

# Top Quark Physics (part 1)

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# Topics in This Talk

- Motivation for top physics
- Top production cross section
- Top quark mass (constraint on Higgs mass)
- Single top production (top partial width,  $V_{tb}$ , spin effects...)
- Forward-backward/charge asymmetry in  $t\bar{t}$ -bar
- Top spin effects (Top-antitop spin correlations)
- FCNC in  $t\bar{t}$ -bar production

# Top quark -heaviest SM elem. particle

- **Top quark**: discovered at Fermilab (CDF + D0) in 1995
- Completed the 3<sup>rd</sup> generation of SM fermions

leptons	Q	T <sub>3</sub>	quarks	Q	T <sub>3</sub>
$\nu_e \ \nu_\mu \ \nu_\tau$	0	1/2	u c <b>t</b>	2/3	1/2
<b>e</b> $\mu$ $\tau$	-1	-1/2	d s b	-1/3	-1/2

SM fundamental fermions,  $Q \equiv$  electric charge,  
 $T_3 \equiv$  3<sup>rd</sup> comp. of weak isospin

- Top quark mass ( $m_{\text{top}}$ ):  $173.5 \pm 0.6 \pm 0.8$  GeV ( $35 \times m_b$ )
- Main object of study at Fermilab
  - ✓ final sample  $10\text{fb}^{-1} \Rightarrow 70\,000$   $t\bar{t}$ -pair produced
- A very important object of study for LHC -
  - ✓ Top factory:  $\approx 1.5$  M,  $\approx 10$ M  $t\bar{t}$ -bar per  $10\text{fb}^{-1}$  for 7 and 14 TeV, resp.

# Top quark physics: Motivation

- Very high mass: near EWSB scale  $\eta$

Top Yukawa coupling  $\lambda_t = \sqrt{2}m_{\text{top}}/\eta \approx 1$

- $t\bar{t}$ -bar production X-sections: test of QCD

→  $t$  is produced at very small distances  $1/m_t \Rightarrow \alpha_S(m_{\text{top}}) \approx 0.1$ : pert. expansion converges rapidly

- Top decays before hadronization

$$\frac{1}{m_t} < \frac{1}{\Gamma_t} < \frac{1}{\Lambda} < \frac{m_t}{\Lambda^2}$$

Production time < Lifetime < Hadronization time < Spin decorrelation time

→ study of spin characteristics (test of V-A)

- Cross sections sensitive to new physics

→ resonant production of  $t\bar{t}$ , decay:  $t \rightarrow Hb$

- Important background for Higgs studies

Top is special!

Stringent tests of SM  
+  
Search for New physics

$$\eta = 246 \text{ GeV}$$

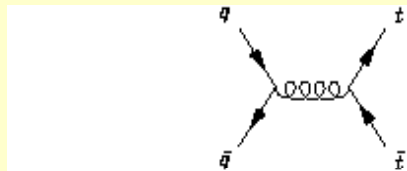
$$\Lambda \approx 250 \text{ MeV}$$

# Top Quark Production

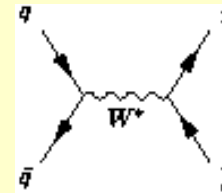
LHC  $\sqrt{s} = 7-14$  TeV vs Tevatron  $\sqrt{s} = 1.96$  TeV

Strong  $t \bar{t}$  pair production

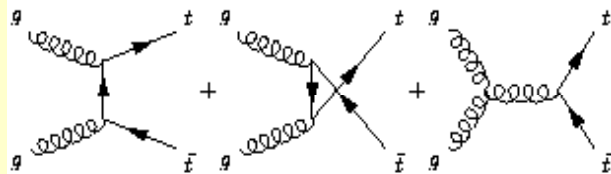
EW single top quark production



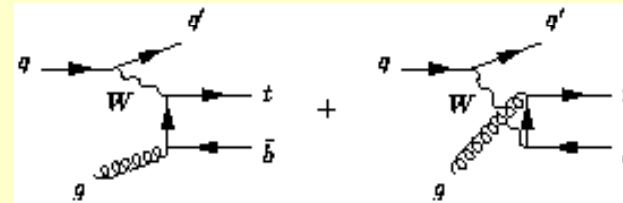
$q\bar{q} \rightarrow t\bar{t}$   
(13%, 85%)



$q\bar{q} \rightarrow t\bar{b}$   
(Drell-Yan)



$gg \rightarrow t\bar{t}$   
(87%, 15%)



$qg \rightarrow q' t\bar{b}$   
(W-g fusion)

Tevatron: X-sec  $\approx 7$  pb (5%)

X-sec  $\approx 3$  pb ( $t + t\text{-bar}$ )

LHC /7TeV:  $\approx 165$  pb

$\approx 85$  pb

/8TeV:  $\approx 230$  pb  $\epsilon \leq 5\%$

/14TeV:  $\approx 870$  pb

$\approx 320$  pb

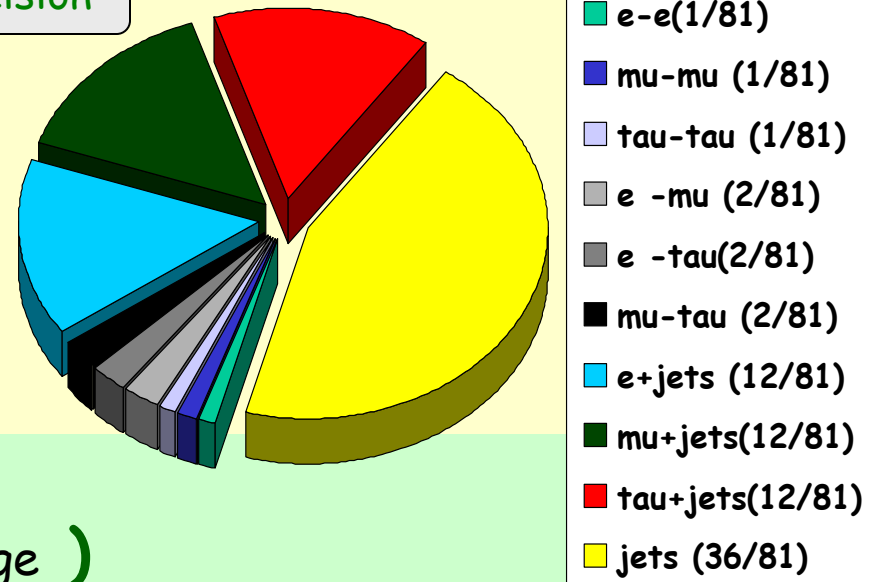
# Top Quark Decay

$\alpha_s^2$  precision

SM: by far dominant  $t \rightarrow bW$

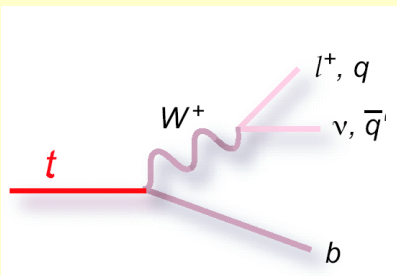
$$\frac{\Gamma(t \rightarrow bW)}{|V_{tb}|^2} \approx 0.807 \times \frac{G_F m_t^3}{8\pi\sqrt{2}} = 1.42 \text{ GeV}$$

$$\tau_{\text{top}} \approx 5 \times 10^{-25} \text{ sec} \ll \tau_{\text{hadr}} (10^{-23} \text{ sec})$$

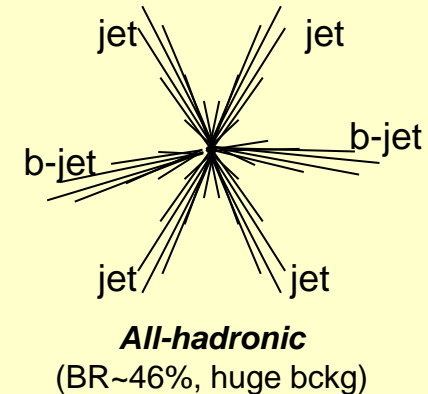
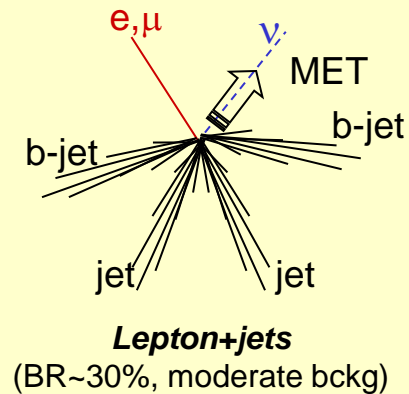
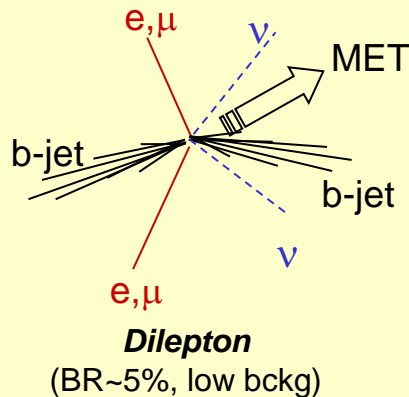


Top decays before hadronization !!!

- No  $t\bar{t}$ -bar bound states ( gluon exchange )
- $t, W$  helicity from SM V-A (no depolarization via hadronization)



$t\bar{t}$ -bar samples  
via W decays



# Cross Section of Top Quark production

# $t\bar{t}$ Production Cross Section

Top quark X-section: Experiment vs Theory

Factorization theorem:

$$\sigma = \sum_{i,j} \int dx_1 dx_2 \underbrace{F_i^{(1)}(x_1, \mu_F) F_j^{(2)}(x_2, \mu_F)}_{\text{Parton Distribution Functions (PDFs)}} \hat{\sigma}_{ij}(s; \mu_F, \mu_R)$$

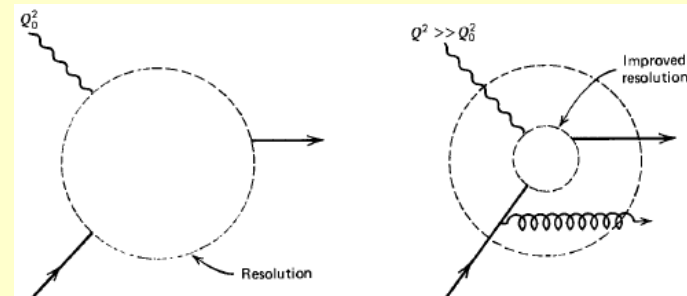
experiment theory

$F_i^{(\lambda)}(x_\lambda, \mu_F) \equiv$  probability density to observe a parton  $i$  with longitudinal momentum fraction  $x_\lambda$  in incoming hadron  $\lambda$ , when probed at a scale  $\mu_F$

$\mu_F \equiv$  factorization scale (a free parameter) - it determines the proton structure if probed (by virtual photon or gluon) with  $q^2 = -\mu_F^2$

$\mu_R \equiv$  renormalization scale - defines size of strong coupling constant

Usual choice:  $\mu_F = \mu_R = \mu \in (m_t/2, 2m_t)$



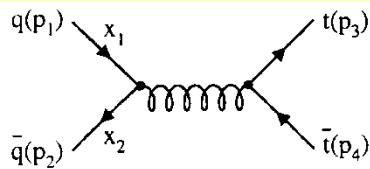


# $t \bar{t}$ Production Cross Section

The LO top quark pairs cross section (Born term):

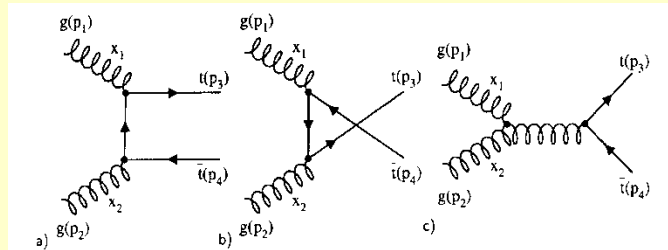
$$d\hat{\sigma} = \frac{1}{2(p_1 + p_2)^2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \delta(p_1 + p_2 - p_3 - p_4) \overline{|M|^2}$$

Quark -antiquark annihilation



$$\overline{|M|^2}(q\bar{q} \rightarrow t\bar{t}) = (4\pi\alpha_s)^2 \frac{8}{9} \left( 2 \frac{(p_1 \cdot p_3)^2 + (p_2 \cdot p_3)^2}{(p_1 \cdot p_2)^2} + \frac{m_t^2}{(p_1 + p_2)^2} \right)$$

Gluon fusion



Averaged over initial and summed over final color and spin state

$$\overline{|M|^2}(gg \rightarrow t\bar{t}) = (4\pi\alpha_s)^2 \left( \frac{(p_1 + p_2)^4}{24(p_1 \cdot p_3)(p_2 \cdot p_3)} - \frac{8}{9} \right)$$

$$\times \left( 4 \frac{(p_1 \cdot p_3)^2 + (p_2 \cdot p_3)^2}{(p_1 \cdot p_2)^4} + \frac{4m_t^2}{(p_1 + p_2)^2} - \frac{m_t^4 (p_1 + p_2)^4}{(p_1 \cdot p_3)^2 (p_2 \cdot p_3)^2} \right)$$

**Experiment:**

LO  $t\bar{t}$  Xsec is not sufficient!

Higher orders are needed

# $t\bar{t}$ Production Cross Section

Theory for top X-section is at NNLO:

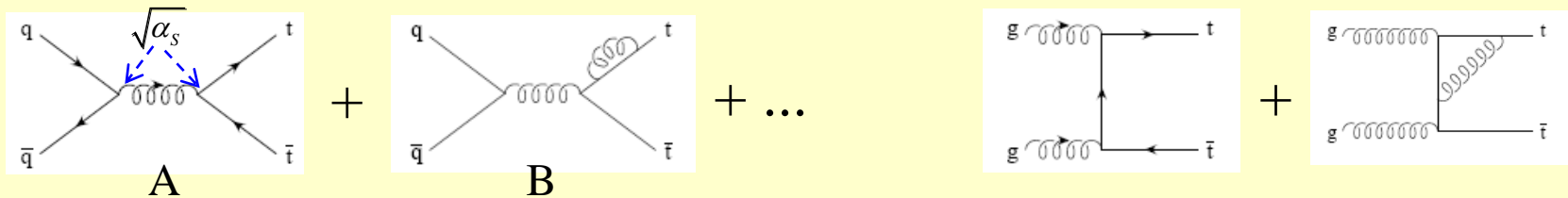
Xsec is expanded into series of strong coupling constant:

$$\sigma_{ij} \left( \beta, \frac{\mu^2}{m^2} \right) = \frac{\alpha_s^2}{m^2} \left\{ \sigma_{ij}^{(0)} + \alpha_s \left[ \sigma_{ij}^{(1)} + L \sigma_{ij}^{(1,1)} \right] + \alpha_s^2 \left[ \sigma_{ij}^{(2)} + L \sigma_{ij}^{(2,1)} + L^2 \sigma_{ij}^{(2,2)} \right] + O(\alpha_s^3) \right\}$$

$$LO \sim \alpha_s^2, \quad NLO \sim \alpha_s^3, \quad NNLO \sim \alpha_s^4 \dots \quad \beta = \sqrt{1 - 4m^2/s} \quad L \equiv \text{big log term}$$

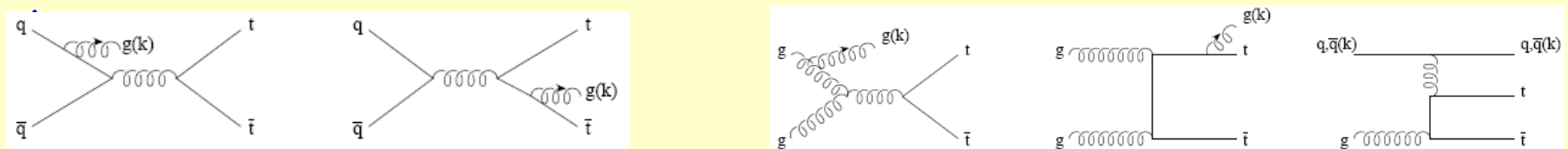
NLO: virtual and real corrections are added to LO

Virtual corrections:



Taking  $|A+B|^2 = \dots + AB^* + \dots$ ,  $AB^* \sim \alpha_s^3$

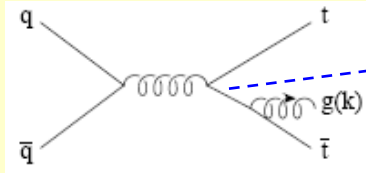
Real corrections - with real gluons ( $\sim \alpha_s^3$ ):



# A few top Cross Section issues

Higher order real and virtual corrections exhibit IR and UV divergences:

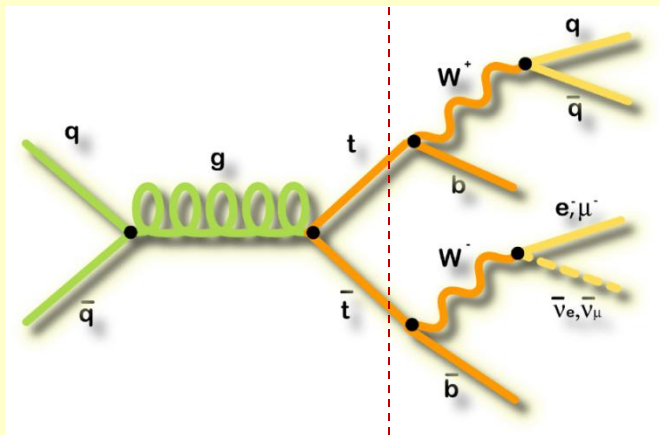
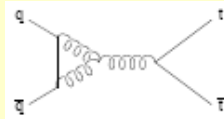
Example:



$$\text{propagator} = \frac{1}{(p+k)^2} = \frac{1}{2E_p E_k} \cdot \frac{1}{1 - \beta_p \cos \theta}, \quad \beta_p = \sqrt{1 - m^2/E_p^2}$$

✓ **IR singularity:**  $E_k \rightarrow 0$  and  $1 - \beta_p \cos \theta \rightarrow 0 \Rightarrow$  cancelled when Xsec of virtual and real emission are summed also mass singularities are cancelled  $\Rightarrow$  Cancellation is not full  $\Rightarrow$  **presence of big logs (L) in Xsec terms!**

✓ UV singularities in loops (



In real we observe  $t\bar{t}$  decay products not  $t\bar{t}$   
Factorization is used based on the **narrow width approximation:**

- ✓ polarized top quarks are produced on mass shell
- ✓ polarized on-shell top quarks decay

Narrow width app. vs direct  $pp \rightarrow WWbb$ :

For LHC 7TeV/DIL: Xsec(fb) 837 vs 841 also done for 14 and 1.96TeV

# Top cross section - measurement

Selection criteria: trigger + offline selection  $\Rightarrow$  candidate events

□ Depend on the analysed channel:  $t\bar{t}$  production

- lepton+jets (LJ), dilepton (DL) and all hadronic mode (AH)

- $lv2b2j$                        $2(l\nu)2b$                        $2b4j$       all:  $+1j, 2j\dots$

- LJ: single lepton high- $p_T$  ( $E_T$ ) trigger applied + Reconstructed level:

- ✓ 1 high- $p_T$  lepton +  $\geq 4$  high- $p_T$  jets (1-2b-tagged) + high  $E_T$

- ✓ Restricted on pseudo-rapidity,  $p_T$  ( $E_T$ )  $> 20$  GeV  $E_T > 20$  GeV

- What are selection criteria for DL and AH?

□ Background processes - non  $t\bar{t}$  events also pass Selection criteria:

- Basic bkgd processes for LJ channel:

- ✓ W+jets, Z+jets, diboson, single top quark, QCD multijets

- Bkgd processes: studied using MC + data driven techniques

$$\sigma_{t\bar{t}} = \frac{N_{obs} - N_{bkg}}{A \cdot \varepsilon \int L dt}$$

$N_{obs}(N_{bkg}) \equiv$  observed (expected bkgd) events

$A \equiv$  acceptance,  $\varepsilon \equiv$  trigger efficiency,  $L \equiv$  luminosity

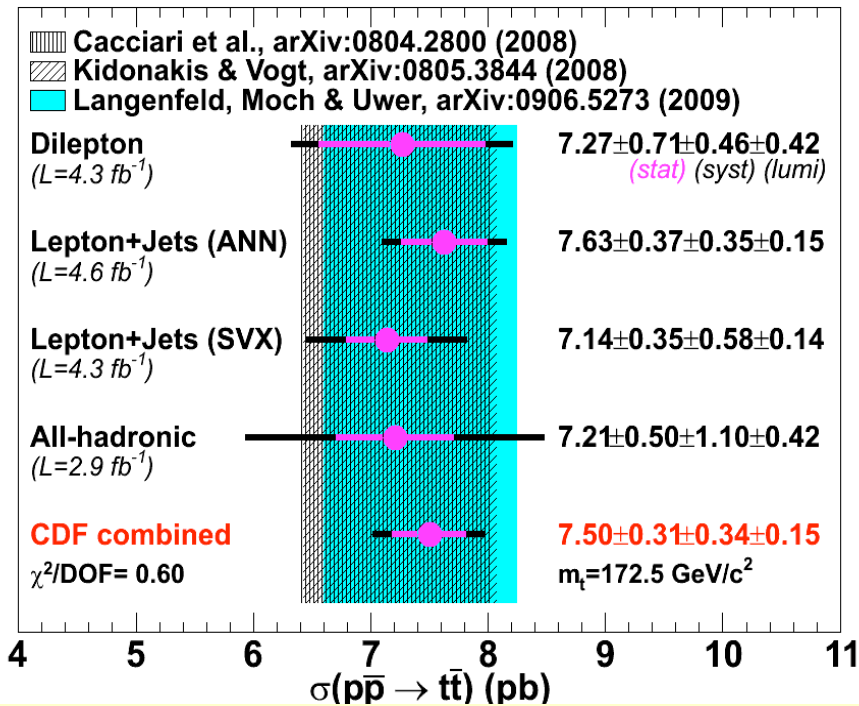
# Theory (NNLO) vs Experiment

	Tevatron	LHC 8GeV	
TOPIXS 1.0	$7.00^{+0.21+0.29}_{-0.31-0.25}$	$229.8^{+16.5+9.7}_{-16.7-9.0}$	} theory
top++ 1.3	$7.00^{+0.20+0.29}_{-0.31-0.24}$	$230.2^{+15.3+9.8}_{-15.2-9.0}$	
HATHOR	$7.07^{+0.31+0.29}_{-0.40-0.24}$	$246.8^{+13.4+10.8}_{-17.7-9.9}$	
TopNNLO	$6.59^{+0.07+0.63}_{-0.41-0.47}$	$220.0^{+11.7+19.0}_{-11.8-18.5}$	
Kidonakis 2010	$7.08^{+0.00+0.36}_{-0.24-0.27}$		
D0 2011	$7.56^{+0.63}_{-0.56}$		} experiment
CDF 2009	$7.50^{+0.48}_{-0.48}$		
CMS 2012		$227^{+15}_{-15}$	

Theory: Tevatron  $\approx 5\%$ , experiment: CDF  $\approx 6.4\%$ , CDF+D0  $\approx 5.5\%$   
 LHC  $\approx 4\%$  CMS  $\approx 6.3\%$   
 Events: CDF: 1 200 vs CMS: 7 000

# Tevatron (2TeV): Top pair cross sections

## CDF combination

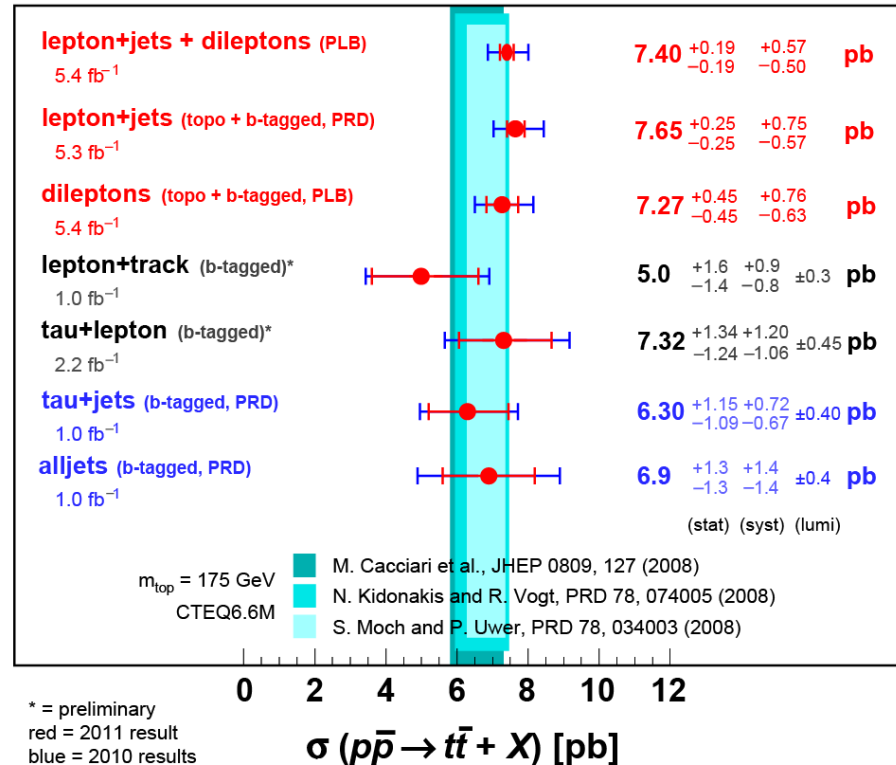


## Main uncertainties

- ✓ JES for all
- ✓ b-tagging for SVX analysis
- ✓ Generator for all

## DØ Run II

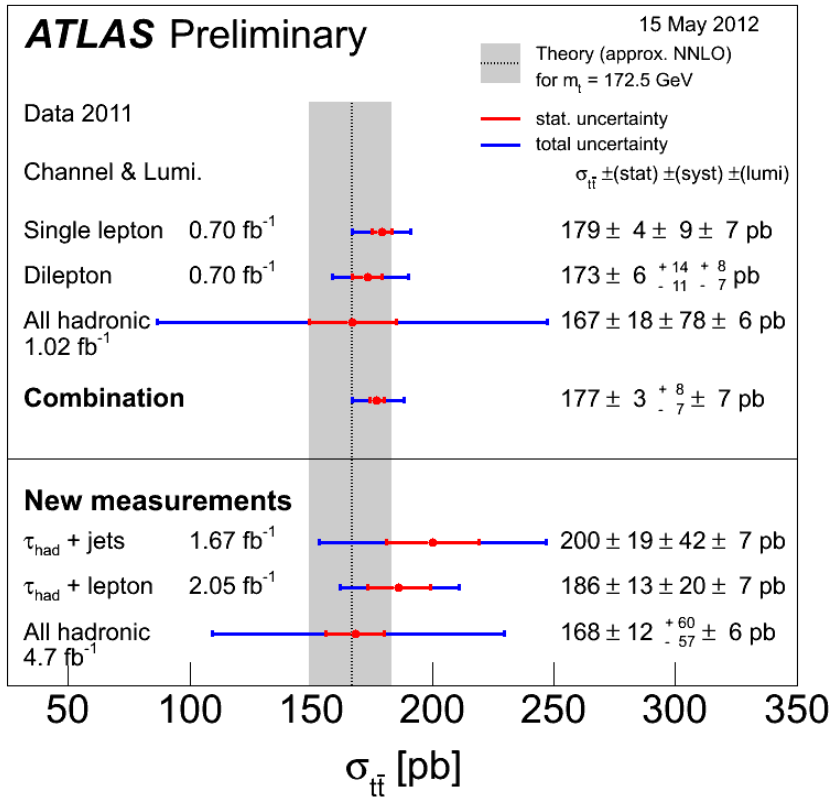
July 2011



- ❖ All channels are compatible
- ❖ Exper. error  $\approx$  theo. Error
- ❖ Full NNLO needed!

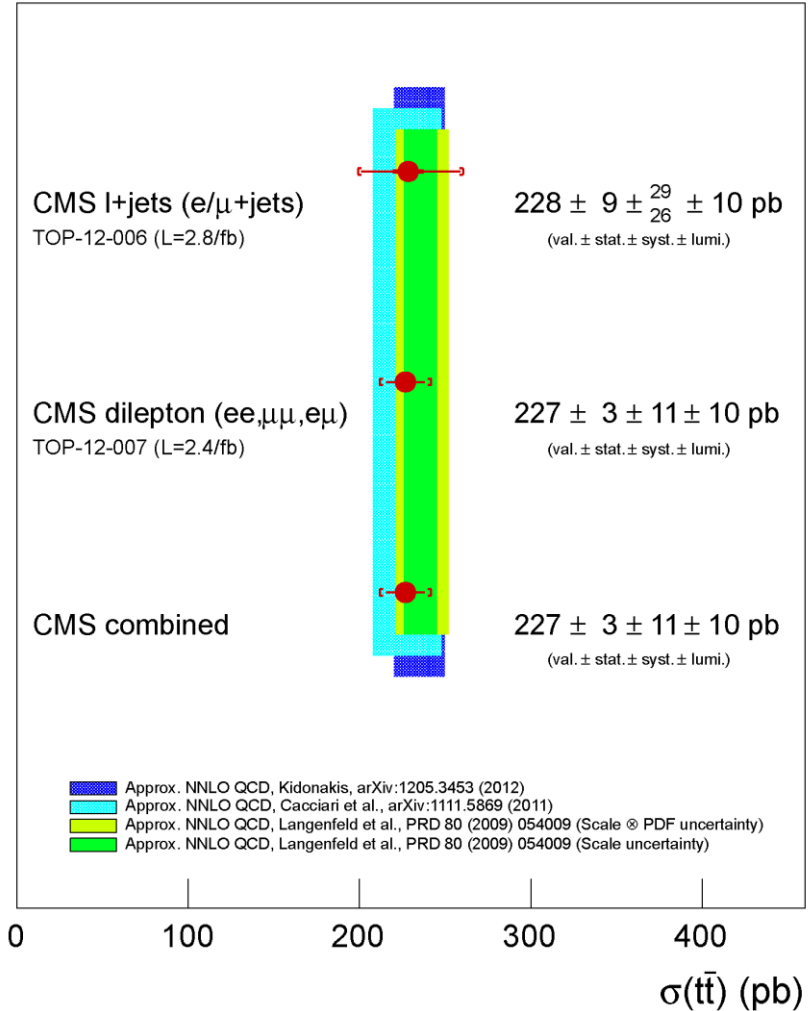
# LHC ttbar cross section measurement

Atlas,  $\sqrt{s}=7$  TeV



- ✓ A good agreement with theory
- ✓ Statistical error plays no role ...

CMS Preliminary,  $\sqrt{s}=8$  TeV



# Top Quark Mass Reconstruction

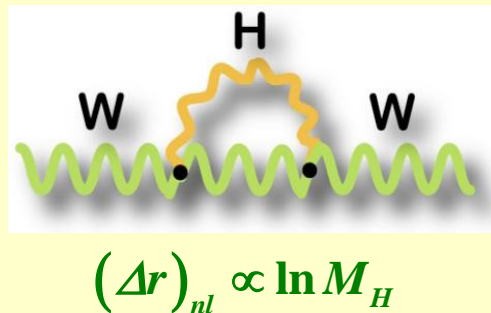
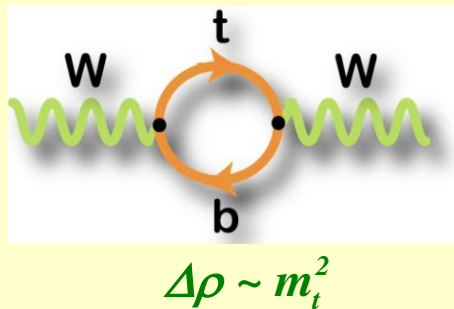


# Top mass and EW precision physics

Masses of top, W and Higgs are bounded by

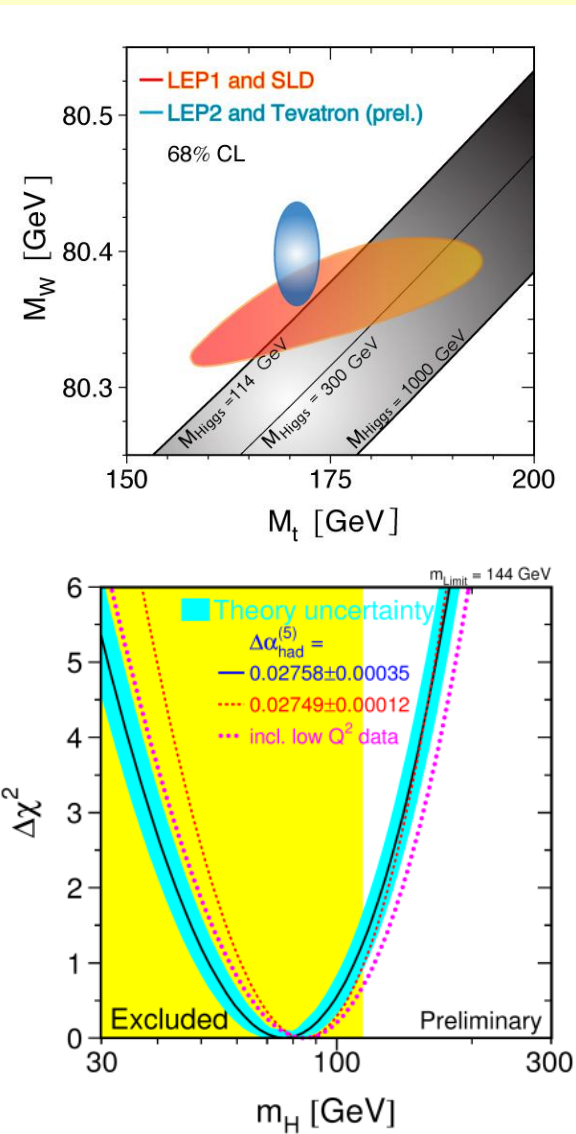
$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_F} (1 + \Delta r), \quad \Delta r = \Delta\alpha + \frac{s_W}{c_W} \Delta\rho + (\Delta r)_{nl}$$

From rad. Corrections to W-boson propagator (any process, e.g.  $\mu^- \rightarrow \nu_\mu W^- \rightarrow \nu_\mu e^- \bar{\nu}_e$ ):

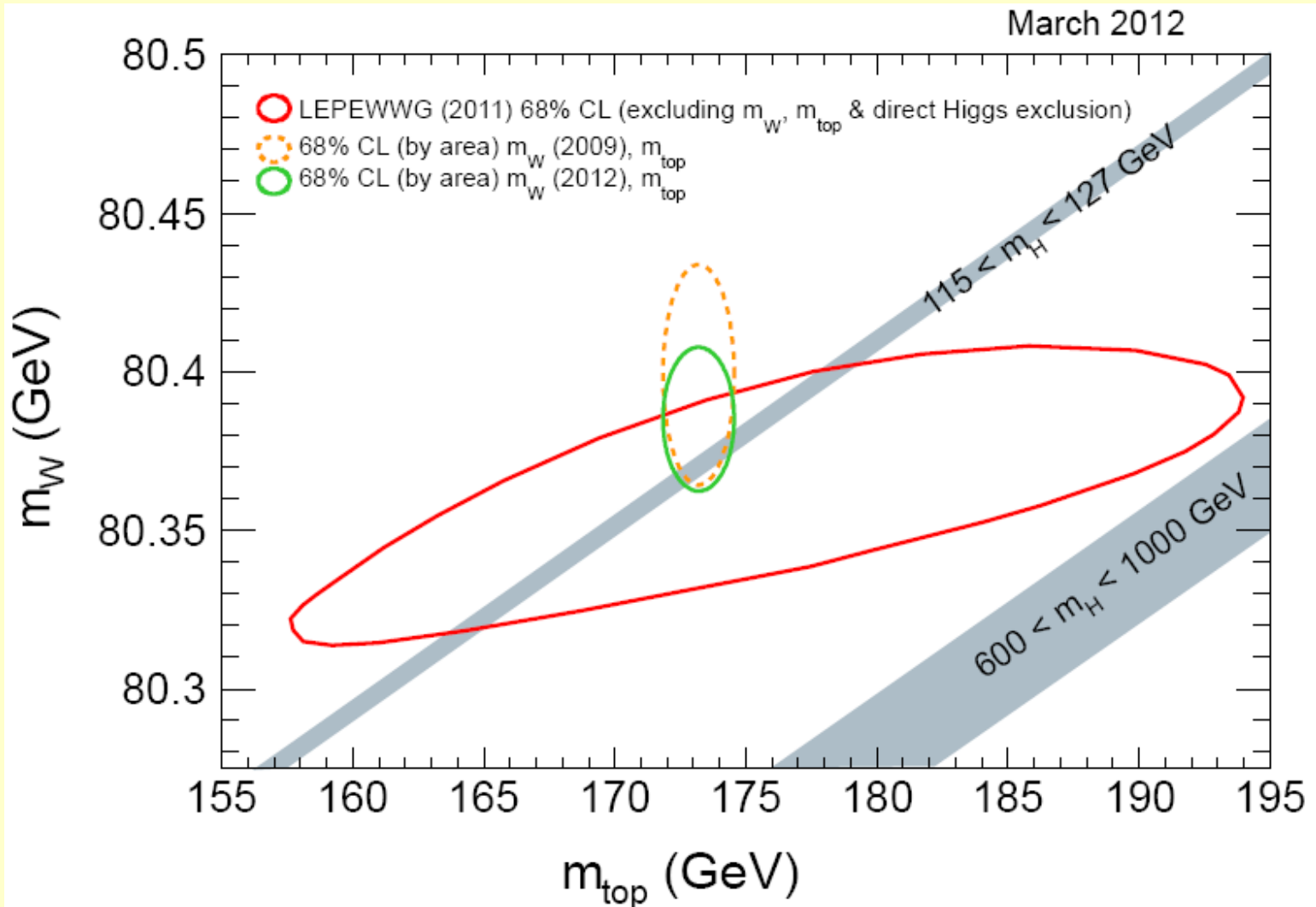


Precise  $M_W$  and  $m_t \Rightarrow$  constraint on  $M_H$ !

- ✓ LHC can improve:  $\Delta m_t$  and  $\Delta M_W$
- ✓ Stringent consistency test of SM



# ...an another Higgs restriction plot



What will be happen with the green ellipse after 20-30  $\text{fb}^{-1}$ ?...

# How to measure top mass?

Top quark mass can be reconstructed in all  $t\bar{t}$  topologies (LJ, DL AH)

Best results usually in lepton + jets topology

- Different approaches are used - usually:
  - ✓ Template methods - different variants
  - ✓ Matrix element methods - use dependence of top pair production  $X_{\text{sec}}$  on top quark mass.
  - ✓ Any variable correlated with top quark mass can be used for determination of top mass - e.g. mean lepton  $p_T$  (LJ, DL)
- To retrieve top mass usually event kinematic should be reconstructed

# Top quark mass template method

Basic idea of template method - L+jets topology :

- ✓ to find invariant mass of top decay products:  $t \rightarrow bq\bar{q}, \bar{t} \rightarrow \bar{b}lv, t \leftrightarrow \bar{t}$
- ✓ Using reconstructed objects of candidate events a **kinematic fitter** is used to find 4-momenta of top decays products .
- ✓ Kinematic fitter minimizes  $\chi^2$  function, e.g.:

$$\chi^2 = \sum_{i=l,4 \text{ jets}} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=1,2} \frac{(U_j^{fit} - U_j^{meas})^2}{\sigma_j^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{lv} - M_W)^2}{\Gamma_W^2} + \frac{(M_{bjj} - m_t^{rec})^2}{\Gamma_t^2} + \frac{(M_{blv} - m_t^{rec})^2}{\Gamma_t^2}$$

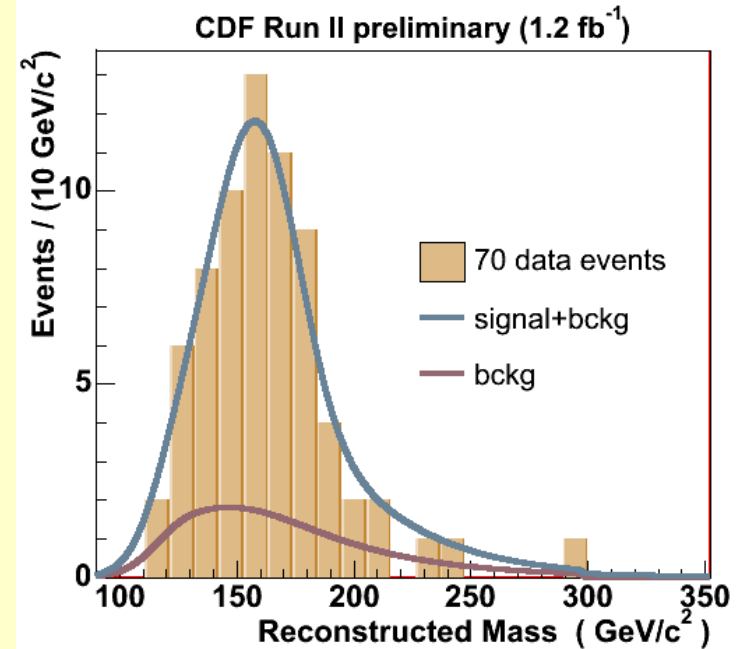
**Problem:** for candidate event we can have several event configurations - connected with different assignments of jets to quarks - without b-tagging: **12 configurations per a LJ event (and for 1 or 2-btags?)**

- ✓ The  $\chi^2$  fit is applied to all the event configurations
- ✓ KF gives for each event comb.  $m_t^{rec}$  and  $\chi^2$  - correct  $m_t^{rec} \Leftrightarrow$  minimal  $\chi^2$

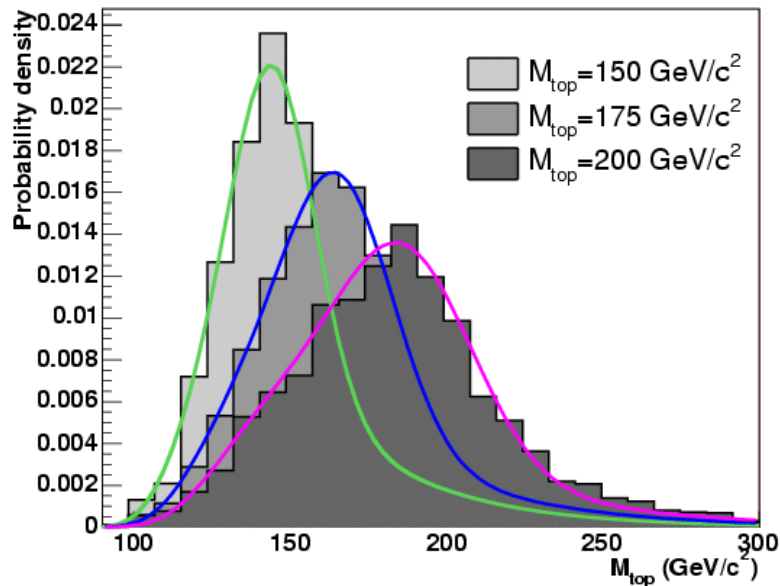
Using MC for a given input top mass - expected rec. mass distribution (**template**) can be found - data mass distr. is compared with mass templates

# Top mass in DIL channel: templates

- ✓ Top quark mass measured in dilepton channel using a template method.
- ✓ Due to 2 neutrinos  $M_{\text{top}}$  reconstruction from dilepton events is underconstrained  $\Rightarrow p_z^{t\bar{t}}$  fixed to solve event kinematics
- ✓ The sample is separated into b-tagged and non-tagged samples.



B-tagged signal templates



For each event  $M_{\text{top}}^r$  reconstructed assum.:

$$M_{W^\pm} = 80.4 \text{ GeV}/c^2, M_t = M_{\bar{t}} \text{ and } p_z^t + p_z^{\bar{t}} = 0$$

$$M_{\text{top}} = 169.7_{-4.9}^{+5.2} \text{ (stat.) } \pm 3.1 \text{ (syst.) GeV}/c^2$$

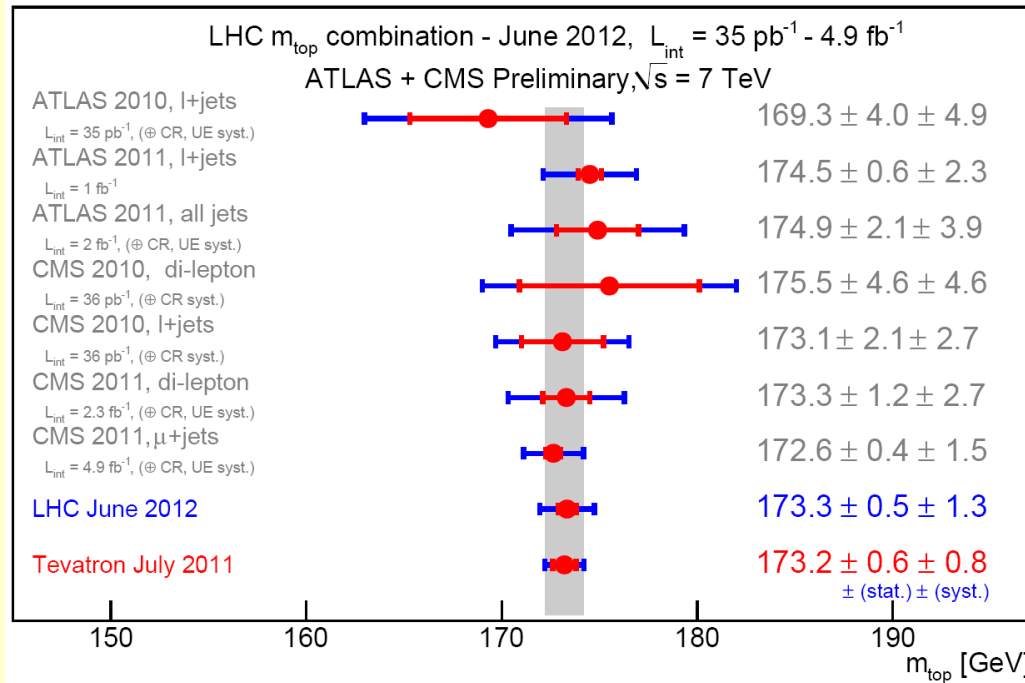
using a cross-section constraint

$$M_{\text{top}} = 170.7_{-3.9}^{+4.2} \text{ (stat.) } \pm 2.6 \text{ (syst.) } \pm 2.4 \text{ (th.) GeV}/c^2$$

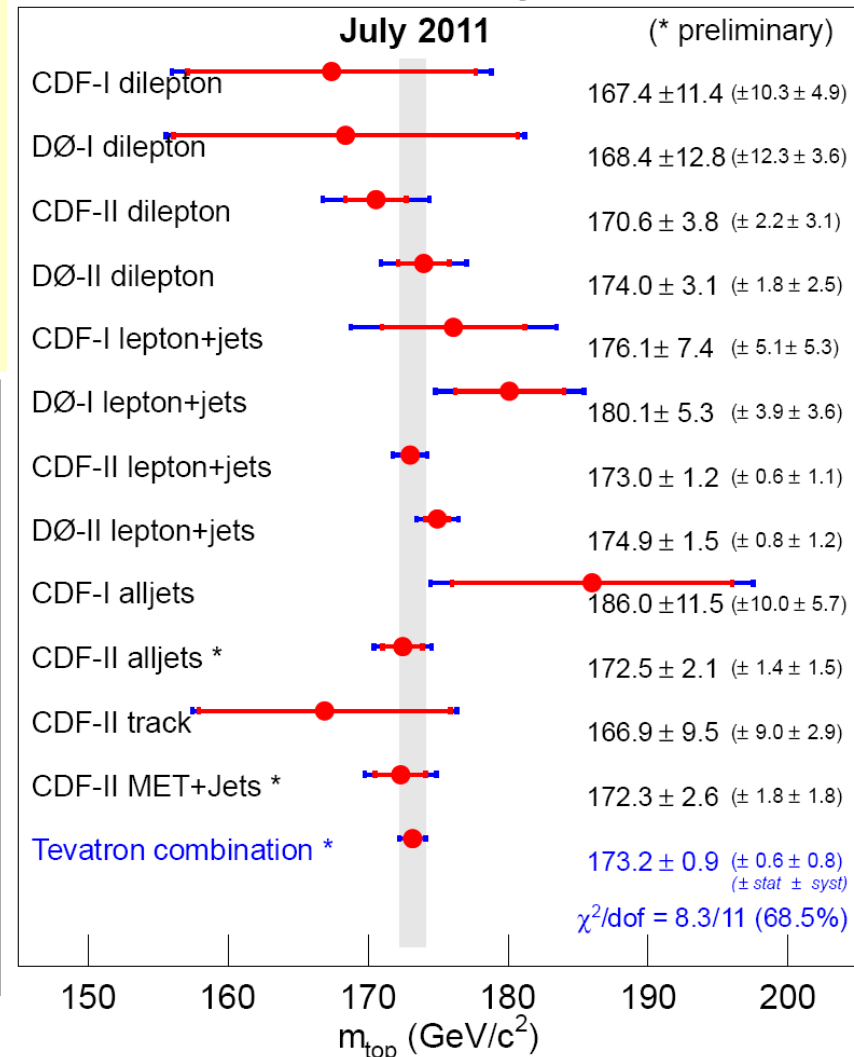
# Top quark mass

Tevatron (CDF, D0)  $\implies$

L  
H  
C } ATLAS  
CMS



## Mass of the Top Quark

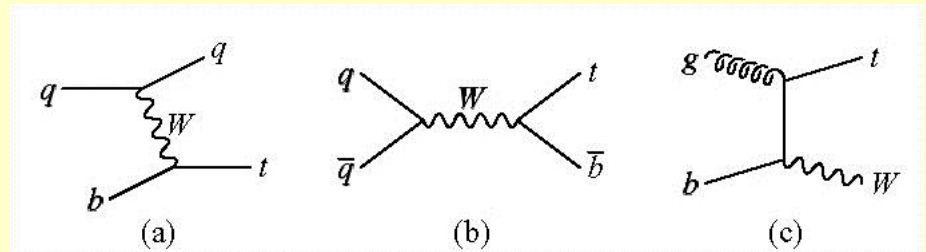


# Single Top Quark production

# Single top quark production

## Production via weak forces

- $\sigma \sim |V_{tb}|^2$   
(direct measurement of  $V_{tb}$ )
- Significant bckgd to Higgs signal
- Single top -100% polarization  
(test of V-A structure of EW)
- Possible new physics



t-channel      s-channel      assoc. prod.

	Tevatron	7 TeV LHC	14 TeV LHC
$t(\bar{t})$ "t"-ch	1.2	40 (20)	150 (100)
$t(\bar{t})$ "s"-ch	0.55	2.5 (1.4)	7 (4)
$tW^-$	0.15	8	45

## Signature of Single Top Event

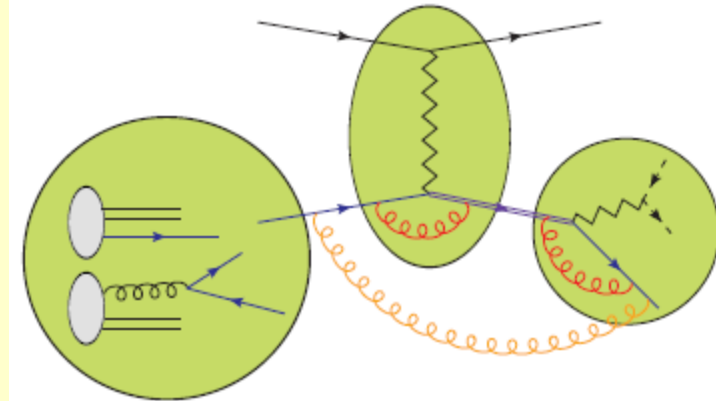
- ✓ Only 1 isolated high  $p_T$  lepton ( $e$  or  $\mu$ ):  $p_T > 20 \text{ GeV}$
- ✓ High miss- $p_T$  ( $E_T$ )  $> 25 \text{ GeV}$
- ✓ 2 or 3 high  $p_T$  jets:  $p_T > 20 \text{ GeV}$
- ✓  $\geq 1$  b-tagged jet



# Single top quark cross section

## Present status:

- ✓ Production and decay are factorized
- ✓ NLO corrections in production
- ✓ resummation of soft logs
- ✓ top decay, at LO/NLO, spin correlations
- ✓ off-shell effects / non-factorizable corrections
- ✓  $b$  quark issues ( $m_b$  mass) ...



Variety of theoretical approaches:  
Go to NNLO taking big logs

## Single top: s-channel

Kidonakis  $m_+ = 173 \text{ GeV}$

$$\sigma_{TeV} = 0.523^{+0.001+0.030}_{-0.005-0.028} \text{ pb}$$

$$\sigma_{LHC} = 3.170^{+0.06+0.13}_{-0.06-0.10} \text{ pb}$$

Zhu et al.  $m_+ = 173.2 \text{ GeV}$

$$\sigma_{TeV} = 0.467^{+0.010}_{-0.010} \text{ pb}$$

$$\sigma_{LHC} = 2.81^{+0.16}_{-0.10} \text{ pb}$$

# Single top: t-channel and assoc.prod

Single top: t-channel, calculated at  $m_t = 173 \text{ GeV}$

Kidonakis [1103.2792]

$$\sigma_{TeV} = 1.04_{-0.02}^{+0.00} \pm 0.06 \text{ pb}$$

$$\sigma_{LHC7} = 41.7_{-0.2}^{+1.6} \pm 0.8 \text{ pb}$$

$$\sigma_{LHC14} = 151_{-1}^{+4} \pm 3 \text{ pb}$$

Zhu et al. [1010.4509]

$$\sigma_{TeV} = 0.982 \text{ pb}$$

$$\sigma_{LHC7} = 40.9_{-0.1}^{+0.1} \text{ pb}$$

$$\sigma_{LHC14} = 152.4_{-1.0}^{+0.4} \text{ pb}$$

An excellent compatibility of the theoretical calculations

**W t production** : Kidonakis [1005.4451], at  $m_t = 173 \text{ GeV}$

$$\sigma(tW^-) = 7.8 \pm 0.2_{-0.02}^{+0.00} \text{ pb}$$

- ✓ NLO → 'N'NLO: 8% increase at 7 TeV LHC
- ✓ At LHC assoc. production gives a noticeable contribution

# Single top: experimental analysis

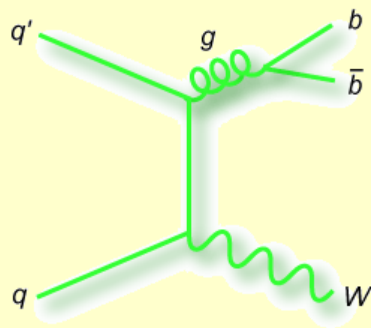
Single top quark production first observed by D0 and CDF in 2009

Main problem in experiment:  
huge background - an example from  
CDF analysis at  $L=3.2 \text{ fb}^{-1}$

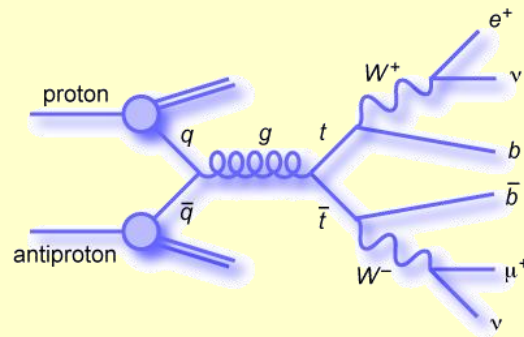
Single top	145.7	± 21.4
Total background	2119.3	± 350.9
Total prediction	2265.0	± 375.4
Observed	2229	

## Main Backgrounds

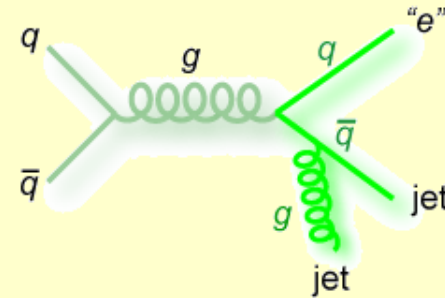
### W + jets



### top-antitop pairs



### QCD multijet production



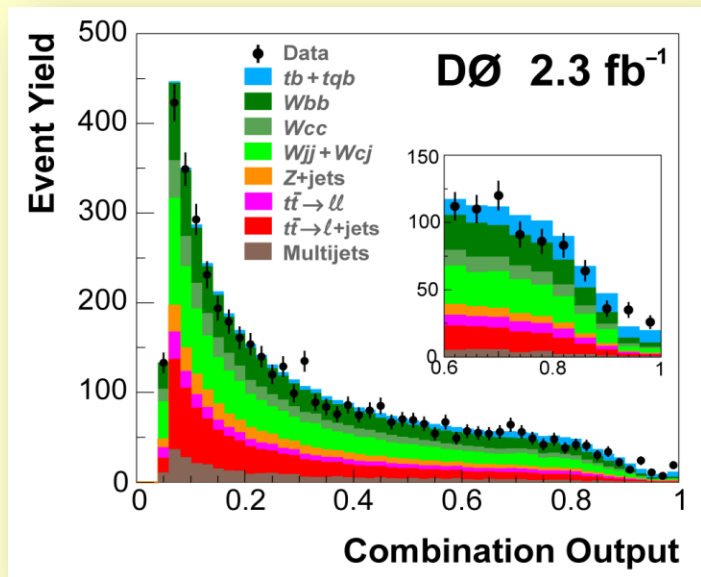
# Multivariate techniques

To cope with background Multivariate techniques (MVT) are used:

- ✓ Neural Networks (NN)
- ✓ Boosted Decision Tree (BDT)
- ✓ Matrix Element (ME)
- ✓ Likelihood Discriminants (LD)

Basic idea: a set of different kinematic variables ( $M_{l\nu b}$ ,  $H_T$ ,  $M_{jj}$ ,  $M_T$ ...) is used as input for MVT which employ them to optimize Signal vs Background.

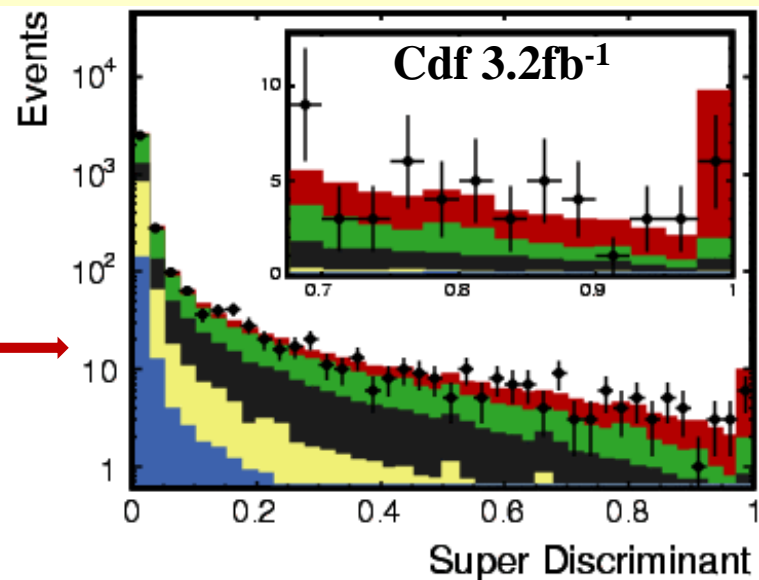
Output of MVT: **output discriminant** - a variable in (0,1) or (-1, 1)



p-value:

$$2.5 \times 10^{-7}$$

$$3.1 \times 10^{-7}$$



# ATLAS: single top quark

Xsec of single top quark production in the t-channel,  $L=1.04 \text{ fb}^{-1}$ , pp collision data at  $\sqrt{s} = 7 \text{ TeV}$

SM expectation:  $\sigma_t = 64^{+2.7}_{-2.0} \text{ pb}$ ,  $\sigma_{Wt} = 15.7 \pm 1.1 \text{ pb}$ ,  $\sigma_s = 4.6 \pm 0.2 \text{ pb}$

**Event selection:** exactly one charged lepton (e or  $\mu$ ), two or three jets, and  $E_T > 25 \text{ GeV}$ ,  $m_T(W) > (60 \text{ GeV} - E_T)$

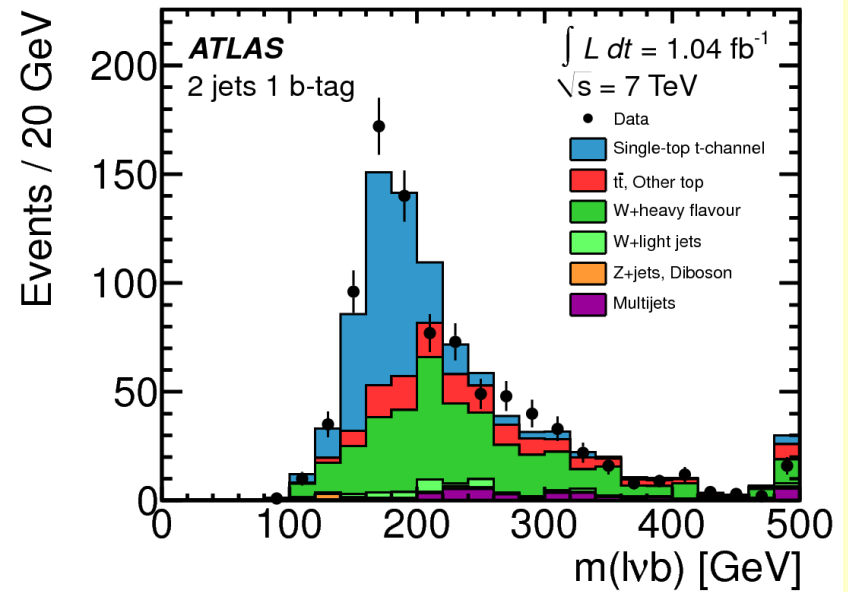
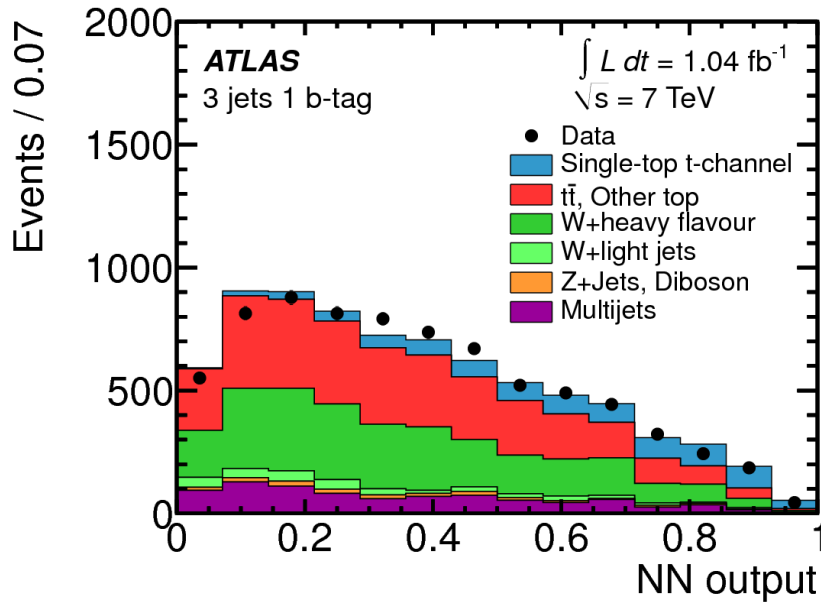
	Electron		Muon	
	2-jet	3-jet	2-jet	3-jet
single-top t-channel	$447 \pm 11$	$297 \pm 7$	$492 \pm 12$	$323 \pm 8$
<i>tt</i> , other top	$785 \pm 52$	$1700 \pm 120$	$801 \pm 53$	$1740 \pm 130$
W+light jets	$350 \pm 100$	$128 \pm 56$	$510 \pm 150$	$209 \pm 91$
W+heavy flavour jets	$2600 \pm 740$	$1100 \pm 400$	$3130 \pm 880$	$1270 \pm 480$
Z+jets, diboson	$158 \pm 63$	$96 \pm 44$	$166 \pm 61$	$80 \pm 31$
Multijet	$710 \pm 350$	$580 \pm 290$	$440 \pm 220$	$270 \pm 140$
Total expected	$5050 \pm 830$	$3900 \pm 520$	$5530 \pm 930$	$3900 \pm 520$
Data	5021	3592	5592	3915

Higher # events than in CDF: 18300 vs 2200

Higher signal % : 8.5% vs 6.5%

**NN discriminant:** 12 input variables in the jet data set:  $m(\ell vb)$ , the highest  $p_T$  untagged jet  $|\eta(j_u)|$  and  $E_T(j_u)$  - most important

# ATLAS: single top quark



Measured  $X_{sec}$  in the t-channel, simultaneous measurement in the 2-jet and 3-jet channels:

$$\sigma_t = 83 \pm 4 (\text{stat})_{-19}^{+20} (\text{syst}) \text{ pb} = 83 \pm 20 \text{ pb}$$

Significance:  $7.2\sigma$

$|V_{tb}|^2$  is extracted: ratio of the observed  $\sigma_t$  and SM expectation:

$$|V_{tb}| = 1.13_{-19}^{+20} \quad + \text{ the 95\% C.L. lower limit } |V_{tb}| \text{ is } 0.75.$$

(see arXiv.1205.3130, sub. Phys. Lett.B)

Thank you!