From form factors to generalized parton distributions

Markus Diehl
Deutsches Elektronen-Synchroton DESY

DIS 2013, Marseille
24 April 2013

HELMHOLTZ
GEMEINSCHAFT

DESY

Motivation

- Generalized parton distributions provide unique information about structure of nucleon
  - transverse spatial distribution of partons correlated with their longitudinal momentum
  - access to orbital angular momentum interpretation via
    - Ji's sum rule for total angular momentum
    - shift of spatial parton density induced by transverse proton polarization

Electromagnetic form factors

- related to GPDs via sum rules

$$F_1^q(t) = \int_0^1 dx\, H_v^q(x,t) \qquad\qquad F_2^q(t) = \int_0^1 dx\, E_v^q(x,t)$$

$$H_v^q(x,t) = H^q(x,\xi=0,t) + H^q(-x,\xi=0,t)\,, \ \text{idem for } E_v^q$$

- complementary to determinations from exclusive proc's
  - ★ precise data up to high |t|
  - ★ sensitive to large x range (from ~ 10^{-3} to above 0.6 in our fit)
  - ★ naturally at ξ=0
    → probability interpretation, simple positivity bounds
  - ★ but no model-indepentent separation of x and t dependence
- will present GPD extraction in MD and P Kroll, arXiv:1302.4604
  previous studies: MD, Feldmann, Jakob, Kroll 2004;
  Guidal, Polyakov, Radyushkin, Vanderhaeghen 2004

Form factor data: proton

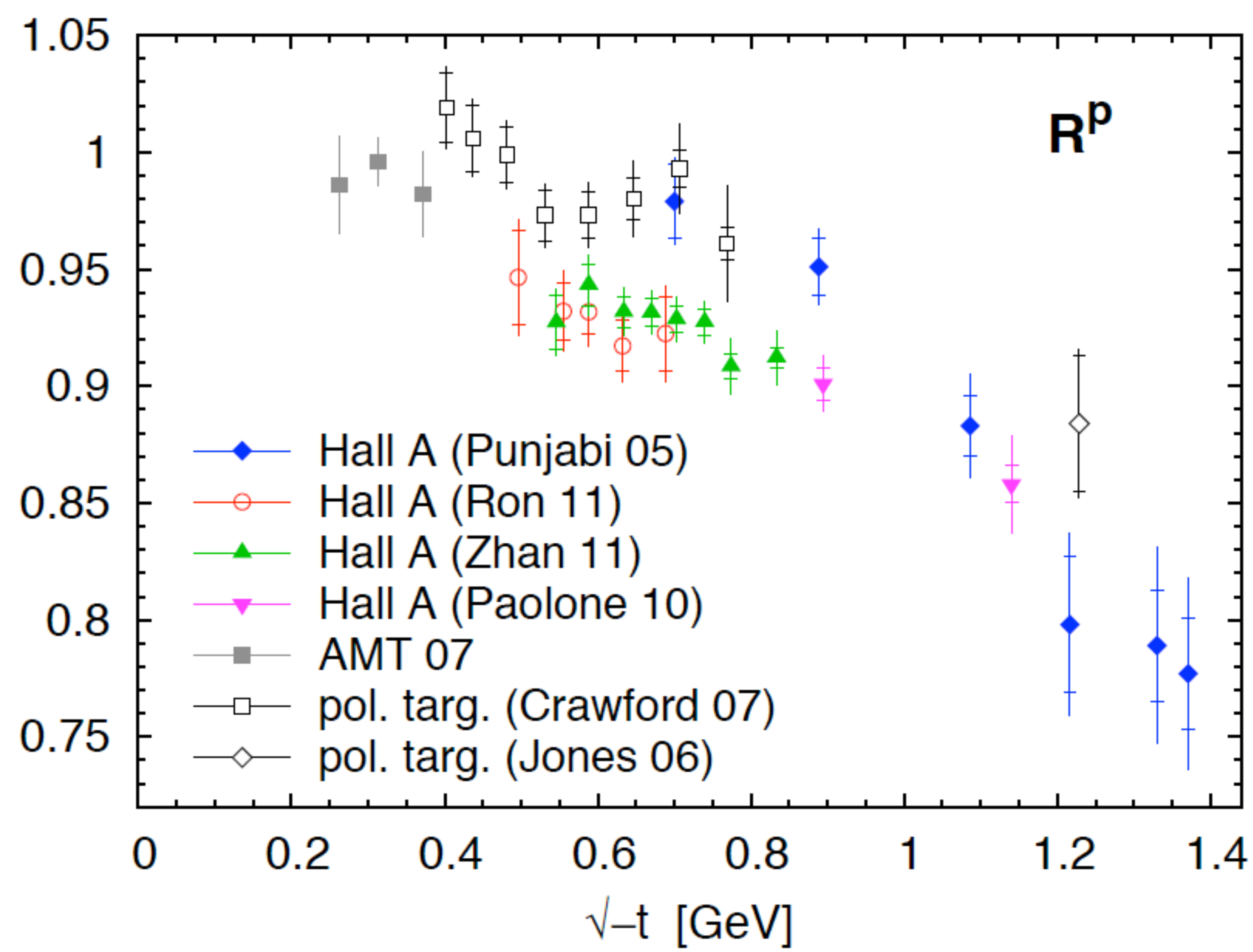
G_M^p

experimental errors at % level
2γ exchange corrections essential

use Arrington, Melnitchouk, Tjon 2007
consistent with other recent extractions

$R^p = G_E^p / (G_M^p / \mu_p)$

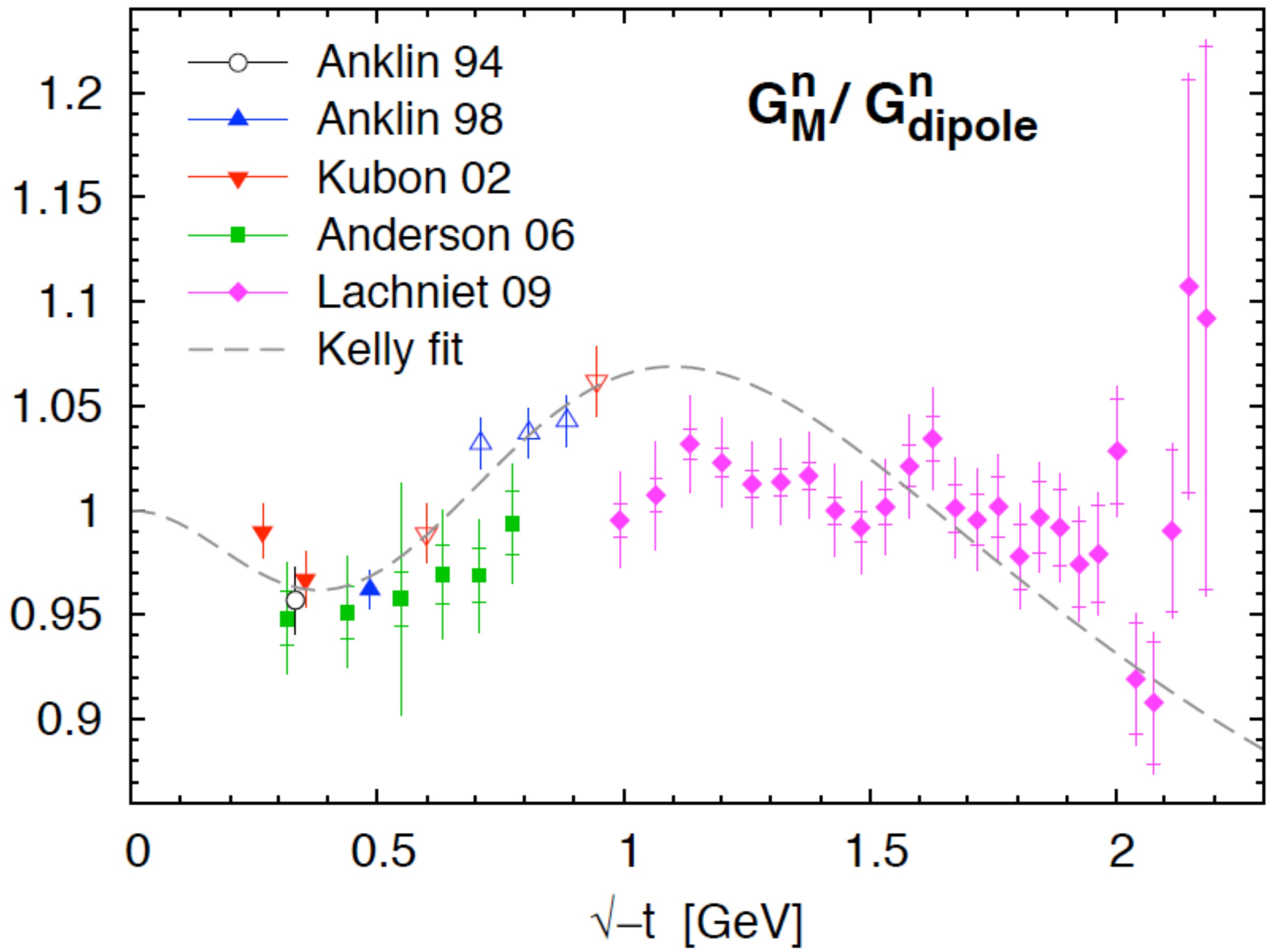
only use polarization data
inconsistency between
Hall A results ≥ 2010 and
older Hall A + polarized target data
we omit pol. target results



Form factor data: neutron

G_M^n

omit older data (open symbols), which are in tension with recent JLab results



$R^n = G_E^n / (G_M^n / \mu_n)$

overall consistent data set llimited to $|t| \leq 3.4$ GeV^2

also take neutron electric radius

$r^2_{nE} = 0.1161(22)$ fm

from neutron scatt. on shell electrons

do not include proton radius because of discrepancy between electronic and muonic hydrogen Lambshift

Form factors: some studies

- include model estimate for strangeness F_1^s and F_2^s checked against lattice and parity violating elast. scatt.
  - ★ find strangeness form factors of similar size as uncertainties on u and d quark form factors
- perform global fit to selected form factor data

$$\frac{F_i^q(t)}{F_i^q(0)} = \left(1 - a_{iq}\,\frac{t}{p_{iq}}\right)^{-p_{iq}} \left(1 - b_{iq}\,\frac{t}{q_{iq}}\right)^{-q_{iq}} \qquad \text{for } i = 1, 2 \text{ and } q = u, d$$

  - ★ 16 parameters, fix 3 and fit 13
  - ★ good overall χ^2 = 122.3 for 178 data pts
- interpolate Sachs form factors to common values of t and calculate Dirac form factors in quark flavor basis up to $|t| = 3.4\ \text{GeV}^2$

plots later

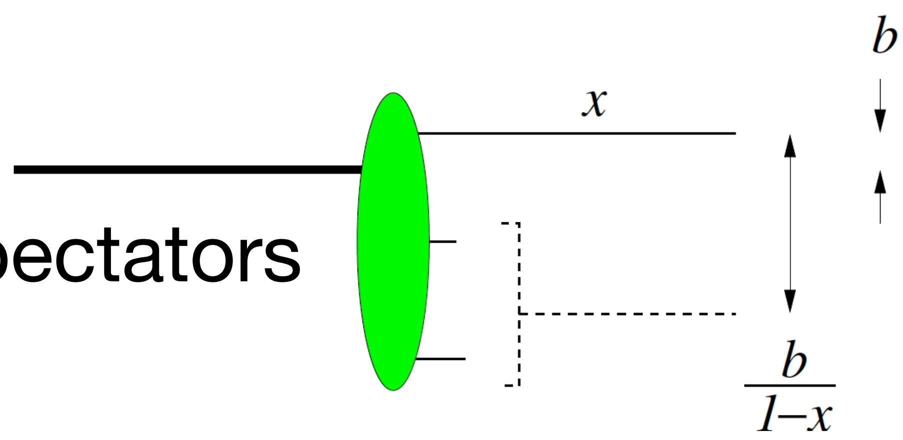
Fit of GPDs: Ansatz for H

- assume exponential t-dependence

$$H_v^q(x,t) = q_v(x)\, \exp\big[t f_q(x)\big]$$

- with $f_q(x) = \alpha'_q(1-x)^3 \log\frac{1}{x} + B_q(1-x)^3 + A_q x(1-x)^2$

$$\langle \boldsymbol{b}^2 \rangle = 4 f_q(x) \propto (1-x)^2 \text{ for } x \to 1$$



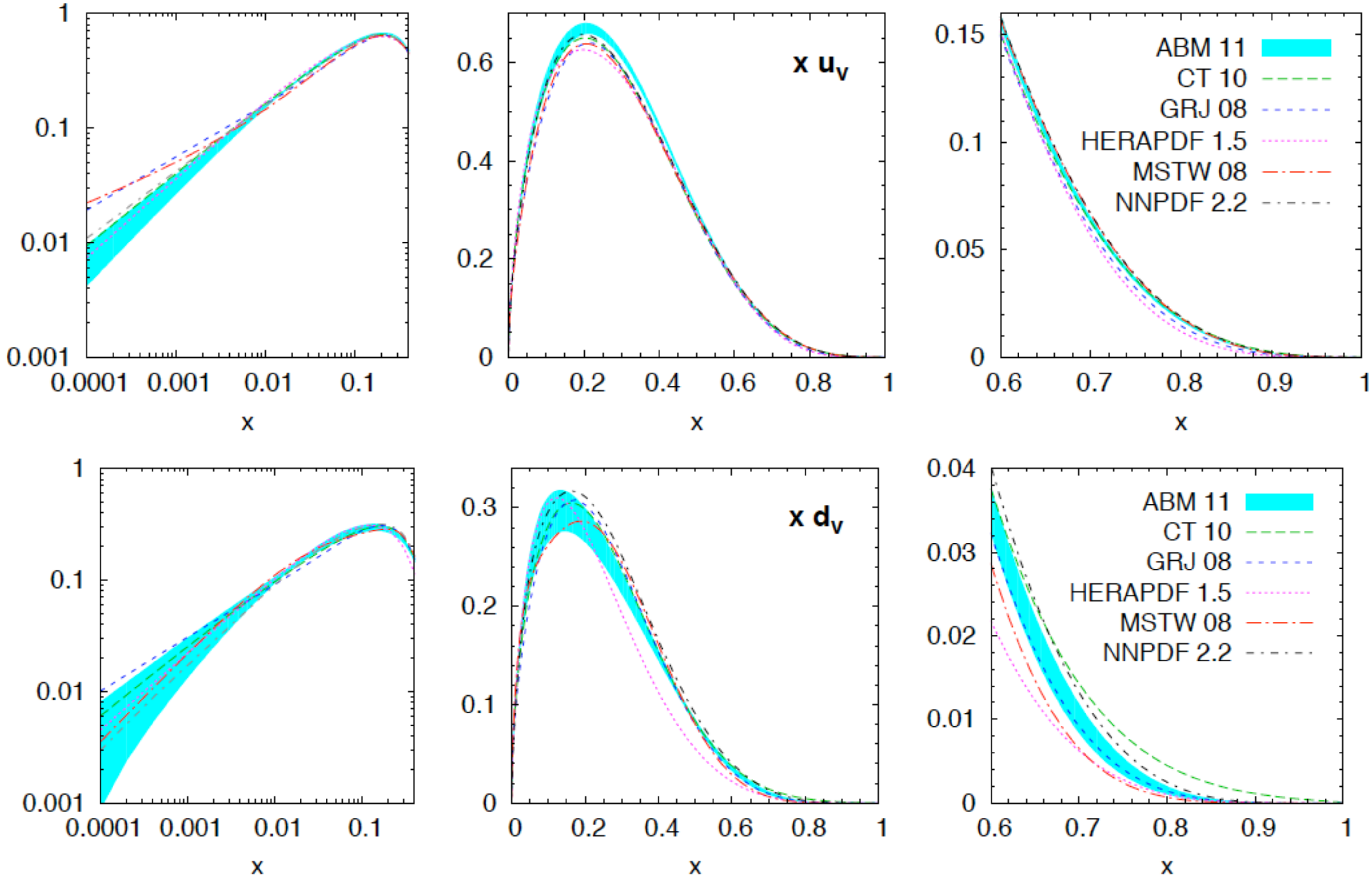
★ b/(1-x) = distance from quark to spectators remains finite for x→1

★ for small x have behavior as in Regge theory from meson trajectories expect α' ~ 0.8 to 1.0 GeV^{-2}

★ A relevant for small x, B for large x

Parton densities used in fit

- use NLO densities at μ = 2 GeV
- notable differenes between modern PDF sets at small and large x
  take ABM 11 as default, others for cross checks



Fit of GPDs: Ansatz for E

- same form as for H

$$E_v^q(x,t) = e_v^q(x)\ \exp\big[t g_q(x)\big]$$

- for exponent g(x) same form as f(x) with indep. parms.
- forward limit is unkown, take as

$$e_v^q(x) \propto x^{-\alpha_q} (1-x)^{\beta_q}\ \big(1+\gamma_q \sqrt{x}\big)$$

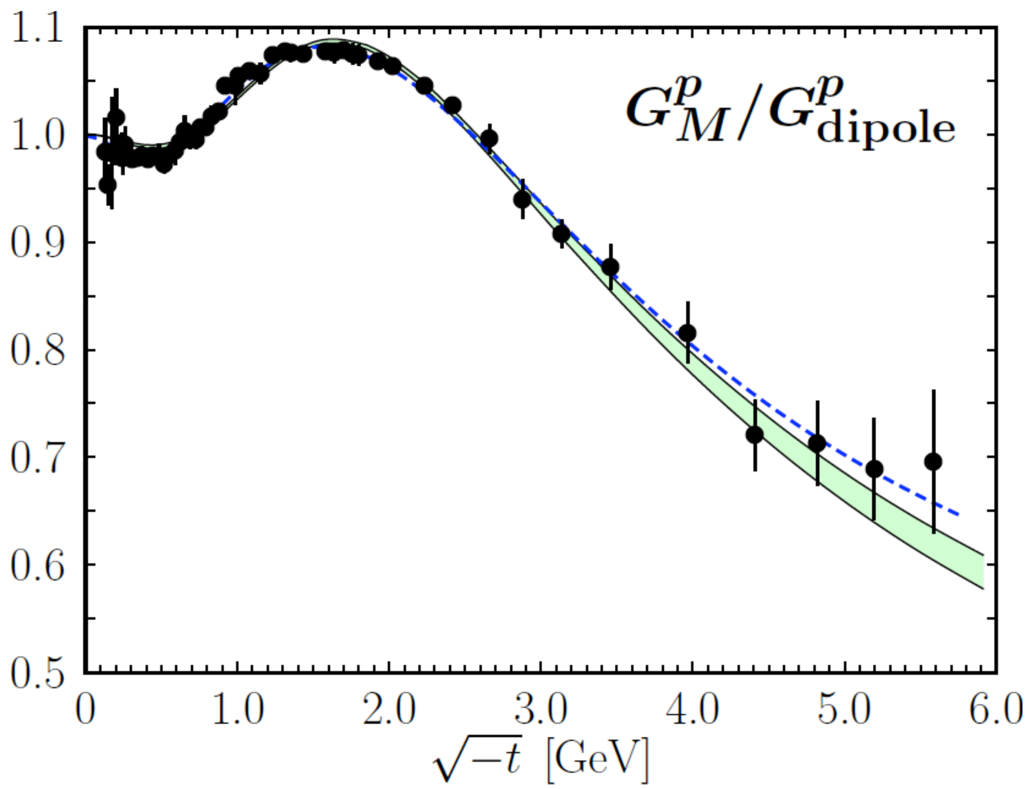
  - ★ normalization fixed by anomalous magnet. moments
- Fourier trf of E describes shift of spatial distribution in tranversely polarized proton
  - ★ have positivity constraint on E in terms of H

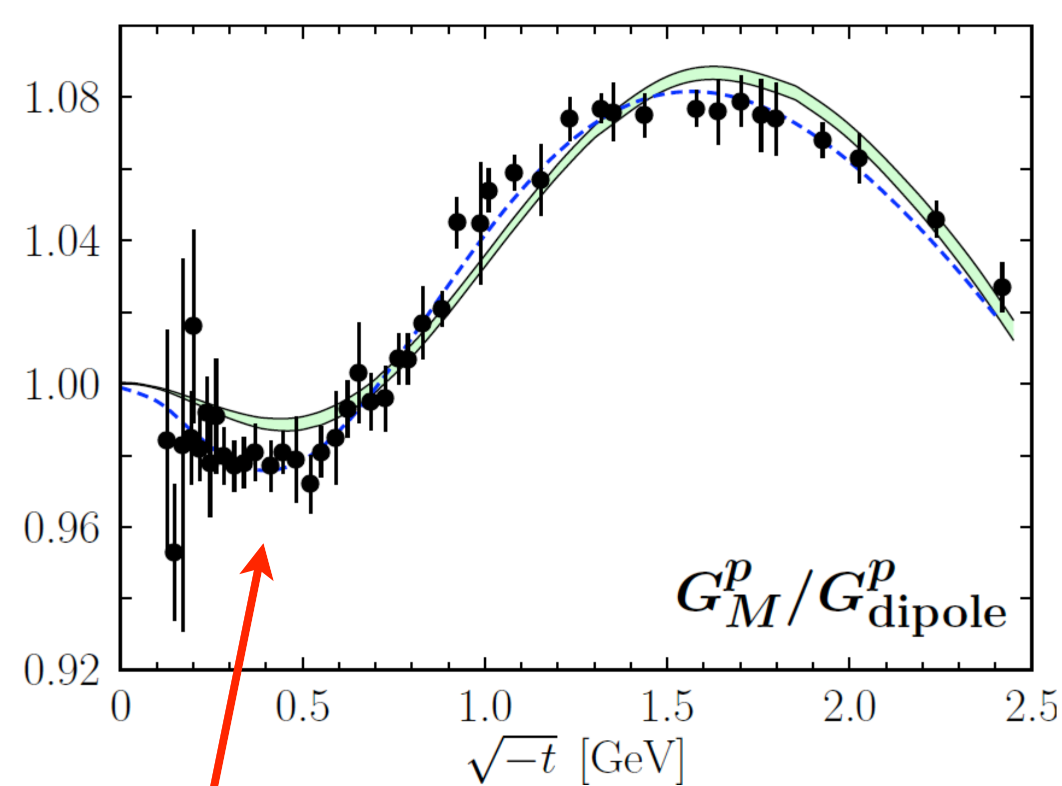
M Burkardt 2003

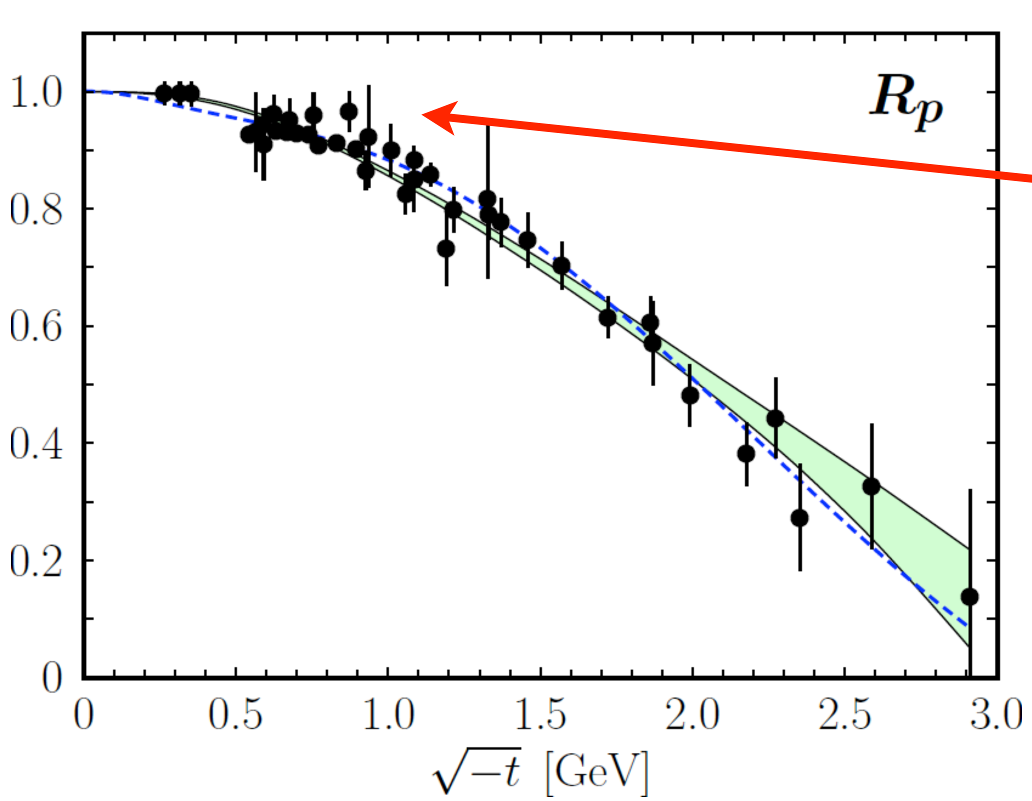
Fit of GPDs

- fix β_u, β_d, γ_u, γ_d (optimize by scanning values)
- fit $\alpha_u = \alpha_d$ and parameters in $f_u(x)$, $f_d(x)$, $g_u(x)$, $g_d(x)$
  - ★ $\alpha'_d \sim 0.68$ to 0.9 GeV^{-2} in line with Regge pheno.
  - ★ $\alpha'_u - \alpha'_d > 0$ preferred by r^2_{nE} and R^n
  - ★ $\alpha \sim 0.6$ to 0.7 larger than small-x power in $q_v(x)$
  - ★ positivity bound on E^q strong influence in fit
    best results if take β_q as small as possible
    $\beta_u \sim 4.5$ to 5 and $\beta_d \sim 5$ to 6
- good overall fit
  - ★ $\chi^2 = 221.2$ for 174 data pts (with ABM 11 PDF)

Description of data: proton





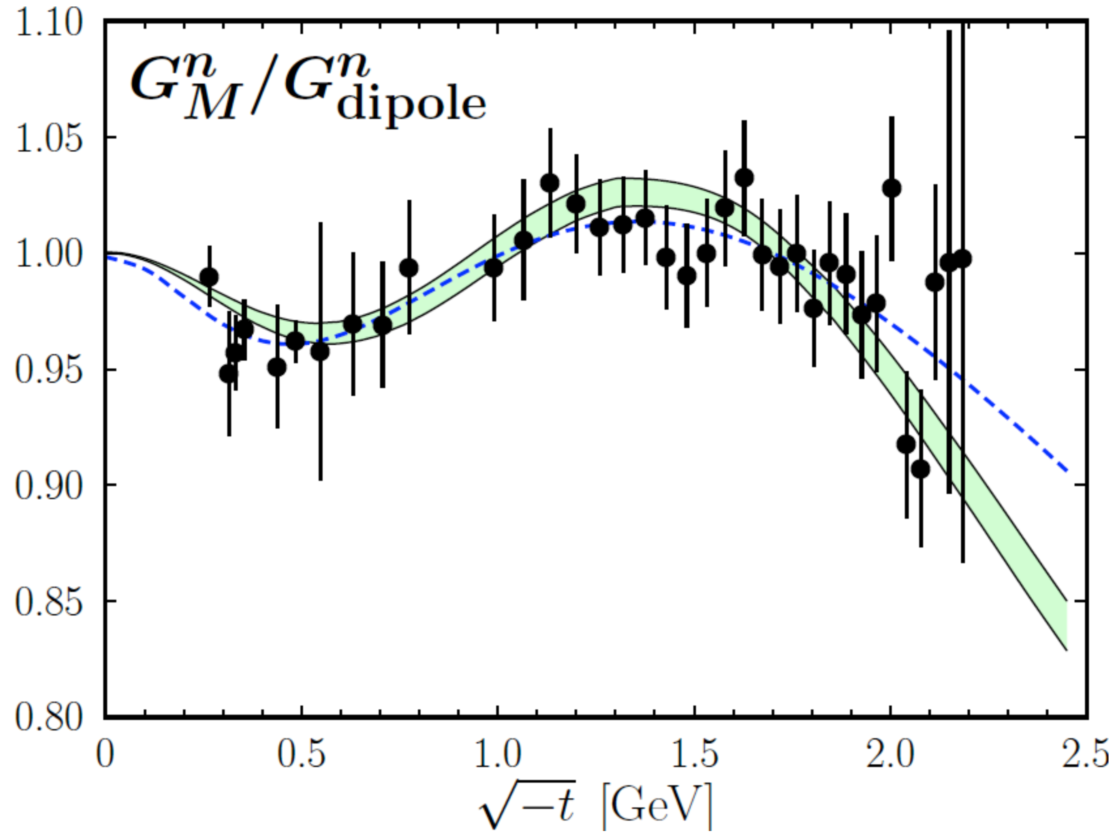


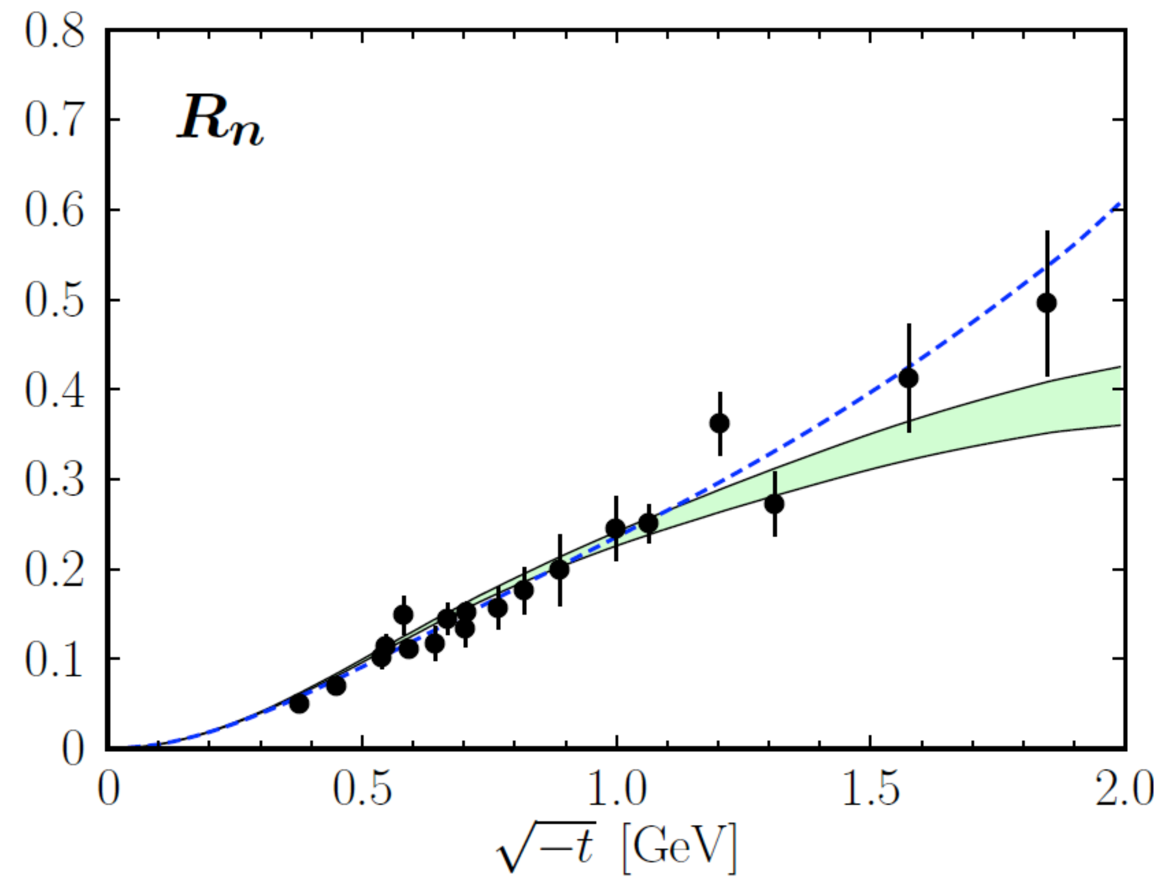
description not perfect

band: GPD fit

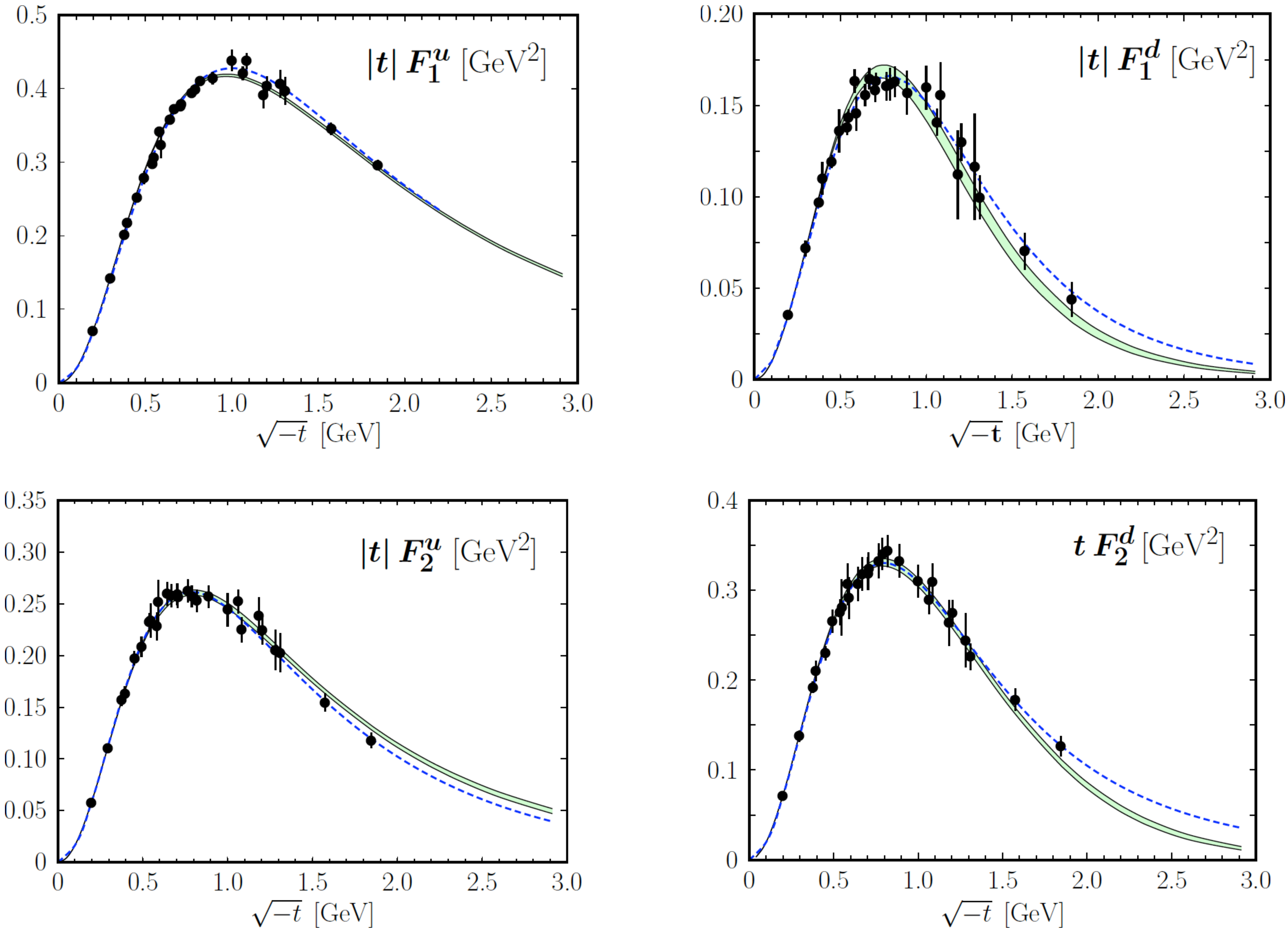
blue dashed line: power law fit

Description of data: neutron

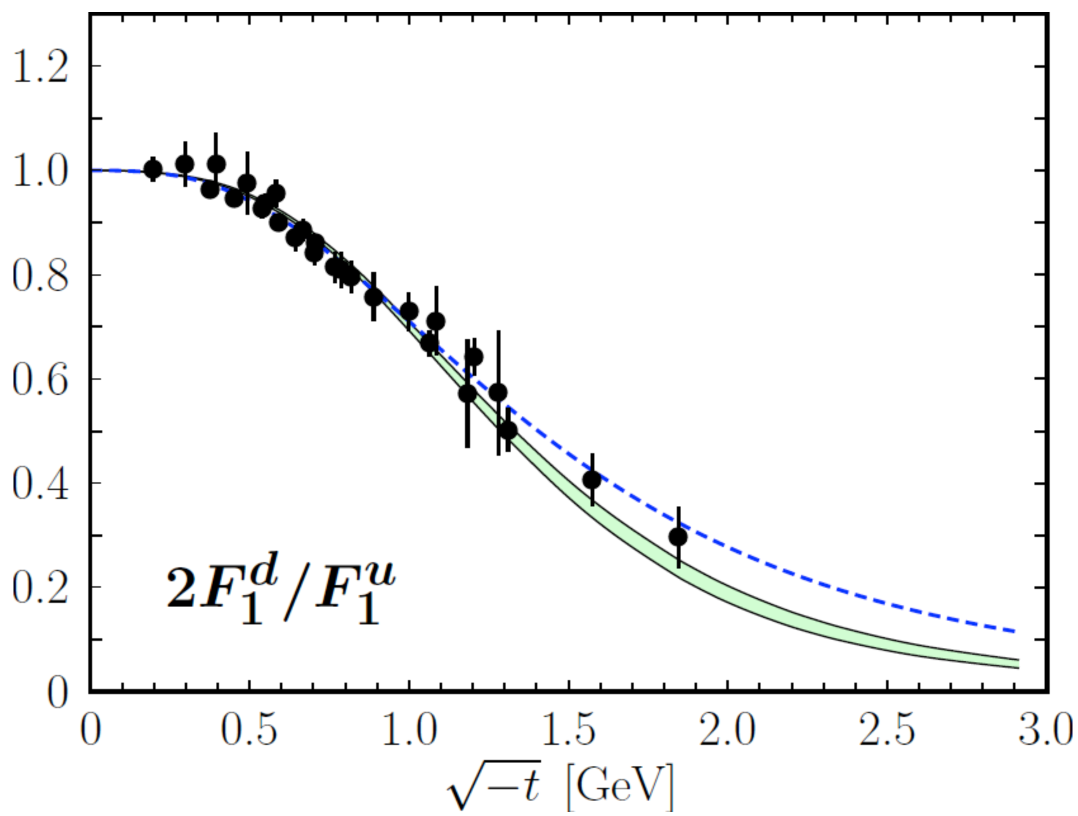




Dirac form factors in quark flavor basis (interpolated data)



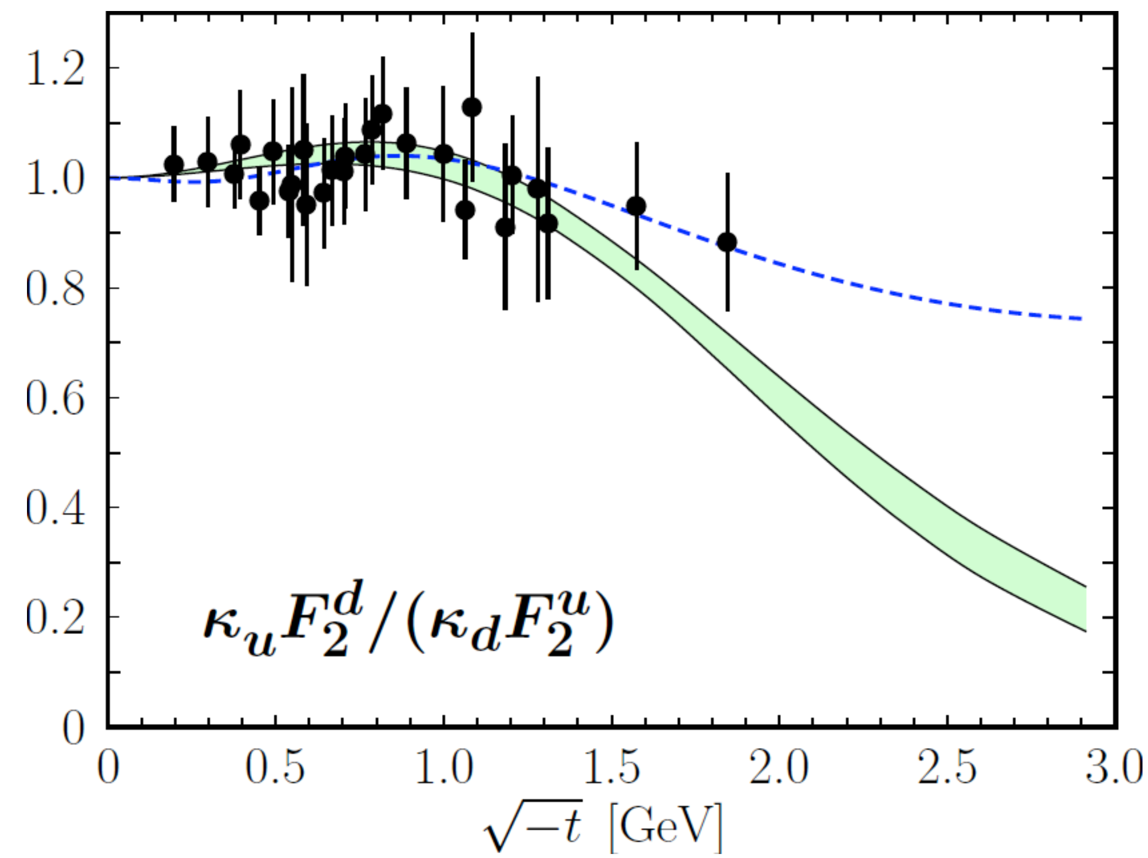
Form factor ratios



with $H_v(x,t) = q_v(x) \exp[\, t\, f(x)\,]$
and $f(x) \sim (1-x)^2$ at large x

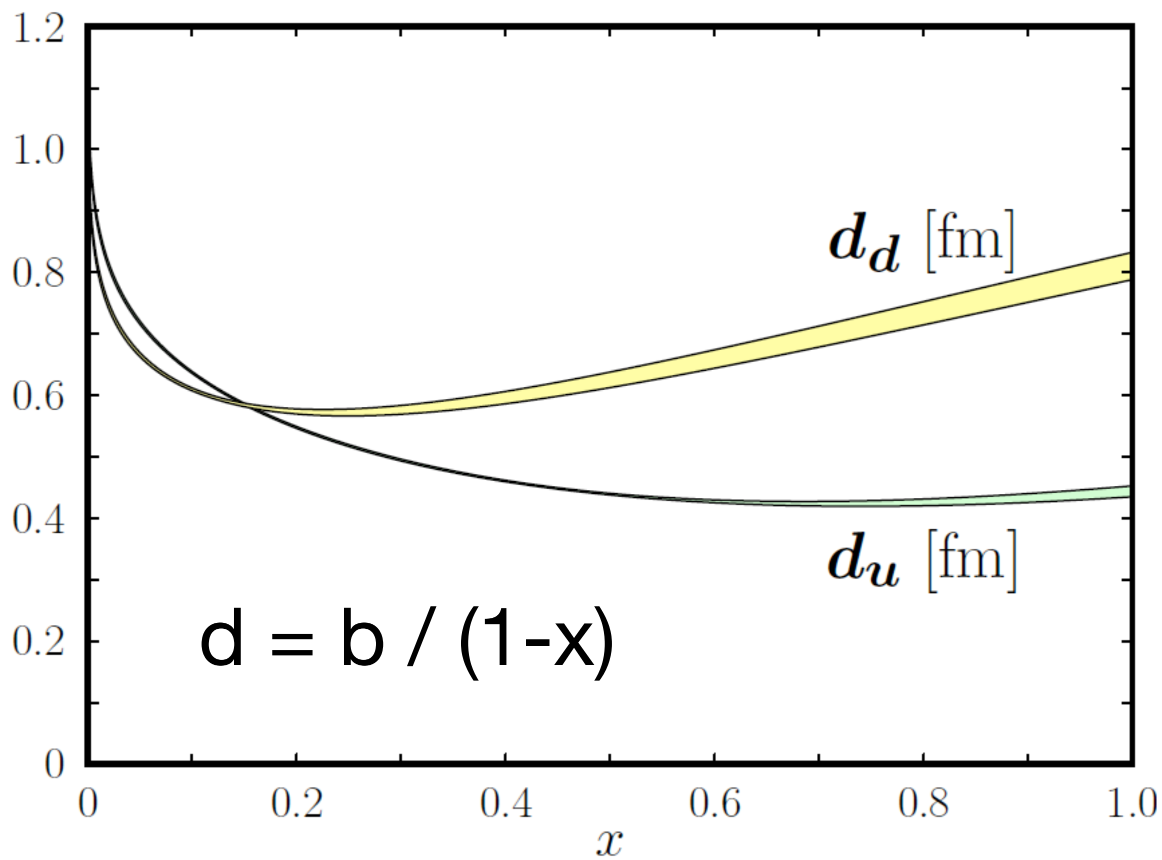
large t behavior of F_1
$\leftrightarrow$ large x behavior of $q_v(x)$

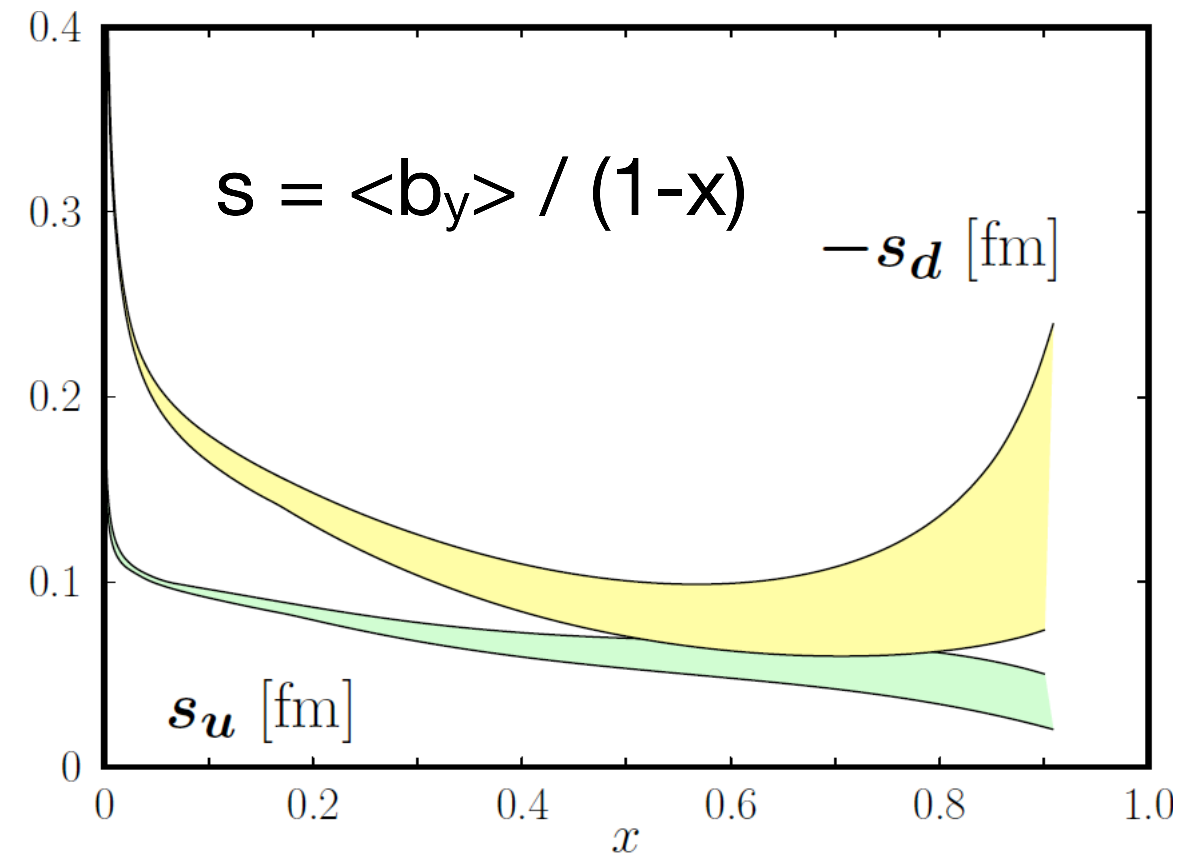
$\rightarrow$ $F_1{}^d$ falls faster than $F_1{}^u$

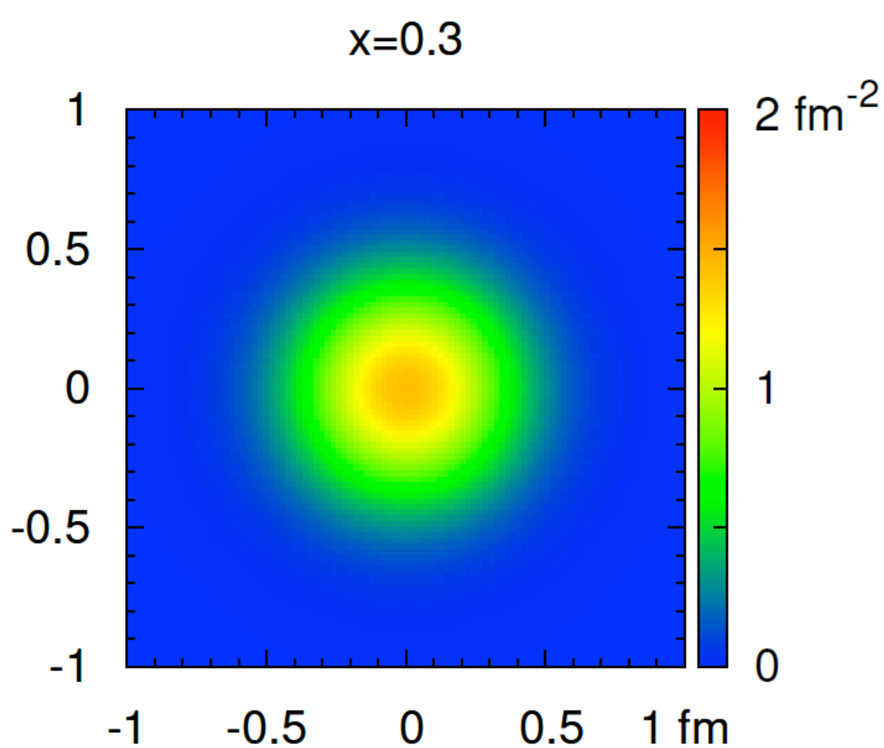


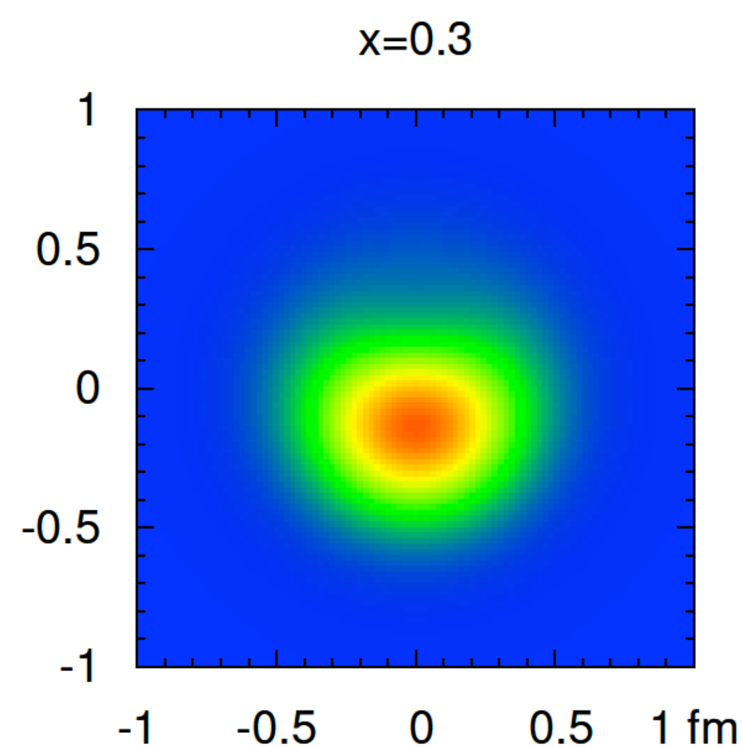
same trend not (yet?)
seen in $F_2{}^d$ vs $F_2{}^u$ data

Results for transverse spatial distributions









$\langle b_y \rangle$ =
average sideways shift if proton polarized along x

Results for total angular momentum

- Ji's sum rule, valence quark contribution only

$$J_v^q = \int_0^1 dx\, x \big[q_v(x) + e_v^q(x) \big]$$

- obtain at μ = 2 GeV

$$J_v^u = 0.230^{+0.009}_{-0.024} \qquad J_v^d = -0.004^{+0.010}_{-0.016}$$

  - ★ error estimate includes scan over PDFs used in fit, alternative data set for R^p and estimate of F_1^s, F_2^s
  - ★ J_v^d consistent with zero

- within errors consistent with determination from Sivers distrib. and model for chromodynamic lensing:

$$J_v^u = 0.214^{+0.009}_{-0.013} \qquad J_v^d = -0.029^{+0.021}_{-0.008}$$

Bacchetta, Radici 2011; I added their errors in quadarature

Results for total angular momentum

- Ji's sum rule, valence quark contribution only

$$J_v^q = \int_0^1 dx\, x\big[q_v(x) + e_v^q(x)\big]$$

- obtain at μ = 2 GeV

$$J_v^u = 0.230^{+0.009}_{-0.024} \qquad J_v^d = -0.004^{+0.010}_{-0.016}$$

- if take from DSSV 2009 fit

$$\Delta u_v = 0.371 \qquad \Delta d_v = -0.118$$

then get orbital angular momentum

$$L_v^u = -0.141 \qquad L_v^d = 0.114 \qquad L_v^{u+d} = -0.027$$

strong cancellation in L_v between u and d quarks

Importance of antiquarks in x moments

- for E do not know
- sea quark contribution to momentum sum rule:

PDF	$u-\bar{u}$	$d-\bar{d}$	$2\bar{u}$	$2\bar{d}$	$2(\bar{u}+\bar{d})$	$s+\bar{s}$	$s-\bar{s}$
ABM 11	0.297	0.115	0.062	0.077	0.139	0.035	0
CT 10	0.287	0.118	0.058	0.072	0.130	0.040	0
GJR 08	0.280	0.116	0.064	0.080	0.144	0.021	0
HERAPDF 1.5	0.284	0.105	0.074	0.091	0.165	0.044	0
MSTW 08	0.282	0.115	0.064	0.076	0.140	0.033	0.0019
NNPDF 2.2	0.290	0.124	0.059	0.074	0.133	0.020	0.0029

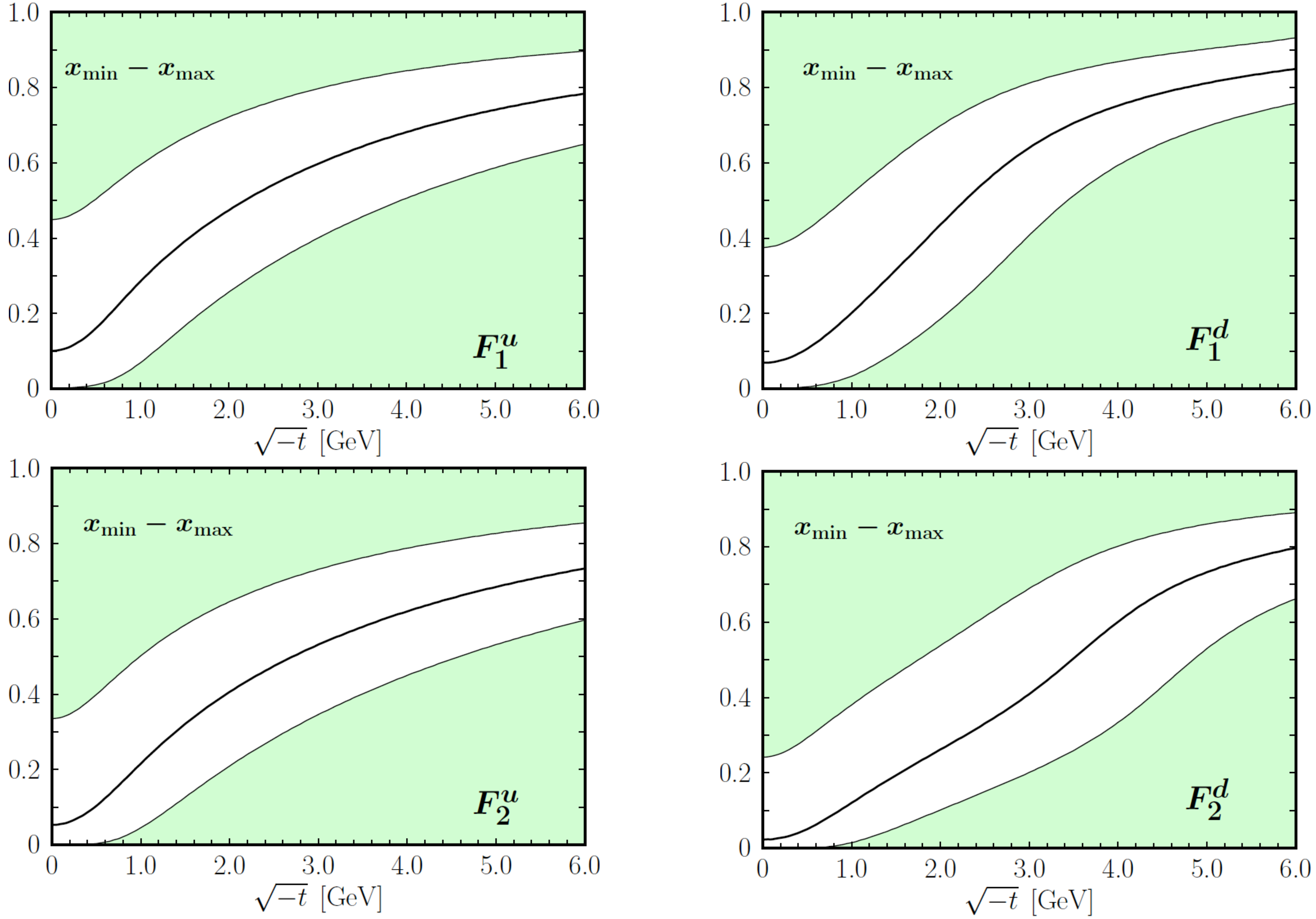
all at $\mu = 2$ GeV

Summary

- experimental precision of e.m. form factors matters!
  several outstanding issues:
  - ★ inconsistencies in R^p data
  - ★ strangeness form factors (future experiments? lattice?)
- good global fit to form based on power-laws
- interpolated set of form factor data for $|t| \leq 3.4\ \mathrm{GeV}^2$
- good fit of data to ansatz for GPDs
  - ★ positivity bound in E essential in fit
  - ★ extracted GPDs can be used for many applications:
    - ✦ proton tomography (transverse images)
    - ✦ Ji's sum rule: angular momentum of valence quarks
    - ✦ not shown here: generalized form factors, wide-angle Compton scatt.

Backup slides

- sensitive x range in form factor integrals
- white region: 90% of integral, central line: median x



Backup: strange form factors

- our model (using MSTW or NNPDF strangeness PDFs) compared with data (HAPPEX, A4, G0) and lattice results (Leinweber, Doi, Wang)