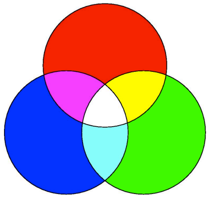

Universality of TMD distribution functions

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Content

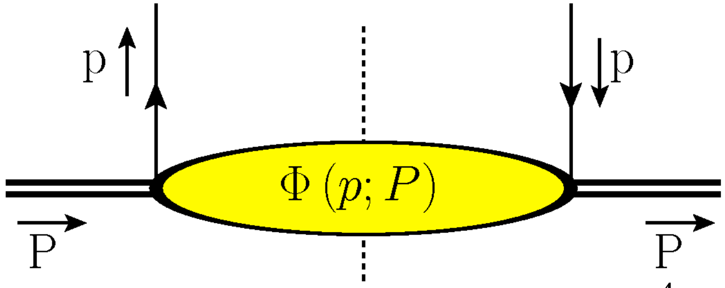
- Overview of TMD correlators

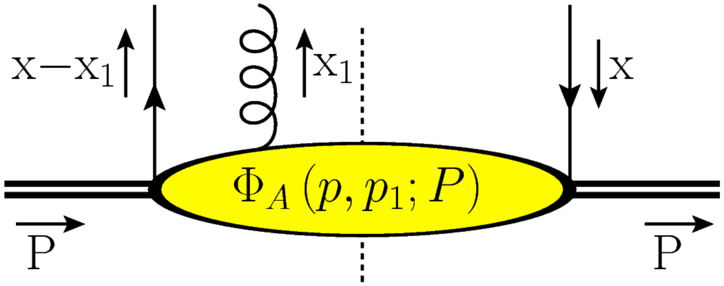
- Transverse momentum weightings
  - Formalism
  - Results for gluons
  - Example

- Summary and conclusions

Correlators

- Quark correlators can be written as matrix elements

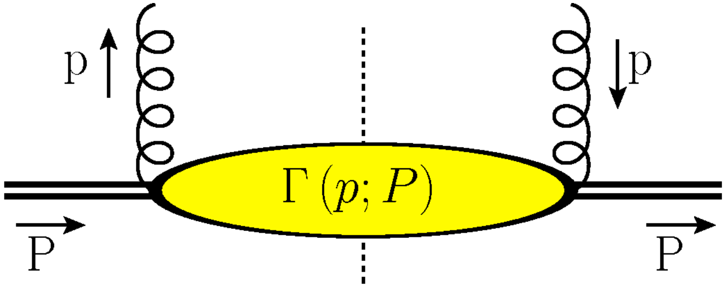




$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}_j(0)\, \psi_i(\xi) \middle| P \right\rangle$$

$$\Phi^{\alpha}_{A;ij}(p-p_1, p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi + ip_1.\eta} \left\langle P \middle| \bar{\psi}_j(0)\, A^{\alpha}(\eta)\, \psi_i(\xi) \middle| P \right\rangle$$

- Similarly, the gluon correlator can be written as a matrix element.



$$\Gamma^{\alpha\beta}(p\,|\,p) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| F^{n\alpha}(0)\, F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

Correlators and TMDs

- The gluon TMD correlator can be parametrized as

$$2x\,\Gamma^{[U]}(x,p_T) = -\,g_T^{\mu\nu} f_1^g(x,p_T^2) + g_T^{\mu\nu}\frac{\varepsilon_T^{p_T S_T}}{M} f_{1T}^g(x,p_T^2)$$

$$+\, i\varepsilon_T^{\mu\nu} g_{1S}^g(x,p_T) + \left(\frac{p_T^\mu p_T^\nu - \frac{1}{2}p_T^2 g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g}(x,p_T^2)$$

$$-\,\frac{\varepsilon_T^{p_T\{\mu}\, p_T^{\nu\}}}{2M^2} h_{1S}^{\perp g}(x,p_T) - \frac{\varepsilon_T^{p_T\{\mu}\, S_T^{\nu\}} + \varepsilon_T^{S_T\{\mu}\, p_T^{\nu\}}}{4M} h_{1T}^{\perp g}(x,p_T^2)$$

- In which $g_{1S}^g(x,p_T) = S_L\, g_{1L}^g(x,p_T^2) - \dfrac{p_T\cdot S_T}{M} g_{1T}^g(x,p_T^2)$

$$S^\mu = S_L P^\mu + S_T^\mu + M^2 S_L n^\mu$$

- The parametrization contains three types of contributions

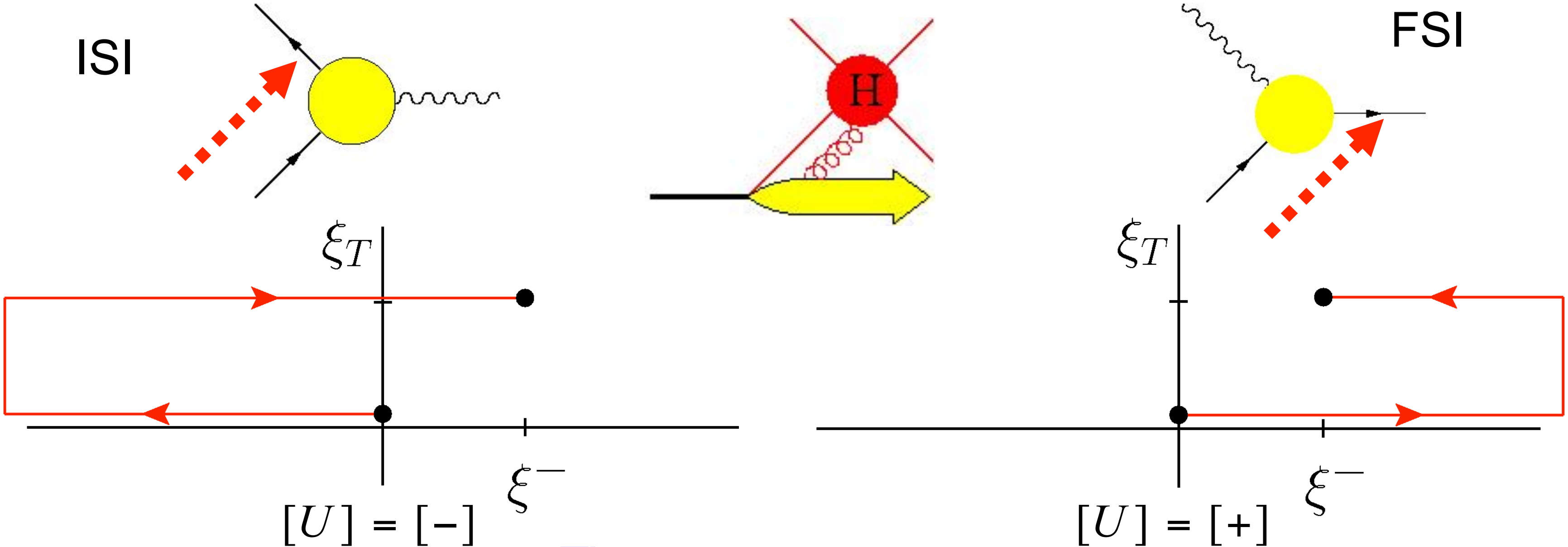
Mulders, Tangerman, Boer
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 0702 (2007) 093

Gauge invariance for quark TMDs

- Gauge links are required to make the nonlocal combinations of fields gauge invariant

$$U_{[0,\xi]} = \mathcal{P}\exp\left(-ig\int_0^{\xi} ds^{\mu} A_{\mu}\right)$$

- Introduces a path dependence for correlators



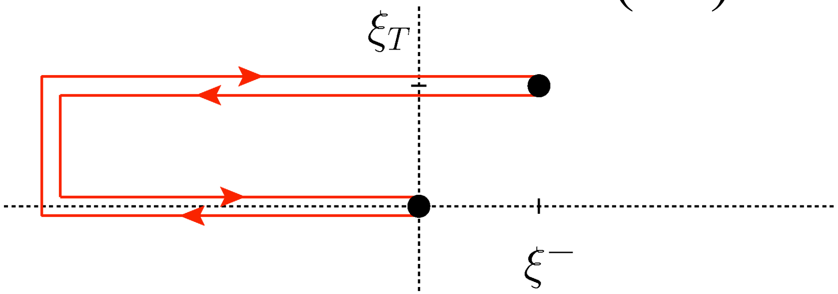
Time-reversal couple

A.V. Belitsky, X. Ji, F. Yuan, NP B 656, (2003) 165
D. Boer, P.J. Mulders & F. Pijlman, NP B 667 (2003) 2012

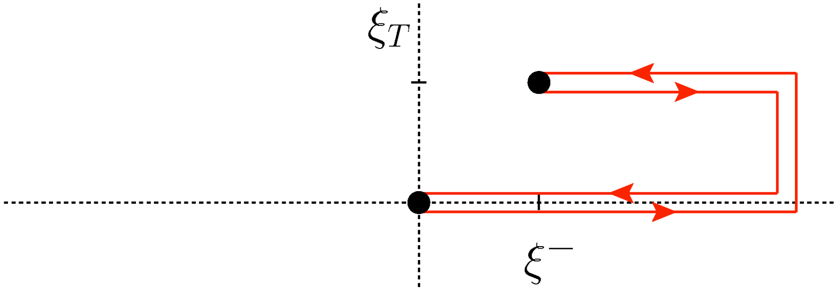
Gauge invariance for gluon TMDs

- For gluons more gauge link structures exist.

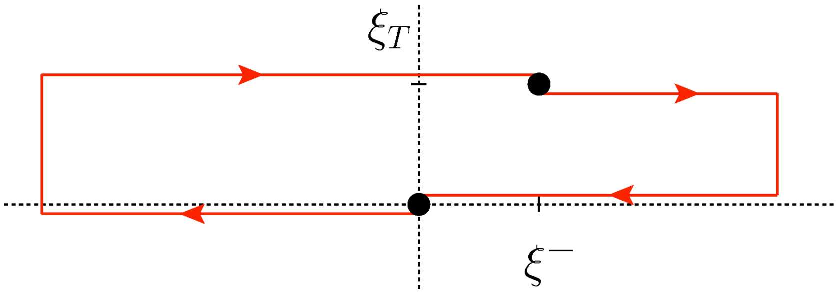
$$\Gamma^{\alpha\beta[U,U']}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\middle|F^{n\alpha}(0)U_{[0,\xi]}F^{n\beta}(\xi)U_{[\xi,0]}\middle|P\right\rangle_{\xi.n=0}$$



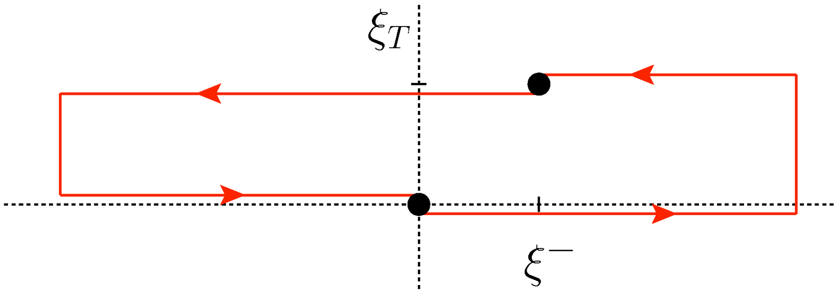
$[U,U'] = [-,-]$



$[U,U'] = [+,+]$



$[U,U'] = [-,+]$



$[U,U'] = [+,-]$

- More complicated structures arise for complicated processes

Mellin moments

- Collinear functions

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\,\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) (\partial^n)^{N-1} U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U^{[n]}_{[0,\xi]} (D^n)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

$$\Phi^{(N)} = \left\langle P \middle| \overline{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

- All operators have same twist since dim(D^n) = 0

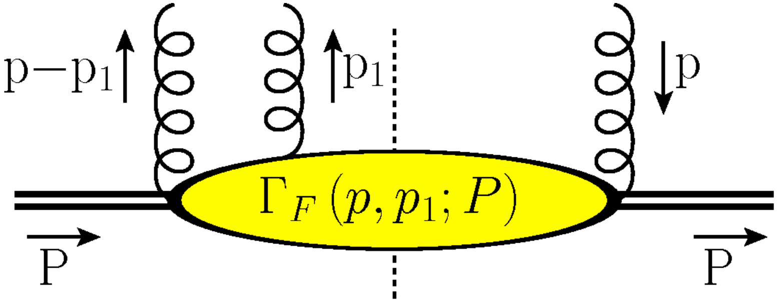
Transverse moments

- Transverse moments are considered for transverse weightings

$$p_T^{\alpha}\Gamma^{[U,U']}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\left|F^{n\alpha}(0)\,i\,\partial_T^{\alpha}U_{[0,\xi]}F^{n\beta}(\xi)U_{[\xi,0]}\right|P\right\rangle_{\xi.n=0}$$

- Due to transverse directions, partonic operators show up. For the simplest gauge links one finds

$$i\,\partial_T^{\alpha}U_{[0,\xi]}^{[\pm]}=U_{[0,\xi]}^{[\pm]}\left(i\,D_T^{\alpha}(\xi)-gA_T^{\alpha}(\xi)\pm\pi\tilde{G}^{n\alpha}(\xi)\right)$$



$$\Gamma_D^{\alpha}(x)=\int dx_1\,\Gamma_D^{\alpha}(x-x_1,x_1\,|\,x)$$

$$\Gamma_A^{\alpha}(x)=\mathrm{PV}\int dx_1\,\frac{1}{x_1}\Gamma_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

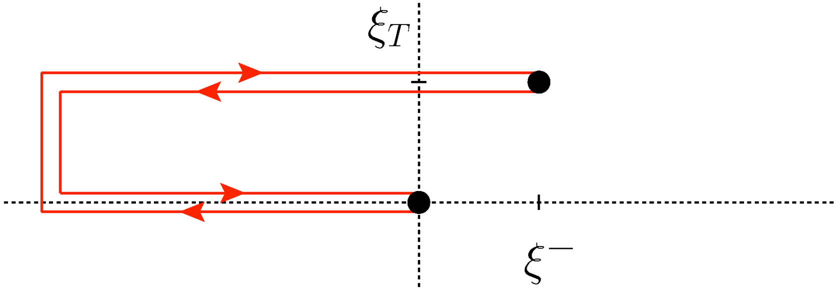
$$\tilde{\Gamma}_{\partial}^{\alpha}(x)=\Gamma_D^{\alpha}(x)-\Gamma_A^{\alpha}(x)$$

$$\Gamma_G^{\alpha}(x)=\Gamma_F^{n\alpha}(x,0\,|\,x)$$

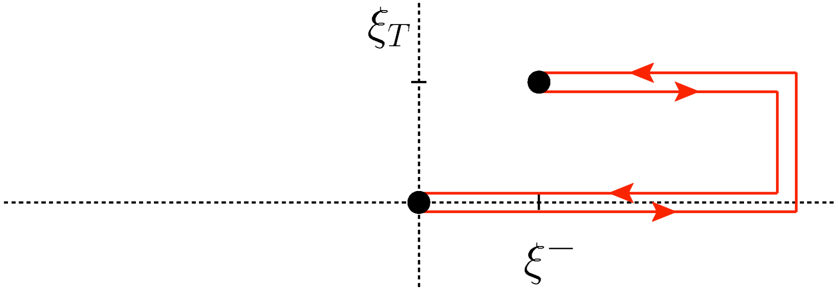
Bomhof, PJM, 2007; Dominguez, Xiao, Yuan (2011); MGAB, A Mukherjee, PJM, in preparation

Color structures

- This gives the operator combinations

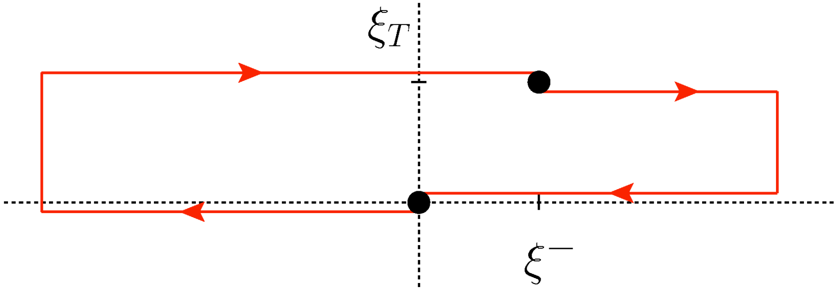


$[U, U'] = [-, -]$

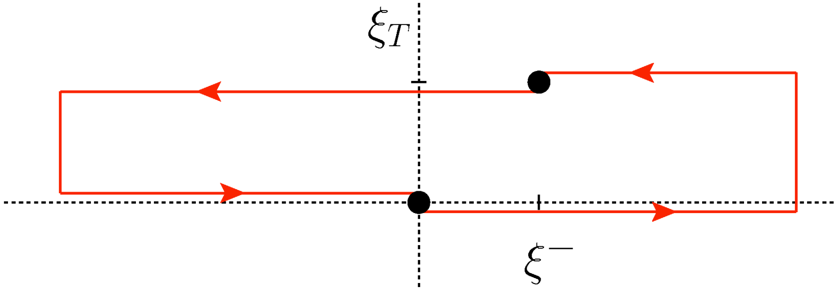


$[U, U'] = [+, +]$

$$\Gamma^{\alpha}_{G,c=1} = \ldots \left\langle P \middle| \mathrm{Tr}_{\mathrm{c}} \left(F^{n\alpha}(0) U_{[0,\xi]} \left[G_T^{n\alpha}(\xi), F^{n\beta}(\xi) \right] U_{[\xi,0]} \right) \middle| P \right\rangle_{\xi.n=0}$$



$[U, U'] = [-, +]$

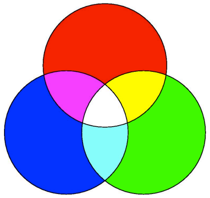


$[U, U'] = [+, -]$

$$\Gamma^{\alpha}_{G,c=2} = \ldots \left\langle P \middle| \mathrm{Tr}_{\mathrm{c}} \left(F^{n\alpha}(0) U_{[0,\xi]} \left\{ G_T^{n\alpha}(\xi), F^{n\beta}(\xi) \right\} U_{[\xi,0]} \right) \middle| P \right\rangle_{\xi.n=0}$$

Bomhof, PJM, 2007; Dominguez, Xiao, Yuan (2011); MGAB, A Mukherjee, PJM, in preparation

Classifying the nonuniversality



- All contributing matrix elements can be classified according to their rank and behavior under time reversal symmetry

$$\int d^2p_T\; p_T^\alpha\,\Gamma^{[U]}(x,p_T;n) = \underset{\underset{\text{T-even}}{\uparrow}}{\tilde{\Gamma}_\partial^\alpha(x)} + \sum_c \pi\, C_{G,c}^{[U]}\,\underset{\underset{\text{T-odd (gluonic pole or ETQS m.e.)}}{\uparrow}}{\Gamma_{G,c}^\alpha(x)}$$

$$\boxed{\tilde{\Gamma}_\partial^\alpha(x) = \Gamma_D^\alpha(x) - \Gamma_A^\alpha(x)} \qquad \boxed{\Gamma_{G,c}^\alpha(x) = \Gamma_{F,c}^{n\alpha}(x,0\,|\,x)}$$

- Behavior of TMDs under time reversal symmetry and rank can be used to identify their corresponding matrix element

- E.g. $g_T^{\mu\nu}\dfrac{\varepsilon_T^{p_T S_T}}{M} f_{1T}^{g}(x,p_T^2) \longrightarrow$ T-odd & rank 1

$$\left(\frac{p_T^\mu p_T^\nu - \tfrac{1}{2}p_T^2 g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g}(x,p_T^2) \longrightarrow \text{T-even \& rank 2}$$

Efremov & Teryaev 1982; Qiu and Sterman 1991
Boer, Mulders & Pijlman, NP B667 (2003) 201;
Bomhof, Mulders and Pijlman, EPJ C 47 (2006) 147

Ranking the contributions

- All contributing matrix elements can be classified according to their rank

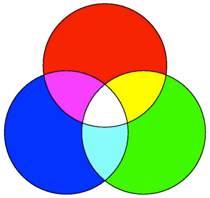
GLUONIC POLE RANK			
0	1	2	3
$\Gamma(x,p_T^2)$	$C^{[U]}_{G,c}\Gamma_{G,c}$	$C^{[U]}_{GG,c}\Gamma_{GG,c}$	$C^{[U]}_{GGG,c}\Gamma_{GGG,c}$
$\widetilde{\Gamma}_\partial$	$C^{[U]}_{G,c}\widetilde{\Gamma}_{\{\partial G\},c}$	$C^{[U]}_{G,c}\widetilde{\Gamma}_{\{\partial GG\},c}$	$\ldots$
$\widetilde{\Gamma}_{\partial\partial}$	$C^{[U]}_{G,c}\widetilde{\Gamma}_{\{\partial\partial G\},c}$	$\ldots$	$\ldots$
$\widetilde{\Gamma}_{\partial\partial\partial}$	$\ldots$	$\ldots$	$\ldots$

- The identification with the gluon PDFs yields

PDFs FOR GLUONS			
f_1^g, g_{1L}^g	$f_{1T}^{\perp g(Ac)}$, $h_{1T}^{g(Ac)}$	$h_1^{\perp g(Bc)}$	$h_{1T}^{\perp g(Bc)}$
g_{1T}^g	$h_{1L}^{\perp g(Ac)}$		
$h_1^{\perp g(A)}$	$h_{1T}^{\perp g(Ac)}$		

MGAB, Mukherjee, PJM, PR D86 (2012) 074030; MGAB, Mukherjee, PJM, in preparation

TMD correlators for gluons



- Gluon TMD correlator can be expanded as

$$\Gamma^{[U]}(x,p_T) = \frac{1}{2x}\left(-g_T^{\mu\nu} f_1^{g}(x,p_T^2) + \left(\frac{p_T^{\mu} p_T^{\nu} - \frac{1}{2} p_T^2 g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g[U]}(x,p_T^2) + ...\right)$$

- Depending on polarizations, multiple contributing PDFs

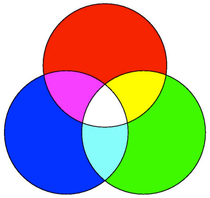
gluon polarization

nucleon pol.

	U	L	Lin.
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

T-odd

TMD correlators for gluons



- Gluon TMD correlator can be expanded as

$$\Gamma^{[U]}(x,p_T) = \frac{1}{2x}\left(-g_T^{\mu\nu} f_1^g(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu - \frac{1}{2} p_T^2 g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g[U]}(x,p_T^2) + ...\right)$$

- Depending on polarizations, multiple contributing PDFs

gluon polarization

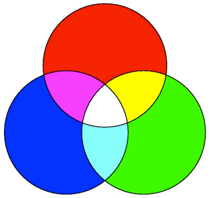
nucleon pol.

	U	L	Lin.
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Process dependent

$h_1^{\perp g}$ is T-even and process-dependent

Color contributions



- Depending on the process, TMDs contain multiple contributions

$$f_{1T}^{\perp g[U]}(x) = \sum_{c=1}^{2} C_{G,c}^{[U]} f_{1T}^{\perp g(Ac)}(x)$$

$$h_{1T}^{g[U]}(x) = \sum_{c=1}^{2} C_{G,c}^{[U]} h_{1T}^{g(Ac)}(x)$$

$$h_{1L}^{\perp g[U]}(x) = \sum_{c=1}^{2} C_{G,c}^{[U]} h_{1L}^{\perp g(Ac)}(x)$$

gluon polarization

nucleon pol.	U	L	Lin.
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g $h_{1T}^{\perp g}$

$$h_{1}^{\perp g[U]}(x) = h_{1}^{\perp g(A)}(x) + \sum_{c=1}^{4} C_{GG,c}^{[U]} h_{1}^{\perp g(Bc)}(x)$$

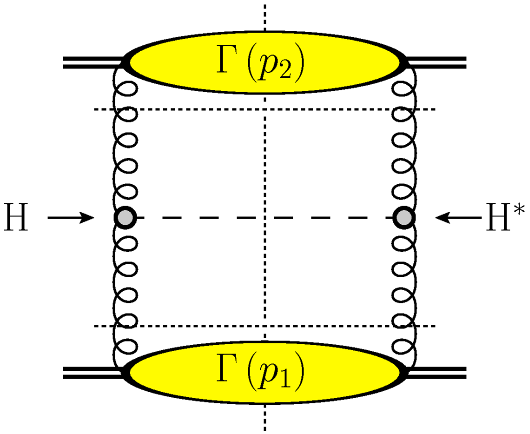
$$h_{1T}^{\perp g[U]}(x) = \sum_{c=1}^{2} C_{G,c}^{[U]} h_{1T}^{\perp g(Ac)}(x) + \sum_{c=1}^{7} C_{GGG,c}^{[U]} h_{1T}^{\perp g(Bc)}(x)$$

Bomhof, Mulders, 2007
MGAB, A Mukherjee, PJM, in preparation

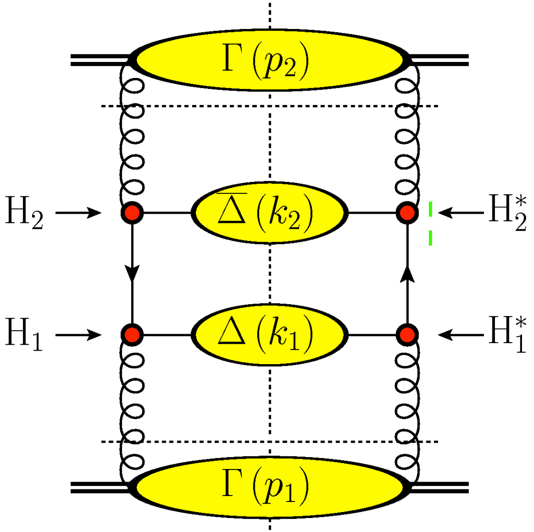
Processes: example for $h_1^{\perp g}$

- gg → H and a specific gg → $q\bar{q}$ process have different gauge link structures and different contributions to PDFs
- Gauge link contributions:

$$[U,U']_{g(\bar{q})} = -\frac{1}{N_c^2-1}\Big[+,+\Big] + \frac{N_c^2}{N_c^2-1}\Big[-,+(\Box)\Big]$$



$$[U,U'] = [+,+]$$



$$[U,U']_{g(q)} = -\frac{1}{N_c^2-1}\Big[+,+\Big] + \frac{N_c^2}{N_c^2-1}\Big[+,-(\Box^\dagger)\Big]$$

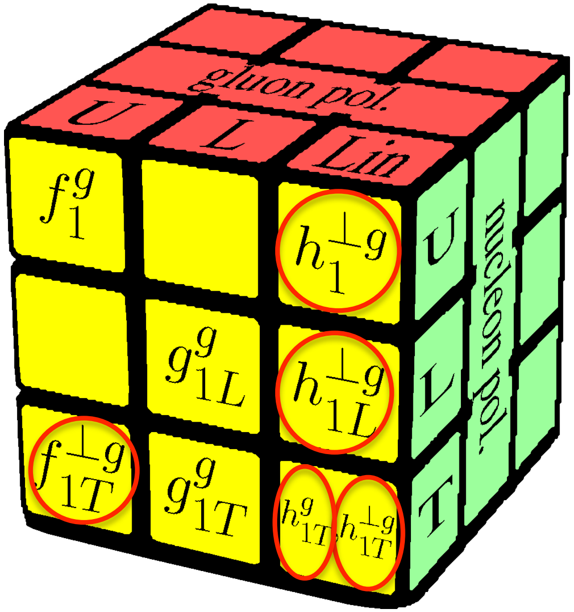
- TMD contributions

$$\boxed{h_1^{\perp g}(x) = h_1^{\perp g(A)}(x) + h_1^{\perp g(B1)}(x)}$$

$$h_1^{\perp g(q/\bar{q})}(x) = h_1^{\perp g(A)}(x) - \frac{1}{N_c^2-1}h_1^{\perp g(B1)}(x) + \frac{N_c^2}{N_c^2-1}h_1^{\perp g(B2)}(x) + \frac{N_c^2}{N_c^2-1}h_1^{\perp g(B3)}(x)$$

MGAB, A Mukherjee, PJM, in preparation

Summary and conclusion



- Transverse weightings involve multiple matrix elements.
- Separation in T-even and T-odd parts is no longer enough to isolate the process-dependence part.
- TMD PDFs can be written down as combinations of universal functions.
- There are multiple universal functions for $h_1^{\perp g}$, $h_{1L}^{\perp g}$, $f_{1T}^{\perp g}$, h_{1T}^{g} and $h_{1T}^{\perp g}$.

Further reading:

- Phys.Rev. **D86**, 074030 (MGAB, Mukherjee, Mulders), 1207.3221 [hep-ph]
- Results for gluons: forthcoming

Spare slides

Transverse moments Quarks

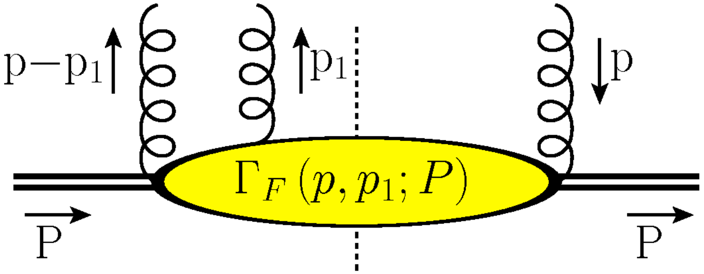
- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\middle|\bar{\psi}(0)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

- Transverse moments involve collinear twist-3 multi-parton correlators F_D and F_F built from non-local combination of three parton fields

$$p_T^\alpha\Phi^{[\pm]}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\middle|\bar{\psi}(0)UD_T^\alpha(\pm\infty)U\psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x)=\int\frac{d\xi.P\,d\eta.P}{(2\pi)^2}e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\bar{\psi}(0)F^{n\alpha}(\eta)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$



$$\Phi_D^\alpha(x)=\int dx_1\,\Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x)=\mathrm{PV}\int dx_1\,\frac{1}{x_1}\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

TMD correlators for quarks

- Quark TMD correlator can be expanded as

$$\int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\pi\Phi_G^\alpha(x)$$

$$\int d^2p_T\; p_T^\alpha p_T^\beta \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG}^{[U]}\pi^2\Phi_{GG}^{\alpha\beta}(x) + C_G^{[U]}\pi\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

- For the T-even Pretzelocity this implies

$$\boxed{h_{1T}^{\perp(2)[U]}(x) = \ldots\, h_{1T}^{\perp(2)(A)}(x) + \ldots \sum_c C_{GG,c}^{[U]}\, h_{1T}^{\perp(2)(Bc)}(x)}$$

MGAB, Mukherjee, PJM
PR D86 (2012) 074030

		quark polarization		
		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Process dependent

← T-even