



Electroweak interactions of quarks

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**PART 1 : Hadron decay,
history of flavour mixing**

**PART 2 : Oscillations and
CP Violation**

**PART 3 : Top quark and
electroweak physics**

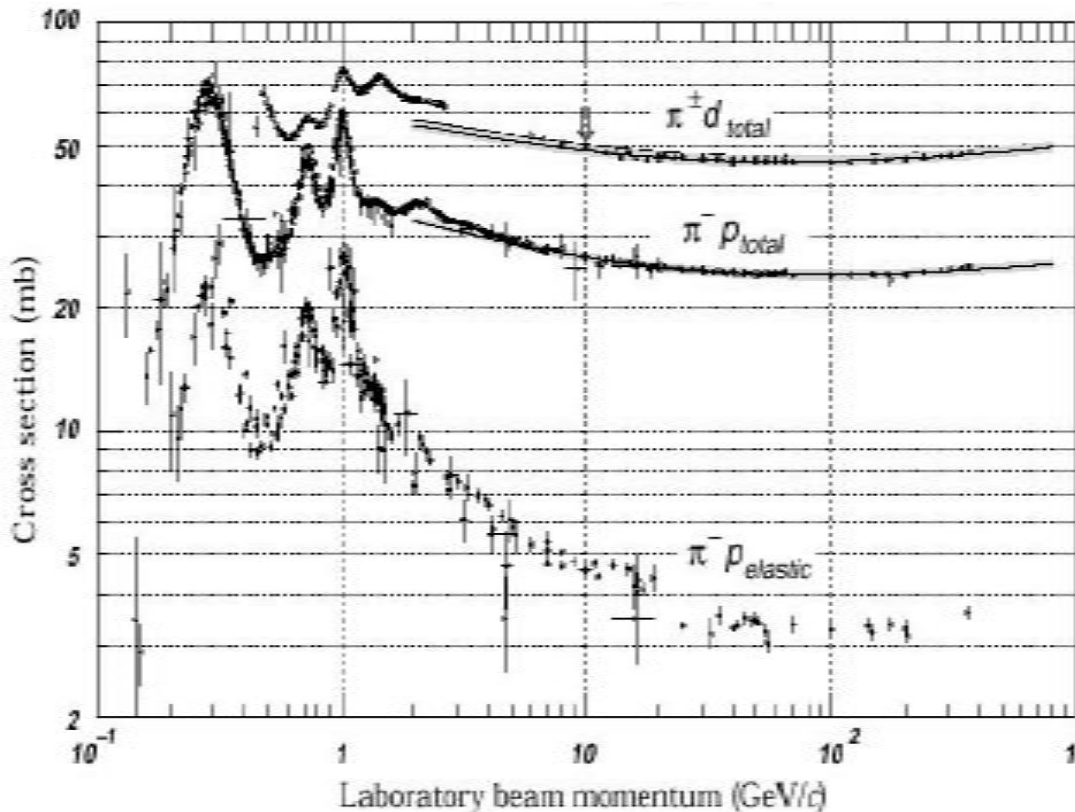
PART 1

Hadron decays

History of flavour mixing

Hadron spectroscopy

1950/1960 : lots of « hadronic states » observed
resonances in inelastic diffusion of nucleons or pions (pp,
 $\rho\pi$, np,...)



Interpreted as bound
states of strong force

Several particles

- almost same mass

- same spin

- same behavior wrt.
strong interaction

- **different electric charge**

Multiplet structure associated with SU(2) group symmetry₄

Strange hadrons

A few particles does not fit into this scheme :

Name	Mass (MeV)	Decay	Lifetime
Λ	1116	$N\pi$	$1 \cdot 10^{-10} \text{s}$
Ξ^0, Ξ^-	1320	$\Lambda\pi$	$3 \cdot 10^{-10} \text{s}$
Σ^+, Σ^-	1190	$N\pi$	$8 \cdot 10^{-11} \text{s}$
Σ^0	1200	$\gamma\Lambda$	$1 \cdot 10^{-20} \text{s (EM)}$

Decay time characteristic from **weak interaction**

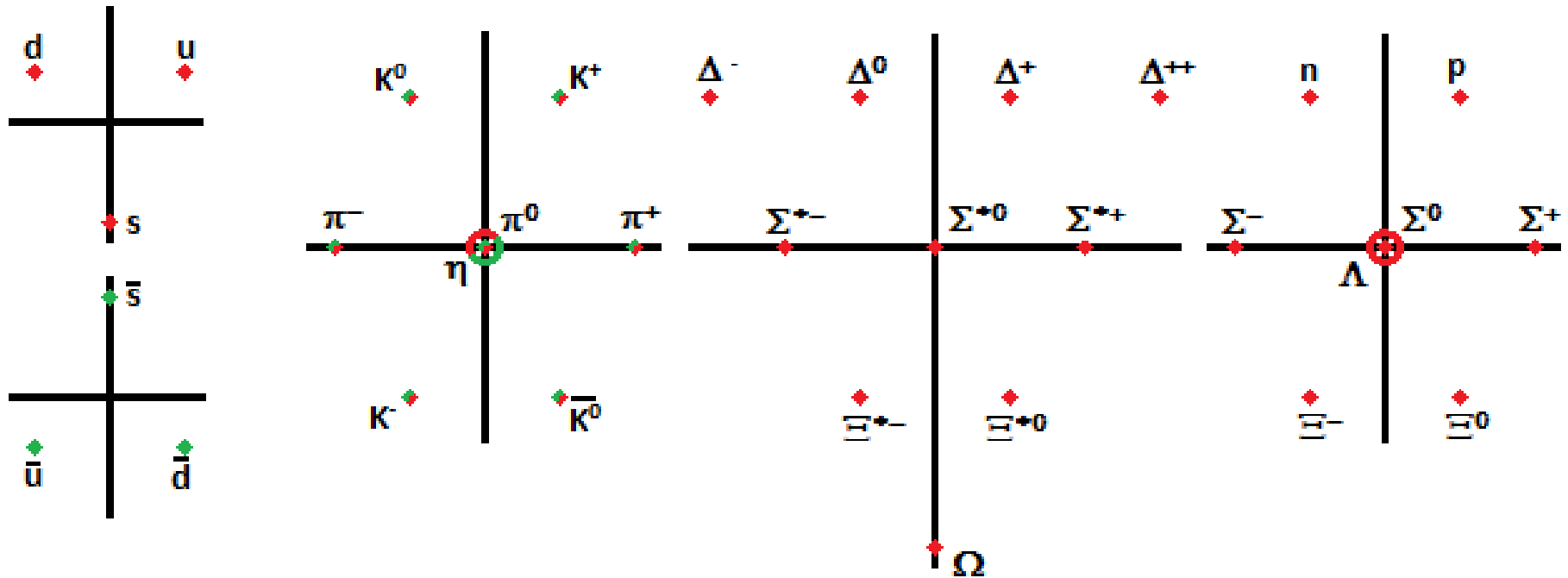
Particles stable wrt. strong/EM coupling

Conserved quantum number : strangeness

Quark model

Gell-Mann (Nobel 69) and Zweig, 1963 :

hadrons are build from 3 quarks **u**, **d** and **s**.



Strangeness : content in strange quarks

Only weak interaction can induce change of flavour

$$s \rightarrow W u \quad \text{or} \quad s \rightarrow \cancel{Z d} \quad \text{eg.} \quad \frac{BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{BR(K^+ \rightarrow \pi^0 \mu^+ \nu)} < 10^{-5}$$

No Flavor Changing Neutral Currents (FCNC) in SM ₆

Electroweak lagrangian

$$\begin{aligned} \mathcal{L}_{GSW} = & \sum_{\ell=e,\mu,\tau} [i\bar{\psi}_\ell \gamma^\mu \partial_\mu \psi_\ell + i\bar{\psi}_{\nu_\ell} \gamma^\mu \partial_\mu \psi_{\nu_\ell}] - \frac{1}{2} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) (\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\ & - \frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu) (\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu) (\partial^\mu A^\nu - \partial^\nu A^\mu) + \frac{1}{2} \partial_\mu h \partial^\mu h \\ & - \sum_\ell \frac{\lambda_\ell v}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell - \left(\frac{gv}{2}\right)^2 W_\mu^+ W^{-\mu} - \frac{1}{2} \left(\frac{gv}{2 \cos \theta_W}\right)^2 Z_\mu Z^\mu - \frac{1}{2} (-2m^2)^2 h^2 \end{aligned}$$

$$\begin{aligned} & -g_{em} \sum_\ell \bar{\psi}_\ell \gamma^\mu \psi_\ell A_\mu + \frac{g}{2\sqrt{2}} \sum_\ell [\bar{\psi}_{\nu_\ell} \gamma^\mu (1 - \gamma^5) \psi_\ell W_\mu^+ + \bar{\psi}_\ell \gamma^\mu (1 - \gamma^5) \psi_{\nu_\ell} W_\mu^-] \\ & + \frac{g}{4 \cos \theta_W} \sum_\ell [\bar{\psi}_{\nu_\ell} \gamma^\mu (1 - \gamma^5) \psi_{\nu_\ell} Z_\mu - \bar{\psi}_\ell \gamma^\mu (1 - 4 \sin^2 \theta_W - \gamma^5) \psi_\ell Z_\mu] - \sum_\ell \frac{\lambda_\ell}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell h \end{aligned}$$

$$\begin{aligned} & +ig_{em} [\partial_\mu A_\nu W^{-\mu} W^{+\nu} + \partial_\mu W_\nu^+ W^{-\nu} A^\mu + \partial_\mu W_\nu^- W^{+\mu} A^\nu - \partial_\mu A_\nu W^{-\nu} W^{+\mu} \\ & \quad - \partial_\mu W_\nu^+ W^{-\mu} A^\nu - \partial_\mu W_\nu^- W^{+\mu} A^\mu] \\ & +ig \cos \theta_W [\partial_\mu Z_\nu W^{-\mu} W^{+\nu} + \partial_\mu W_\nu^+ W^{-\nu} Z^\mu + \partial_\mu W_\nu^- W^{+\mu} Z^\nu - \partial_\mu Z_\nu W^{-\nu} W^{+\mu} \\ & \quad - \partial_\mu W_\nu^+ W^{-\mu} Z^\nu - \partial_\mu W_\nu^- W^{+\mu} Z^\mu] + \frac{g^2 v}{2} W_\mu^+ W^{-\mu} h + \frac{g^2 v}{4 \cos^2 \theta_W} Z_\mu Z^\mu h - \lambda v h^3 \\ & +g_{em}^2 [W_\nu^+ W^{-\mu} A_\nu A^\mu - W_\mu^+ W^{-\mu} A_\nu A^\nu] + g^2 \cos^2 \theta_W [W_\nu^+ W^{-\mu} Z_\nu Z^\mu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu] \\ & +g^2 \cos \theta_W \sin \theta_W [2W_\mu^+ W^{-\mu} Z_\nu A^\nu - W_\mu^+ W^{-\nu} A_\nu Z^\mu - W_\mu^+ W^{-\nu} A^\mu Z_\nu] \\ & +\frac{g^2}{2} [W_\mu^- W^{-\mu} W_\nu^+ W^{+\nu} - W_\mu^- W^{+\mu} W_\nu^- W^{-\nu}] + \frac{g^2}{4} W_\mu^+ W^{-\mu} h^2 + \frac{g^2}{8 \cos^2 \theta_W} Z_\mu Z^\mu h^2 - \frac{\lambda}{4} h^4 \end{aligned}$$

$$g_{em} = g \sin \theta_W \quad , \quad v^2 = \frac{-m^2}{\lambda} \quad , \quad m^2 < 0 \quad , \quad \lambda > 0$$

OK for
leptons

What
happens for
quarks ?

$SU(2)_L$ and quarks

$SU(2)_L$ symmetry :

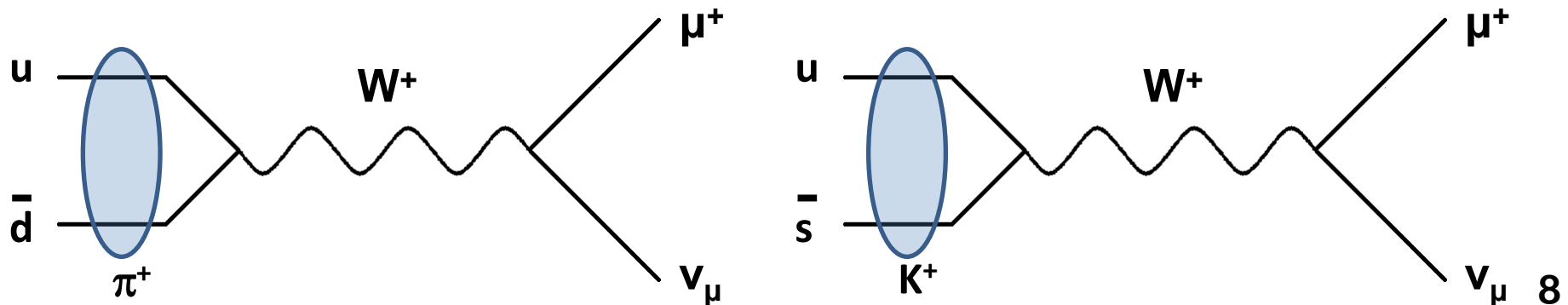
doublets of left handed fermions with $\Delta Q=1$:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \text{ and } s_L \quad \text{or} \quad \begin{pmatrix} u \\ s \end{pmatrix}_L \text{ and } d_L$$

singlets for right handed fermions : u_R, d_R, s_R

But both Wud and Wus vertices happen !

Eg. Leptonic pion and kaon decays :



Flavour mixing

strong interaction eigenstates (mass eigenstates)
may be **different** from
weak interaction eigenstates

Some mixing of d and s :
$$\begin{pmatrix} d_c \\ s_c \end{pmatrix} = U \begin{pmatrix} d \\ s \end{pmatrix}$$

universality of weak interaction : conserve the
overall coupling : $U^\dagger U = 1$

U is 2x2 rotation matrix , **1 parameter**

$$U = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$
 θ_c is the **Cabibbo angle (1963)**

Back to lagrangian

1 doublet $\begin{pmatrix} u \\ d_C \end{pmatrix}_L$ and 4 singlets $u_R, d_{CR}, s_{CR}, s_{CL}$

Charged currents :

$$\begin{aligned} L_{CC} &= \frac{ig}{2\sqrt{2}} \bar{\psi}_u \gamma^\mu (1 - \gamma^5) \psi_{d_C} W_\mu^+ + h.c. \\ &= \frac{ig \cos \theta_C}{2\sqrt{2}} \bar{\psi}_u \gamma^\mu (1 - \gamma^5) \psi_d W_\mu^+ \\ &\quad + \frac{ig \sin \theta_C}{2\sqrt{2}} \bar{\psi}_u \gamma^\mu (1 - \gamma^5) \psi_s W_\mu^+ + h.c. \end{aligned}$$

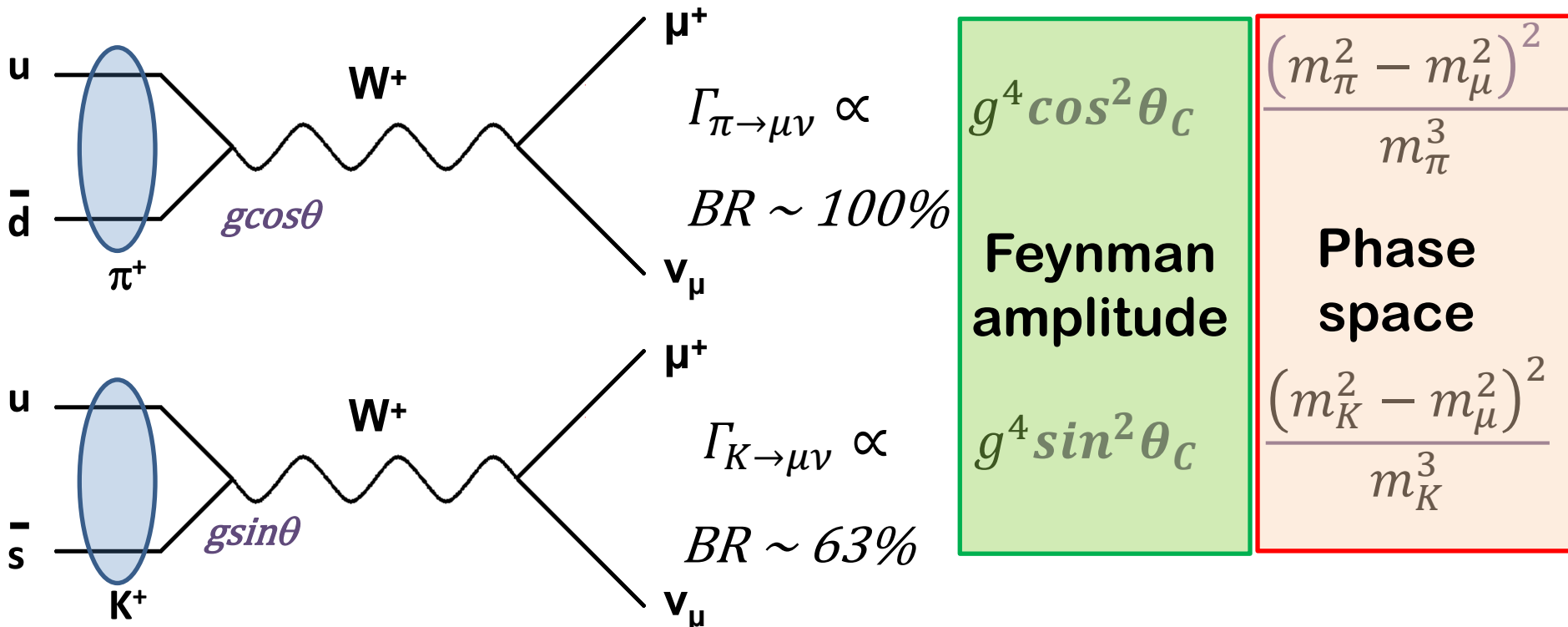
Vertices :

$$\mathbf{Wud} : -\frac{ig \cos \theta_C}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5) \quad \mathbf{Wus} : -\frac{ig \sin \theta_C}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5)$$

Naive estimation of θ_c

From pion and kaon lifetimes

$$\tau_\pi = 2.603 \times 10^{-8} \text{ s} \quad \tau_K = 1.23 \times 10^{-8} \text{ s}$$



$$\tan^2 \theta_c = 0.63 \frac{\tau_\pi m_K^3 (m_\pi^2 - m_\mu^2)^2}{\tau_K m_\pi^3 (m_K^2 - m_\mu^2)^2} \Rightarrow \begin{cases} \sin \theta_c = 0.265 \\ \cos \theta_c = 0.964 \end{cases}$$

FCNC troubles

Neutral currents (d and s quark only)

$$L_{NC} = \frac{ig}{2\cos\theta_W} \left(\bar{\psi}_{d_C} \gamma^\mu (\widehat{c}_V - \widehat{c}_A \gamma^5) \psi_{d_C} Z_\mu + \bar{\psi}_{s_C} \gamma^\mu (\widehat{c}_V - \widehat{c}_A \gamma^5) \psi_{s_C} Z_\mu \right)$$

$$\widehat{c}_V = \widehat{T}^3 - 2\sin^2\theta_W \widehat{Q} : \widehat{c}_V \psi_{d_C} = \left(-\frac{1}{2} + \frac{2}{3} \sin^2\theta_W \right) \psi_{d_C} ; \widehat{c}_V \psi_{s_C} = \left(\frac{2}{3} \sin^2\theta_W \right) \psi_{s_C}$$

$$\widehat{c}_A = \widehat{T}^3 : \widehat{c}_A \psi_{d_C} = -\frac{1}{2} \psi_{d_C} ; \widehat{c}_A \psi_{s_C} = 0$$

Introducing mass eigenstates :

$$L_{NC} = \frac{ig}{2\cos\theta_W} \left(\bar{\psi}_{d_C} \gamma^\mu c_Z^1 \psi_{d_C} Z_\mu + \bar{\psi}_{s_C} \gamma^\mu c_Z^2 \psi_{s_C} Z_\mu \right)$$

$$= \frac{ig}{2\cos\theta_W} \left(\bar{\psi}_d \gamma^\mu (\cos^2\theta_C c_Z^1 + \sin^2\theta_C c_Z^2) \psi_d Z_\mu + \bar{\psi}_s \gamma^\mu (\sin^2\theta_C c_Z^1 + \cos^2\theta_C c_Z^2) \psi_s Z_\mu \right)$$

$$+ \frac{ig \cos\theta_C \sin\theta_C}{2\cos\theta_W} \left(\bar{\psi}_d \gamma^\mu (c_Z^1 - c_Z^2) \psi_s Z_\mu + \bar{\psi}_s \gamma^\mu (c_Z^1 - c_Z^2) \psi_d Z_\mu \right)$$

FCNC inducing term

GIM and charm

Natural solution proposed by Glashow, Iliopoulos, Maiani in 1970 (**GIM mechanism**)

Add a 4th quark to restore the symmetry : **charm**
2 $SU(2)_L$ doublets + right singlets

$$\begin{pmatrix} u \\ d_C \end{pmatrix}_L, \begin{pmatrix} c \\ s_C \end{pmatrix}_L, u_R, d_{CR}, c_R, s_{CR}$$

Then the coupling to the Z becomes :

$$c_Z^1 = c_Z^2 = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W - \frac{1}{2} \gamma^5$$

And the **FCNC terms cancel out** :

$$\frac{ig \cos \theta_c \sin \theta_c}{2 \cos \theta_W} \left(\bar{\psi}_d \gamma^\mu \overset{=0}{(c_Z^1 - c_Z^2)} \psi_s Z_\mu + \bar{\psi}_s \gamma^\mu \overset{=0}{(c_Z^1 - c_Z^2)} \psi_d Z_\mu \right)$$

Top and bottom

Generalization to 6 quarks : $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$

Complex 3x3 unitary matrix :

Kobayashi & Maskawa in 1973 (Nobel in 2008)

Cabibbo-Kobayashi-Maskawa or CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Lepton vertex : $-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$

$W^+q_u\bar{q}_d$ vertex : $-\frac{igV_{quqd}}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$

$W^-q_u\bar{q}_d$ vertex : $-\frac{igV_{quqd}^*}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$

CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

First approximation : Diagonal matrix $V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

no family change : **Wud**, **Wcs** and **Wtb** vertices

Second approximation : Block matrix $V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & 0 \\ V_{cd} & V_{cs} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

submatrix is almost the Cabibbo matrix

$$V_{ud} \approx V_{cs} \approx \cos\theta_C \quad \text{and} \quad V_{us} \approx V_{cd} \approx \sin\theta_C$$

Top quark only decays to bottom quark

Charm quark mostly decays to strange quark

Bottom and Strange decays are CKM suppressed

PART 2

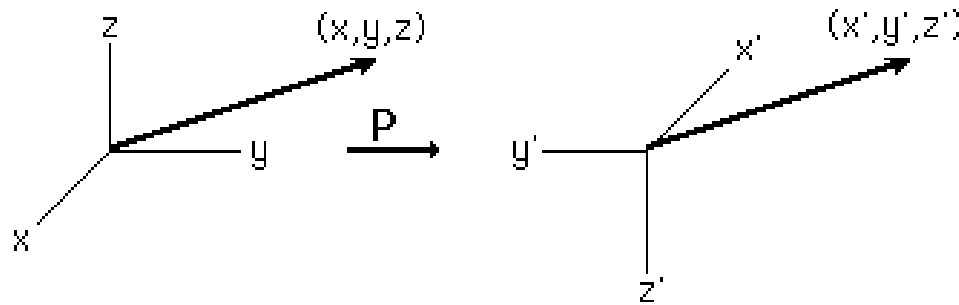
Oscillations and CP Violation

Discrete symmetries

3 discrete symmetries, such as $\hat{S}^2=1$

Affects : coordinates, operators, particles fields

\hat{P} = Parity : space coordinates reversal : $\vec{x} \rightarrow -\vec{x}$



\hat{C} = Charge conjugation : particle to antiparticle transformation (i.e. inversion of all conserved charges, lepton and baryon numbers)

eg. $e^- \xleftrightarrow{\hat{C}} e^+$, $u \xleftrightarrow{\hat{C}} \bar{u}$, $\pi^- (d\bar{u}) \xleftrightarrow{\hat{C}} \pi^+ (u\bar{d})$, $K^0 (d\bar{s}) \xleftrightarrow{\hat{C}} \bar{K}^0 (s\bar{d})$

\hat{T} = Time : time coordinate reversal : $t \rightarrow -t$

Parity

Weak interaction is not invariant under parity :

maximum violation of parity
(C.S.Wu experiment on ^{60}Co beta decay)

Strong and EM interaction are OK.

Parity does not change the nature of particles parity eigenstates : $\hat{P}|p\rangle = \eta|p\rangle$

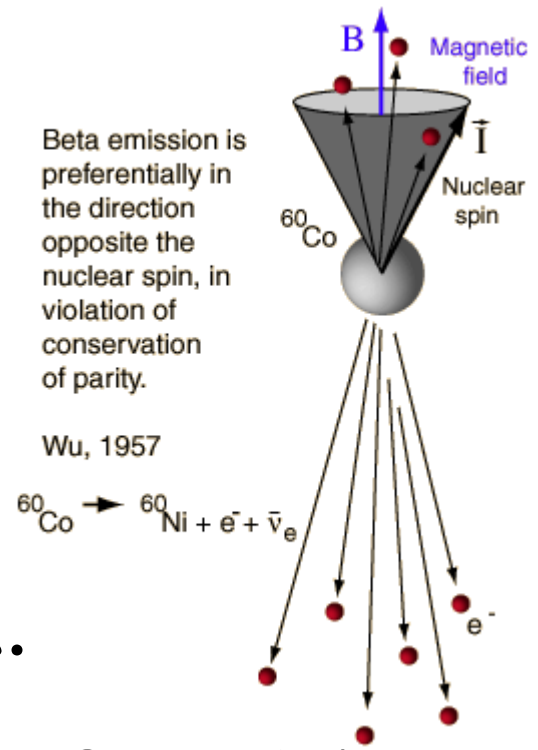
η : Intrinsic parity

since $\hat{P}^2=1$, $\eta=\pm 1$

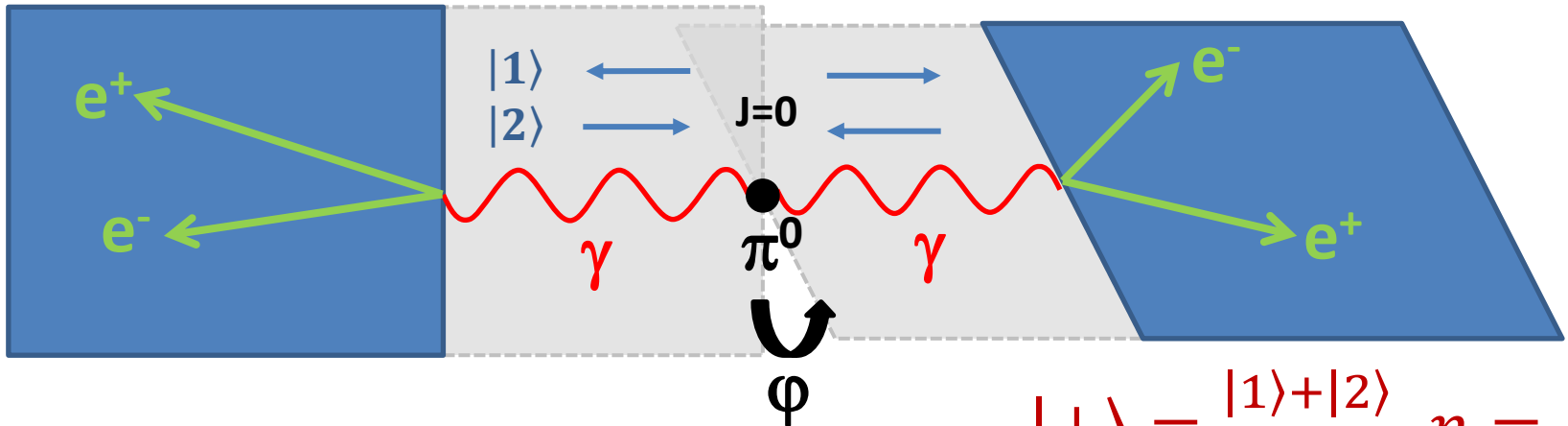
Under \hat{P} : $\vec{E} \rightarrow -\vec{E}$, vector; $\vec{B} \rightarrow \vec{B}$, pseudovector

4 -potential : $(\phi, \vec{A}) \rightarrow (\phi, -\vec{A})$ so : $\eta_{photon} = -1$

Strong and EM interaction conserves parity.



Parity of the pion



EM decay : $\pi^0 \rightarrow \gamma\gamma$, conserves parity

Parity eigenstates for photon pair :

Pions can only decay in one of these states

$$|+\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}, \eta = 1$$

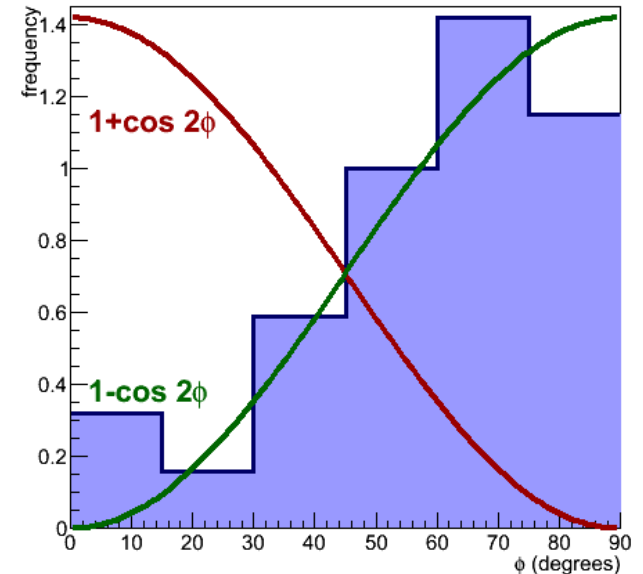
$$|-\rangle = \frac{|1\rangle - |2\rangle}{\sqrt{2}}, \eta = -1$$

Measure angular distribution
of e^+e^- pairs :

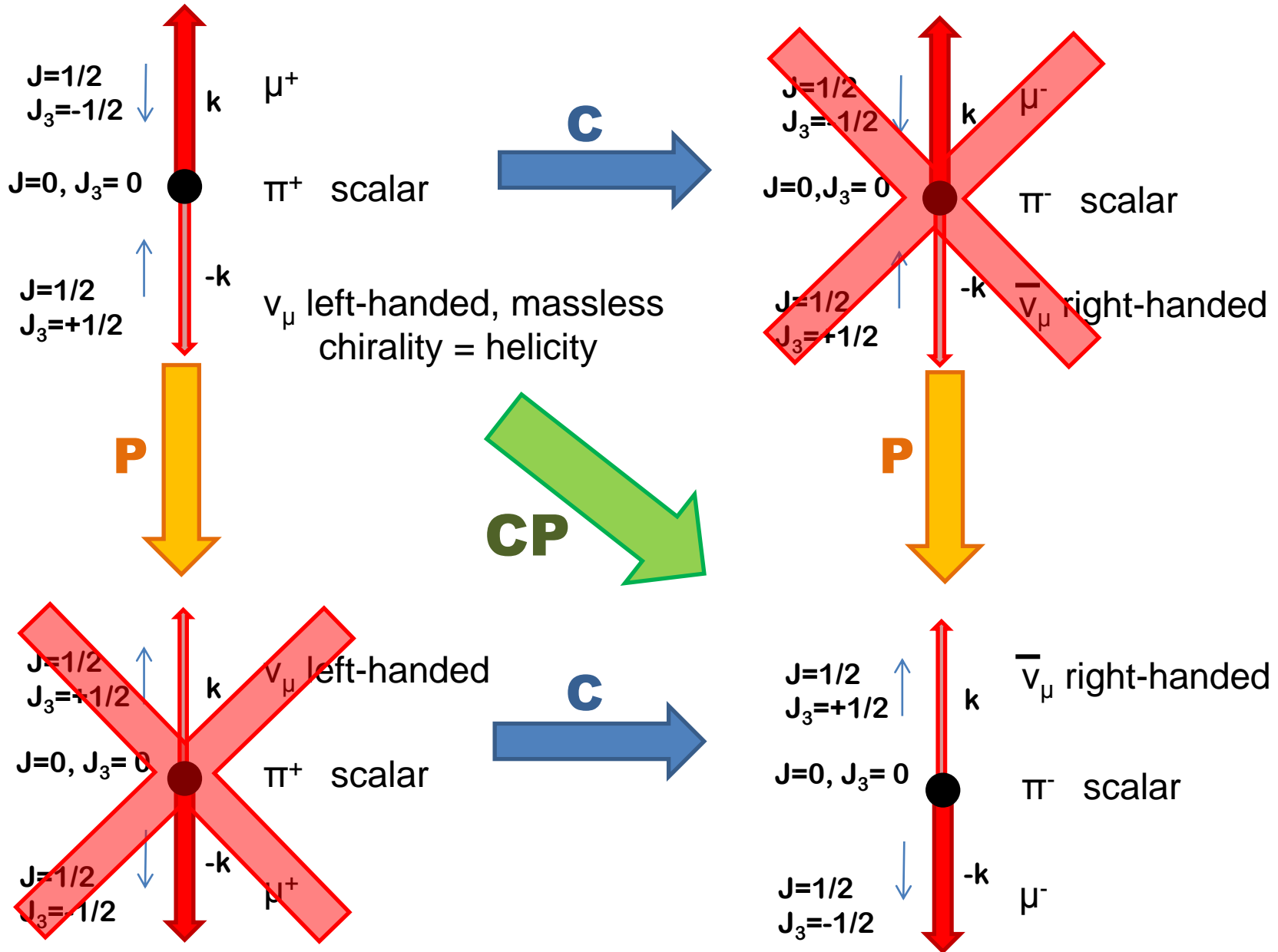
$$|+\rangle : 1 + \cos 2\phi$$

$$|-\rangle : 1 - \cos 2\phi$$

Experimentally : $\eta_{\pi^0} = -1$



CP symmetry : pion decay



CP symmetry : neutral kaons

Kaons are similar to pions :

→ same SU(3) octet pseudoscalar mesons

$$\hat{P}|K^0\rangle = -|K^0\rangle \quad \text{and} \quad \hat{P}|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

K^0 (=u \bar{s}) and \bar{K}^0 (=u \bar{s}) are antiparticle of each other

$$\hat{C}|K^0\rangle = |\bar{K}^0\rangle \quad \text{and} \quad \hat{C}|\bar{K}^0\rangle = |K^0\rangle$$

So: $\hat{C}\hat{P}|K^0\rangle = -|\bar{K}^0\rangle$ and $\hat{C}\hat{P}|\bar{K}^0\rangle = -|K^0\rangle$

Then CP-eigenstates are :

$$|K_1^0\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}, \eta_{CP} = 1 \quad |K_2^0\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}, \eta_{CP} = -1$$

Does weak currents conserve CP ?

CP violation in Kaon decays

If CP is conserved by weak interactions then only

$$|K_1^0\rangle \rightarrow \pi\pi, \quad (\eta_{CP} = 1) \quad \text{and} \quad |K_2^0\rangle \rightarrow \pi\pi\pi, \quad (\eta_{CP} = -1)$$

With a longer lifetime for $|K_2^0\rangle$ (more vertices)

$$\text{Experimentally : } \tau_1 = 0.9 \times 10^{-10} \text{ s} \quad \tau_2 = 5.2 \times 10^{-8} \text{ s}$$

After $t \gg \tau_1$ only the long-lived component remains
but **a few 2 pions decays are still observed!**

Cronin & Fitch, 1964, Nobel 1980

Conclusion : the long lived component isn't a pure CP eigenstate : **small CP violation!**

$$\text{Physical states : } |K_L^0\rangle = \frac{|K_1^0\rangle - \varepsilon |K_2^0\rangle}{\sqrt{1 + \varepsilon^2}}, \quad |K_S^0\rangle = \frac{\varepsilon |K_1^0\rangle + |K_2^0\rangle}{\sqrt{1 + \varepsilon^2}}$$
$$\varepsilon = 2.3 \times 10^{-3}$$

Meson oscillations

Neutral pseudoscalar mesons $|P\rangle$ and $|\bar{P}\rangle$ (K^0 , D^0 , B^0 , B_s)

Propagation/strong interaction Hamiltonian : \hat{H}_0

$$\hat{H}_0|P\rangle = m_P|P\rangle, \quad \hat{H}_0|\bar{P}\rangle = m_P|\bar{P}\rangle \quad (\text{in rest frame})$$

Interaction (weak) states \neq propagation states.

Decay is allowed : effective hamiltonian \hat{H}_W is not hermitian

$$\hat{H}_W = \hat{M} - i\frac{\hat{\Gamma}}{2} \text{ with } \hat{M} = \frac{\hat{H}_W + \hat{H}_W^\dagger}{2} \quad \hat{\Gamma} = \hat{H}_W - \hat{H}_W^\dagger \text{ hermitian}$$

$$\text{For eigen states : } \hat{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \hat{\Gamma} = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}$$

And the time evolution of state 1 is :

$$|P_1(t)\rangle = e^{-im_1t} e^{-\frac{\gamma_1 t}{2}} |P_1(0)\rangle \quad I(t) = |\langle P_1(0)|P_1(t)\rangle|^2 = e^{-\gamma_1 t}$$

Exponential decay : \hat{M} mass matrix, $\hat{\Gamma}$ decay width

Interaction eigenstates

In the basis $|P\rangle, |\bar{P}\rangle$, the 2 states have the same properties (CPT symmetry) : $M_{11} = M_{22} \equiv M_0$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma_0$

If we assume $\hat{M}, \hat{\Gamma}$ to be real : $M_{12} = M_{21} \equiv \tilde{M}$ and $\Gamma_{12} = \Gamma_{21} \equiv \tilde{\Gamma}$

Then :
$$\hat{H}_W = \begin{pmatrix} M_0 - i\frac{\Gamma_0}{2} & \tilde{M} - i\frac{\tilde{\Gamma}}{2} \\ \tilde{M} - i\frac{\tilde{\Gamma}}{2} & M_0 - i\frac{\Gamma_0}{2} \end{pmatrix}$$

From perturbation theory :
 $\tilde{\Gamma} \approx -\Gamma_0$

In new basis :

$$\begin{aligned} |P_L\rangle &= \frac{|P\rangle + |\bar{P}\rangle}{\sqrt{2}} \\ |P_S\rangle &= \frac{|P\rangle - |\bar{P}\rangle}{\sqrt{2}} \end{aligned} \quad \hat{H}_W = \begin{pmatrix} M_0 + \tilde{M} - i\frac{\Gamma_0 + \tilde{\Gamma}}{2} & 0 \\ 0 & M_0 - \tilde{M} - i\frac{\Gamma_0 - \tilde{\Gamma}}{2} \end{pmatrix}$$

$|P_L\rangle$: long lived, mass $M_L = M_0 + \tilde{M}$ and width $\Gamma_L = \Gamma_0 + \tilde{\Gamma} \ll \Gamma_0$

$|P_S\rangle$: short lived, mass $M_S = M_0 - \tilde{M}$ and width $\Gamma_S = \Gamma_0 - \tilde{\Gamma} \approx \Gamma_0$

Time evolution

General state : mixing of $|P_L\rangle$ and $|P_S\rangle$:

$$|\tilde{P}(t)\rangle = c_L(t)|P_L\rangle + c_S(t)|P_S\rangle$$

Coefficients c_L and c_S satisfy the Schrödinger eq. :

$$i \frac{d}{dt} \begin{pmatrix} c_L(t) \\ c_S(t) \end{pmatrix} = \begin{pmatrix} M_L - i\frac{1}{2}\Gamma_L & 0 \\ 0 & M_S - i\frac{1}{2}\Gamma_S \end{pmatrix} \begin{pmatrix} c_L(t) \\ c_S(t) \end{pmatrix}$$

Then time evolution is :

$$|\tilde{P}(t)\rangle = e^{-iM_0 t} \left(c_L(0) e^{i\tilde{M}t - \frac{\Gamma_L t}{2}} |P_L\rangle + c_S(0) e^{-i\tilde{M}t - \frac{\Gamma_S t}{2}} |P_S\rangle \right)$$

Time evolution

For a pure $|P\rangle$ initial state : $c_L(0) = c_S(0) = \frac{1}{\sqrt{2}}$.

And the intensity after a given time is :

$$I(t) = |\langle P|P(t)\rangle|^2 = \frac{1}{4} \left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(2\tilde{M}t) \right)$$

Oscillations $|P_L\rangle \leftrightarrow |P_S\rangle$ in time, with a frequency equal to the mass difference :

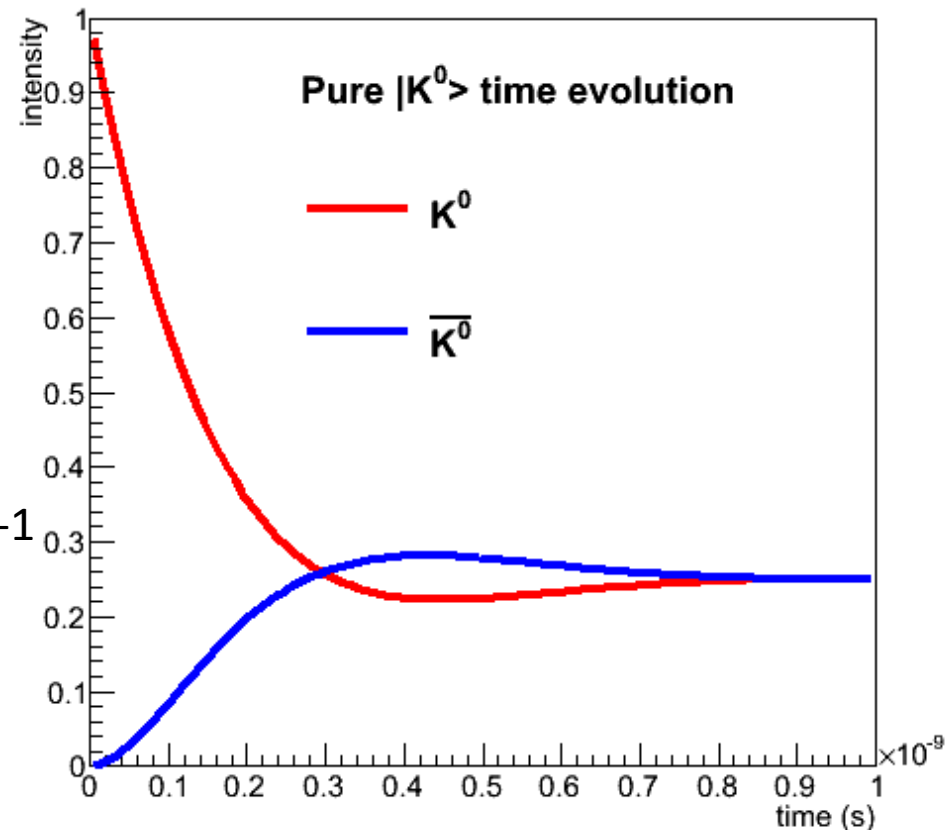
$$\Delta m = M_L - M_S = 2\tilde{M}$$

For the Kaon system :

$$\Delta m = 3.48 \times 10^{-12} \text{ MeV} = 5.29 \text{ ns}^{-1}$$

$$\tau_S = 89.6 \text{ ps}$$

$$\tau_L = 51.2 \text{ ns}$$



CP violation

Previously we assumed \hat{M} and $\hat{\Gamma}$ to be real
 Interaction eigenstates = CP-eigenstates

But if complex $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$

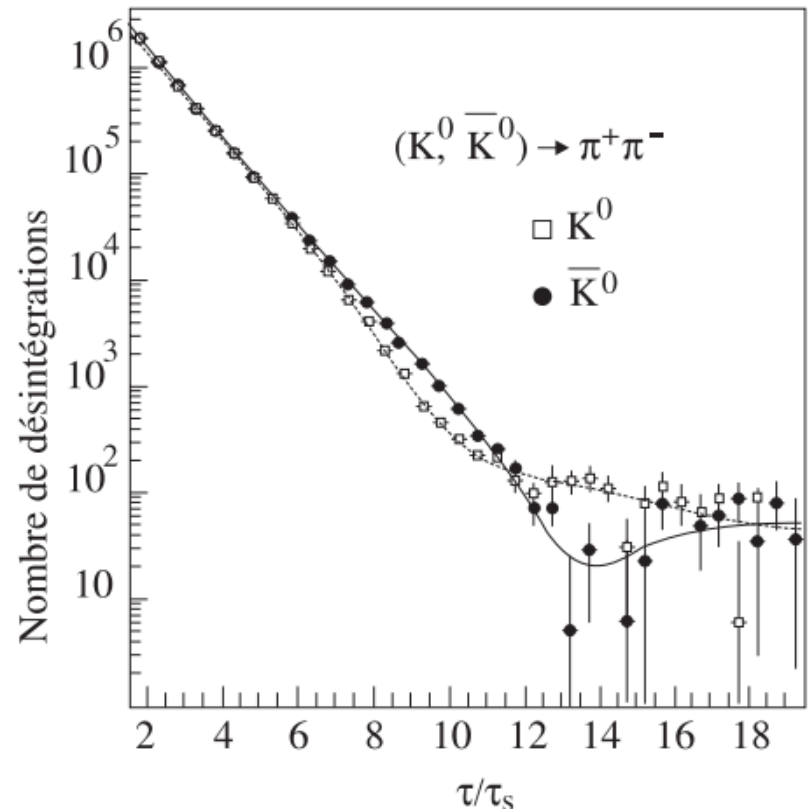
Then the eigenstates are :

$$|P_L\rangle = N(|P\rangle + \varepsilon|\bar{P}\rangle)$$

$$|P_S\rangle = N(|P\rangle - \varepsilon|\bar{P}\rangle)$$

$$\varepsilon = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

$$N^{-2} = 1 + \frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \text{Im}(\Gamma_{12}^* M_{12})}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - \text{Im}(\Gamma_{12}^* M_{12})}$$



Complex mass matrix induce CP-violation

Parametrization of CKM

3x3 complex unitary matrix has 4 parameters

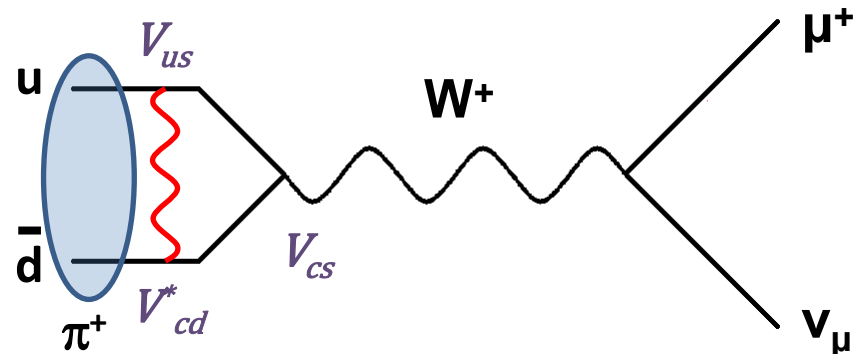
3 angles :

$$\alpha = \varphi_{13} = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \beta = \varphi_{23} = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \gamma = \varphi_{12} = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

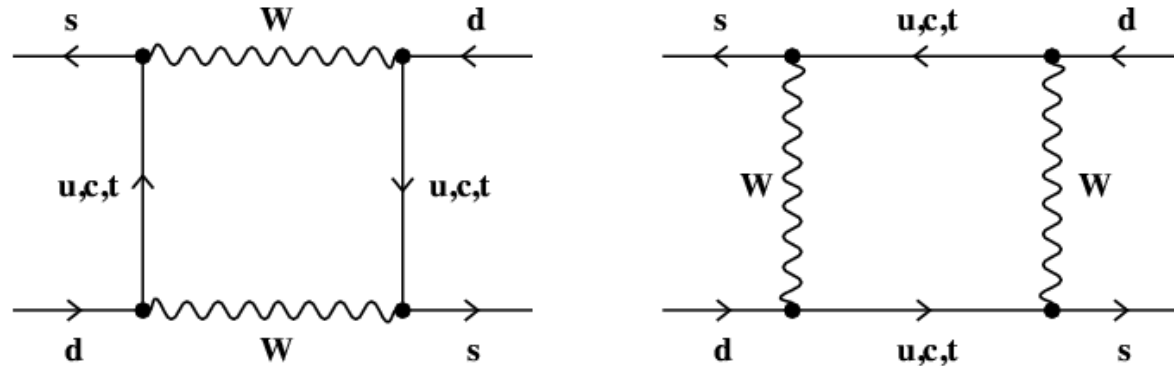
1 complex phase : δ

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

Complex phase
allows CP violation



Oscillation and boxes



Quark interpretation : box diagram

The mass matrix off-diagonal elements becomes:

$$\begin{aligned}
 M_{12} &= C(V_{us}^*V_{ud}m_u + V_{cs}^*V_{cd}m_c + V_{ts}^*V_{td}m_t)^2 \\
 &= C(V_{us}^*V_{ud}(m_u - m_c) + V_{ts}^*V_{td}(m_t - m_c))^2
 \end{aligned}$$

Complex because of phase in CKM matrix :

CP violation :

can only happen if at least **3 families**.

only happens in **loop diagrams** : rare decays and oscillations.

Unitarity triangles

Unitarity of CKM matrix :

$$V_{CKM}^\dagger V_{CKM} = 1$$

$$V_{ud}V_{ud}^* + V_{us}V_{us}^* + V_{ub}V_{ub}^* = 1$$

$$V_{cd}V_{cd}^* + V_{cs}V_{cs}^* + V_{cb}V_{cb}^* = 1$$

$$V_{td}V_{td}^* + V_{ts}V_{ts}^* + V_{tb}V_{tb}^* = 1$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$V_{us}V_{ud}^* + V_{cs}V_{cd}^* + V_{ts}V_{td}^* = 0$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

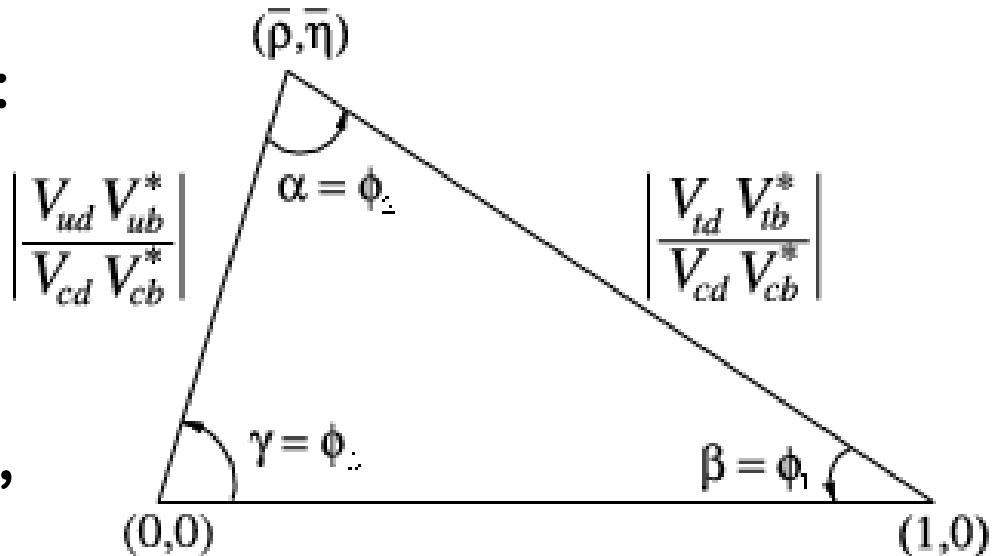
$$V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$$

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$

Sum of 3 complex numbers :
triangle in complex plane

Complex only if CP violating
phase is large

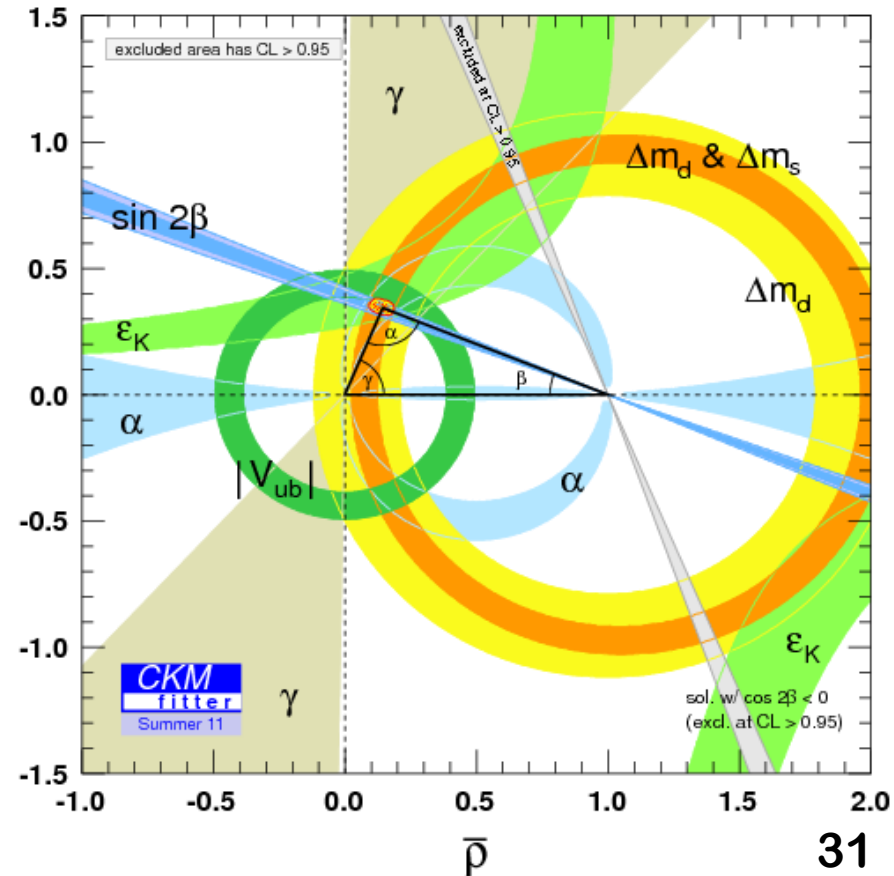
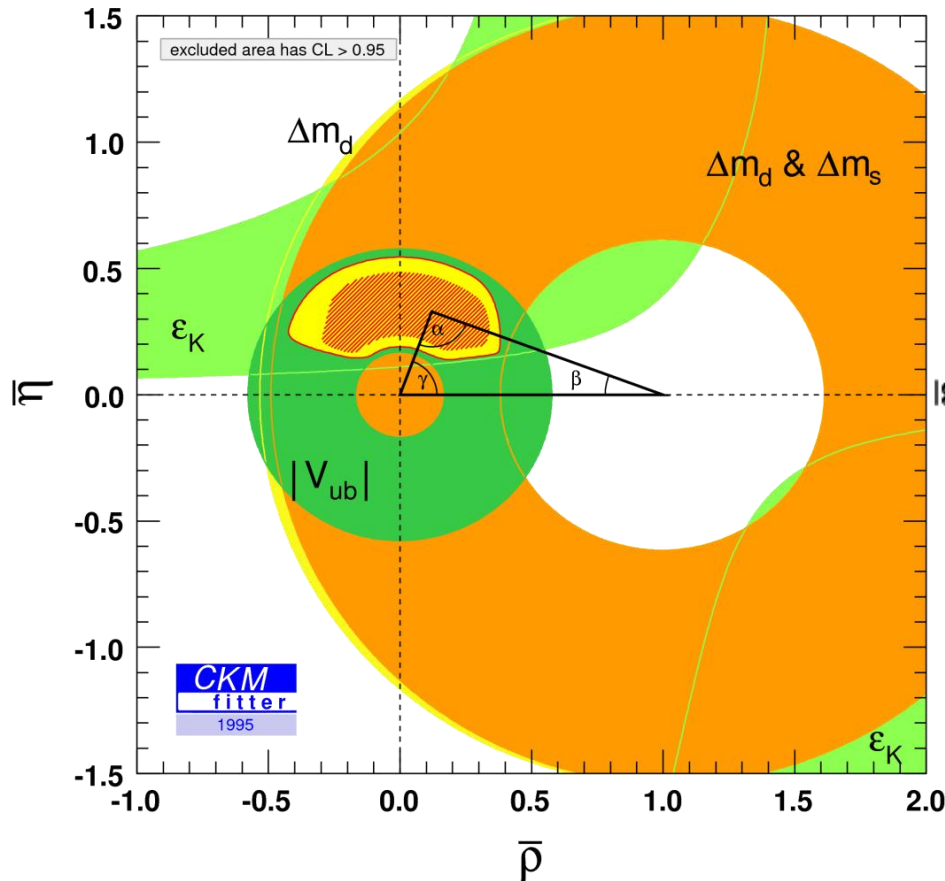
Most triangle are almost flat,
except one.



Unitarity triangles

Many decay processes, including rare decays of strange/charmed/beauty hadrons

- Some sensitive to matrix elements V_{xy}
- Some sensitive mass differences (oscillations)
- Some sensitive to angles (CP-violation)



Experiments

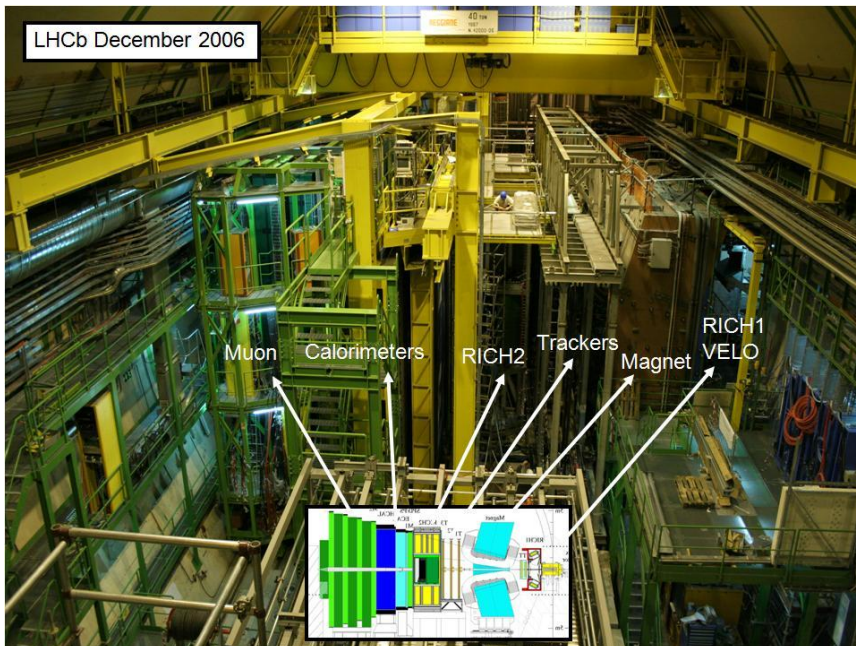
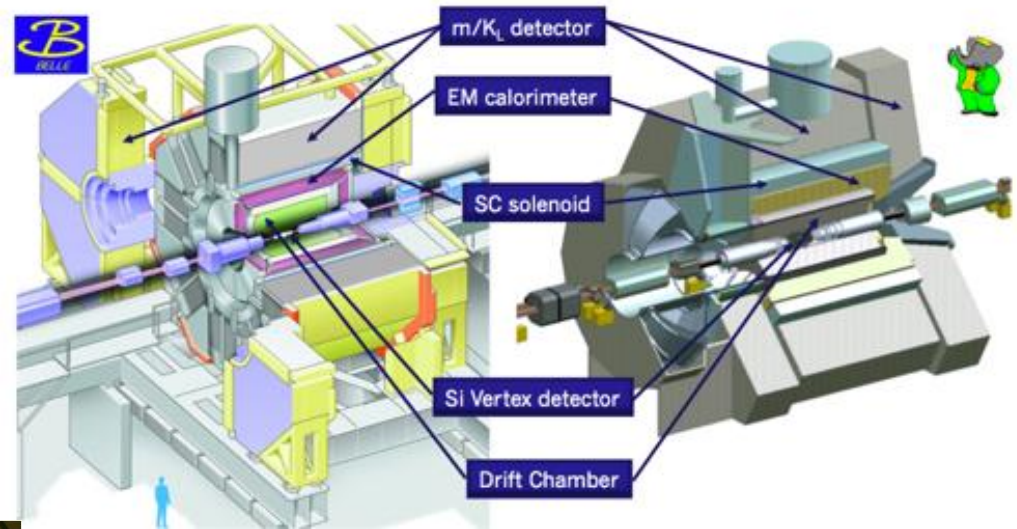
PAST :

e^+e^- , at $\psi(4S)$ resonance
(bb state)

BELLE (KEK, Japan)

BaBAR (SLAC, US)

pp : CDF and D0



PRESENT pp : LHCb
(+ ATLAS/CMS)

FUTURE : e^+e^- SuperBELLE (Japan) SuperB (Italy)

PART 3

Top quark and electroweak physics

Top quark decays

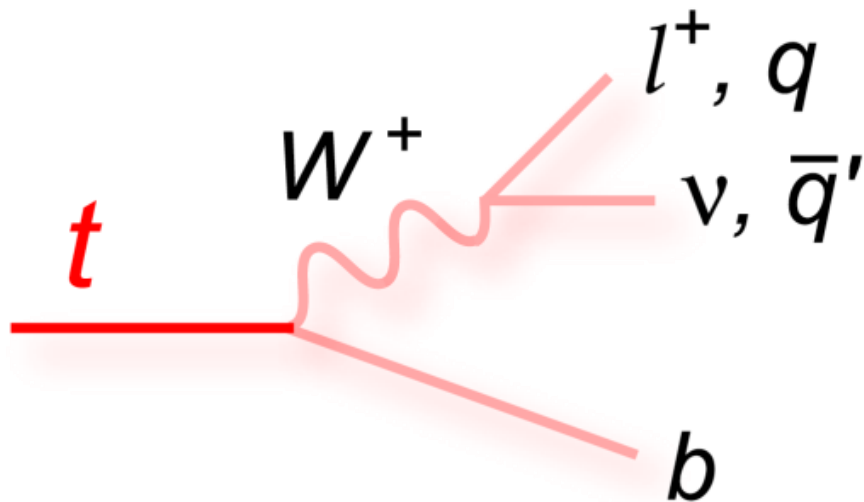
Top quark only decays through weak interaction

$V_{tb} \sim 1$: $t \rightarrow Wb$ at 100%

$m_t > m_W + m_b$: only quark that decays with a **real W**

Coupling **not « weak »** : $\tau_{top} \sim 10^{-25} \text{s} \gg \tau_{hadronization}$

No loss of polarization.



Top quark signature :

1 central b-quark jet, with high p_T (~ 70 GeV)

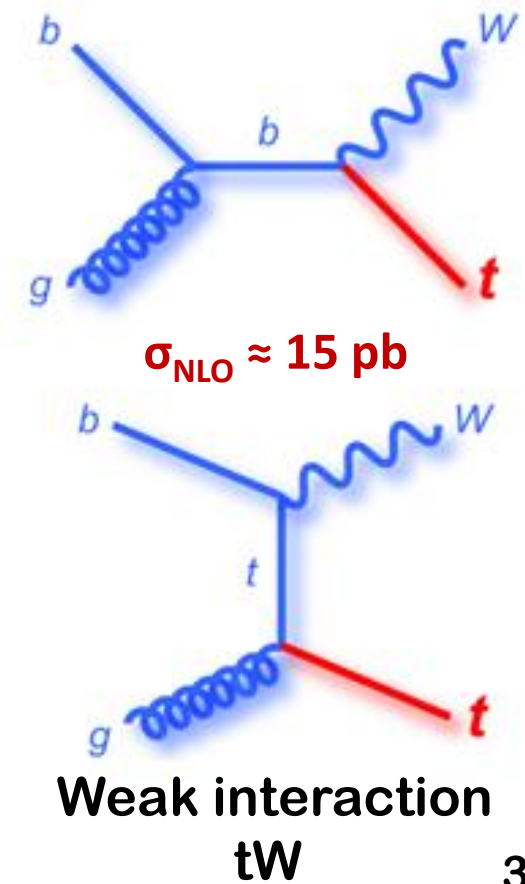
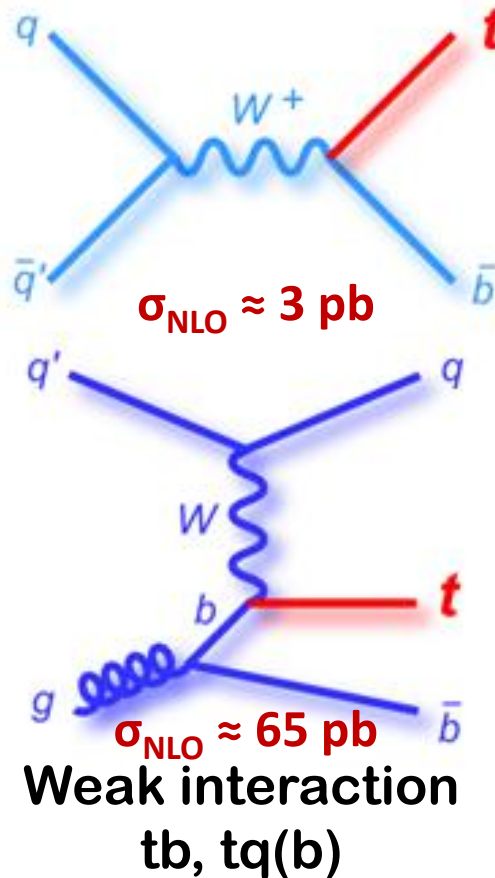
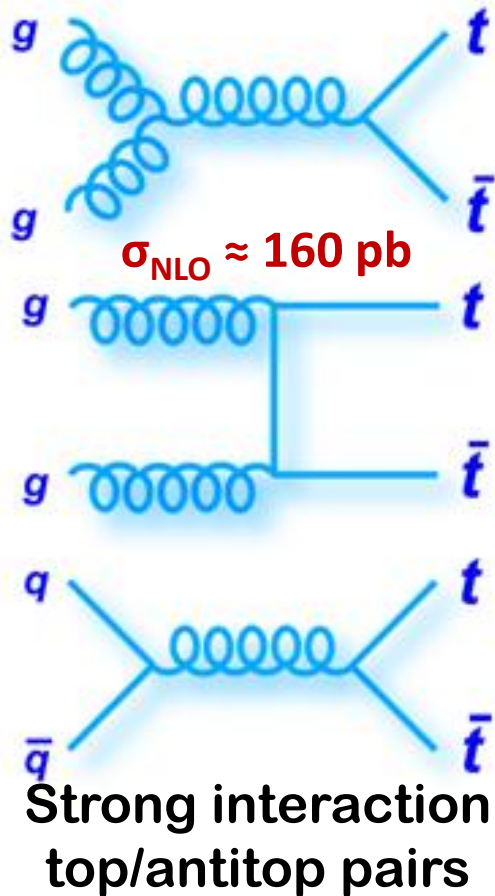
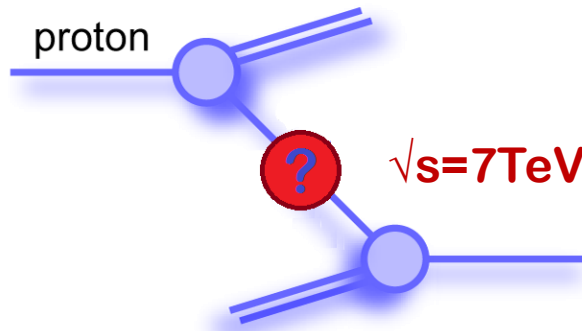
1 on-shell W boson :

1 isolated lepton

and E_T (~ 35 GeV)

or 2 jets (~ 35 GeV)

Top quark production



Top quark and electroweak

Top quark decays through Wtb vertex

- Use pair production (largest cross-section)
- Probe V-A theory in top coupling
- Measure V_{tb} assuming 3 generations

Electroweak production of the top quark

- lower cross-section, more background
- cross-section gives access to V_{tb} without assumptions on unitarity
- sensitivity to new physics (W' , 4th generation...)

Precision measurement of mass and coupling

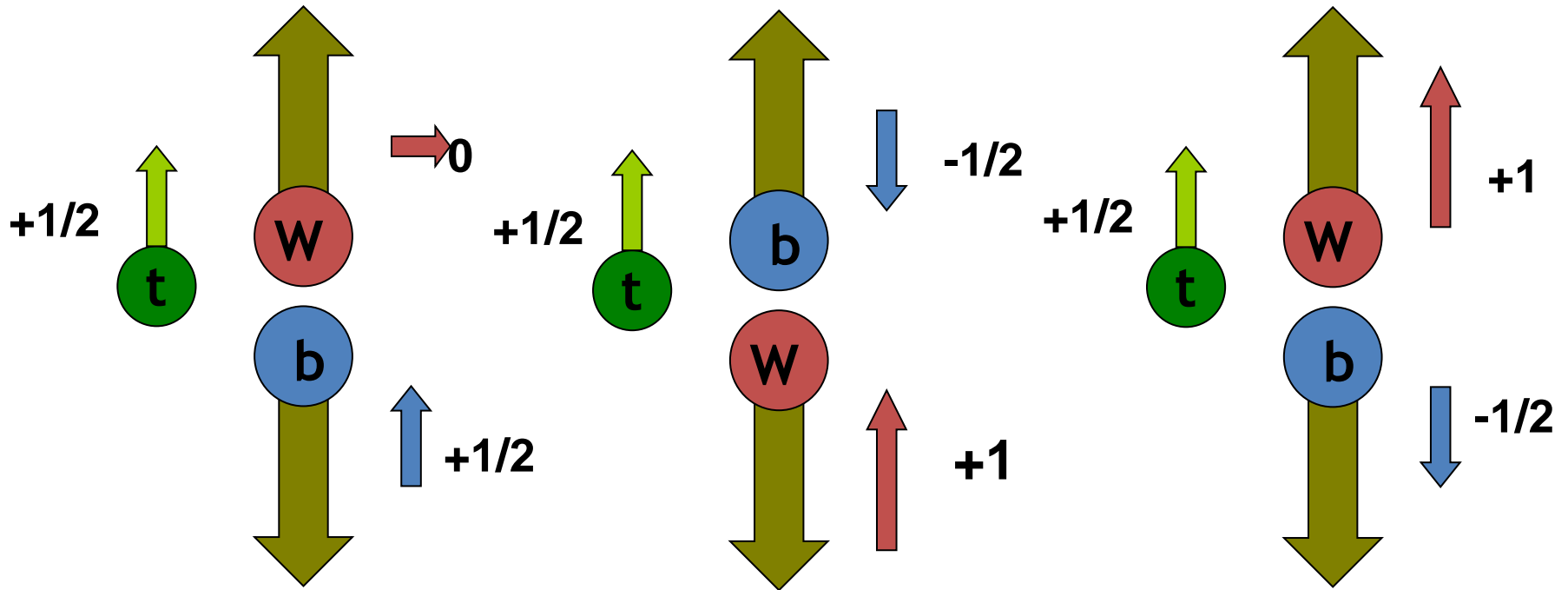
- all electroweak quantities are linked at higher order
- Sensitivity to the Higgs sector (high mass)

W helicity in top decays (1)

Longitudinal W
 $F_0 \approx 0.7$

Left-handed W
 $F_L \approx 0.3$

Right-handed W
 $F_R \approx 0$



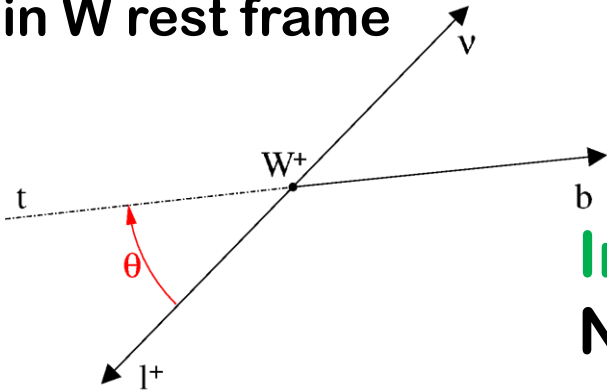
Like for muon decays : $m_b \ll m_t, m_W$

\Rightarrow chirality = helicity

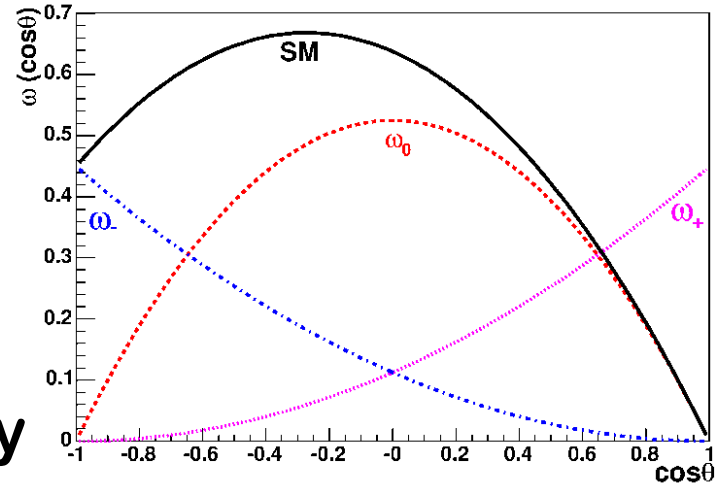
\Rightarrow right-handed W is strongly suppressed

W helicity in top decays (2)

Angle between top and lepton
in W rest frame

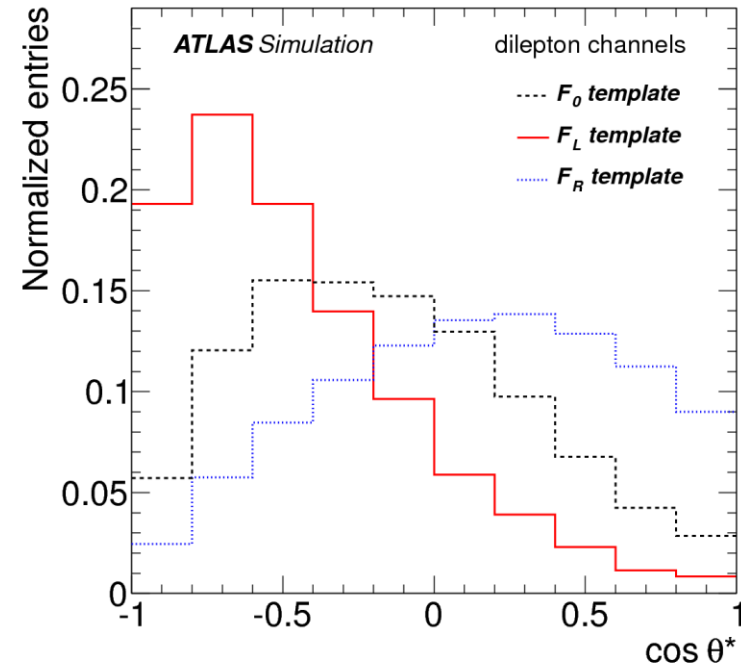
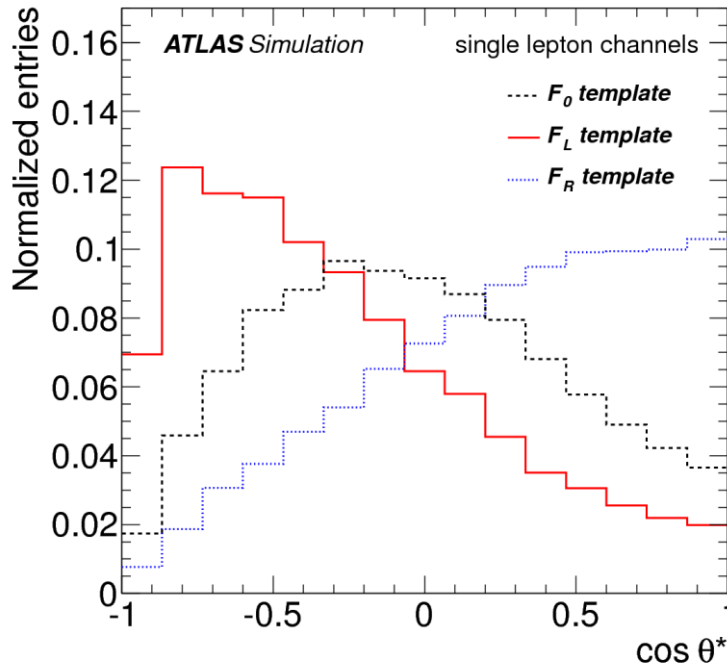


In theory :
Nice and easy



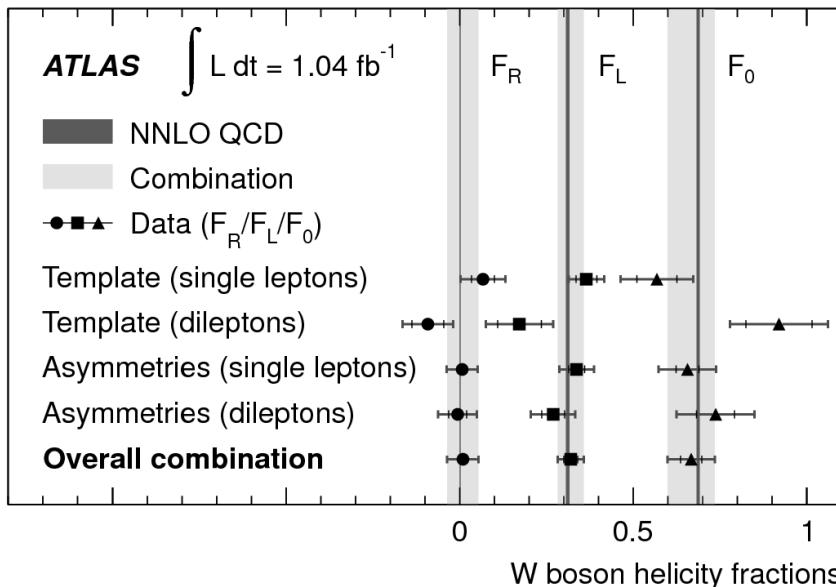
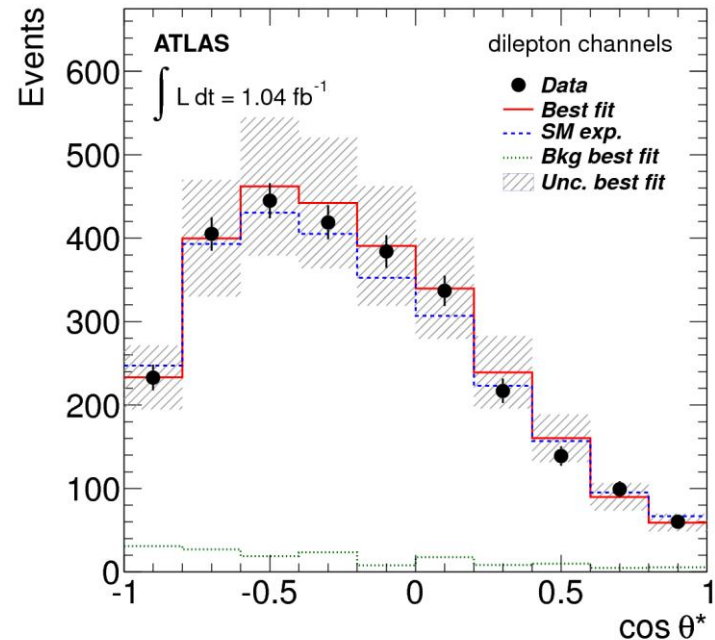
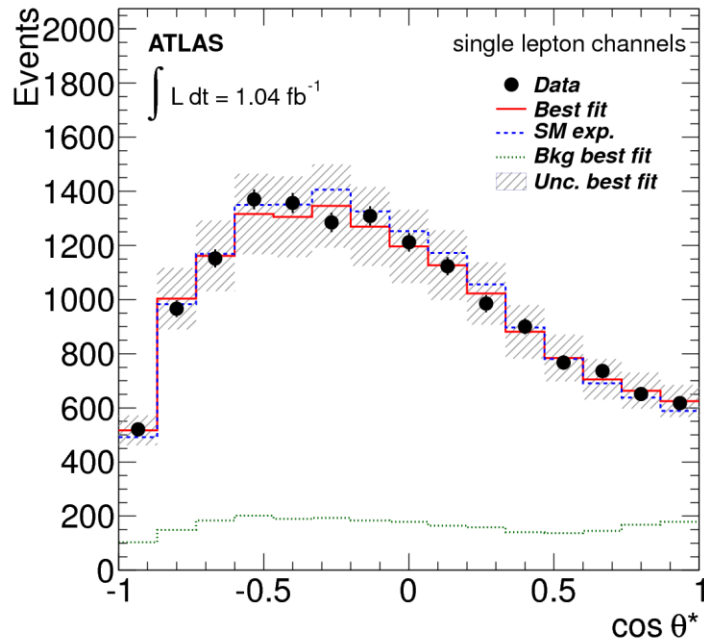
In practice :

Expected
shapes for
ATLAS
experiment



Also possible : unfolding...

ATLAS results 1fb⁻¹



The **Model** stays boringly **Standard**

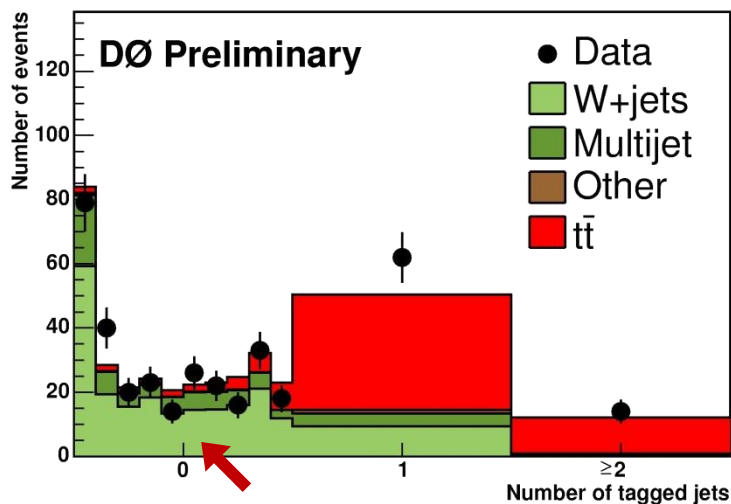
Top decays and V_{tb}

B-jets can be identified : long lifetime of b-hadrons, larger mass, fragmentation...

$$\epsilon_{\text{btag}} \sim 60\%, \quad \text{mistag} \sim 0.1\%$$

Use events with **0 b-tag**, **1 b-tag** and **2 b-tag**
 Simultaneous measurement of $\sigma_{t\bar{t}}$ and R

$$R = \frac{B(t \rightarrow Wb)}{B(t \rightarrow Wq)} = \frac{|V_{tb}|^2}{|V_{ts}|^2 + |V_{td}|^2 + |V_{tb}|^2} = |V_{tb}|^2$$

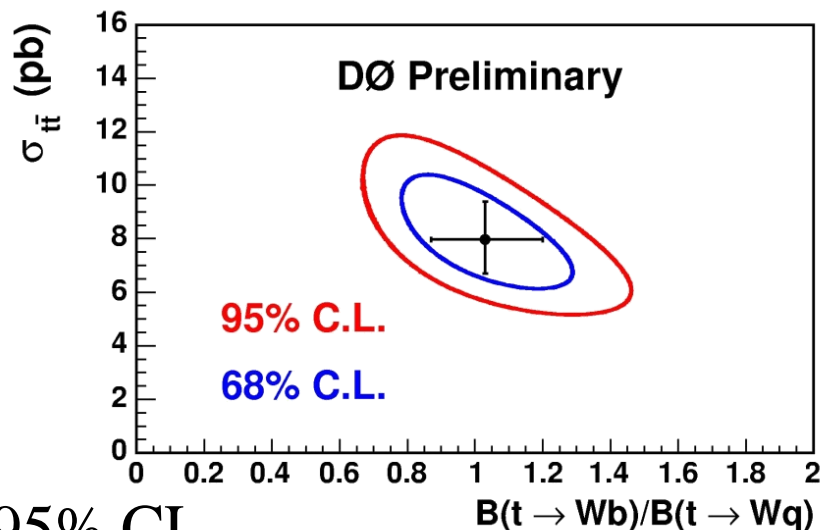


Likelihood discriminant

Very old DØ
 result
 Lepton+jets
 ($\sim 230 \text{ pb}^{-1}$)

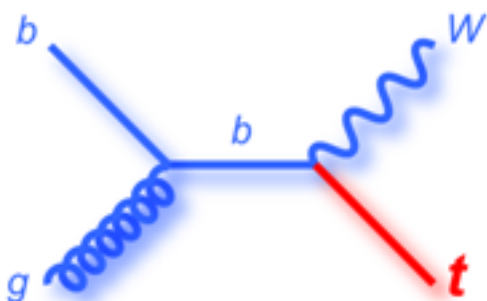
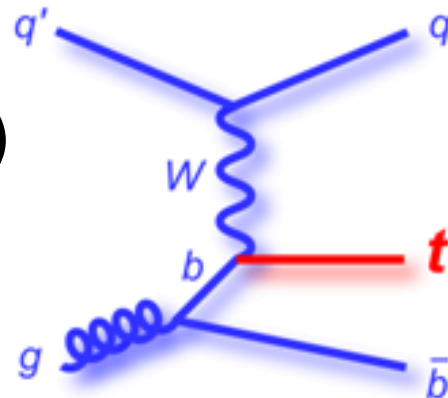
$$R = 1.03^{+0.19}_{-0.17}$$

$R > 0.64$ @ 95% CL

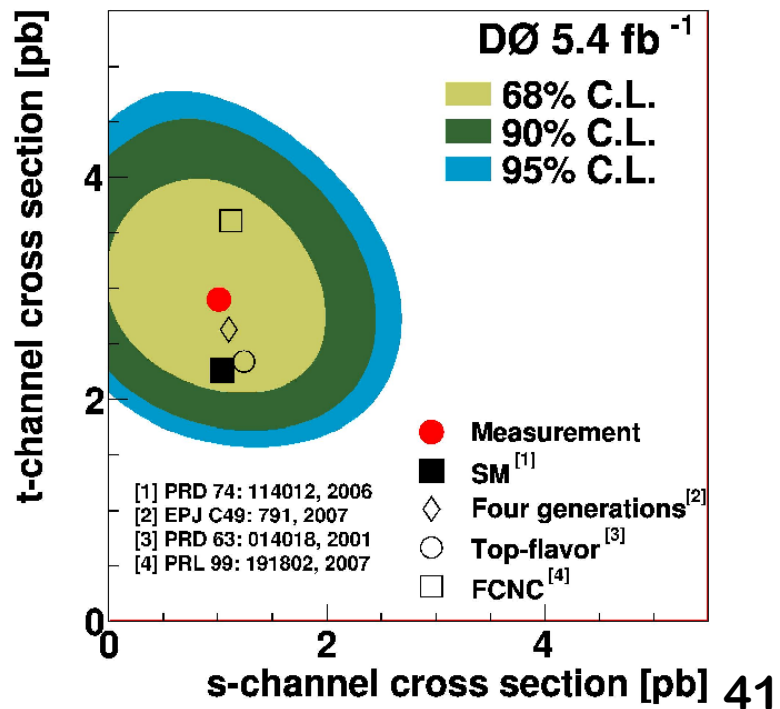
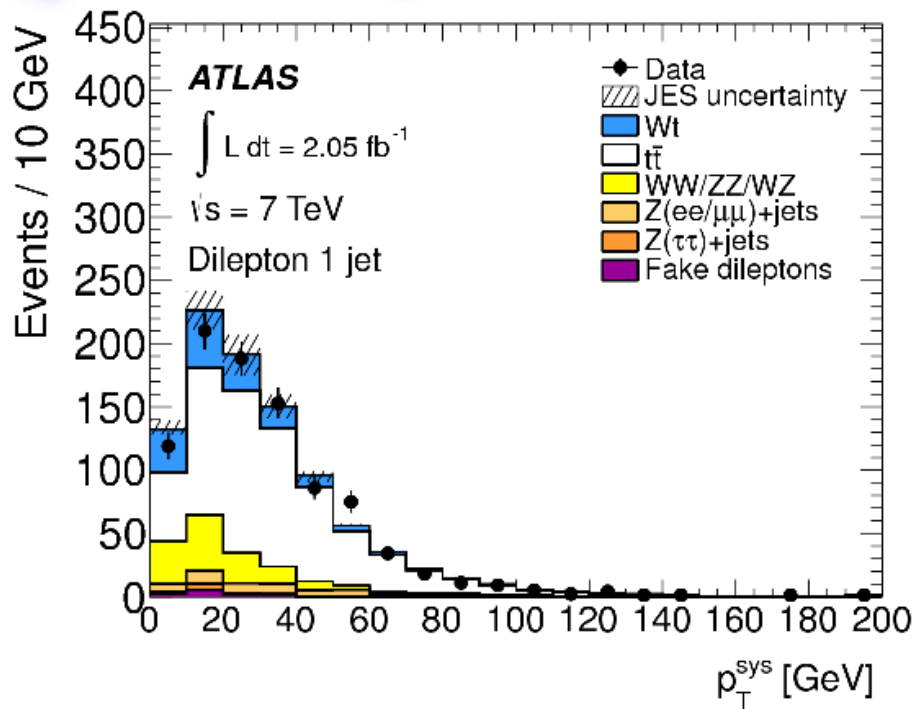


Single top

Only t-channel has been clearly ($>5\sigma$) seen at Tevatron and LHC.



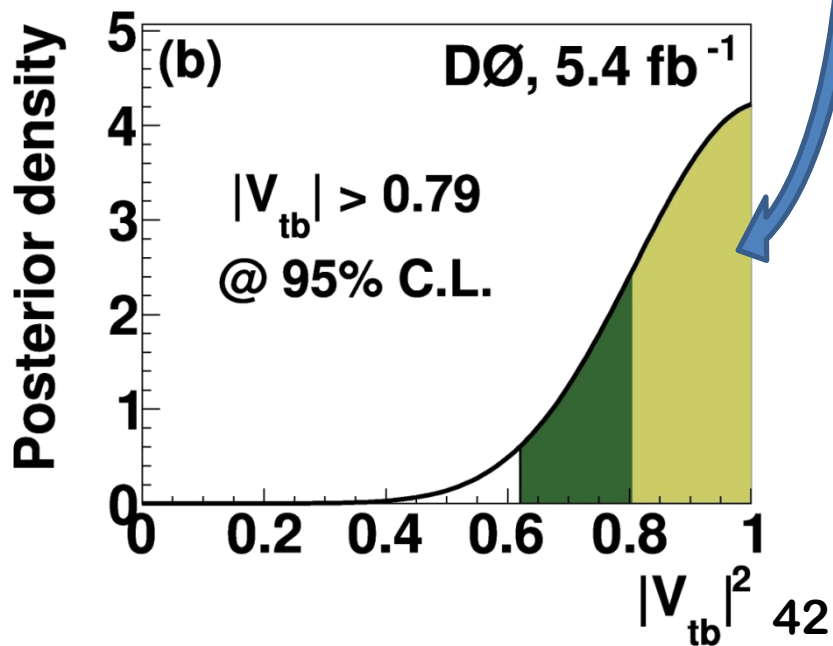
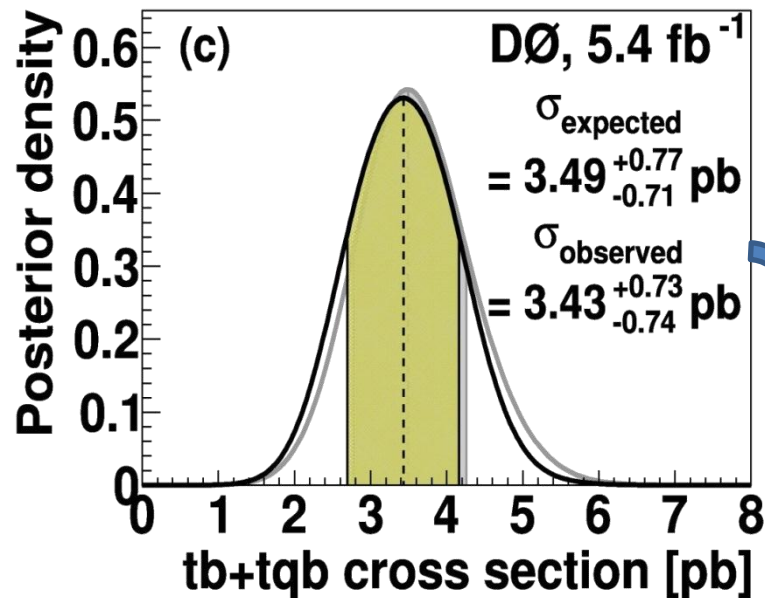
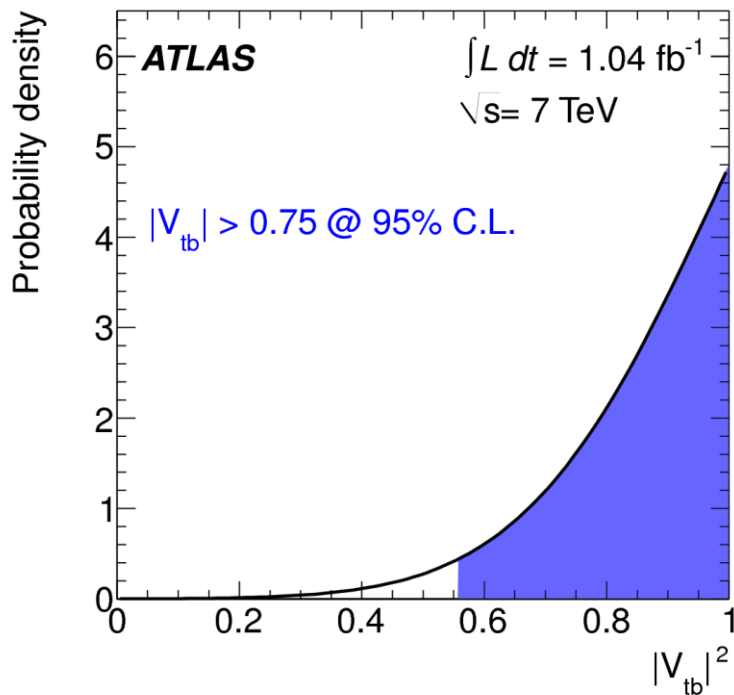
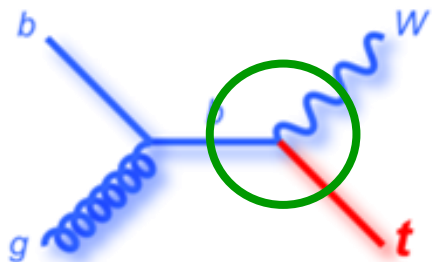
First 3σ evidence for tW (ATLAS)



Direct V_{tb} measurement

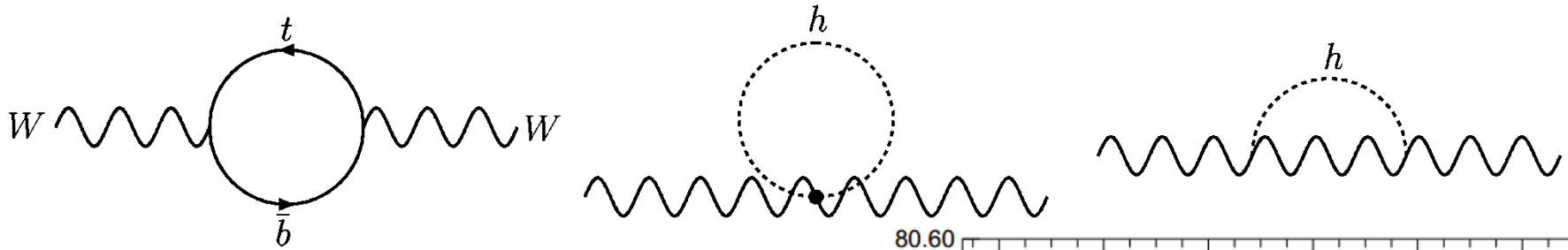
All 3 diagrams feature
1Wtb vertex

$$\sigma \propto |V_{tb}|^2$$



Standard Model consistency

Top mass is one of the ingredients of the electroweak fit
indirect constraints on the Higgs mass



**Radiative corrections (1 loop)
to the W mass**

$$m_W = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W} \frac{1}{(1 + \Delta r)}$$

$$\Delta r_{\text{top}} \approx -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2 \tan^2 \theta_W} \propto m_t^2$$

$$\Delta r_{\text{Higgs}} \approx \frac{11G_F M_Z^S \cos^2 \theta_W}{24\sqrt{2}\pi^2} \ln \frac{m_h^2}{M_Z^2} \propto \ln(m_h)$$

