

ROOT Tutorial

Simulation and fitting in ROOT



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Outline

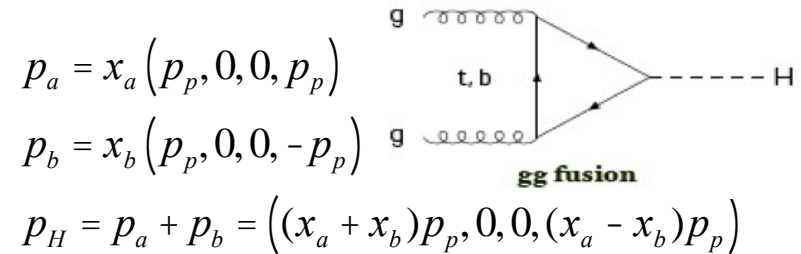
- Today is my personal opinion of what should be in the toolbox of every physicist!
- Making simulations:
 - Random number generation in RooT
 - Build the TTree we will use in the hands-on session
- Fitting a model to data
- ...and how to do all of that in a simpler way using RooFit
 - Or more complicated way, it is a matter of taste...
- Macros for today exercises in **RootTutorial2.zip** in the indico page of the lecture.



What we want to learn

- How the Higgs boson decay in two photons looks like in a LHC detector?

- Produced gluon-gluon fusion with gluons, carrying fractions x_a and x_b of protons 4-momentum



- It is a narrow resonance: its mass is well defined $m_H^2 = p_H^2 = x_a x_b (4p_p^2) = x_a x_b s$
- It has rapidity $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \frac{1}{2} \ln \frac{x_a}{x_b}$

- It decays into two γ 's isotropically distributed

$$\frac{1}{N} \frac{dN}{dW^*} = \frac{1}{4\rho} \Rightarrow \frac{1}{N} dN = \frac{df^*}{2\rho} \frac{d\cos q^*}{2}$$

- Each photon is observed if:

- With a finite energy resolution
- If it is within the geometrical acceptance ($|\eta| < 2.4$ for ATLAS)
- If it passes acceptance cuts ($p_T > 40$ GeV for leading, $p_T > 30$ GeV for sub-leading)



TRandom

- Class **TRandom** is the basic random number generator in ROOT

<http://root.cern.ch/root/html/TRandom.html>

- More refined generators **TRandom2**, **TRandom3** inherit from **TRandom**
→ same interface

Constructor: argument is seed.
Seed=0: use system clock [s]

BreitWigner (resonance)
distribution

Gaussian distribution

Poisson distribution

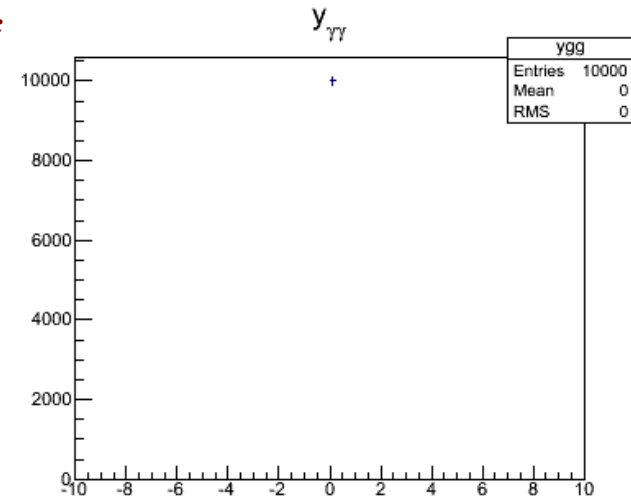
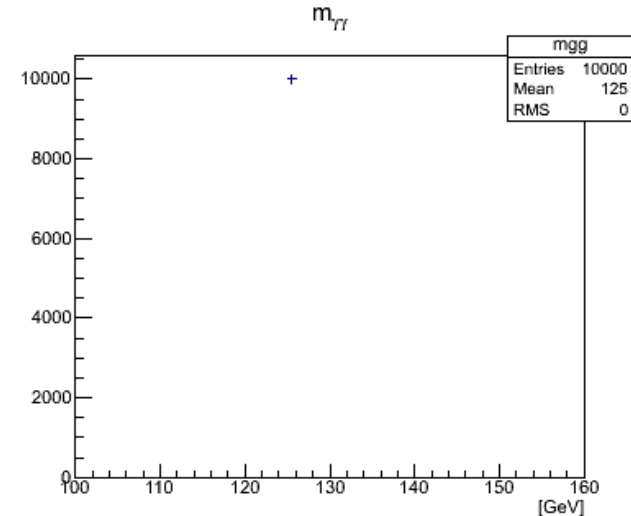
Uniform distribution

```
public:
    TRandom (UInt_t seed = 65539)
    TRandom (const TRandom&)
    virtual ~TRandom ()
    virtual Int_t Binomial (Int_t ntot, Double_t prob)
    virtual Double_t BreitWigner (Double_t mean = 0, Double_t gamma = 1)
    virtual void Circle (Double_t& x, Double_t& y, Double_t r)
    static TClass* Class ()
    virtual Double_t Exp (Double_t tau)
    virtual Double_t Gaus (Double_t mean = 0, Double_t sigma = 1)
    virtual UInt_t GetSeed () const
    virtual UInt_t Integer (UInt_t imax)
    virtual TClass* IsA () const
    virtual Double_t Landau (Double_t mean = 0, Double_t sigma = 1)
    TRandom& operator= (const TRandom&)
    virtual Int_t Poisson (Double_t mean)
    virtual Double_t PoissonD (Double_t mean)
    virtual void Rannor (Float_t& a, Float_t& b)
    virtual void Rannor (Double_t& a, Double_t& b)
    virtual void ReadRandom (const char* filename)
    virtual Double_t Rndm (Int_t i = 0)
    virtual void RndmArray (Int_t n, Float_t* array)
    virtual void RndmArray (Int_t n, Double_t* array)
    virtual void SetSeed (UInt_t seed = 0)
    virtual void ShowMembers (TMemberInspector& insp)
    virtual void Sphere (Double_t& x, Double_t& y, Double_t& z, Double_t r)
    virtual void Streamer (TBuffer& b)
    void StreamerNVirtual (TBuffer& b)
    virtual Double_t Uniform (Double_t x1 = 1)
    virtual Double_t Uniform (Double_t x1, Double_t x2)
    virtual void WriteRandom (const char* filename)
```



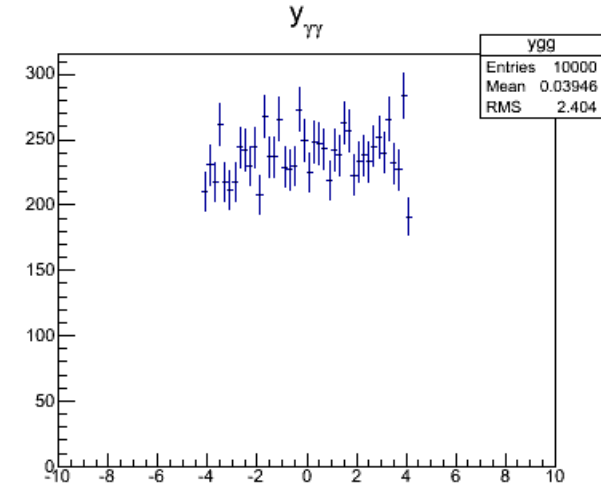
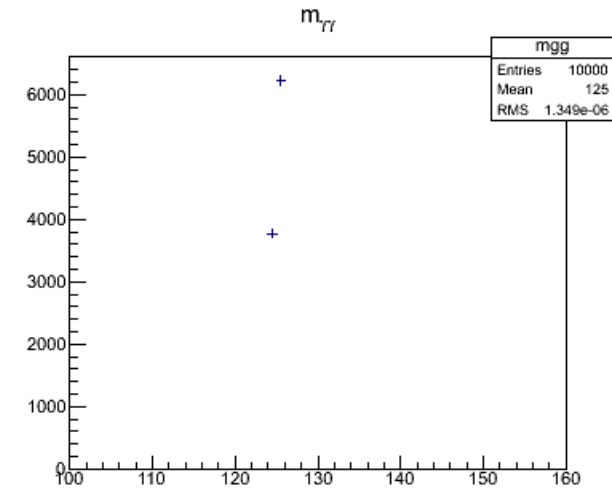
Higgs decay

```
{Double_t mH = 125.;
TRandom3 gen(0);
TH1F* mgg = new TH1F("mgg" ,"m_{#gamma#gamma}", 60,100.,160.);
TH1F* ygg = new TH1F("ygg" ,"y_{#gamma#gamma}", 100,-10.,10.);
for (Int_t i=0; i<events; i++) {
  Double_t phis = gen.Uniform(0.,TMath::TwoPi());
  Double_t coss = gen.Uniform(-1.,1.);
  Double_t sins = sqrt(1-coss*coss);
  TLorentzVector g1(cos(phis)*sins,sin(phis)*sins,coss,1.);
  g1*=(mH/2.);
  TLorentzVector g2(-cos(phis)*sins,-sin(phis)*sins,-coss,1.);
  g2*=(mH/2.);
  TLorentzVector H=g1+g2;
  mgg->Fill( H.M() );
  ygg->Fill( H.Rapidity() );
}
TCanvas myCanvas("myCanvas","Higgs properties",400,700);
myCanvas.Divide(1,2);
myCanvas.cd(1); mgg->Draw("e");
myCanvas.cd(2); ygg->Draw("e"); }
```



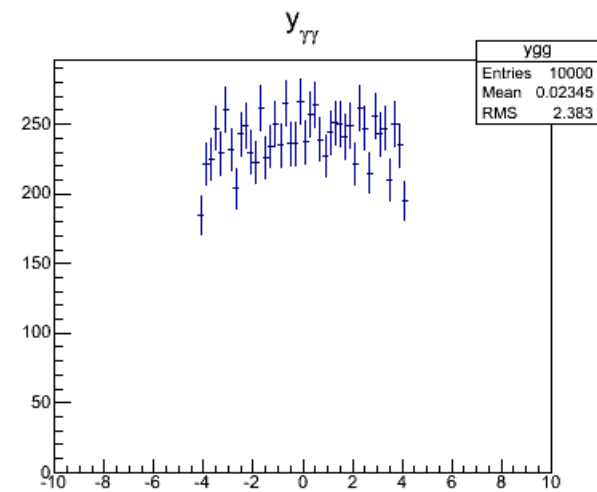
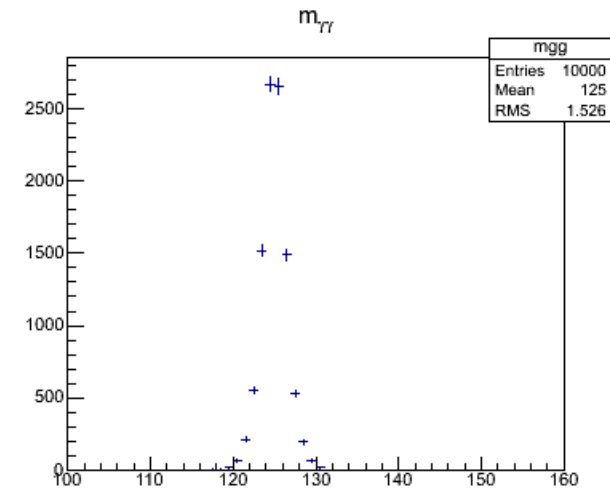
Higgs production

```
{  
  ...  
  Double_t sqrts = 8000.; // center of mass energy  
  Double_t ymax = log      (sqrts/mH);  
  for (Int_t i=0; i<events; i++) {  
    ...  
    Double_t y = gen.Uniform(-ymax,ymax);  
    TLorentzVector beta(0.,0.,tanh(y),1.);  
    g1.Boost(beta.BoostVector());  
    g2.Boost(beta.BoostVector());  
    ...  
  }  
  ...  
}
```



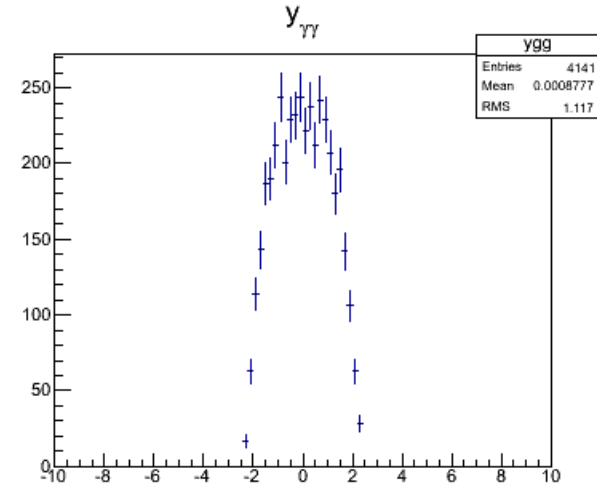
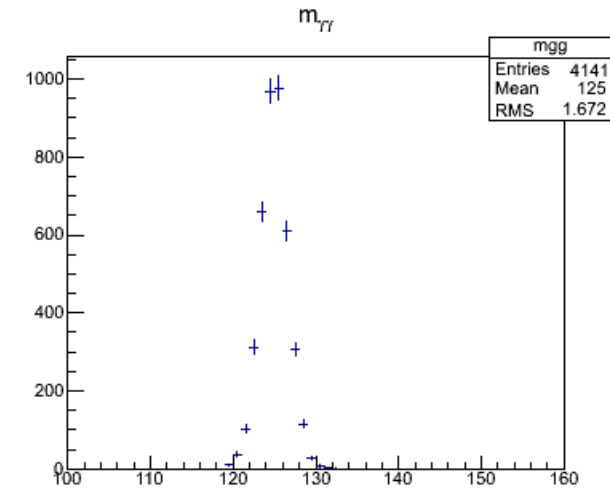
Energy resolution

```
{  
  ...  
  Double_t A = 0.01;  
  Double_t B = 0.15;  
  for (Int_t i=0; i<events; i++) {  
    ...  
    g1*=(gen.Gaus(1.,sqrt(A*A+B*B/g1.E())));  
    g2*=(gen.Gaus(1.,sqrt(A*A+B*B/g2.E())));  
    ...  
  }  
  ...  
}
```



Detector and selection acceptance

```
{  
  ...  
  for (Int_t i=0; i<events; i++) {  
    ...  
    if ( fabs(g1.Eta())>2.4 || fabs(g2.Eta())>2.4 ) continue;  
    if ( g1.ET(<40. && g2.ET(<40. ) continue;  
    if ( g1.ET(<30. || g2.ET(<30. ) continue;  
    ...  
  }  
  ...  
}
```



Putting everything into a TTree

```
{  
  ...  
  TFile* myFile = TFile::Open("myHiggs.root","RECREATE");  
  TTree *tree = new TTree("tree","Di-photons");  
  Double_t Et1, eta1, phi1, Et2, eta2, phi2;  
  tree->Branch("Et1" ,&Et1 ,"Et1/D" );  
  tree->Branch("eta1",&eta1,"eta1/D");  
  tree->Branch("phi1",&phi1,"phi1/D");  
  tree->Branch("Et2" ,&Et2 ,"Et1/D" );  
  tree->Branch("eta2",&eta2,"eta2/D");  
  tree->Branch("phi2",&phi2,"phi2/D");  
  for (...) {  
    ...  
  }  
  ...  
  tree->Write();  
  mgg->Write();  
  ygg->Write();  
  myFile->Close()  
}
```



```
for (...) {  
  ...  
  Et1 =g1.Et();  
  eta1=g1.Eta();  
  phi1=g1.Phi();  
  Et2 =g2.Et();  
  eta2=g2.Eta();  
  phi2=g2.Phi();  
  tree->Fill();  
  ...  
}
```



Move to a compiled macro

```
#include "TRandom3.h"
#include "TH1F.h"
#include "TMath.h"
#include "TTree.h"
#include "TFile.h"
#include "TLorentzVector.h"
#include "TCanvas.h"

#include <cmath>

void generator_simple(Int_t events, char* filename) {
    ...
}
```

All used classes need their corresponding *header* file

Define interface to interactively change number of events and output file

Interpreted macro:

```
root -l
.L generator_simple.C
generator_simple(100000,"file.root")
```

Compiled macro:

```
root -l
.L generator_simple.C+
generator_simple(100000,"file.root")
```

Check the execution time difference



For other distributions

- The **TF1** and the **TH*** (including multi-dimensional histograms) classes provide a **GetRandom** method to generate random numbers according to the given distributions.

- **Example:**

use for the photons a forward-peaked distribution, instead of an uniform one:

$$dN \propto \frac{d \cos \theta^*}{\sin \theta^*}$$

```
TF1* myPDF = new TF1("myPDF", "1./sqrt(1-x*x)", -0.99, 0.99);  
for (Int_t i=0; i<events; i++) {  
    ...  
    Double_t coss = myPDF->GetRandom();  
    ...  
}
```



Fitting

- Determine a set of parameters of a model that better describe the measurements.
- But also:
 - Does the model described the data well enough?
 - Which are the uncertainties on the best set of parameters?
 - Correlations among the parameters
- Different techniques and methods:
 - Binned vs. unbinned data
 - χ^2 vs. log-Likelihood



Example: χ^2 fit

- Minimize deviations normalized by their uncertainties.

- Binned data**

- Compare bin content of an histogram with expectation from model.

- Example:** data normally distributed

$$N_i^{\text{exp}} = A e^{-\frac{1}{2} \frac{(x_i - \bar{x})^2}{s^2}}$$

$$\chi^2 = \sum_{i=\text{bins}} \frac{(N_i^{\text{obs}} - N_i^{\text{exp}})^2}{N_i^{\text{obs}}} = \sum_{i=\text{bins}} \frac{N_i^{\text{obs}} - A \exp\left[-\frac{1}{2} \frac{(x_i - \bar{x})^2}{s^2}\right]}{N_i^{\text{obs}}}$$

Poisson statistics

Exercise:
figure out the connection between A and number of events in Gaussian

- Unbinned data**

- Compare each individual measurement with model

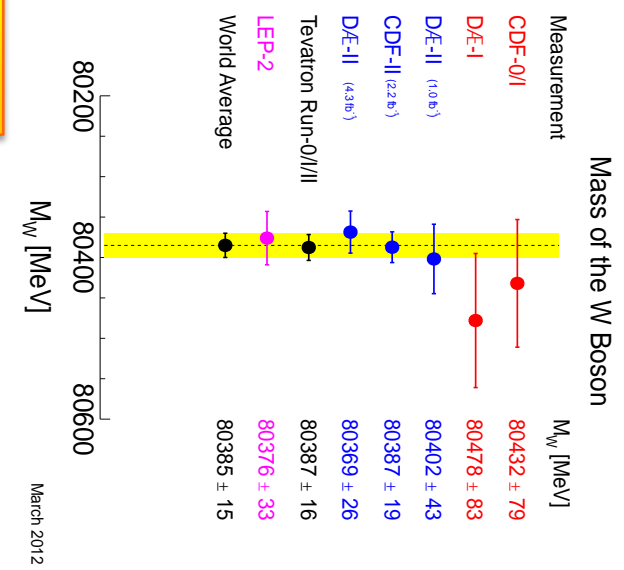
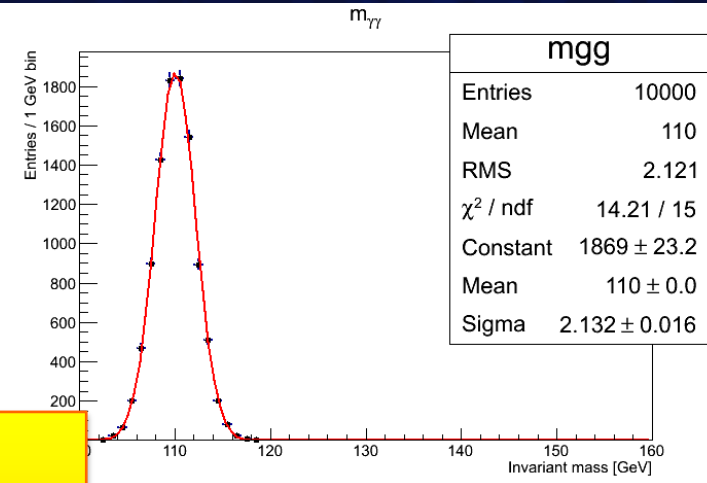
- Example:** different measurements of the same physics quantity α

$$\chi^2 = \sum_{i=\text{measurements}} \frac{(x_i - a)^2}{s_i^2}$$

- Gets back the well known weighted mean:

$$a = \frac{\sum_i x_i / s_i^2}{\sum_i 1 / s_i^2}$$

- 1 σ uncertainty:** increase of χ^2 by 1



Example: Likelihood fit

- Use the pdf of observables and maximize the product of likelihoods.

- In practice: minimize the $-2 \cdot \log$ of the likelihoods

- Binned data**

- Use Poisson pdf for bin content
- **Example:** data normally distributed

$$\mathcal{L} = \prod_{i=\text{bins}} \text{Poisson}(N_i^{\text{obs}}, A \exp(-\frac{1}{2} \frac{(x_i - \bar{x})^2}{s^2}))$$

- Unbinned data**

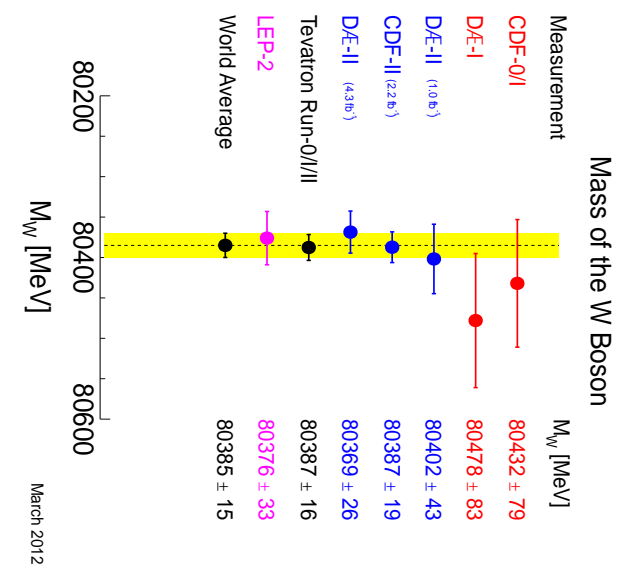
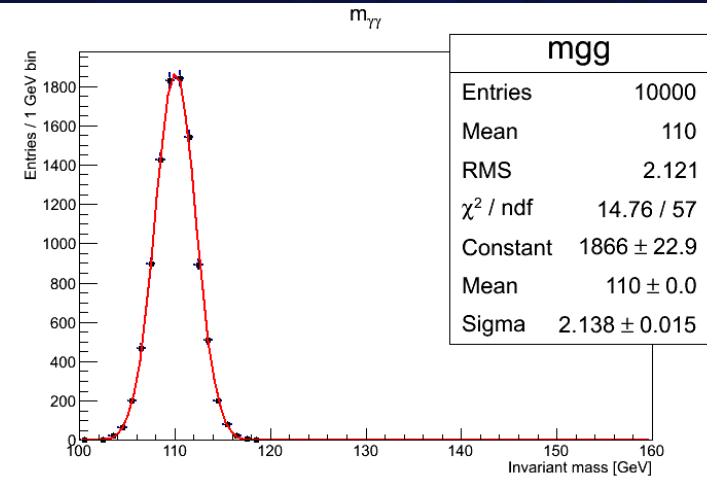
- **Example:** normally distributed points about α

$$\mathcal{L} = \prod_{i=\text{measurements}} \frac{1}{\sqrt{2\pi} s_i} \exp(-\frac{1}{2} \frac{(x_i - a)^2}{s_i^2})$$

$$-2 \ln \mathcal{L} = \sum_{i=\text{measurements}} \left(\frac{(x_i - a)^2}{s_i^2} + 2 \ln(\sqrt{2\pi} s_i) \right)$$

- It looks familiar, doesn't it?

- 1 σ uncertainty:** increase of $-2 \ln \mathcal{L}$ by 1

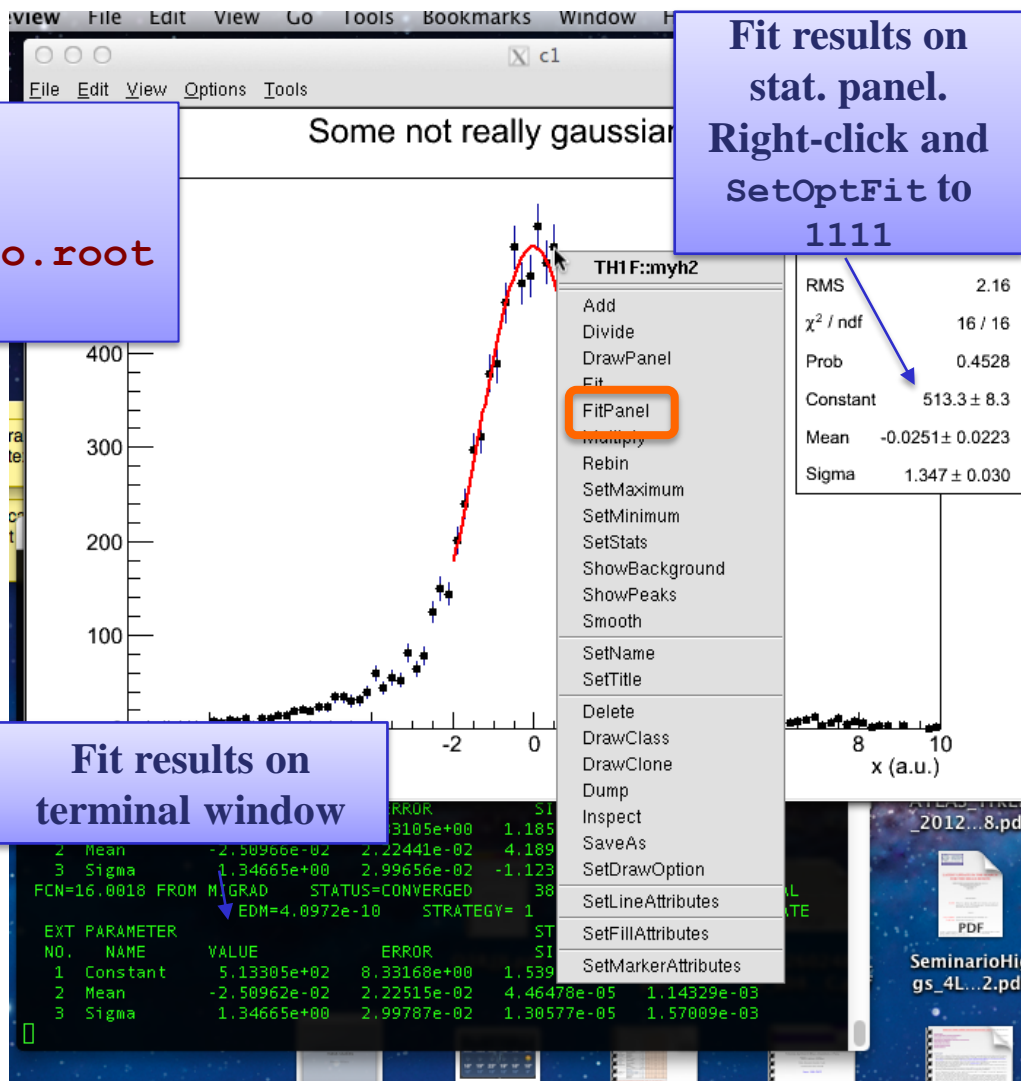


Fitting in the GUI

Sample histogram

in `myHisto.root`:

```
> root -l myHisto.root  
> myh2->Draw();
```



Fit results on stat. panel.
Right-click and SetOptFit to 1111

Fit results on terminal window

Fit Panel

Data Set: TH1F::myh2

Fit Function
Type: Predef-1D | gaus

Operation
 Nop Add Conv

gaus
Selected: gaus

General
Fit Settings
Method: Chi-square | User-Defined...
 Linear fit Robust: 0.95

Fit Options
 Integral Use range
 Best errors Improve fit results
 All weights = 1 Add to list
 Empty bins, weights=1 Use Gradient

Draw Options
 Do not store/draw | Advanced...

X: -10.00 | 10.00

Update | Fit | Reset | Close

TH1F::myh2 | LIB Minuit | MIGRAD | ltr: 0 | Prn: DEF

Function choice

Range setting

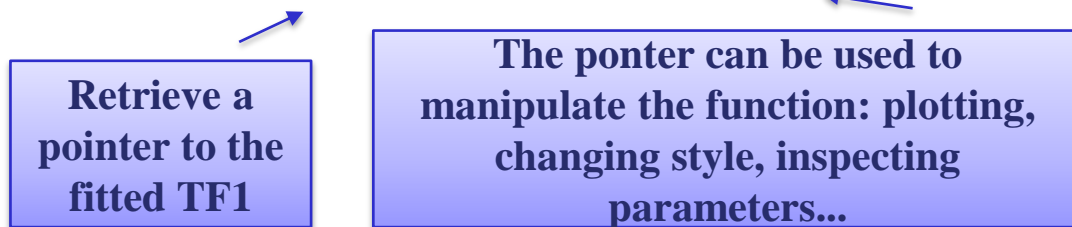


Simple fits

```
myh2->Fit("gaus","","",-2.,2.);
```



```
myh2->GetFunction("gaus")->Draw();
```



```
TFitResultPtr fitres = myh2->Fit("gaus","S");
```

```
fitres->Parameter(2);
```

```
fitres->ParError(2);
```

```
fitres->Chi2();
```

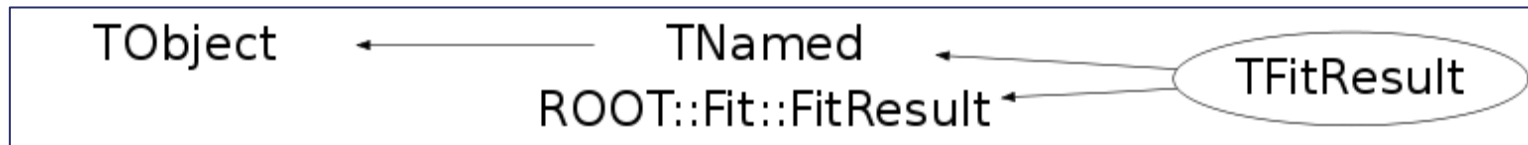
```
fitres->Ndf();
```

- Some predefined fit functions available:
 - Exponential
 - Gaussian
 - Landau
 - Polynomial (up to 9th degree)
- Most common options:
 - "L" likelihood fit
 - "WL" likelihood fit with weighed points
 - "Q" quiet mode: do not print info on screen
 - "S" save the results in a TFitResultPtr object.



TFitResult

- The **TFitResultPtr** is a class that behaves like a pointer to a **TFitResult** object.
 - The latter inherits from a **ROOT::Fit::FitResult**



- Many accessor methods, full documentation at http://root.cern.ch/root/html/ROOT__Fit__FitResult.html



TFitResult

```
root [38] TFitResultPtr fitres = myh2->Fit("gaus","S","", -2., 2.)
FCN=16.5644 FROM MIGRAD      STATUS=CONVERGED      71 CALLS      72 TOTAL
          EDM=6.18403e-09    STRATEGY= 1      ERROR MATRIX ACCURATE

EXT  PARAMETER              STEP              FIRST
NO.   NAME                VALUE              ERROR              SIZE              DERIVATIVE
  1   Constant            5.12179e+02      8.19111e+00      1.21137e-02      2.10983e-06
  2   Mean                 -1.99953e-02     2.13804e-02     4.36950e-05     -5.06752e-03
  3   Sigma                1.35596e+00     2.78382e-02     1.20495e-05     -1.87226e-03

root [39] fitres->Print(cout)

*****
Minimizer is Minuit / Migrad
Chi2          =          16.5644
Ndf           =              17
Edm           =    6.18403e-09
NCalls        =              72
Constant      =          512.179  +/-    8.19111
Mean          =    -0.0199953  +/-    0.0213804
Sigma         =          1.35596  +/-    0.0278382          (limited)
```



TFitResult

```
root [40] fitres->PrintCovMatrix(cout)
```

Covariance Matrix:

	Constant	Mean	Sigma
Constant	67.094	0.0048267	-0.1576
Mean	0.0048267	0.00045712	-2.9505e-05
Sigma	-0.1576	-2.9505e-05	0.00077498

Correlation Matrix:

	Constant	Mean	Sigma
Constant	1	0.027561	-0.69116
Mean	0.027561	1	-0.049571
Sigma	-0.69116	-0.049571	1

```
root [41]
```



Inlined functions

- Not always you can find the function
...or the parameterization that you want to use.
- In this case we want to try to fit our distribution as the *sum of two Gaussians*.
- So you can define it yourself providing its formula:

```
TF1* gauss2 = new TF1("gauss2",  
    "([0]/[2])*exp(-0.5*(x-[1])**2/[2]**2)+([3]/[4])*exp(-0.5*(x-[1])**2/[4]**2)",  
    -10.,10.);
```

- And use in fitting:

```
TFitResultPtr fitres = histo->Fit("gauss2","S");
```

- Just remember in between:

```
gauss2->SetParameter(0,100.);  
gauss2->SetParameter(1,0.);  
gauss2->SetParameter(2,1.);  
gauss2->SetParameter(3,100.);  
gauss2->SetParameter(4,2.);
```



C-functions

- Sometimes you cannot put everything on a line...
- ...use a normal C-function:

```
Double_t myTwoGauss(Double_t *x, Double_t *p) {  
    Double_t norm = 1./sqrt(TMath::TwoPi());  
    Double_t g1 = exp(-0.5*pow((x[0]-p[1])/p[2],2));  
    g1*=(norm*p[0]/p[2]);  
    Double_t g2 = exp(-0.5*pow((x[0]-p[1])/p[4],2));  
    g2*=(norm*p[3]/p[4]);  
    return g1+g2;  
}
```

- So you can define it yourself providing its formula:

```
TF1* gauss3 = new TF1("gauss3", myTwoGauss, -10., 10., 5);
```

- And use in fitting:

```
TFitResultPtr fitres2 = histo->Fit("gauss3","S");
```

- Just remember to set the parameters!

Let's run the `FitTwoGaussian.C` macro





...and when things starts to become really complicated

ROOFIT



Introduction to RooFit

- **Disclaimer:**

I am not a RooFit expert: I learnt it last week because of **you!**
Only few of the features will be presented here!

- So you can get much better material from the official sources:

- RooFit web page: <http://roofit.sourceforge.net/>

- RooFit tutorials:

- <http://root.cern.ch/root/html/tutorials/roofit/index.html>

- RooFit entry in the ROOT reference guide:

- http://root.cern.ch/root/html/ROOTFIT_Index.html

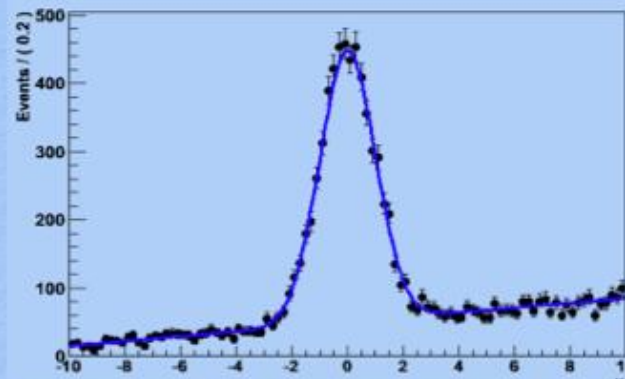
- In particular I appreciated a lot the tutorial at the *Desy 2012 School of Statistics* by L. Moneta and S. Kreiss:

<https://twiki.cern.ch/twiki/bin/view/RooStats/WebHome#Resources>
many slides here by them and W. Werverke



RooFit

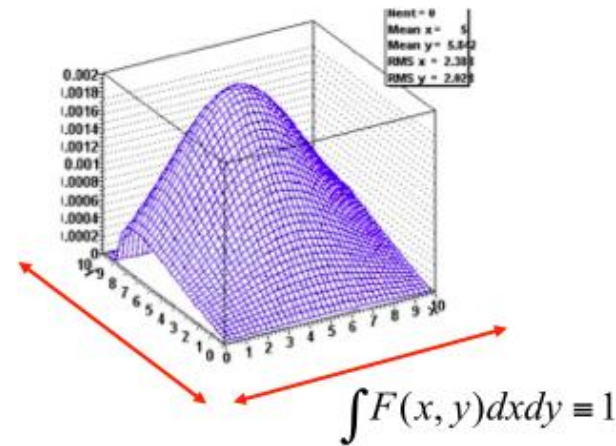
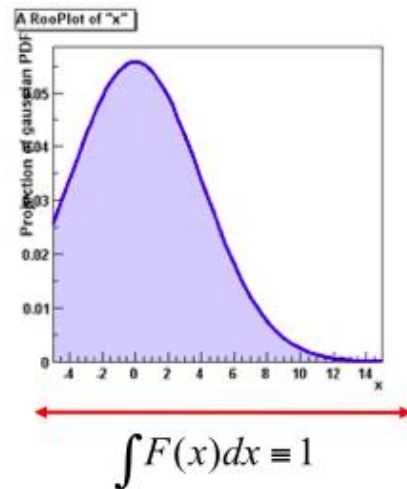
- Toolkit for data modeling
 - developed by *W. Verkerke and D. Kirkby*
- model distribution of observable x in terms of parameters p
 - probability density function (pdf): $\mathcal{P}(x; p)$
- pdf are normalized over allowed range of observables x with respect to the parameters p



Mathematic – Probability density functions

- Probability Density Functions describe probabilities, thus
 - All values must be >0
 - The total probability must be 1 *for each* p , i.e.
 - Can have any number of dimensions

$$\int_{\bar{x}_{\min}}^{\bar{x}_{\max}} g(\bar{x}, \bar{p}) d\bar{x} \equiv 1$$



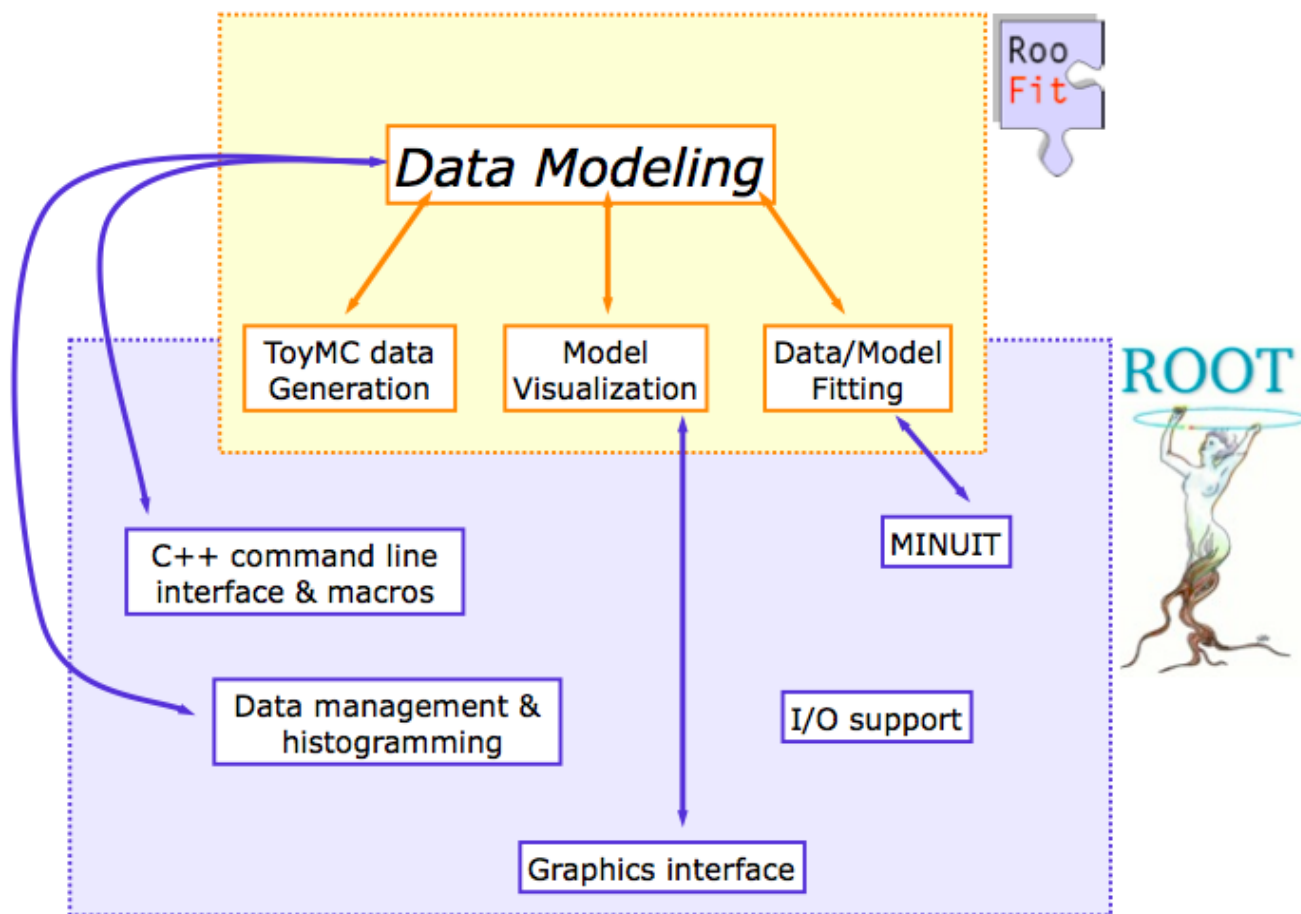
- Note distinction in role between *parameters* (p) and *observables* (x)
 - Observables are measured quantities
 - Parameters are degrees of freedom in your model

RooFit

- RooFit provides functionality for building the pdf's
 - complex model building from standard components
 - composition with addition product and convolution
- All models provide the functionality for
 - maximum likelihood fitting
 - toy MC generator
 - visualization
- Extension of ROOT functionality

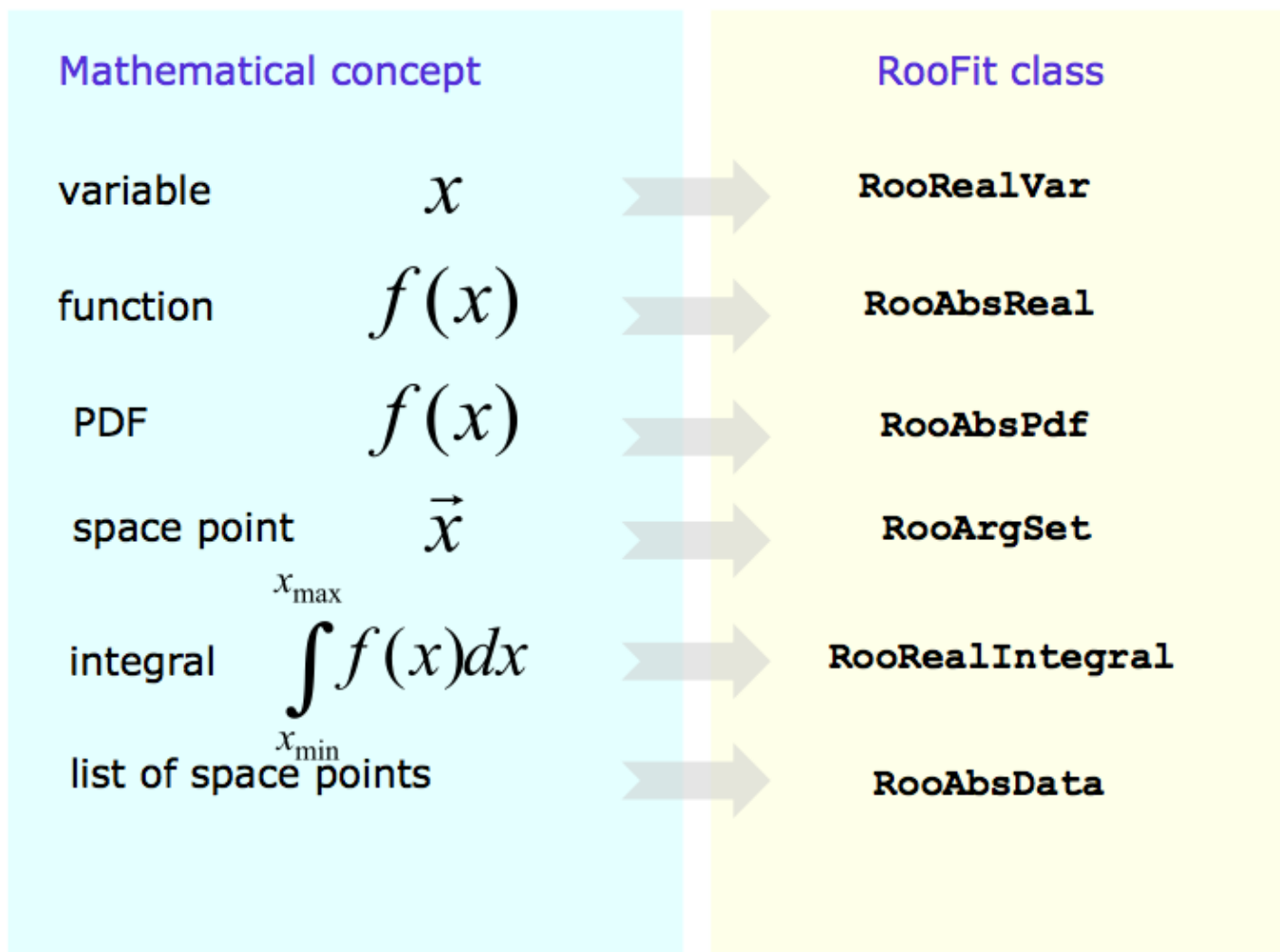
Introduction – Relation to ROOT

Extension to ROOT – (Almost) no overlap with existing functionality



RooFit core design philosophy

- Mathematical objects are represented as C++ objects



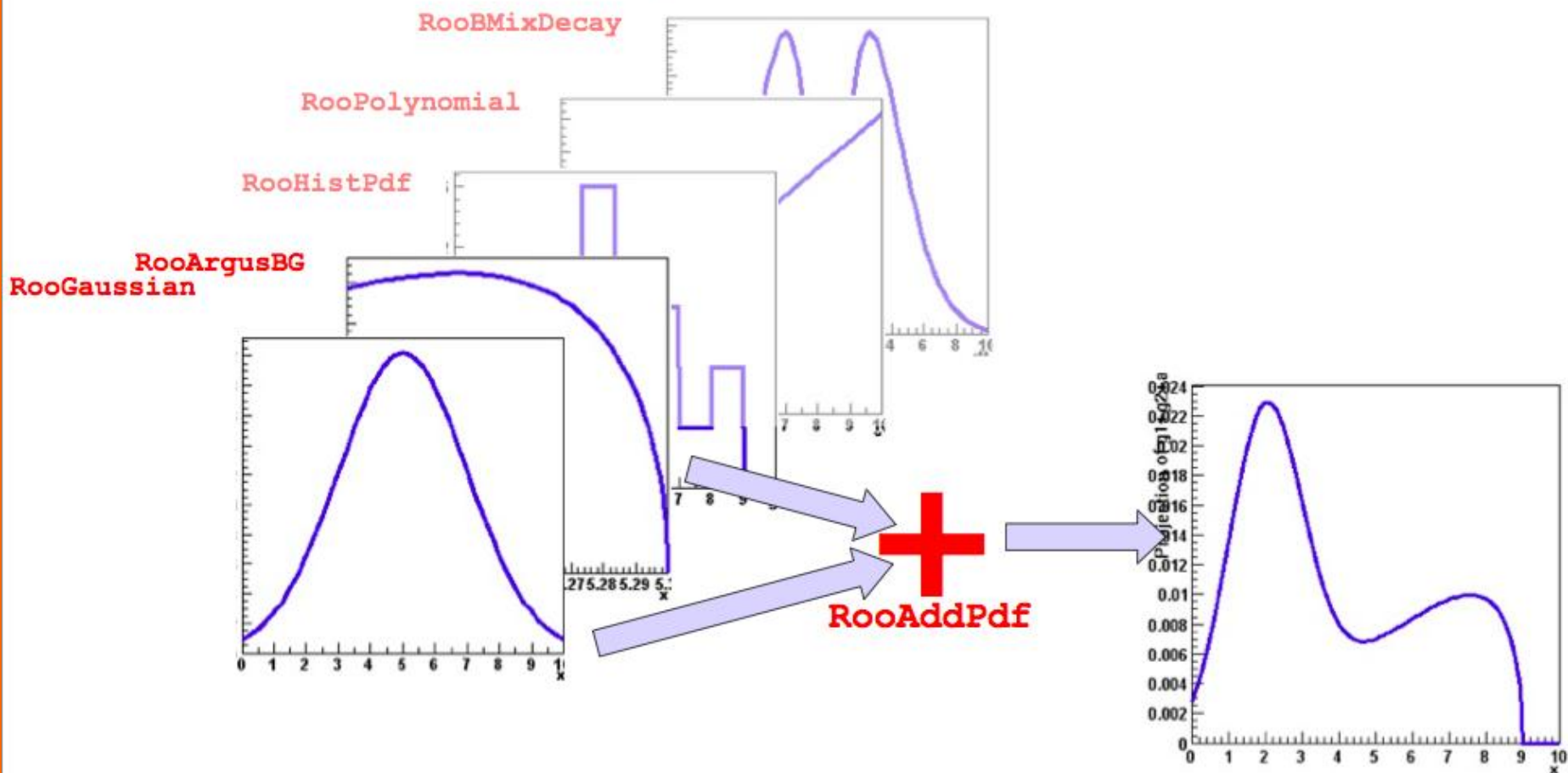
Some of the available pdf

RooExponential	Exponential PDF	RooSpHarmonic	SpHarmonic polynomial
RooFunctor1DBinding	RooAbsReal binding to a ROOT::Math	RooStepFunction	Step Function
RooFunctor1DPdfBinding	RooAbsPdf binding to a ROOT::Math	RooTFnBinding	RooAbsReal binding to ROOT TF[123] functions
RooFunctorBinding	RooAbsReal binding to a ROOT::Math	RooTFnPdfBinding	RooAbsPdf binding to ROOT TF[123] functions
RooFunctorPdfBinding	RooAbsPdf binding to a ROOT::Math	RooUnblindCPAsymVar	CP-Asymmetry unblinding transformation
RooGExpModel	Gauss (x) Exponential resolution model	RooUnblindOffset	Offset unblinding transformation
RooGamma	Gaussian PDF	RooUnblindPrecision	Precision unblinding transformation
RooGaussModel	Gaussian Resolution Model	RooUnblindUniform	Uniform unblinding transformation
RooGaussian	Gaussian PDF	RooUniform	Flat PDF in N dimensions
RooHistConstraint	Your description goes here...	RooVoigtian	Voigtian PDF (Gauss (x) BreitWigner)
RooIntegralMorph	Linear shape interpolation operator p.d.f.		
RooJeffreysPrior	Sum of RooAbsReal objects		
RooKeysPdf	One-dimensional non-parametric kernel estimation p.d.f.		
RooLandau	Landau Distribution PDF		
RooLegendre	Legendre polynomial		
RooLognormal	log-normal PDF		
RooMomentMorph	Your description goes here...		
RooMultiBinomial	Simultaneous pdf of N Binomial distributions with associated efficiency functions		
RooNDKeysPdf	General N-dimensional non-parametric kernel estimation p.d.f.		
RooNonCPEigenDecay	PDF to model CP-violating decays to final states which are not CP eigenstates		
RooNonCentralChiSquare	non-central chisquare pdf		
RooNovosibirsk	Novosibirsk PDF		
RooParamHistFunc	Your description goes here...		
RooParametricStepFunction	Parametric Step Function Pdf		
RooPoisson	A Poisson PDF		
RooPolynomial	Polynomial PDF		



Model building – (Re)using standard components

- Most realistic models are constructed as the sum of one or more p.d.f.s (e.g. signal and background)
- Facilitated through **operator p.d.f RooAddPdf**



Two Gaussians in RooFit

- This is the story behind the histogram used for the fit study:

```
RooRealVar x("x","Bad Gaussian",-10.,10.);
RooRealVar mean("mean","mean of Gaussian",0.,-10.,10.) ;
RooRealVar sigma1("sigma1","width of narrow Gaussian",1.2,0.1,10.) ;
RooRealVar sigma2("sigma2","width of wide Gaussian",3.4,2.,10.) ;
RooRealVar fraction("fraction","fraction of narrow Gaussian",2./3.,0.,1.);

RooGaussian gauss1("gauss1","Narrow Gaussian",x,mean,sigma1);
RooGaussian gauss2("gauss2","Wide Gaussian",x,mean,sigma2);

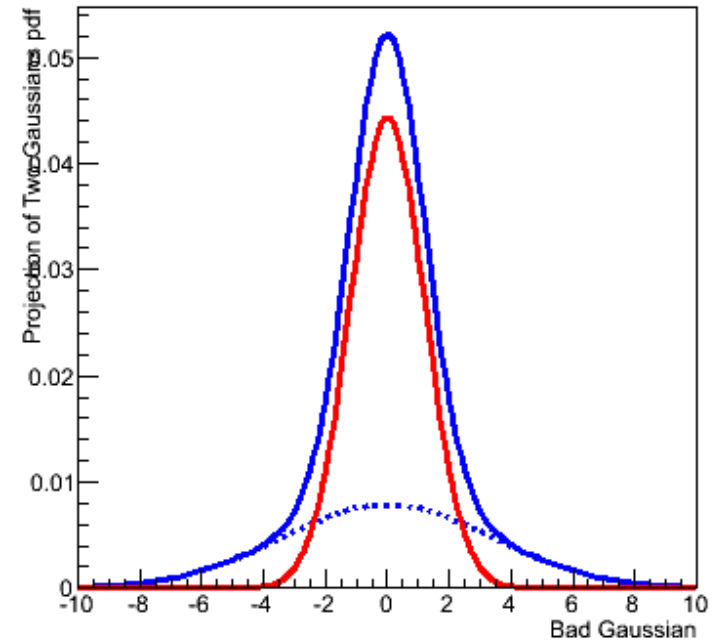
RooAddPdf twogauss("twogauss","Two Gaussians pdf",
                    RooArgList(gauss1,gauss2),fraction);
```

Now twogaussian is pdf:
what can we do with it?



Drawing a pdf

A RooPlot of "Bad Gaussian"



- Simply plotting a pdf is a bit more elaborate:

- Create a frame to show a certain variable

```
RooPlot* xframe = x.frame();
```

- Plot the pdf in that frame

```
twogauss.plotOn(xframe)
```

- But also can plot individual components, setting the style on the fly:

```
twogauss.plotOn(xframe, Components("gauss2"), LineStyle(kDashed));
```

```
twogauss.plotOn(xframe, Components("gauss1"), LineColor(kRed));
```

- Finally draw the frame

```
xframe.Draw();
```



Generate events according to a pdf

- From a pdf it is possible to create datasets:

- Create a dataset of 10000 sampling of variable x

```
RooDataSet* mydata = twogauss.generate(x,10000);
```

- Plot the dataset in a frame

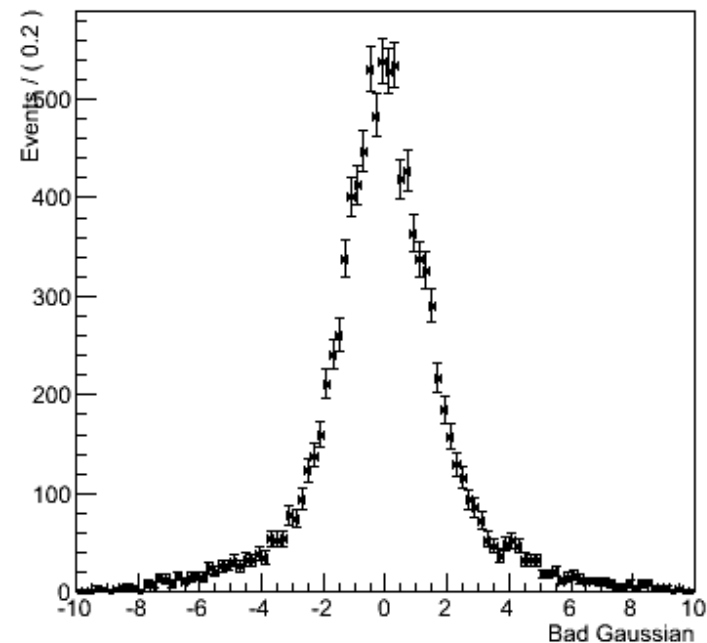
```
RooPlot* xframe = x.frame();
```

```
mydata->plotOn(xframe)
```

- Finally draw the frame

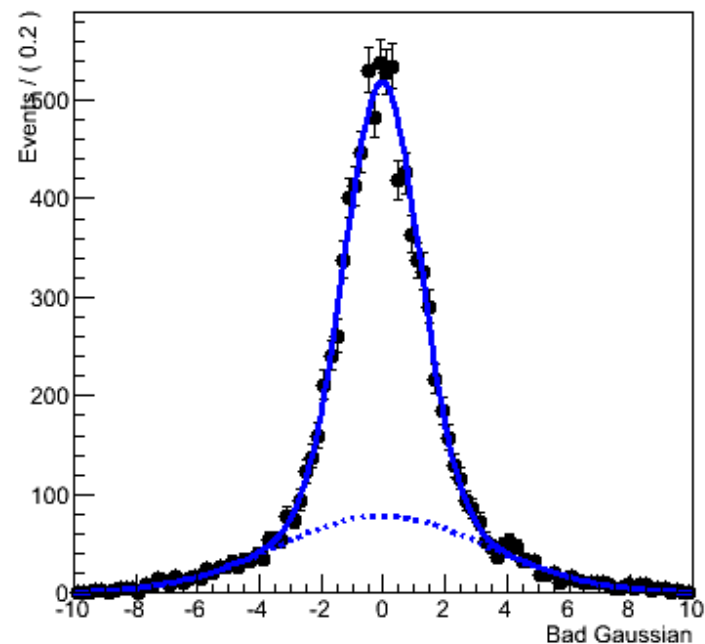
```
xframe.Draw();
```

A RooPlot of "Bad Gaussian"



Fitting a dataset

A RooPlot of "Bad Gaussian"



- A pdf can be fitted to a dataset:

- This is very simple:

```
twogauss.fitTo(*mydata);
```

- Plot the resulting dataset and pdf in a frame

```
mydata->plotOn(xframe);
```

```
twogauss.plotOn(xframe);
```

```
twogauss.plotOn(xframe, Components("gauss2"), LineStyle(kDashed));
```

- Like for normal fits, it is possible to store results in a `RooFitResult`:

```
RooFitResult* fitres = twogauss.fitTo(*mydata, Save());
```

```
fitres->Print();
```

```
fitres->correlationMatrix()->Print();
```

To fix a parameter:

```
mean=0.;
```

```
mean.setConstant(true);
```



Datasets <-> TTree, TH1

- It is possible to convert a TTree or a TH1 into a RooDataSet for fitting

```
RooDataSet mytreedata("mytreedata","imported  
data",x,Import(*myTree));
```

```
RooDataHist myhistdata("myhistdata","imported  
data",x,Import(*myTH1));
```

- And also the other way around (this is how our test histogram was created)

```
TH1F* myh = x.createHistogram("myh","Entries");  
TH1* myh2 = mydata->fillHistogram(myh,x);
```

```
TTree* myTree = mydata->tree();
```



Another way of summing

- In many application we are interested to know how many event are in the narrow Gaussian (**N1**) or in the wide one (**N2**):

```
RooRealVar x("x","Bad Gaussian",-10.,10.);
RooRealVar mean("mean","mean of Gaussian",0.,-10.,10.) ;
RooRealVar sigma1("sigma1","width of narrow Gaussian",1.2,0.1,10.) ;
RooRealVar sigma2("sigma2","width of wide Gaussian",3.4,2.,10.) ;

RooGaussian gauss1("gauss1","Narrow Gaussian",x,mean,sigma1);
RooGaussian gauss2("gauss2","Wide Gaussian",x,mean,sigma2);

RooRealVar N1("N1","events in narrow Gaussian",6000.);
RooRealVar N2("N2","events in wide Gaussian",3000.);
RooAddPdf twogauss("twogauss","Two Gaussians pdf",
                    RooArgList(gauss1,gauss2),RooArgList(N1,N2));
```

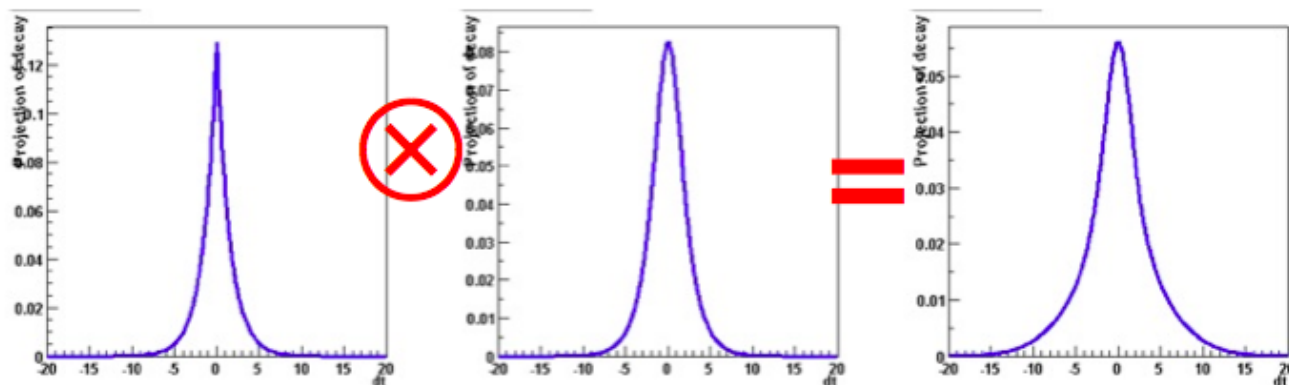
It will be useful in tomorrow exercises.



Convolution

- Model representing a convolution of a theory model and a resolution model often useful

$$f(x) \otimes g(x) = \int_{-\infty}^{+\infty} f(x)g(x - x')dx'$$



- But numeric calculation of convolution integral can be challenging. No one-size-fits-all solution, but 3 options available
 - Analytical convolution (BW \otimes Gauss, various B physics decays)
 - Brute-force numeric calculation (slow)
 - FFT numeric convolution (fast, but some side effects)

Example of convolution: BW+Gauss

- Let's assume we want to describe the widening of the Z resonance peak (Breit-Wigner shape) due to detector resolution (Gaussian).

```
RooRealVar m("m","mumu invariant mass",91.,66.,116.);
```

```
RooRealVar mZ("mZ","Z mass",91.1876);
```

```
RooRealVar GZ("GZ","Z width",2.4952);
```

```
RooBreitWigner peak("peak","Z peak",m,mZ,GZ);
```

```
RooRealVar mres("mres","auxiliary variable",0.,-20.,20.);
```

```
RooRealVar sigma("sigma","mass resolution",2.,0.,10.);
```

```
RooGaussian gauss("gauss","Resolution function",m,RooConst(0.),sigma);
```

```
RooFFTConvPdf conv("conv","Reconstructed peak",m,peak,gauss);
```

```
RooVoigtian voigt("voigt","Reconstructed peak",m,mZ,GZ,sigma);
```

Let's run the Convolution.C macro.



Comparing FFT vs Analytical

```
TCanvas cfun("cfun","Plot of a function",400,400);
```

```
RooPlot* xframe = m.frame(Range(66.,116.));
```

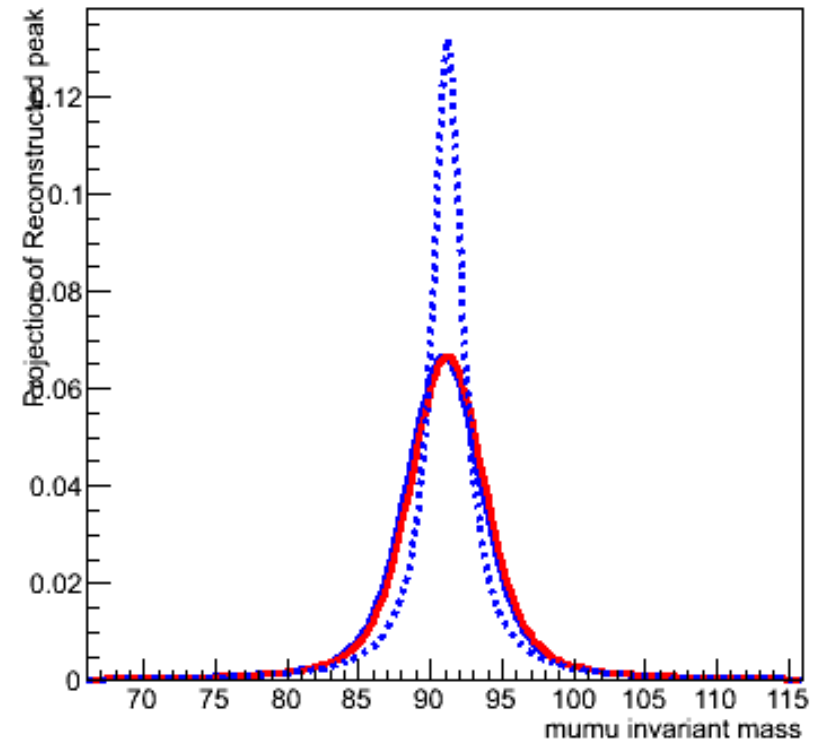
```
conv.plotOn(xframe);
```

```
voigt.plotOn(xframe,LineColor(kRed));
```

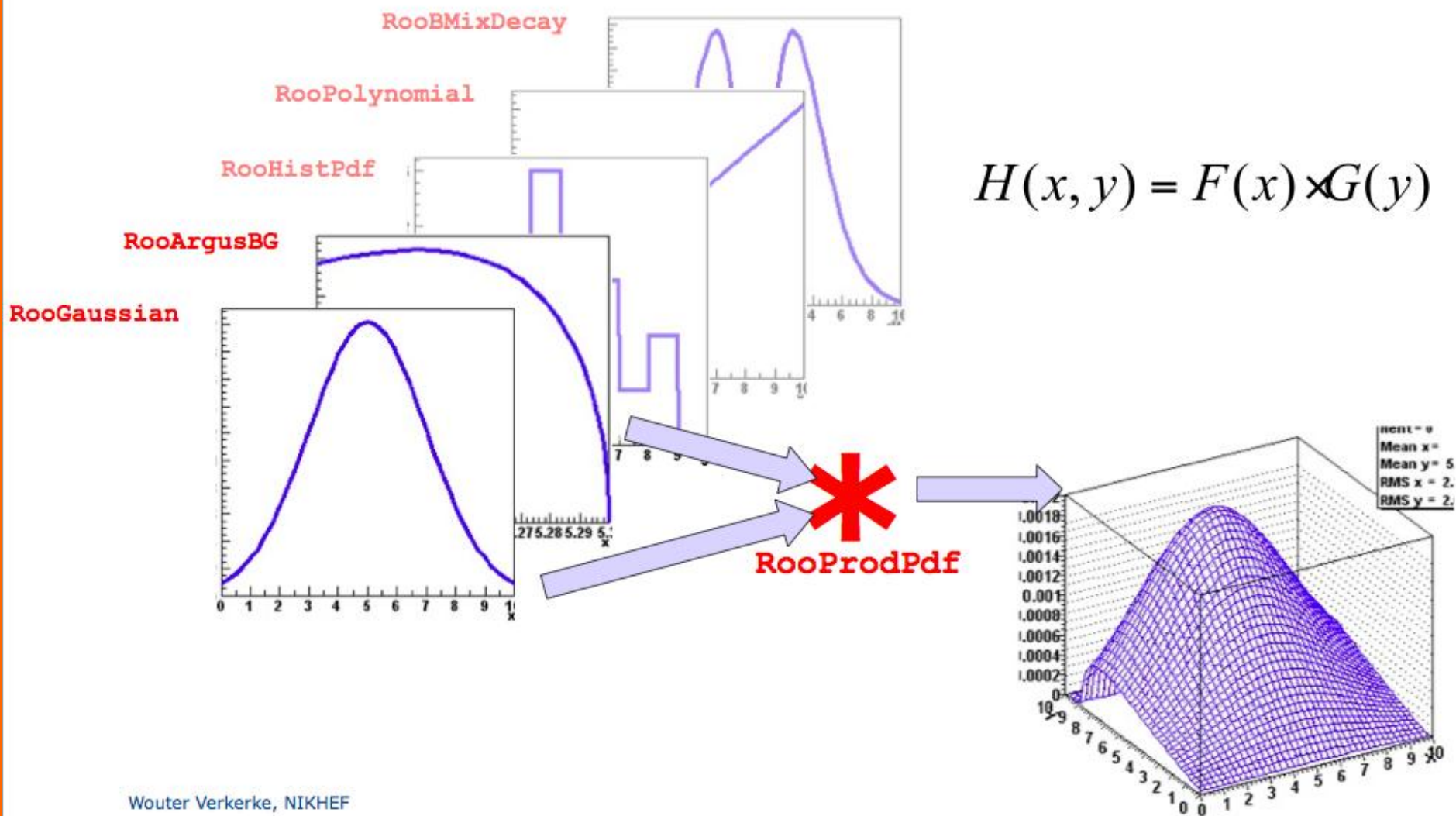
```
peak.plotOn(xframe,LineStyle(kDashed));
```

```
xframe.Draw();
```

A RooPlot of "mumu invariant mass"



Model building – Products of ~~uncorrelated~~ p.d.f.s



Product of PDF: Poisson with uncertainty on μ

- We expect data to be distributed according to a Poisson distribution
...but we have some uncertainty on value of μ .
- We can describe the problem in RooFit as product of two probability distribution, one *conditional* on the other.

```
RooRealVar Nobs("Nobs","Observed events",1,0.,14.);
RooRealVar mu("mu","Expected events",0.,6.);
RooPoisson observed("observed","Observed events",Nobs,mu);

RooGaussian mupdf("mupdf","Mu value",mu,RooConst(2.5),RooConst(0.9));

RooProdPdf modPoisson("modPoisson","Poisson with uncertainty on mu",
                      mupdf,Conditional(observed,Nobs));
```

Let's run the `PoissonComposition.C` macro.



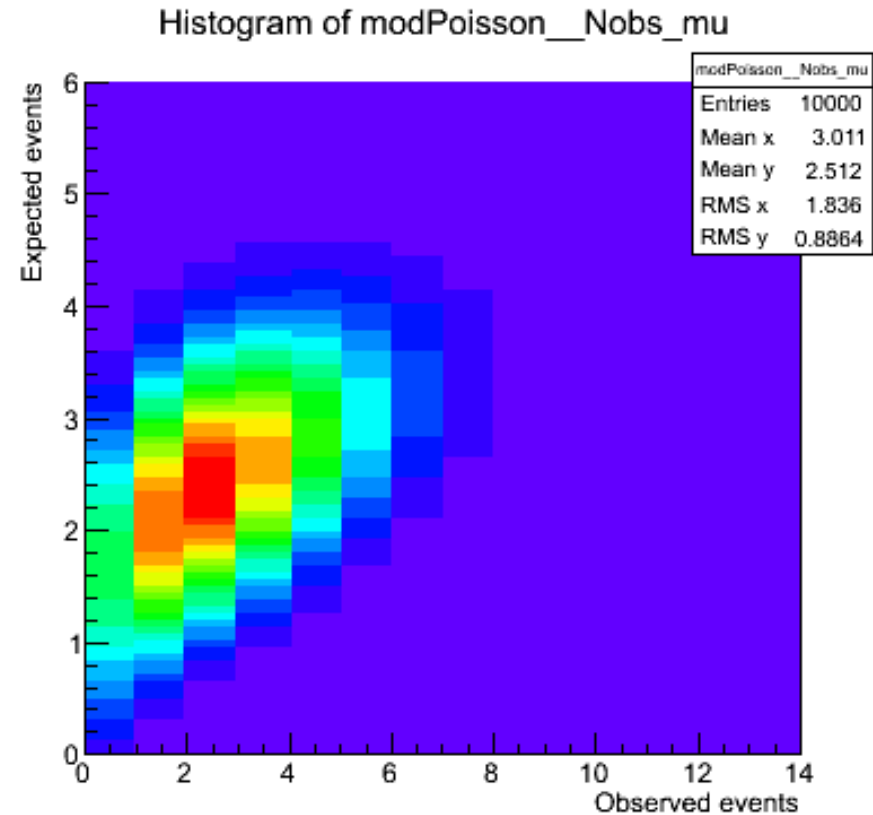
Plotting the 2D pdf

```
RooDataSet* mydata = modPoisson.generate(RooArgSet(Nobs,mu),10000);
```

```
TCanvas c2D("cfun","2D distribution",400,400);  
TH2* hh = modPoisson.createHistogram("Nobs,mu");  
hh->Draw("COL");
```

```
TCanvas cmu("cmu","mu distribution",400,400);  
RooPlot* frame1 = mu.frame();  
mydata->plotOn(frame1);  
frame1.Draw();
```

```
TCanvas cN("cN","Nobs distribution",400,400);  
RooPlot* frame2 = Nobs.frame();  
mydata->plotOn(frame2);  
frame2.Draw();
```



Plotting the N_{obs} pdf

```
RoDataSet* mydata = modPoisson.generate(RooArgSet(Nobs,mu),10000);
```

```
TCanvas c2D("cfun","2D distribution",400,400);  
TH2* hh = modPoisson.createHistogram("Nobs,mu");  
hh->Draw("COL");
```

```
TCanvas cmu("cmu","mu distribution",400,400);  
RooPlot* frame1 = mu.frame();  
mydata->plotOn(frame1);  
frame1.Draw();
```

```
TCanvas cN("cN","Nobs distribution",400,400);  
RooPlot* frame2 = Nobs.frame();  
mydata->plotOn(frame2);  
frame2.Draw();
```

A RooPlot of "Observed events"

