Effective model for deconfinement at high temperature

Vladimir Skokov

Department of Physics, Brookhaven National Laboratory, Upton, NY, 11973 USA

E-mail: vskokov@quark.phy.bnl.gov

Abstract. In this talk I consider the deconfining phase transition at nonzero temperature in a SU(N) gauge theory, using a matrix model. I present some results including the position of the deconfining critical endpoint, where the first order transition for deconfinement is washed out by the presence of massive, dynamical quarks, and properites of the phase transition in the limit of large N. I show that the model is soluble at infinite N, and exhibits a Gross-Witten-Wadia transition.

1. The matrix model

In my talk, I addressed the properties of the thermodynamics of pure $SU(N)$ gauge theory in the so-called semi-QGP region for temperatures about or a few times that for the deconfining phase transition T_d.

A commonly accepted order parameter for the deconfinement phase transition is the Wilson line defined by

$$\mathrm{L}(\vec{x}) = \mathrm{P}\exp\left(ig\int_0^\beta A_0(\vec{x},\tau)d\tau\right). \tag{1}$$

An effective theory is constructed by expanding about a constant background field for the vector potential,

$$A_0^{ij} = \frac{2\pi T}{g}\, q_i\, \delta^{ij}, \tag{2}$$

where the field A_0 was diagonalized by a proper gauge transformation and where $i,j = 1\cdots N$ and the eigenvalues q_i are subject to the $SU(N)$ constraint $\sum_i^N q_i = 0$. In the matrix model q_i are the fundamental variables characterizing the transition. It is assumed that after integrating out the other components of gluon field A_i, that we obtain an effective potential for q_i ($\tilde{V} = V/N^2$) [1, 2, 3, 4, 5, 6]:

$$\tilde{V}_{eff}(q) = -d_1(T)\tilde{V}_1(q) + d_2(T)\tilde{V}_2(q), \tag{3}$$

$$\tilde{V}_n(q) = \sum_{i,j=1}^{N} |q_i - q_j|^n \left(1 - |q_i - q_j|\right)^n. \tag{4}$$

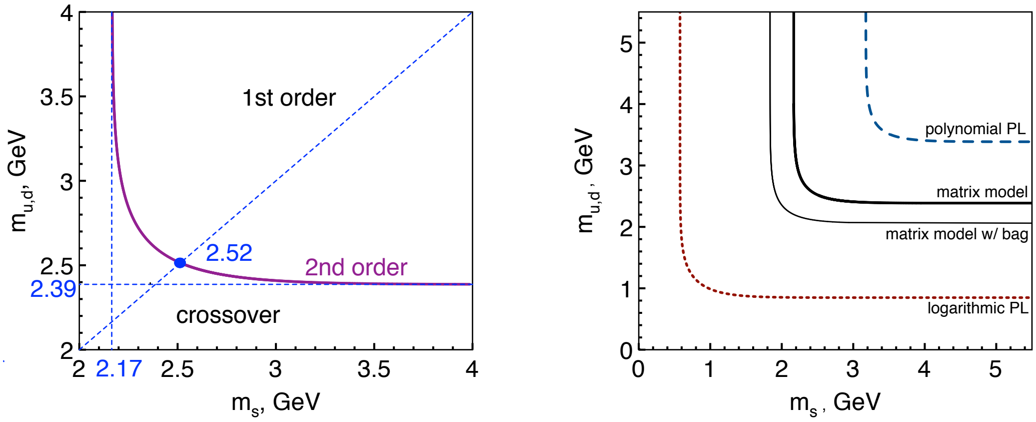

Figure 1. The left panel: The phase diagram for the deconfining phase transition in the matrix model, assuming $T_d \sim 270$ GeV. The right panel: The phase diagram of the deconfining phase transition for the matrix model without/with bag constant (bold/thin solid line) and the Polyakov loop models with the polynomial (dashed line) and logarithmic (dotted line) potentials. The lines correspond to the second order deconfining phase transition.

The potential includes both perturbative ($\tilde{V}_2$) and non-perturbative ($\tilde{V}_1$) contributions. The temperature-dependent functions d_1 and d_2 are given by

$$d_1(T) = \frac{2\pi}{15}\, c_1\, T^2\, T_d^2,\ d_2(T) = \frac{2\pi}{3}\left(T^4 - c_2 T^2 T_d^2\right). \tag{5}$$

The constants are fixed by comparison to the simulations of SU(3) on the lattice [4]. They are not important for further discussion.

2. Critical point of the deconfinement phase transition

When dynamical quarks are added one needs to compute the one-loop perturbative analogous contribution which they make to the q-dependent potential. This has been computed in Ref. [1]. For quarks of mass m, the one loop potential is given by

$$\ln\det(\gamma^\mu\partial_\mu + q\delta^{\mu 4} + im) = 2\ln\det\left[\left(\partial_0 + 2\pi T q\right)^2 - \vec{\partial}^2 + m^2\right], \tag{6}$$

As fermions, in the Matsubara formalism, the frequencies are odd multiples of πT. The straightforward calculations a single flavor of massive quark lead to the contribution to the potential

$$V^{qk}_{\rm pert}(\mathbf{q}) = \frac{2\,m^2\,T^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\,K_2\left(\frac{n\,m}{T}\right)\sum_{i=1}^{N}\cos(2\pi n q_i)\,. \tag{7}$$

For the pure glue theory, by a $Z(3)$ rotation we can assume that the Polyakov loop is real. With dynamical quarks, this remains true if all of the quark masses are real and the quark chemical potential vanishes. In the left panel of Fig. (1) the results are presented for the matrix model. The critical temperature changes very little from the pure gauge theory,

$$T_{\rm de} = .995\,T_d\,. \tag{8}$$

The results are compared for different models in the right panel of Fig. (1). For the logarithmic Polyakov loop model [7], the masses are light, $m^{de} \sim 1$ GeV. The temperature for the deconfining critical endpoint is significantly less than for the pure glue theory,

$$T_{de} = 0.90\, T_d\ . \tag{9}$$

A polynomial Polyakov loop model [8, 9] gives a very large mass, $m^{de} \sim 3.5$ GeV. The critical temperature is very close to the pure glue theory, $T_{de} \sim 0.996\, T_d$.

The sensitivity of T_{de} and m^{de} to the models' assumptions may allow to single out the model with the most appropriate description of lattice $SU(3)$ simulations. The detailed description can be found in Ref. [5].

3. Large N limit

In the infinite N limit it is convenient to introduce a continuous variable $x = \frac{i}{N}$. Labeling the eigenvalue $q_i \to q(x)$, one gets

$$\rho(q) = \lim_{N\to\infty} \frac{1}{N} \sum_i^N \delta(q - q_i) = \int_0^1 dx \delta[q - q(x)] = \frac{dx}{dq}\ . \tag{10}$$

At finite N, the identities

$$\sum_i^N 1 = N,\ \ \sum_i^N q_i = 0 \tag{11}$$

become, at infinite N,

$$\int dq \rho(q) = 1,\ \ \int dq \rho(q) q = 0. \tag{12}$$

The potential is proportional to N^2,

$$\begin{aligned} \tilde{V}_n(q) &= N^2 V_n(q) = N^2 \int dx\ dy\ |q(x) - q(y)|^n \left(1 - |q(x) - q(y)|\right)^n \\ &= N^2 \int dq\ dq'\ \rho(q)\ \rho(q')\ |q - q'|^n \left(1 - |q - q'|\right)^n . \end{aligned} \tag{13}$$

This representation transforms the potential into a polynomial in q.

The minimum of Eq. (13) was found in Ref. [6]:

$$\rho(q) = 1 + b \cos dq, \quad -q_0 < q < q_0, \tag{14a}$$

$$d = 2\sqrt{\frac{3 d_2}{d_1}}. \tag{14b}$$

$$\cot(dq_0) = \frac{d}{3}\left(\frac{1}{2} - q_0\right) - \frac{1}{d\,(1/2 - q_0)}, \tag{14c}$$

$$b^2 = \frac{d^4}{9}\left(\frac{1}{2} - q_0\right)^4 + \frac{d^2}{3}\left(\frac{1}{2} - q_0\right)^2 + 1. \tag{14d}$$

For $T > T_d$, $d > 2\pi$ and $q_0 < \frac{1}{2}$. The eigenvalues do not span the full range between $-\frac{1}{2}$ and $\frac{1}{2}$. The density is discontinuous at the end points $\rho(\pm q_0) > 0$. For $T = T_d^+$, $q_0 = \frac{1}{2}$ and $d = 2\pi$, the density is continuous for all values of q in $[-1/2, 1/2]$. In particular it vanishes at the end points $\rho(\pm q_0) = 0$. For $T < T_d$, the theory is in confined phase, with a uniform distribution of eigenvalues, Eq. (14a) with $q_0 = \frac{1}{2}$ and $b = 0$.

The Polyakov loop jumps at the transition from $L_c = 0$ to $L_d = 0.5$. Assuming that $\delta d \sim T_d - T$, one obtains that as $T \to T_d^+$,

$$L(T) - \frac{1}{2} \sim (T_d - T)^\beta \quad , \quad \beta = 2/5 \; . \tag{15}$$

Thus, near the transition $L(T)$ exhibits a power like behavior which is characteristic of a second order phase transition, although $L(T_d^+) \neq 0$. Therefore, the system exhibits a Gross-Witten-Wadia transition. That is, it exhibits aspects of both first order and second order phase transitions; thus it can be termed "critical first order" [10].

Other critical exponents besides β can be calculated, as was demonstrated in Ref. [6], they satisfy the usual Griffths scaling relation,

$$2 - \alpha = \beta(1 + \delta) \; . \tag{16}$$

The critical first order transition described above is clearly special to infinite N. At finite N, one expects a first order phase transition, and a smoothing of the critical behavior. This leads to a natural question: how large must N be to see such putative critical behavior? The present model can be solved numerically for $\infty > N \geq 4$. In Ref.[6] the behavior of the numerical solution for the specific heat, divided by $N^2 - 1$, was shown for different values of N. To observe the divergence in the specific heat, moderate values of N do not suffice. Instead, it is necessary to go to rather large values, $N \geq 40$. However, the recent results of Ref. [11] imply that to detect a manifestation of the Gross-Witten-Wadia phase transitions, it is enough to calculate the interface tension up to $N = 10$.

The present matrix model suggests that *very* near T_d, a novel transition may arise at large N.

4. Acknowledgement

I thank my co-authors, R. Pisarski, S. Lin and K. Kashiwa, with whom I have been working on the subject of this talk. This work was supported by Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy.

[1] Meisinger P N, Miller T R and Ogilvie M C 2002 *Phys.Rev.* **D65** 034009 (*Preprint* `hep-ph/0108009`)
[2] Meisinger P N and Ogilvie M C 2002 *Phys.Rev.* **D65** 056013 (*Preprint* `hep-ph/0108026`)
[3] Dumitru A, Guo Y, Hidaka Y, Altes C P K and Pisarski R D 2011 *Phys.Rev.* **D83** 034022 (*Preprint* `1011.3820`)
[4] Dumitru A, Guo Y, Hidaka Y, Altes C P K and Pisarski R D 2012 *Phys.Rev.* **D86** 105017 (*Preprint* `1205.0137`)
[5] Kashiwa K, Pisarski R D and Skokov V V 2012 *Phys.Rev.* **D85** 114029 (*Preprint* `1205.0545`)
[6] Pisarski R D and Skokov V V 2012 *Phys.Rev.* **D86** 081701 (*Preprint* `1206.1329`)
[7] Fukushima K 2004 *Phys.Lett.* **B591** 277–284 (*Preprint* `hep-ph/0310121`)
[8] Pisarski R D 2000 *Phys.Rev.* **D62** 111501 (*Preprint* `hep-ph/0006205`)
[9] Scavenius O, Dumitru A and Lenaghan J 2002 *Phys.Rev.* **C66** 034903 (*Preprint* `hep-ph/0201079`)
[10] Dumitru A, Lenaghan J and Pisarski R D 2005 *Phys.Rev.* **D71** 074004 (*Preprint* `hep-ph/0410294`)
[11] Lin S, Pisarski R D and Skokov V V 2013 (*Preprint* `1301.7432`)