Elements of QCD for hadron colliders

theoretical concepts and phenomenology

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Contents of this course

- Basics of QCD
  - The QCD Lagrangian
  - Perturbation Theory & The running coupling
- Soft & collinear singularities
- The concepts of parton showers and jets
- QCD for processes with incoming protons
- Monte-Carlo event generators

Soft & collinear singularities: recap

I) soft/collinear gluon emission cross section factorizes

$$|\mathcal{M}_{q\bar{q}g}|^2 d\Phi_{q\bar{q}g} \simeq |\mathcal{M}_{q\bar{q}}|^2 d\Phi_{q\bar{q}} d\mathcal{S}$$

where

$$d\mathcal{S} = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$\rightsquigarrow$ divergent as $E \to 0$ and/or $\theta \to 0$

II) very singularities cancel between real & virtual parts

$$\sigma_{tot}(e^+e^- \to q\bar{q}) = \sigma_{q\bar{q}} \left(\underbrace{1}_{\text{LO}} + \underbrace{1.045 \frac{\alpha_s(Q^2)}{\pi}}_{\text{NLO}} + \underbrace{\ldots}_{\text{higher orders}} \right)$$

$\rightsquigarrow$ perturbation theory works well for inclusive cross sections

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$\rightsquigarrow$ perturbation theory works well for inclusive cross sections

$\rightsquigarrow$ **let's look a little more exclusive now**

$\rightsquigarrow$ **estimate the number of emitted gluons**

Multiple gluon emissions

Let's try to integrate emission probability to estimate mean number of gluon emissions off a quark with energy $\sim Q$

$$\langle N_g \rangle \simeq \frac{2\alpha_s C_F}{\pi} \int^{Q} \frac{dE}{E} \int^{\pi/2} \frac{d\theta}{\theta} \Theta(E\theta > Q_0)$$

- diverges for $E \to 0$ & $\theta \to 0$
- cut out transverse momenta ($k_t \simeq E\theta$) smaller than $Q_0 \sim \Lambda_{\mathrm{QCD}}$
  $\rightsquigarrow$ below that the language of quarks & gluons loses its meaning

$$\langle N_g \rangle \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} + \mathcal{O}\left(\alpha_s \ln \frac{Q}{Q_0}\right)$$

assume $Q = 200$ GeV & $Q_0 = 1$ GeV $\rightsquigarrow \ln^2 \frac{Q}{Q_0} \approx 30$

$\rightsquigarrow$ simple expansion in α_s spoiled by large logarithms, $\langle N_g \rangle > 1$

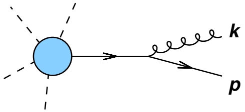
Is 1st order perturbation theory useless beyond total cross sections?

- Could try to calculate next order, and see what happens!
- Can try to approximate higher-order contributions!
- Look for better behaved final-state observables!

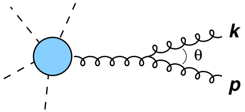
Multiple gluon emissions

Once a gluon is emitted it can itself emit further gluons

- consider collinear (& soft) emissions only [logarithmically enhanced]
- in the small angle limit ($\theta \ll 1$) emissions factorize



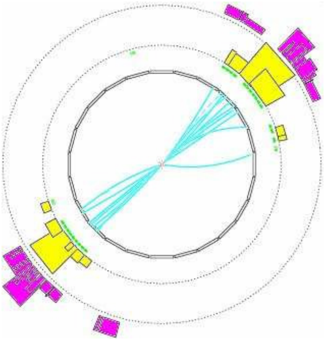
$$\simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

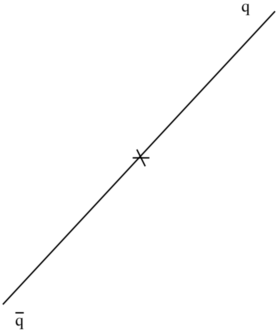


$$\simeq \quad \frac{2\alpha_s C_A}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

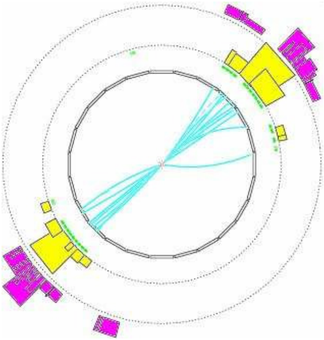
- same divergence structure, independent of who emits
- only difference being the colour factor ($C_F = 4/3,\ C_A = 3$)
  $\rightsquigarrow$ gluons emit more
- expect 1st-order structure ($\alpha_s \ln^2 Q/Q_0$) to appear at each new order

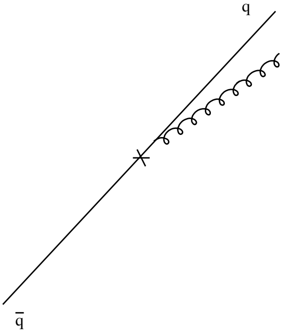
Start out with the $q\bar{q}$ system



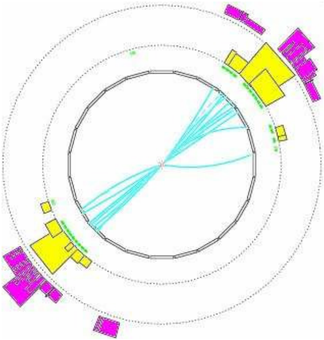


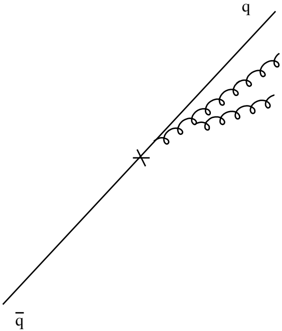
Quark emits small angle gluon





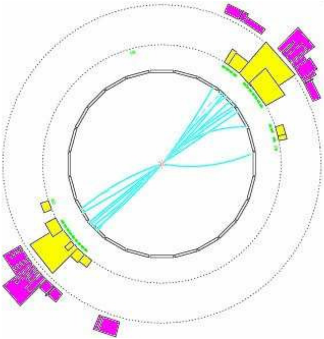
Gluon radiates a further gluon

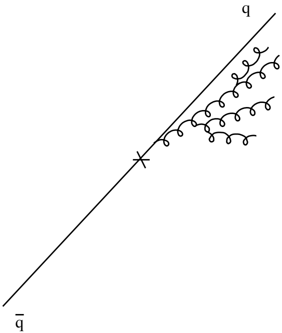




Multiple gluon emissions

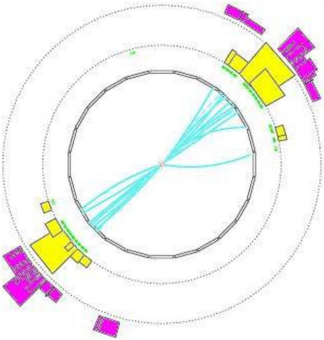
And so on and so forth ...

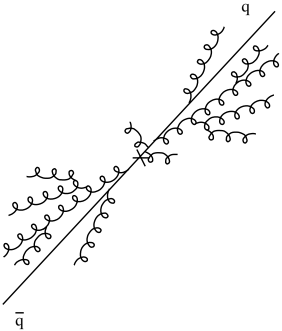




Multiple gluon emissions

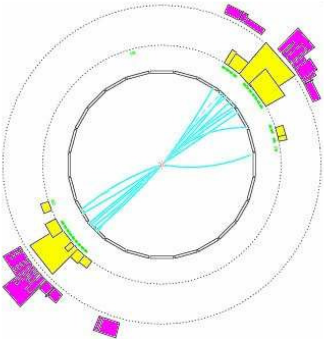
Meanwhile the same happend on the other side

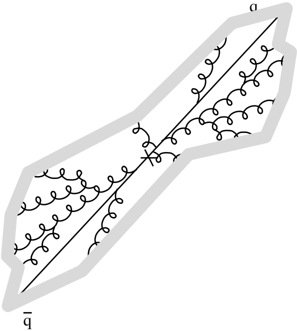




Multiple gluon emissions

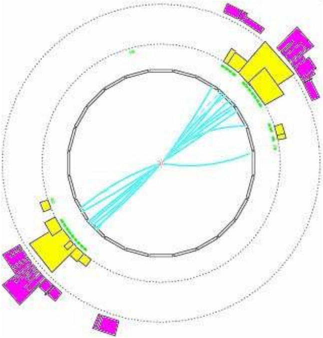
At some point a non-perturbative transition happens

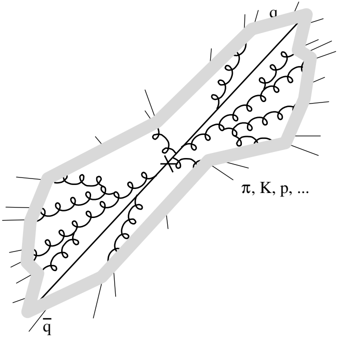




Multiple gluon emissions

Resulting in a pattern of collimated hadrons **[at small angles wrt to the quarks]**





Gluon vs. Hadron multiplicity

gluon multiplicity can be calculated by summing all orders of perturbation theory (n):

$$\langle N_g \rangle \sim \frac{C_F}{C_A} \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \left(\frac{C_A}{2\pi b_0^2 \alpha_s} \right)^n$$

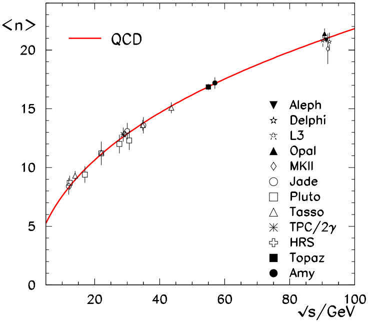
$$\sim \frac{C_F}{C_A} \exp\left(\sqrt{\frac{2C_A}{\pi b_0^2 \alpha_s(Q)}} \right)$$

interprete as a function of $Q \equiv \sqrt{s}$

direct comparison suggests

$$\langle N_{\rm had} \rangle = c_{\rm fit} \langle N_g \rangle$$

charged hadron multiplicity in e^+e^- collisions



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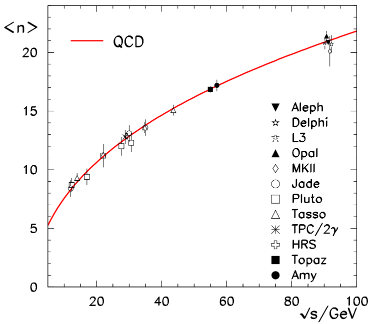


Seems like perturbative QCD can get us quite far!

Using the soft/collinear approximation we can make predictions for events' detailed partonic structure, when supplemented with a model for hadronization for hadronic final states even.

However, we cannot perform analytic calculations for every observable ever be measured. [too many experimenters, too many observables, too few theorists]

The solution: Parton-Shower simulations

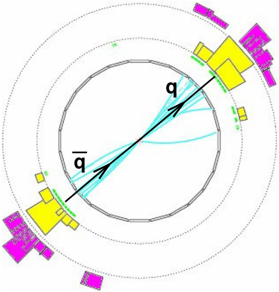
- implement space-time picture of parton evolution [limited to leading logarithms]
- successive parton emissions for arbitrary processes
- Markov-chain Monte Carlo process describing the parton proliferation
- observable/process independent

$\rightsquigarrow$ cornerstone of Monte-Carlo event generators, more soon

The emergent picture: final-state jets

Jet definition (prel.): jets are collimated sprays of hadronic particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons' directions



Counting jets

⇝ near perfect two-jet event

⇝ almost all energy contained in two cones

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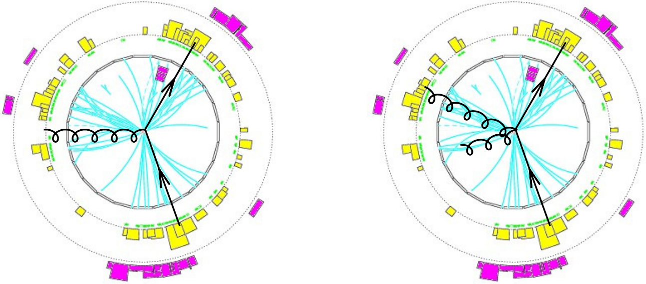


Counting jets

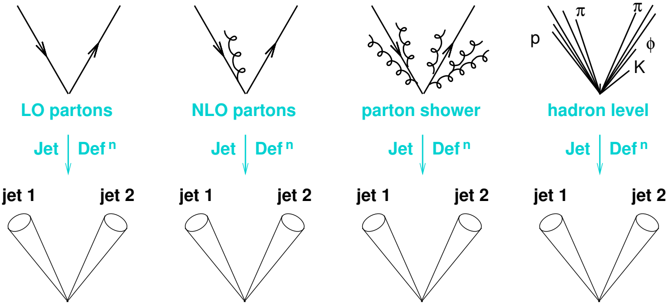
$\rightsquigarrow$ hard emissions can induce more jets

$\rightsquigarrow$ jet counting not obvious, is this a three- or four-jet event?

Defining jets

Jet definition (addendum): jet number shouldn't depend upon just a soft/collinear emission

$\rightsquigarrow$ **Infrared & collinear safety**



Infrared & Collinear safe jet definitions

crucial for comparing theory with experimental results

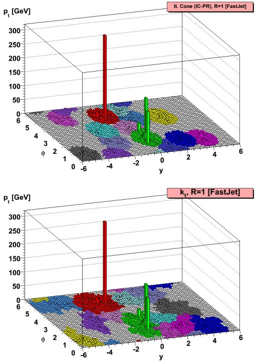
Jet algorithms

Jet definition

- group together particles into a common jets [jet algorithm]
- typical parameter is R, distance in $y - \phi$ space, determines angular reach
- combine momenta of jet constituents to yield jet momentum [recombination scheme]

two generic types of jet algorithms are commonly used:

- cone algorithms
  - widely used in the past at the Tevatron
  - jets have regular/circular shapes
  - some suffer from IR or collinear unsafety
- **sequential recombination algorithms**
  - widely used at LEP [Durham k_T algorithm]
  - jet can have irregular shapes
  - default at the LHC experiments [anti-k_T algorithm]



Sequential recombination algorithms

A generic jet finding algorithm

1. compute a distance measure y_{ij} for each pair of final-state particles
2. determine all distance measures wrt the beam y_{iB}
3. determine the minimum of all y_{ij}'s and y_{iB}'s
    1. if y_{ij} is smallest, **combine** particles ij, sum four-momenta
    2. if y_{iB} is smallest, **remove** particle i, call it a jet
4. go back to step one, until all particles are clustered into jets

in analyses one typically uses

- jets with inter-jet distances $y_{ij} > y_{\rm cut}$ [exclusive mode]
- jets with inter-jet distances $y_{ij} > y_{\rm cut}$ & $E > E_{\rm cut}$ [inclusive mode]

different algorithms use different measures: $\mathbf{y_{ij}}$ & $\mathbf{y_{iB}}$

Sequential recombination algorithms: the k_T algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \to ij$

$$dS \simeq \frac{2\alpha_s C_{A/F}}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}} ,$$

using $\min(E_i, E_j)$ we can avoid specifying which of i and j is soft

The k_T-algorithm distance measure

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{Q^2}$$

- in the collinear limit: $y_{ij} \simeq \min(E_i^2 E_j^2)\theta_{ij}^2/Q^2$
- relative transverse momentum, normalized to total energy
- soft/collinear particles get clustered first
- effectively inverts the sequence of shower emissions

Sequential recombination algorithms: the anti-k_T algorithm

recall the soft and collinear limit of the gluon-emission probability for $a \to ij$

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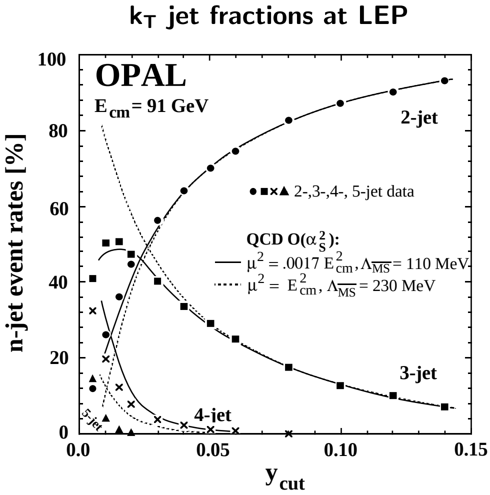
using $\min(E_i, E_j)$ we can avoid specifying which of i and j is soft

The anti-k_T-algorithm distance measure

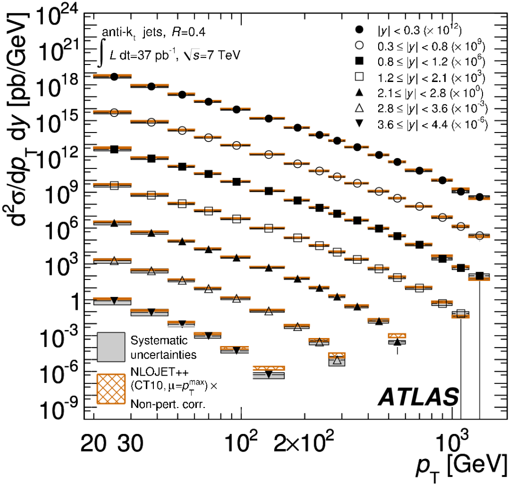
$$y_{ij} = 2Q^2 \min(E_i^{-2}, E_j^{-2})(1 - \cos\theta_{ij})$$

- ⇝ jet-finding starts out with hard objects
- ⇝ softer particles get clustered into hard jets later on
- ⇝ produces nicely regular shaped jets
- ⇝ default in current LHC physics analyses


kT jet fractions at LEP
OPAL
E cm = 91 GeV
2-jet
3-jet
4-jet
5-jet
2-,3-,4-, 5-jet data
QCD O(α s 2):
μ2 = .0017 E 2 cm, ΛMS = 110 MeV
μ2 = E 2 cm, ΛMS = 230 MeV
n-jet event rates [%]
y cut
100
80
60
40
20
0
0.0
0.05
0.10
0.15

Jet algorithms at work: anti-k_T jets at LHC

anti-k_T inclusive jets at LHC



Processes with incoming hadrons

- so far considered processes with final-state hadrons only
- to predict cross sections for processes involving initial-state hadrons, detailed understanding of the *short distance* structure of protons is needed
- at hadron colliders all processes, even of intrinsically electroweak nature, e.g. γ, W, Z, h, are induced by quarks & gluons

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Starting point: the naïve parton model

- quarks deeply bound inside proton
- binding forces responsible for confinement due to soft gluons $\mathcal{O} \simeq \Lambda_{\rm QCD}$
- the exchange of hard gluons would break the proton apart [recoil]

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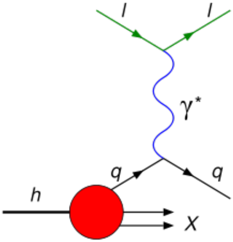
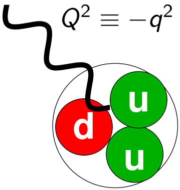
⇝ learn about the proton structure via Deep-Inelastic-Scattering (DIS)

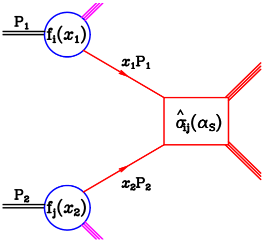




Processes with incoming hadrons: factorization

hadronic cross section in the naïve parton model

$$\sigma(s) = \sum_{ij} \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/p}(x_2)\, \hat{\sigma}_{ij \to X}(x_1 x_2 s)$$



factorized cross section

- assume partons move collinear with the protons: $p_i = x_i P_i$
- partonic cms energy: $\hat{s} = x_1 x_2 s$
- $f_{i/p}$ Parton-Distribution-Funtions parametrize number densities of quarks inside protons

Parton-Distribution-Functions: sum rules

- $|p\rangle = |\, u\ u\ d\rangle$, the valence quark distributions

$$\rightsquigarrow \int\limits_0^1 dx\ \left(f_{u/p}(x) - f_{\bar{u}/p}(x)\right) = 2 \quad \& \quad \int\limits_0^1 dx\ \left(f_{d/p}(x) - f_{\bar{d}/p}(x)\right) = 1$$

- fraction of proton's momentum carried by quarks

$$\sum_q \int\limits_0^1 dx\, x f_{q/p}(x) \simeq 0.5$$

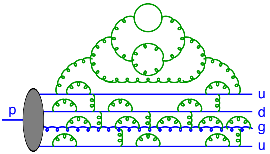
$\rightsquigarrow$ well, we kind of forgot the gluons, carry $\simeq 0.5$ of protons' momentum

$\rightsquigarrow$ gluons appear in splitting processes $q \to qg$

$\rightsquigarrow$ let's better check impact of higher-order QCD corrections

Factorization revised

most fluctuations inside the proton happen at times $t_{had} \sim 1/\Lambda_{QCD}$



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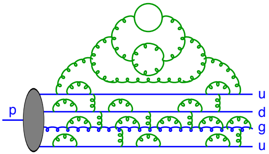
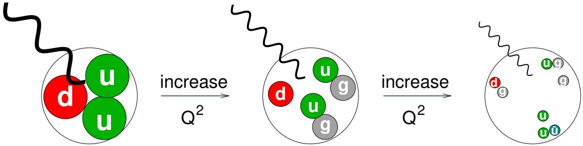
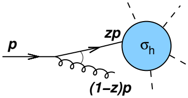


- a hard interaction (e.g. γ^* in DIS) probes much shorter times $t_{\rm hard} \sim 1/Q$
- hard probes take instantaneous snapshots of hadron structure
- PDFs are scale dependent objects: $f_{i/p}(x) \to f_{i/p}(x, Q^2)$



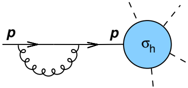
Factorization revised: the factorization scale

consider soft & collinear emissions from an inital-state quark



$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

where we assume σ_h involves momentum transfer $Q \gg k_t$



$$\sigma_{g+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

total cross section receives contributions from both

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2}\frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int_0^1 \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

regulate the singularity in the k_t integral by μ_F, the factorization scale
absorb the singularity into redefined, scale dependent, PDFs

The hadronic cross section revised

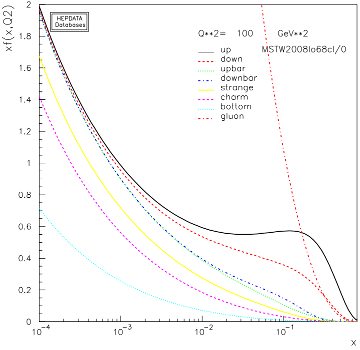
Factorization into hard and soft component (resummed in PDFs)

$$\sigma_{pp\to X_{\rm part}}(s;\mu_R^2,\mu_F^2) \equiv \sum_{ij}\int \mathrm{d}x_1\mathrm{d}x_2\; f_{i/p}(x_1,\mu_F^2)f_{j/p}(x_2,\mu_F^2)\;\mathrm{d}\hat{\sigma}_{ij\to X_{\rm part}}(\hat{s};\{p_X\},\mu_R^2,\mu_F^2)$$

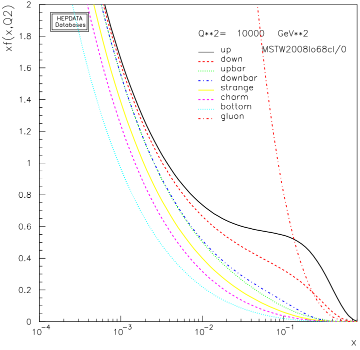
- emissions with $k_t \lesssim \mu_F$ implicitely included in PDFs
- emissions with $k_t \gtrsim \mu_F$ described by the hard process
- change of PDFs wrt to μ_F covered by perturbative QCD, calculable
  [in analogy to the renormalization scale, μ_R]
  $\rightsquigarrow$ only need to extract PDFs at some non-perturbative input scale
- typically we identify μ_F with the inherent process scale, Q

PDFs for the LHC

$Q = 10$ GeV



$Q = 100$ GeV



- current PDF sets extracted from DIS, $p\bar{p}$ & fixed target data
- only since very recently first LHC data gets included in fits
- much, much more to come over the next years

perturbative QCD gets us quite far

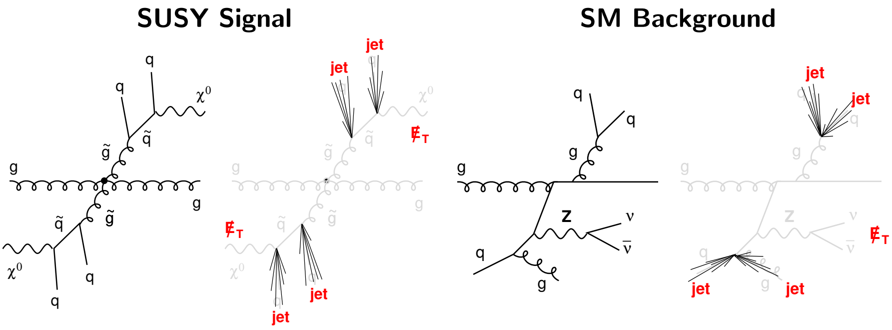
multiple gluon emission & jets

- we can calculate multiple gluon emissions efficiently
- resummation of leading higher-order terms [parton shower]
- giving rise to internal structure of jets
- proper jet definition allows to consistently use jets
    - in fixed-order calculations
    - after parton showering
    - including hadronization corrections

The hadron–hadron cross section

- factorization of soft and hard component
- hard kernel convoluted with non-perturbative PDFs
- need to be extracted from data
- PDFs scale dependent, evolution described by pQCD

Search for New Physics in a busy QCD environment



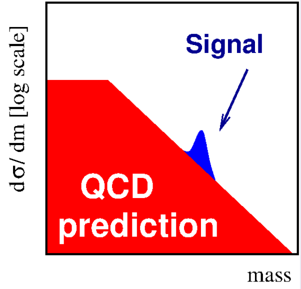
- identify relevant/measureable signatures
  - ⇝ largest cross sections for color-charged particles
- find selection criteria to enhance signal over SM background [$S/B \sim 1$]
  - ⇝ many hard jets, isolated leptons/photons, large $\not{E}_T$
  - ⇝ might need to focus on rare decays, e.g. $h \to \gamma\gamma$
  - ⇝ New Physics encoded in energies, flavors, kinematical edges

What does discovery look like?

Searching for New Physics in collision experiments

Find excess of events over the Standard Model expectation

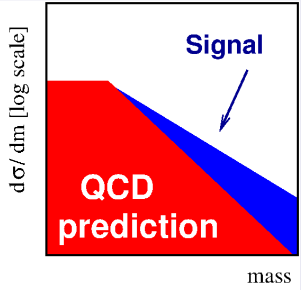
mass peak



- fully reconstructed resonance, e.g. new gauge boson Z'
- simple invariant mass variable

⇝ largely background independent

broad high-mass (high-p_T) excess



- inclusive multi-particle final state, e.g. unreconstructed cascade decay
- sum of all transverse momenta

⇝ knowledge of backgrounds crucial

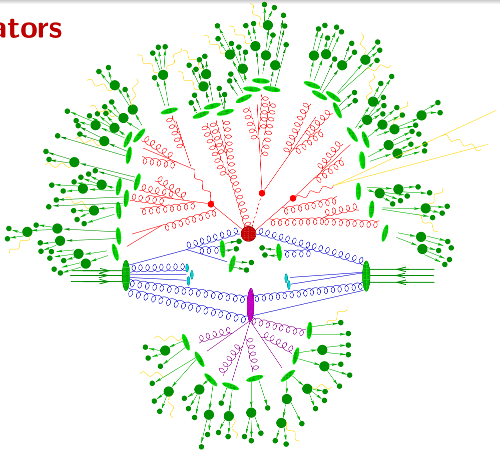
The theory challenge

Precise SM predictions

&

Flexible New Physics simulations

Monte Carlo Event Generators



- **Hard interaction**

  exact matrix elements $|\mathcal{M}|^2$

- **QCD bremsstrahlung**

  parton showers in the initial and final state

- **Multiple Interactions**

  beyond factorization: modelling

- **Hadronization**

  non-perturbative QCD: modelling

- **Hadron Decays**

  phase space or effective theories

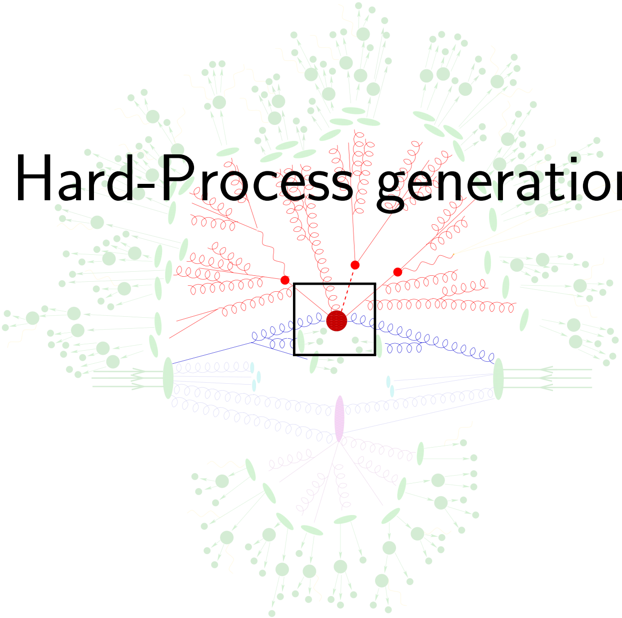
Pythia, Herwig, Sherpa

[Buckley, S. et al. Phys. Rept. **504** (2011) 145]

$\Rightarrow$ **stochastic simulation of pseudo data**
$\Rightarrow$ **fully exclusive hadronic final states**
$\Rightarrow$ **direct comparison with experimental data, e.g. ATLAS, CMS, LHCb, DØ, CDF**

modulo detector simulation

Hard-Process generation



The Hard Process

$$\sigma_{pp\to X_n} = \sum_{ab} \int dx_1 dx_2\; f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2)\; |\mathcal{M}_{ab\to X_n}|^2\; d\Phi_n$$

generic features

- high-dimensional phase space $dim[\Phi_n] = 3n - 4$
- $|\mathcal{M}_{ab\to X_n}|^2$ wildly fluctuating over Φ_n
- steep parton densities [parametrization]

state-of-the-art

- tree-level fully automated [up to $2 \to 8 - 10$]
  - extract Feynman rules from Lagrangian $\mathcal{L}$

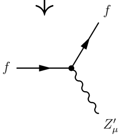
    [FeynRules by Christensen & Duhr Comput. Phys. Commun. **180** (2009) 1614]
  - generate compact expressions for $|\mathcal{M}|^2$
  - self-adaptive Monte-Carlo integrators

  ⇝ e.g. MadGraph, Alpgen, Sherpa
- at NLO QCD first $2 \to 5$ results available

↪ automation of one-loop calculations within reach

$\mathcal{L}_{\mathrm{(B)SM}}$

$\downarrow$



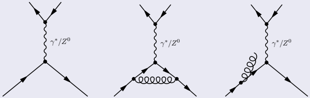
$\downarrow$

$|\mathcal{M}_n|^2 d\Phi_n$

Hard Processes at Next-to-Leading Order QCD

Anatomy of NLO QCD calculations [in dim. regularization $d = 4 - 2\epsilon$]

$$\sigma^{NLO}_{2\to n} = \int_n \mathrm{d}^{(4)}\sigma^B + \int_n \mathrm{d}^{(d)}\sigma^V + \int_{n+1} \mathrm{d}^{(d)}\sigma^R$$



- (UV renormalized) virtual-corrections σ^V $\rightsquigarrow$ IR divergent
- real-emission σ^R $\rightsquigarrow$ IR divergent

  $\rightsquigarrow$ for IR safe observables sum is finite

Hard Processes at Next-to-Leading Order QCD

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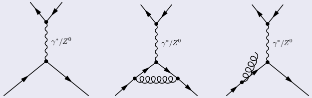


- (UV renormalized) virtual-corrections σ^V ⇝ IR divergent
- real-emission σ^R ⇝ IR divergent

  ⇝ for IR safe observables sum is finite

Dipole subtraction method [Catani, Seymour Nucl. Phys. B **485** (1997) 291]

$$\sigma^{NLO}_{2\to n} = \int_n \left[\mathrm{d}^{(4)}\sigma^B + \int_{\mathrm{loop}} \mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0} + \int_{n+1}\left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right]$$

- subtraction terms yield local approximation for the real emission process
- describe the amplitude in the soft & collinear limits [$1/\epsilon$ and $1/\epsilon^2$ poles]

$$\int_{n+1} \mathrm{d}^{(d)}\sigma^{\mathrm{A}} = \sum_{\mathrm{dipoles}} \int_n \mathrm{d}^{(d)}\sigma^{\mathrm{B}} \otimes \int_1 \mathrm{d}^{(d)} V_{\mathrm{dipole}}$$

spin- & color correlations ⟵ ⟶ universal dipole terms

Hard Processes at Next-to-Leading Order QCD

The emerging picture: a fully differential NLO calculation

$$\sigma_{2\to n}^{NLO} = \int_{n+1}\left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right] + \int_{n}\left[\mathrm{d}^{(4)}\sigma^{B} + \int_{\mathrm{loop}} \mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_{1} \mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0}$$

Monte-Carlo codes

- all the tree-level bits
- subtraction of singularities
- efficient phase-space integration

One-Loop codes

- Loop amplitudes, *i.e.* $2\Re(\mathcal{A}_V\mathcal{A}_B^\dagger)$
- Loop integration

$\rightsquigarrow$ $1/\epsilon$, $1/\epsilon^2$ coefficients & finite terms

some recent NLO calculations by the year:

2009 $W + 3$jets, $t\bar{t}b\bar{b}$

2010 $W + 4$jets, $Z + 3$jets

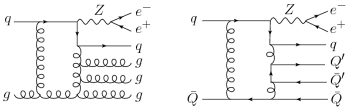
2011 $Z + 4$jets, $t\bar{t} + 2$jets, $b\bar{b}b\bar{b}$, $WW + 2$jets, 4jets

2012 $\gamma + 3$jets

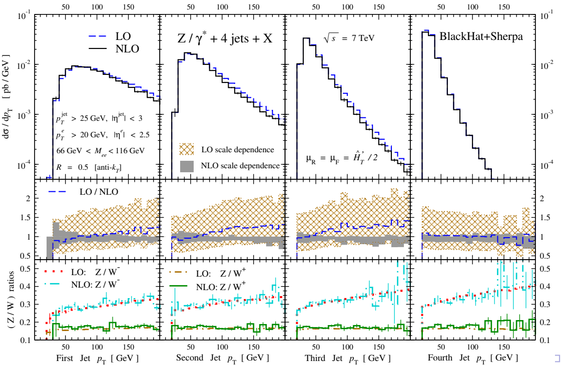
Hard Processes at Next-to-Leading Order QCD

BlackHat+Sherpa: $Z+4$jets LHC predictions [Ita et al. Phys. Rev. D **85** (2012) 031501]

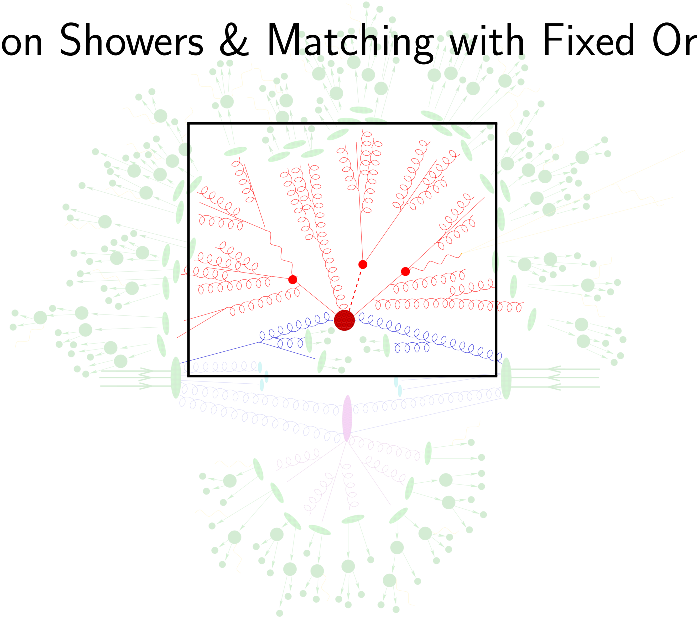
- include one-loop virtual & real emission corrections, e.g.



⇝ reduced scale uncertainties in cross sections & differential distributions



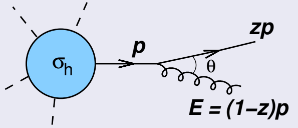
Parton Showers & Matching with Fixed Order



Approximating multi-parton production

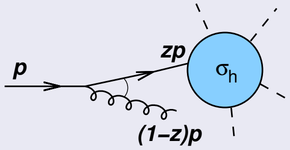
n-parton cross section dominated by soft and/or collinear emissions

final-state splitting



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

initial-state splitting

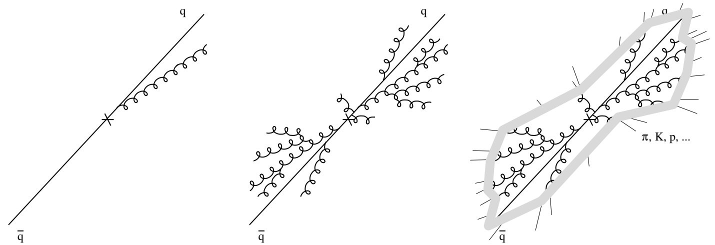


$$\sigma_{g+h}(p) \simeq \sigma_h(zp) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- valid when the gluon is much lower in energy than the emitter, i.e. $z \lesssim 1$
- emission angle θ ($k_t \simeq E\theta$) is much smaller than the angle between the emitter and any other parton in the event [angular ordering, color coherence]

$\rightsquigarrow$ lends itself into simulation: parton shower of subsequent emissions

The QCD Parton Shower picture



- construct explicitly the initial- & final-state partons history/fate
- successive branching of incoming and outgoing legs

  ⇝ exclusive partonic final states

- evolve parton ensemble from high- to low scale $Q_0 \sim \mathcal{O}(1\mathrm{GeV}^2)$

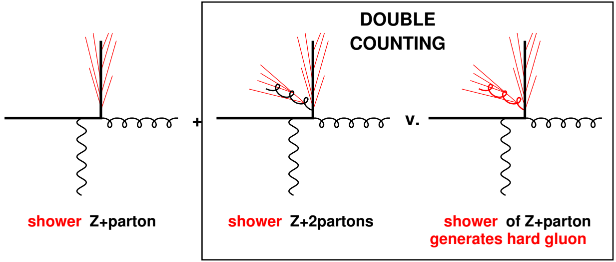
  ⇝ link the hard process to universal hadronization models

- model intra-jet energy flows: jets become multi-parton objects

Matching exact matrix elements with parton showers

The art of combining matrix elements with parton showers

- model (few) hardest emissions by exact matrix elements
- avoid any double counting or dead regions of emission phase space
- preserve fixed-order & logarithmic precision of the calculation
- seminal work:
  - multileg tree-level matching: Catani et al. JHEP **0111** (2001) 063 ⇝ ME+PS
  - NLO + Parton Shower: Frixione, Webber JHEP **0206** (2002) 029 ⇝ MCatNLO



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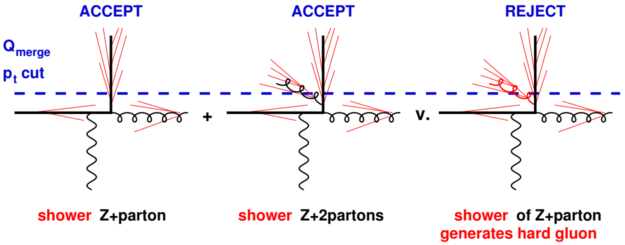
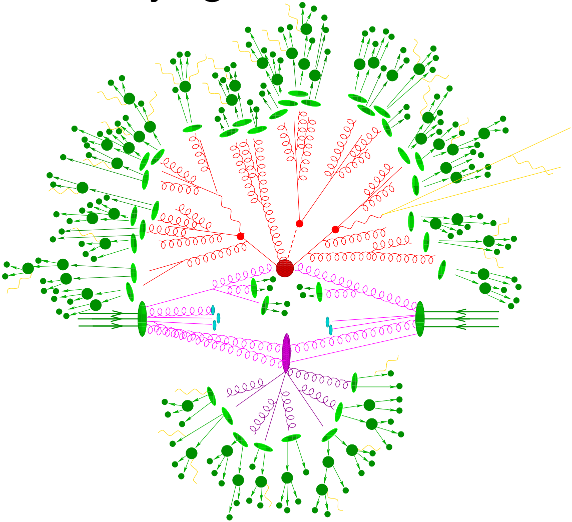


⇝ the new standards for LHC event generation [Alwall et al. Eur. Phys. J. C **53** (2008) 473]

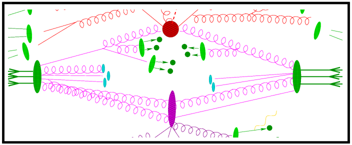
⇝ necessitates truncated showering [Höche, S. et al. JHEP **0905** (2009) 053]

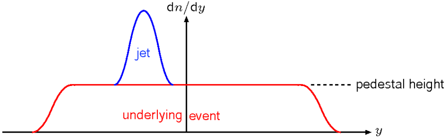
Leaving the perturbative ground: The Underlying Event & Hadronization

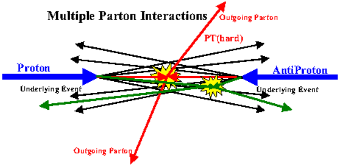


Definition: An attempt

- everything but the hard interaction including showers & hadronization
  ↪ soft & hard remnant-remnant interactions







Definition: An attempt

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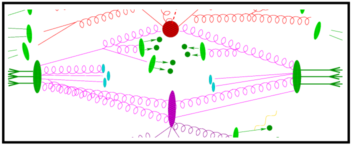
  $\hookrightarrow$ soft & hard remnant-remnant interactions



Beyond factorization: Multiple-Parton Interactions

$$\sigma^{2\to2}_{\rm QCD}(p^2_{T,\rm min}) = \int\limits_{p^2_{T,\rm min}}^{s/4} {\rm d}p_T^2 \frac{{\rm d}\sigma^{2\to2}_{QCD}(p_T^2)}{{\rm d}p_T^2}$$

$$= \int\int\int_{p^2_{T,\rm min}}^{s/4} {\rm d}x_a {\rm d}x_b {\rm d}p_T^2 f_a(x_a, p_T^2) f_b(x_b, p_T^2) \frac{{\rm d}\hat\sigma^{2\to2}_{QCD}}{{\rm d}p_T^2} \sim \frac{\alpha_S^2(p_T^2)}{p_T^4}$$

$\rightsquigarrow$ for low $p_{T,\rm min}$: $\langle\sigma^{2\to2}_{\rm QCD}(p^2_{T,\rm min})/\sigma^{\rm ND}_{pp}\rangle > 1 = \langle n\rangle$

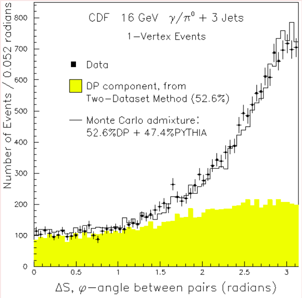
$\rightsquigarrow$ there might be many interactions per event $\mathcal{P}_n = \frac{\langle n\rangle^n}{n!} e^{-\langle n\rangle}$

$\rightsquigarrow$ strong dependence on cut-off $p_{T,\rm min}$ $\rightsquigarrow$ energy dependent!

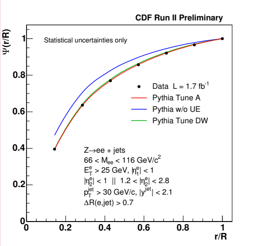
Experimental Evidence

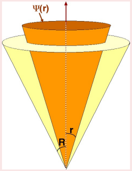
direct: DPS in $\gamma + 3$jets

CDF Phys. Rev. **D56** (1997) 3811



indirect: jet shapes





Multiple Interactions: A simple model

Sjöstrand, Zijl Phys. Rev. D **36** (1987) 2019

- hard process defines scale $p_{T,hard}$
- generate sequence of additional $2 \to 2$ QCD scatterings ordered in p_T

$$\mathcal{P}(p_T) = \frac{1}{\sigma_{ND}} \frac{\mathrm{d}\sigma^{2\to 2}_{QCD}}{\mathrm{d}p_T^2} \exp\left\{ - \int\limits_{p_T^2}^{p^2_{T,hard}} \frac{1}{\sigma_{ND}} \frac{\mathrm{d}\sigma^{2\to 2}_{QCD}}{\mathrm{d}p_T^{2\prime}} \mathrm{d}p_T^{2\prime} \right\}$$

with $\hat{\sigma}^{2\to 2}_{QCD}$ regulated according to

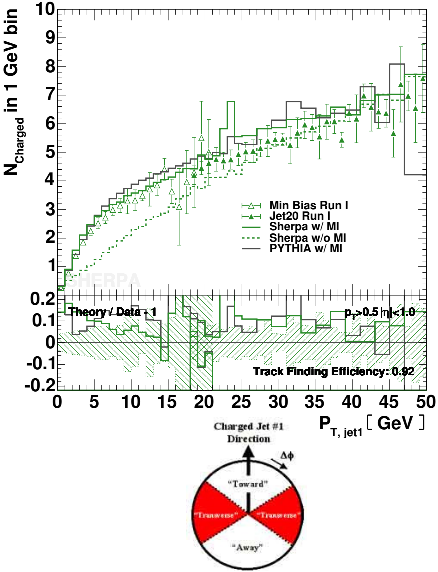
$$\frac{\mathrm{d}\hat{\sigma}^{2\to 2}_{QCD}}{\mathrm{d}p_\perp^2} \to \frac{\mathrm{d}\hat{\sigma}^{2\to 2}_{\mathrm{QCD}}}{\mathrm{d}p_\perp^2} \times \frac{p_\perp^4}{(p_\perp^2 + p_{\perp 0}^2)^2} \frac{\alpha_S^2(p_\perp^2 + p_{\perp 0}^2)}{\alpha_S^2(p_\perp^2)} \quad \text{[parameter } p_{T,0} \approx 2 \text{ GeV]}$$

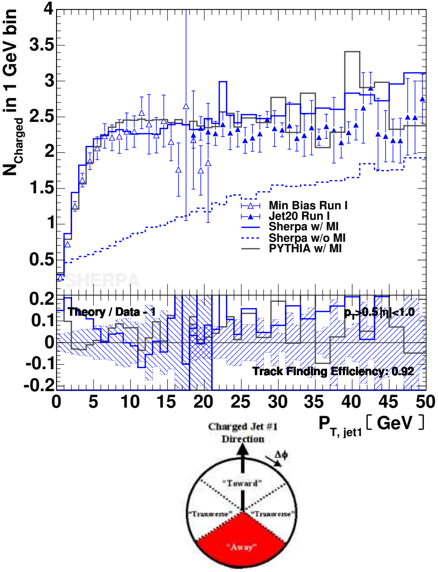
further features

- impact parameter dependence [typically double Gaussian]
- ⇝ central collisions more active, $\mathcal{P}_n$ broader than Poissonian
- use rescaled PDFs taking into account used up momentum
- ⇝ $\mathcal{P}_n$ narrower than Poissonian
- attach parton showers/hadronization

The Underlying Event: comparison to Tevatron data

$N_{charged}$ vs. $p_{\perp,jet1}$ in different $\Delta\phi$ regions w.r.t the leading jet





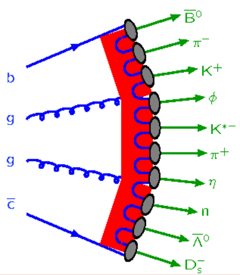
From partons to hadrons: Hadronization Models

Objectives: dynamical hadronization of multi-parton systems

- capture main non-perturbative aspects of QCD
- universality
  $\rightarrow$ robust extrapolation to new machines, higher energies
  $\rightarrow$ should not depend on specifics of the hard process
- model (un)known decays of (un)known hadrons
  $\rightarrow$ hadron multiplicities, meson/baryon ratios
  $\rightarrow$ decay branching fractions
  $\rightarrow$ hadron-momentum distibutions

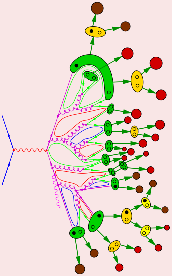
Lund string fragmentation

implemented in PYTHIA



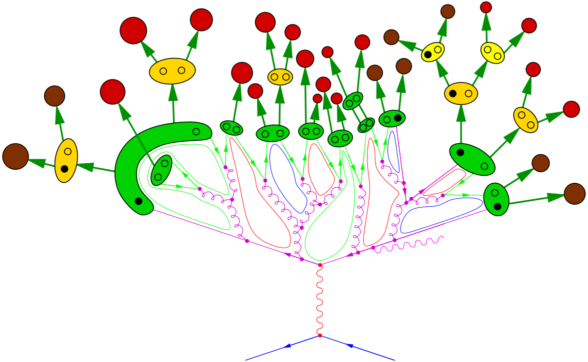
Cluster-hadronization model

implemented in HERWIG & SHERPA



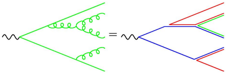
From partons to hadrons: Cluster-Hadronization Model

- **Cluster-formation model**
- **Cluster-decay model**



features

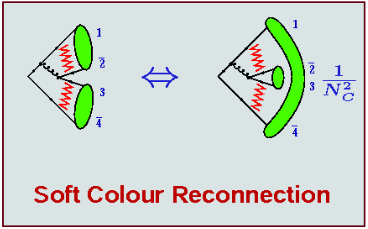
- **preconfinement** **[colour neighboring partons after shower close in phase space]**
- **parametrization of primary-hadron generation**
- **locality and universality**



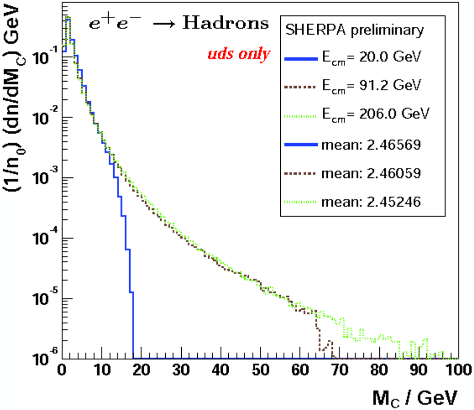
From partons to hadrons: Cluster-Formation Model

Parton shower ends up with colour-ordered parton list

- **Parton masses**
  $\longrightarrow$ constituent masses
- **Enforced gluon splitting**
  $g \to q\bar{q}, \;\; g \to q_1 q_2 \overline{q_1 q_2}$
- **White clusters formed**



Primary cluster mass distribution with CRM

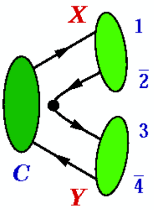


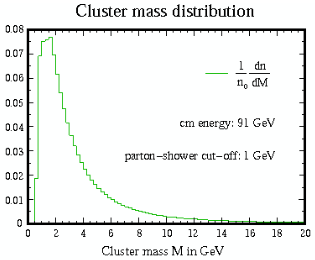
$\rightsquigarrow$ independent of cm energy of the hard process

From partons to hadrons: Cluster-Decay Model

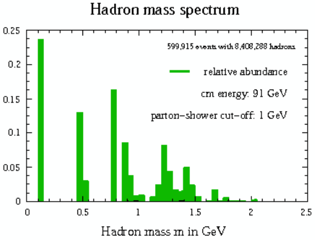
Ansatz: Cluster mass $\Rightarrow$ transition type

- $M_{\mathcal{C}}$ **in hadron regime**
  $\rightarrow$ 1-body decay $\mathcal{C} \rightarrow \mathcal{H}$
- **else** 2-body decay $\mathcal{C} \rightarrow \mathcal{XY}$
  - determine $M_{\mathcal{X}}$ & $M_{\mathcal{Y}}$
  - select channel
    - $\mathcal{C} \rightarrow \mathcal{CC}$ / $\mathcal{C} \rightarrow \mathcal{HH}$
    - $\mathcal{C} \rightarrow \mathcal{CH} / \mathcal{HC}$





kinematics $\Downarrow$ flavour content



Point of reference: LEP @ $\sqrt{s} = 91.2$ GeV

particle multiplicities: HERWIG++

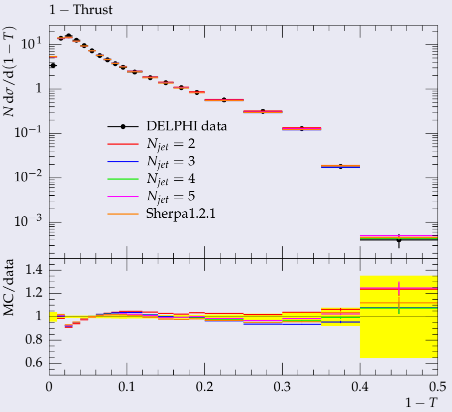
[Gieseke et al. JHEP **0402** (2004) 005]

Particle	Measured LEP	Herwig++
All Charged	20.924 ± 0.117	20.814
γ	21.27 ± 0.6	22.67
π^0	9.59 ± 0.33	10.08
$\rho(770)^0$	1.295 ± 0.125	1.316
$\pi^\pm$	17.04 ± 0.25	16.95
$\rho(770)^\pm$	2.4 ± 0.43	2.14
η	0.956 ± 0.049	0.893
$\omega(782)$	1.083 ± 0.088	0.916
$\eta'(958)$	0.152 ± 0.03	0.136
K^0	2.027 ± 0.025	2.062
$K^*(892)^0$	0.761 ± 0.032	0.681
$K^*(1430)^0$	0.106 ± 0.06	0.079
$K^\pm$	2.319 ± 0.079	2.286
$K^*(892)^\pm$	0.731 ± 0.058	0.657
$\phi(1020)$	0.097 ± 0.007	0.114
p	0.991 ± 0.054	0.947
Δ^{++}	0.088 ± 0.034	0.092
Σ^-	0.083 ± 0.011	0.071
Λ	0.373 ± 0.008	0.384
Σ^0	0.074 ± 0.009	0.091
Σ^+	0.099 ± 0.015	0.077
$\Sigma(1385)^\pm$	0.0471 ± 0.0046	0.0312*
Ξ^-	0.0262 ± 0.001	0.0286
$\Xi(1530)^0$	0.0058 ± 0.001	0.0288*
Ω^-	0.00125 ± 0.00024	0.00144
...	...	...

event shapes: SHERPA

[Sherpa unpublished]

$$T = \max_{|n|=1} \frac{\sum_i n \cdot p_i}{\sum_i |p_i|}$$

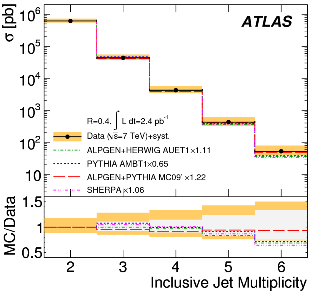


QCD at TeV energies

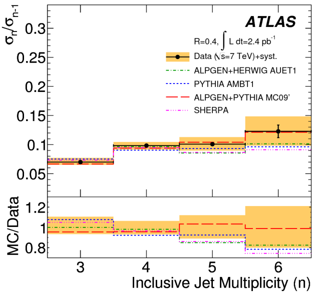
Direct multijet production @ LHC

ATLAS pure jets analysis [G. Aad *et al.* Eur. Phys. J. C **71** (2011) 1763]

inclusive jet rates



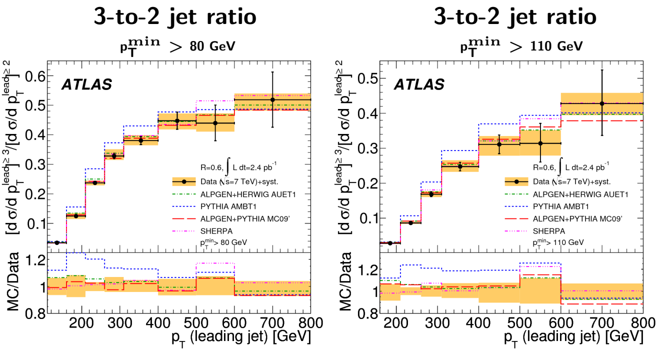
inclusive xsec ratios



⇝ multijet-production rates well under control

Direct multijet production @ LHC

ATLAS pure jets analysis [G. Aad *et al.* Eur. Phys. J. C **71** (2011) 1763]

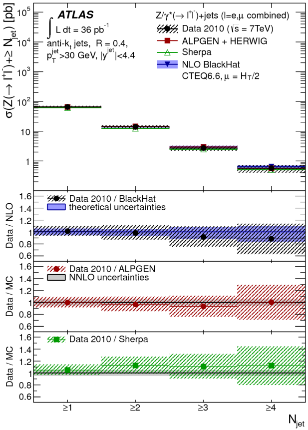


- more differential observables can discriminate calculations
- matrix-element based approaches superior for high-p_T jets

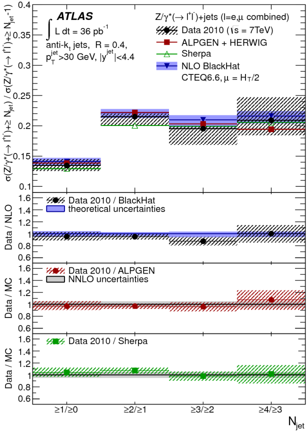
Direct multijet production @ LHC

ATLAS $Z(\to e^+e^-/\mu^+\mu^-)$+jets analysis [G. Aad *et al.* Phys. Rev. D **85** (2012) 032009]

inclusive jet rates



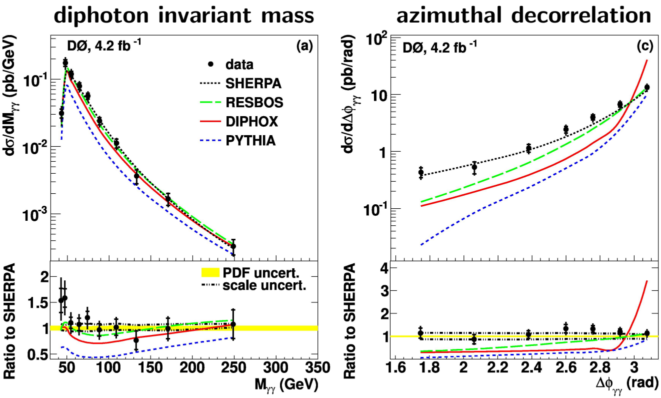
inclusive xsec ratios



'Indirect' multijet sensitivity @ Tevatron

Diphoton analysis of DØ [V. M. Abazov *et al.* Phys. Lett. B **690**]

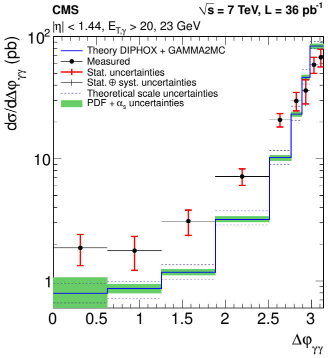
latest plots: `http://www-d0.fnal.gov/Run2Physics/WWW/results/final/QCD/Q10B/`



- ⇝ sophisticated QED⊕QCD matching algorithm [Höche, S., Siegert Phys. Rev. D **81** (2010) 034026]
- ⇝ high-multiplicity matrix elements crucial to describe data

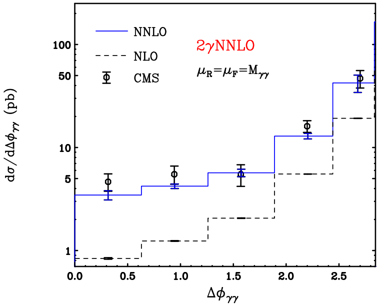
Diphoton analysis of CMS [S. Chatrchyan *et al.* JHEP **1201** (2012) 133]

azimuthal decorrelation



preliminary NNLO

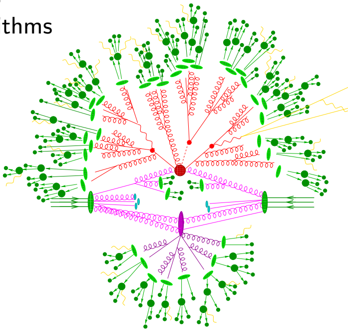
Catani *et al.* Phys. Rev. Lett. **108** (2012) 072001



Summary 3rd lecture

Monte-Carlo generators: Stochastic simulation of exclusive events

- precise predictions for the Standard Model
  - multileg tree-level & one-loop matrix elements
  - sophisticated parton-shower & matching algorithms
- flexible New Physics simulations
  - quick and easy implementation of new ideas
  - generic search strategies



QCD is a very predictive theory

Plenty of interesting phenomena

QCD Monte Carlos are predictive tools for LHC physics