A N_F-Dependent VFNS for Heavy Flavors: Merging the FFNS and VFNS

A. Kusina[1], F. I. Olness, I. Schienbein, J. Y. Yu, T. Stavreva, K. Kovařík, T. Ježo

[1]Southern Methodist University, Dallas, TX 75275, USA

DIS 2013, 22-26 April 2013, Marseille, France

- Motivations
- Generalized VFNS
- Results
- Summary

Problems with FFNS and VFNS

- Doing global analyses we want to include different data in broad energy range.

✗ FFNS: can't be used for LHC energies ($Q \gg m_Q$ unresummed logarithms $\log Q/m_Q$)

Problems with FFNS and VFNS

- Doing global analyses we want to include different data in broad energy range.

✗ FFNS: can't be used for LHC energies ($Q \gg m_Q$ unresummed logarithms $\log Q/m_Q$)

it's like driving on first gear on a highway...

Problems with FFNS and VFNS

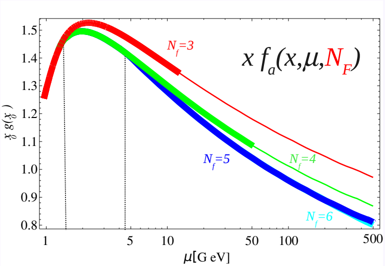
- Doing global analyses we want to include different data in broad energy range.

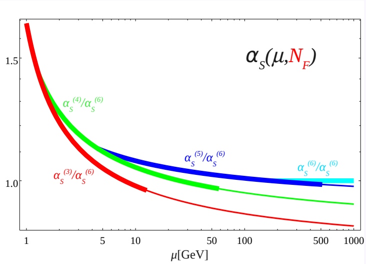
✗ FFNS: can't be used for LHC energies ($Q \gg m_Q$ unresummed logarithms $\log Q/m_Q$)

it's like driving on first gear on a highway...

✗ VFNS: flavor threshold in the middle of (precision) HERA data set potential problem with discontinuities

SOLUTION: VFNS(N_F)





- coexisting N_F-dependent PDFs & α_S
- user can choose when to switch between N_F and $N_F + 1$ scheme

Where is the difference between VFNS(N_F) and VFNS?

✓ both use the same matching conditions

$$f_i(x, \mu, N_F + 1) = A^{ij} \otimes f_j(x, \mu, N_F)$$

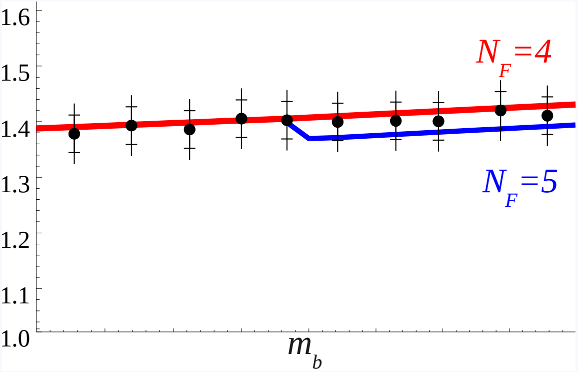
✓ both features

- resummation of logarithms $\log \mu / m_{c,b}$
- can be reliably extended to high scales $1 \lesssim \mu / m_{c,b} \lesssim \infty$
- in the low scales reduces to FFNS

✗ VFNS has no overlap between schemes with different N_F

✗ In VFNS historically switching/transition **always** at $\mu = m_Q$

Why do we want overlap?



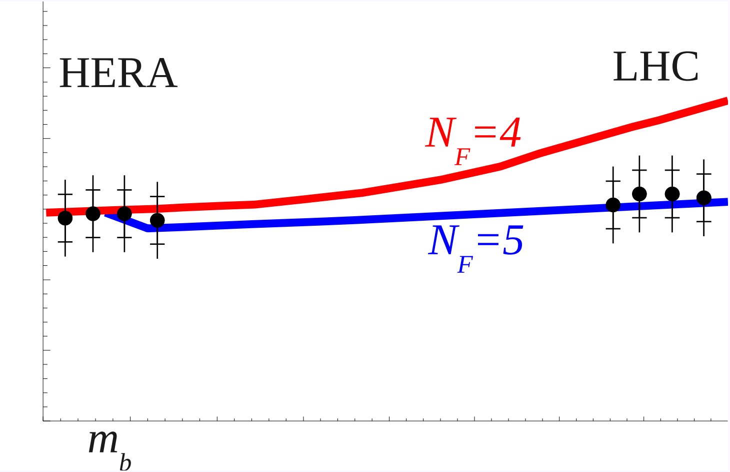
- ✗ at higher orders PDFs/α_S discontinuous
- ✗ discontinuities in perturbatively calculated observables

If we want to analyze HERA data (`1106.1028`): $Q \sim [2, 10]$GeV

- ✗ flavor threshold in the middle of a (precision) data set

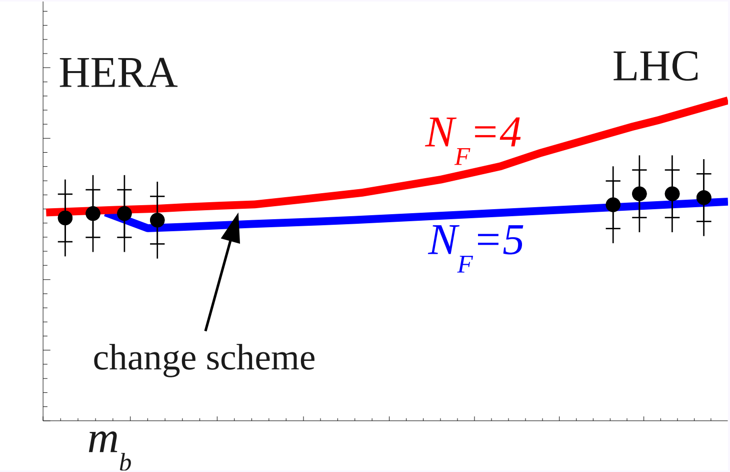
Switching/transition scale choice

Switch between N_F and $N_F + 1$ when convenient



Switching/transition scale choice

Switch between N_F and N_F+1 when convenient



- remember about uncancelled $\log(Q/m_Q)$ in FFNS

N_F Dependent Variable Flavor Number Scheme VFNS(N_F)

We propose a simple generalization of VFNS which however has a big practical advantage!

- coexisting N_f-dependent PDFs
- connected by matching conditions (e.g. in $\overline{MS}$ scheme)
- user can choose when to switch between schemes!
  (he can avoid to do it in the middle of a precision data set, especially when working in higher orders when discontinuities appear)

! Different from phenomenological fits with different number of flavors (e.g. `CTEQ5M hep-ph/9903282`)!

Choice of matching & switching scales

It is rather natural to use the same matching scale as in the traditional VFNS (no logarithmic terms, continues at NLO)

- matching scales $\mu_i^m = m_{Q_i}$

What about

- switching/transition scales $\mu_i^s =???$
  - $\mu_i^s = m_{Q_i}$
  - adjusted to a given data set or process (for HERA $\mu_4^s \simeq 10\text{GeV}$)
  - $\mu_i^s = h(x)$, in particular e.g. $\mu_i^s = W = \sqrt{\frac{Q^2(1-x)}{x}}$

Sketch of a PDF fit in VFNS(N_F)

1. Parametrize PDFs at $\mu_0 \sim 1$ GeV, and generate family of N_F dependent PDFs:
- starting at μ_0 evolve PDFs and α_S using $N_F = 3$ evolution up to μ_{max}, giving: $f_a(x, \mu, N_F = 3)$ and $\alpha_s(\mu, N_F = 3)$
- at $\mu = m_c$ use $\overline{MS}$ matching conditions to compute $N_F = 4$ PDFs and α_s, then use $N_F = 4$ evolution to obtain $f_a(x, \mu, N_F = 4)$ and $\alpha_s(\mu, N_F = 4)$
- at $\mu = m_b$ ...

2. Fit HERA F_2^{charm} data using $N_F = 3$ "FFNS" PDFs and α_S.

3. Fit high-scale LHC data using $N_F = 4, 5$ "VFNS" PDFs and α_S.

4. Minimize χ^2 by adjusting PDF parameters at $\mu = Q_0$.

5. Regenerate new set of VFNS(N_F) PDFs (for $N_F = 3, 4, 5$) and repeat steps 2-4.

RESULTS

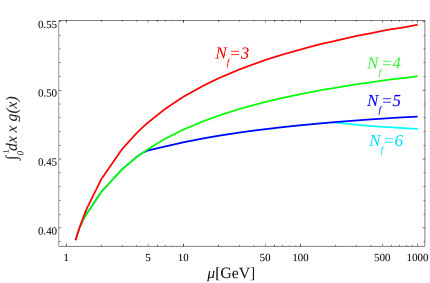
Our N_f-dependent PDFs

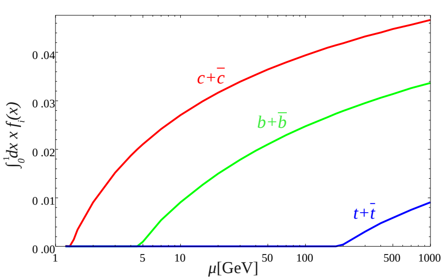
For a demonstration reason of this analyses we produced a benchmark N_F-dependent PDFs, based on `nCTEQ` fit [`0907.2357`], using NLO evolution and initial scale of $\mu_0 = 1.2$GeV.

- matching scales $\mu_i^m = m_{Q_i}$
- switching/transition scales to be chosen by user ($\mu_i^s \sim m_{Q_i}$)

Effect of N_f channels in PDFs

Opening of the new channels (c, b) is compensated by the decreasing gluon $g \to c\bar{c}$





$$f_i(x,\mu,N_F+1) = A^{ij} \otimes f_j(x,\mu,N_F)$$

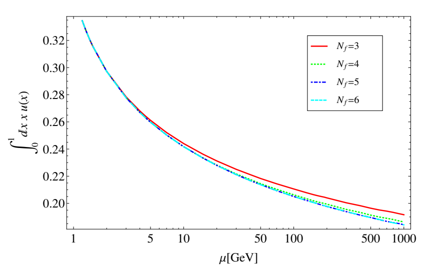
$$A^{ij} = \delta^{ij} + \frac{\alpha_s}{2\pi}\left(a_1^{ij} + b_1^{ij}\ln\left[\frac{\mu}{m}\right]\right) + \left(\frac{\alpha_s}{2\pi}\right)^2\left(a_2^{ij} + b_2^{ij}\ln\left[\frac{\mu}{m}\right] + c_2^{ij}\ln^2\left[\frac{\mu}{m}\right]\right) + \ldots$$

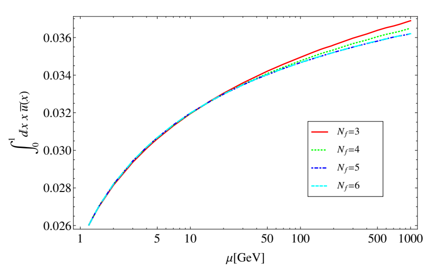
at NLO $a_1^{ij} = 0$

at NNLO $a_2^{ij} \neq 0$

Effect of N_f channels in PDFs

Opening of the new channels (c, b) is compensated by the decreasing gluon $u\bar{u} \to g \to c\bar{c}$





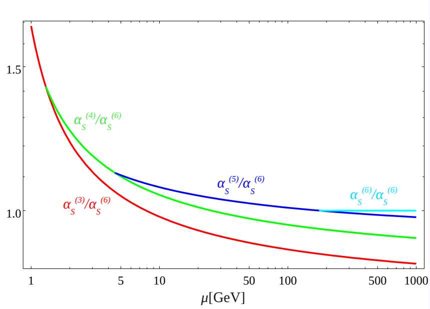
$$f_i(x,\mu,N_F+1) = A^{ij} \otimes f_j(x,\mu,N_F)$$

$$A^{ij} = \delta^{ij} + \frac{\alpha_s}{2\pi}\left(a_1^{ij} + b_1^{ij}\ln\left[\frac{\mu}{m}\right]\right) + \left(\frac{\alpha_s}{2\pi}\right)^2\left(a_2^{ij} + b_2^{ij}\ln\left[\frac{\mu}{m}\right] + c_2^{ij}\ln^2\left[\frac{\mu}{m}\right]\right) + \ldots$$

at NLO $a_1^{ij} = 0$

at NNLO $a_2^{ij} \neq 0$

α_S across flavor thresholds

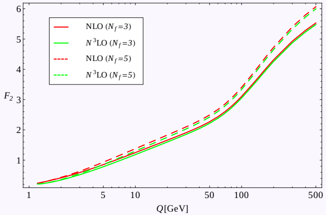


α_S at order $\mathcal{O}\left(\alpha_s^3\right)$ becomes discontinuous

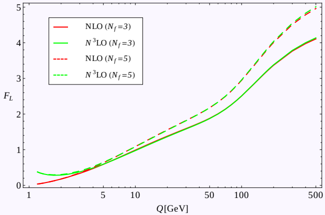
$$\alpha_s(m^2, N_F+1) = \alpha_s(m^2, N_F) + c_{20}\,\alpha_s^3(m^2, N_F)$$

N_f dependence of structure functions (N3LO: 1203.0282)

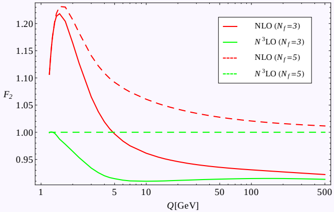
$F_2(x = 10^{-3})$



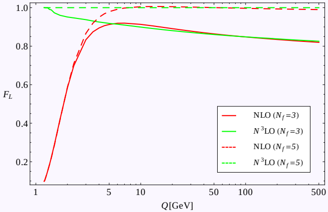
$F_L(x = 10^{-5})$



$F_2/F^{N^3LO}_{2(N_f=5)}$



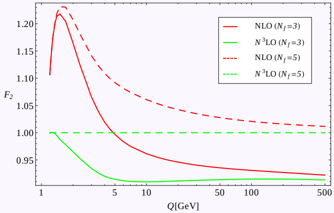
$F_L/F^{N^3LO}_{L(N_f=5)}$



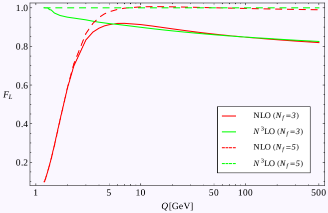
N_f dependence of structure functions (N3LO: 1203.0282)

- $Q < m_Q$:
  - higher order corrections are crucial;
  - $N_F = 3$ and $N_F = 5$ results coincide
- $Q \gg m_Q$:
  - higher order corrections not important;
  - $N_F = 3$ and $N_F = 5$ results diverge due to uncancelled logs $\log(Q/m_q)$
- $Q \geq m_Q$:
  - $N_F = 3$ result can be extended to $Q \sim$ (a few) $\times m_c$

$F_2/F_{2(N_f=5)}^{N^3LO}$

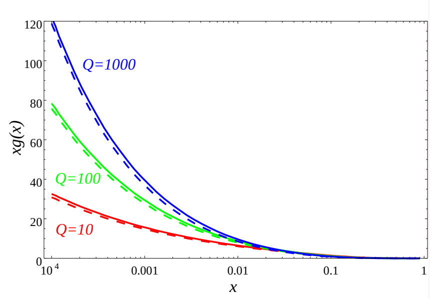


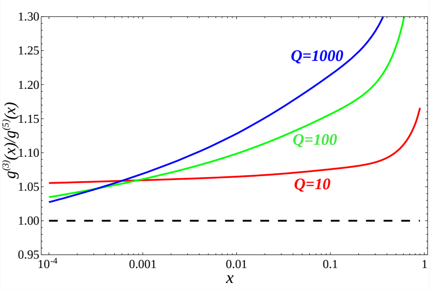
$F_L/F_{L(N_f=5)}^{N^3LO}$



charm production at HERA

Estimate effect of $N_f = 3$ vs. $N_f = 5$ by comparing gluons

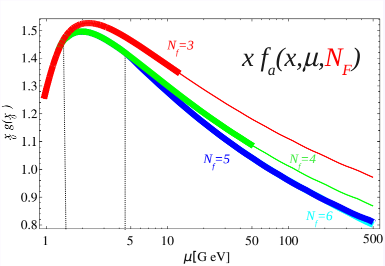


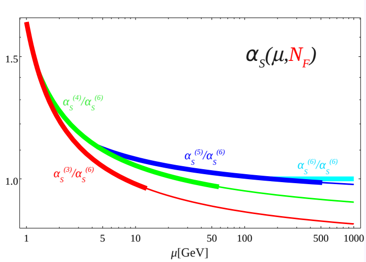


- at low Q (10GeV) the shift is $\sim 6\%$ and flat along x
- `HVQDIS` (`hep-ph/9706334`), used in HERA anlyses, works in $N_F = 3$ FFNS. We can use this ratio to approximate its results in case where only $N_F = 4, 5$ PDFs were available.

Summary

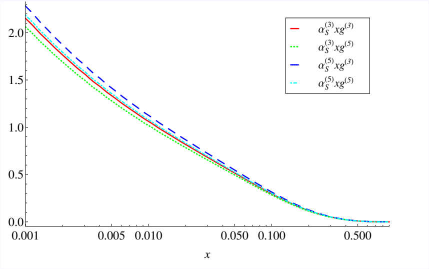
- We proposed a simple generalization of VFNS allowing to take advantage of both FFNS and VFNS.
- Especially interesting for higher order analyses, when PDFs and α_S discontinuous.
- VFNS(N_F) allows to avoid flavors transition if it happens in the middle of a data set.

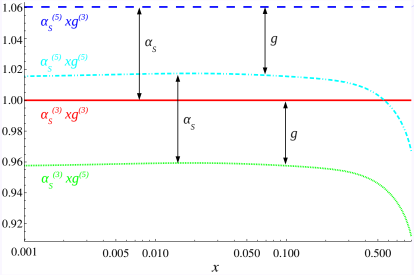




BACKUP SLIDES

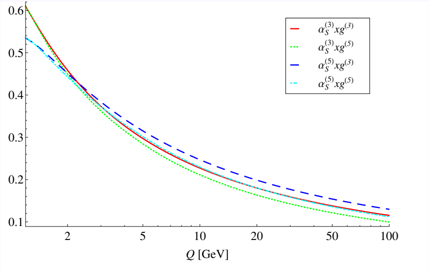
Interplay between $\alpha_S(\mu_R, N_f)$ and $g(x, \mu, N_f)$

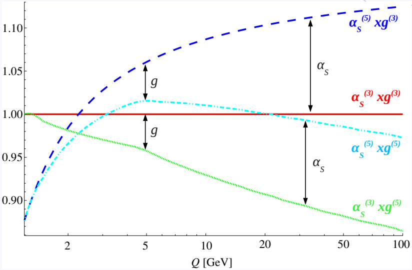




- LO $\sim$ light quark PDFs (not sensitive to change $N_F \to N_F + 1$)
- combination $\alpha_S g$ enters many NLO corrections (leading component)
- for $Q \sim 10$GeV consistent 3-flavor (FFNS) or 5-flavor (VFNS) schemes are comparable: flavor dependence in α_S and g partly cancel

Interplay between $\alpha_S(\mu_R, N_f)$ and $g(x, \mu, N_f)$





- LO ~ light quark PDFs (not sensitive to change $N_F \to N_F + 1$)
- combination $\alpha_S g$ enters many NLO corrections (leading component)
- for $Q \sim 10$GeV consistent 3-flavor (FFNS) or 5-flavor (VFNS) schemes are comparable: flavor dependence in α_S and g partly cancel

Matching conditions for PDFs

$$f_i(x, \mu, N_F + 1) = A^{ij} \otimes f_j(x, \mu, N_F)$$

where A^{ij} can be computed perturbatively as (hep-ph/9601302, hep-ph/9612398)

$$A^{ij} = \delta^{ij} + \frac{\alpha_s}{2\pi} \left(a_1^{ij} + P_0^{ij} \ln \left[\frac{\mu^2}{m^2} \right] \right)$$
$$+ \left(\frac{\alpha_s}{2\pi} \right)^2 \left(a_2^{ij} + \left\{ P_1^{ij} + P_0^{ij} \otimes a_1^{ij} - \beta_0 a_1^{ij} \right\} \ln \left[\frac{\mu^2}{m^2} \right] \right.$$
$$\left. + \frac{1}{2} \left\{ P_0^{ij} \otimes P_0^{ij} - \beta_0 P_0^{ij} \right\} \ln^2 \left[\frac{\mu^2}{m^2} \right] \right) + \ldots$$

α_S evolution

The running of $\alpha_S(\mu, N_F)$

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s) = -\left(b_0\alpha_s^2 + b_1\alpha_s^3 + b_2\alpha_s^4 + ...\right)$$

At the NLO (2-loop) level we obtain

$$\alpha_S(\mu^2, N_F) = \frac{1}{b_0(N_F)\ln(\mu^2/\Lambda^2)}\left(1 - \frac{b_1(N_F)}{b_0^2(N_F)}\frac{\ln\left(\ln(\mu^2/\Lambda^2)\right)}{\ln(\mu^2/\Lambda^2)}\right)$$

where $b_0(N_F) = (33 - 2N_F)/12\pi$ and $b_1(N_F) = (153 - 19N_F)/24\pi^2$.

α_S across flavor thresholds

$$\alpha_s(\mu^2, N_{F+1}) = \alpha_s(\mu^2, N_F) \left[1 + \sum_{k=1}^{\infty} \sum_{\ell=0}^{k} c_{k\ell} \left[\alpha_s(\mu^2, N_F)\right]^k \ln^l \left(\frac{\mu_R^2}{m^2}\right)\right]$$

where $c_{10} = 0$ and $c_{20} = -11/72\pi^2$

Using $\mu = m$ and restricting to $\mathcal{O}\left(\alpha_s^3\right)$ terms

$$\alpha_s(m^2, N_F + 1) = \alpha_s(m^2, N_F) + c_{20}\alpha_s^3(m^2, N_F)$$

ACOT scheme (hep-ph/9312319, hep-ph/9806259)

The key ingredient of ACOT scheme is subtraction term (SUB), at NLO we have

$$\sigma_{TOT} = \sigma_{LO} + \{\sigma_{NLO} - \sigma_{SUB}\}$$

for gluon-initiated processes:

$$\sigma_{SUB} = f_g \otimes \tilde{P}_{g\to Q} \otimes \sigma_{QV\to Q}$$

$\tilde{P}_{g\to Q}$ is the $\overline{MS}$ splitting times the logarithm $\ln(\mu^2/m_Q^2)$

Discontinuities in higher orders

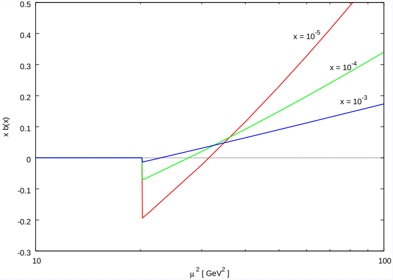
- PDFs – $O(\alpha_s^2)$
- strong coupling – $O(\alpha_s^3)$

- measurable quantities calculated at $O(\alpha_s^n)$

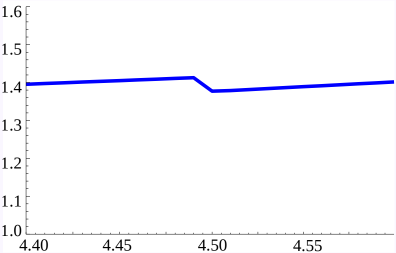
$$\sigma_{TOT}^{N_F=5} = \sigma_{TOT}^{N_F=4} + O(\alpha_s^{n+1})$$

Discontinuities in higher orders

$xb(x)$



$F_L(x = 10^{-5})$



Discontinuities in measurable quantities

Matching condition:

$$f_a^{N_F+1} = A_{ab} \otimes f_b^{N_F}$$

for $\mu = m_b$:

$$\begin{aligned} f_b^5 &= \left\{0 + \frac{\alpha_s}{2\pi} P_{qg} \left(L + a_{qg}\right) + O(\alpha_s^2)\right\} \otimes f_g^4 \\ f_g^5 &= \left\{1 + \frac{\alpha_s}{2\pi} P_{gg} \left(L + a_{gg}\right) + O(\alpha_s^2)\right\} \otimes f_g^4 \end{aligned}$$

where $L = \ln(\mu^2/m_b^2)$

- for $\overline{MS}$ at NLO $a_{qg} = a_{gg} = 0 \rightarrow$ PDFs are continuous

Discontinuities in measurable quantities

What if $a \neq 0$ at NLO:

ACOT for $N_F = 5$ ($\mu > m_b$):

$$\begin{aligned}
\sigma_{LO} &= C^0 \otimes f_b^5 \simeq C^0 \otimes \left\{0 + \frac{\alpha_s}{2\pi} P_{qg} \left(L + a_{qg}\right)\right\} \otimes f_g^4 \\
\sigma_{NLO} &= C^1 \otimes f_g^5 \simeq C^1 \otimes \left\{1 + \frac{\alpha_s}{2\pi} P_{gg} \left(L + a_{gg}\right)\right\} \otimes f_g^4 \\
\sigma_{SUB} &= C^0 \otimes \widetilde{f}_{g\to q} \otimes f_g^5 \simeq C^0 \otimes \left\{\frac{\alpha_s}{2\pi} P_{qg} \left(L + a_{qg}\right)\right\} \\
&\otimes \left\{1 + \frac{\alpha_s}{2\pi} P_{gg} \left(L + a_{gg}\right)\right\} \otimes f_g^4
\end{aligned}$$

to $\mathcal{O}(\alpha_s^1)$ order:

$$\sigma_{TOT}^{N_F=5} = \sigma_{LO} + \sigma_{NLO} - \sigma_{SUB} = C^1 \otimes f_g^4 + O(\alpha_s^2)$$

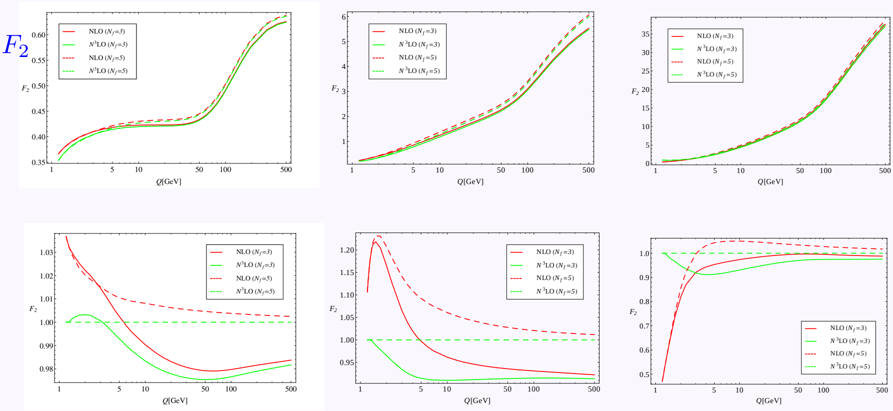
ACOT for $N_F = 4$ ($\mu < m_b$):

$$\sigma_{TOT}^{N_F=4} = \sigma_{NLO} = C^1 \otimes f_g^4 + O(\alpha_s^2)$$

N_f dependence of structure functions: F_2

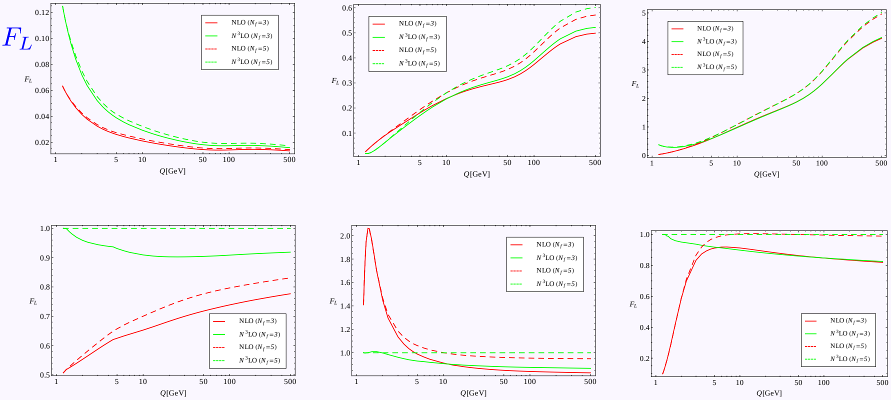
(N3LO calculation: 1203.0282)

$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$



N_f dependence of structure functions: F_L

$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$



Different treatment of α_S in FFNS

- number of flavors entering α_S is fixed and the same as number of flavors entering PDFs: $N_R = N_F$
  `hep-ph/0603143` (A. Martin, W. Stirling, and R. Thorne)
- N_R is changing as in the VFNS (but independently from N_F)
  `hep-ph/0608276` (M. Gluck and E. Reya)