymptotic parton that suffers a scattering with the medium. The remaining two terms correspond to bremsstrahlung associated with the hard scattering followed by the radiated gluon suffering a classical sequential process of rescattering with the medium.This result is a generalization of the probabilistic interpretation of the incoherent limit of the GLV spectrum by including interferences [2].

3.2. Coherent limit

In the case of large formation times $\tau_f \gg L^+$, the gluon spectrum is reduced to [4]

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\tau_f \gg L^+} = \frac{4\,\alpha_s C_F\,\hat{q}}{\pi} \int\limits_0^{L^+} dx^+ \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \Big\{ \frac{1}{(\boldsymbol{\kappa}-\boldsymbol{q})^2} - \frac{1}{\boldsymbol{\kappa}^2} + 2\frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{\kappa}}{\bar{\boldsymbol{\kappa}}^2\boldsymbol{\kappa}^2} - 2\frac{\bar{\boldsymbol{\kappa}}\cdot(\boldsymbol{\kappa}-\boldsymbol{q})}{\bar{\boldsymbol{\kappa}}^2(\boldsymbol{\kappa}-\boldsymbol{q})^2} \Big\} \qquad (8)$$

The first two terms in the last expression are the classical broadening of the gluon emission off the incoming quark. The next two terms are interferences. In the absence of interferences, the independent GLV gluon spectrum gets suppressed due to the LPM effect while in the case studied here some of the interferences among the initial and final parton remain.

3.3. Soft limit

In the soft limit $\omega \to 0$ it is possibe to separate the contributions associated with the initial and final state of the total gluon spectrum (vacuum + medium induced) which reads as [4]

$$\omega \frac{dN^{\rm tot}}{d^3\vec{k}}\bigg|_{\omega\to 0} = \frac{\alpha_s C_F}{(2\pi)^2} \left(\mathcal{P}_{in}^{tot} + \mathcal{P}_{out}^{tot}\right)\big|_{\omega\to 0}\,, \qquad (9)$$

where

$$\mathcal{P}_{in}^{tot} = \big(1-\Delta_{med}\big)\left(\frac{1}{\boldsymbol{\kappa}^2} - \frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{\kappa}}{\bar{\boldsymbol{\kappa}}^2\boldsymbol{\kappa}^2}\right), \qquad \mathcal{P}_{out}^{tot} = \left(\frac{1}{\bar{\boldsymbol{\kappa}}^2} - \big(1-\Delta_{med}\big)\frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{\kappa}}{\bar{\boldsymbol{\kappa}}^2\boldsymbol{\kappa}^2}\right). \qquad (10)$$

[2] An identical definition follows for $\boldsymbol{C}(\boldsymbol{\kappa}-\boldsymbol{q})$ by changing $\boldsymbol{\kappa}\to\boldsymbol{\kappa}-\boldsymbol{q}$.

The fact that we can split the contribution of each emitter to the gluon spectrum allow us to have a probabilistic interpretation: The first term $\mathcal{P}_{in}^{tot}$ is the coherent gluon emission off the incoming quark reduced by the probability Δ_{med} that the emitted gluon interacts with the QCD medium. In addition, the angular ordering property observed in the vaccuum case will be preserved of the case of the initial state radiation. The second term $\mathcal{P}_{out}^{tot}$ accounts for the partial decoherence of the emitted gluon due to the scatterings with the QCD medium which share some similarities observed in the $q\bar{q}$ antenna case [3]. Such decoherence is measured by the probability of an interaction with the medium Δ_{med}. As a consequence, the medium opens the phase space for large angle emissions, a property called antiangular ordering [3].

4. Summary

In this work we consider the impact of the interference pattern between the initial and final state radiation in the presence of a QCD medium. By using semiclassical methods of pQCD we determine the medium induced gluon spectrum of an asymptotic parton which suffers a hard collision and crosses a QCD medium afterwards. The gluon spectrum has three contributions: the independent gluon emissions associated with the incoming and outgoing parton as well as interference terms between both emitters. To shed light on the structure of the spectrum, we consider three asymptotic limits of the gluon spectrum: the incoherent, coherent and soft sector. In the incoherent limit, we find a probabilistic interpretation for the gluon spectrum is found which is a generalization of the GLV spectrum by accounting interferences among the two emitters. For the coherent limit the medium induced gluon spectrum reduces to the classical broadening contribution associated exclusively to the initial state and some of the interferences remain. In the soft limit, we provide a probabilistic interpretation for the radiation pattern of the full gluon spectrum which has two main important features: (i) gluon emissions from the initial state remain coherent as in vacuum but now these are reduced by the probability to interact with the medium measured by $\Delta_{med} \sim \hat{q}/m_D^2$ and (ii) gluon emission off the outcoming parton loses its vacuum coherence, i.e., large angle emissions (antiangular ordering) arise from the medium induced coherence between both emitters.

Our work is a first principles approach for the proper inclusion of coherence effects on observables sensitive to initial state radiation in high energy nuclear collisions.

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