

# **nCTEQ**

## **parton distribution functions**

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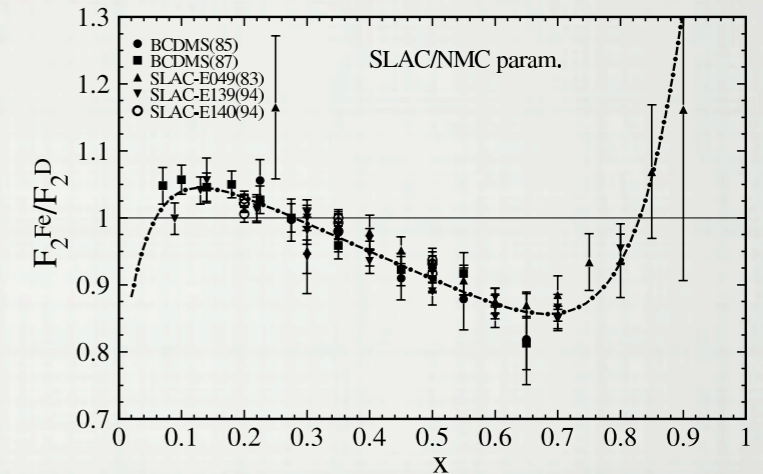
# OUTLINE

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1. Motivation & Review of available nPDFs
2. New results from nCTEQ analysis
3. Conclusions & Outlook

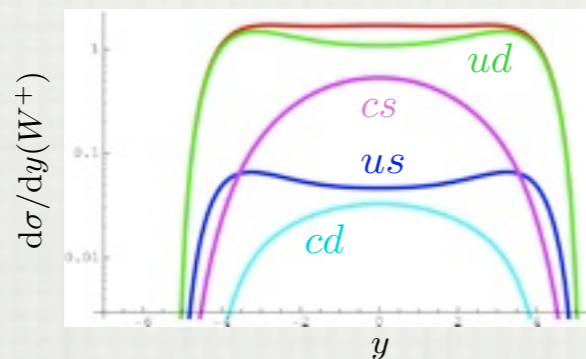
# Motivation

- What are nuclear parton density functions (nPDF) ?
  - parton densities for partons in bound proton & neutron
- Where are nuclear parton density functions useful ?

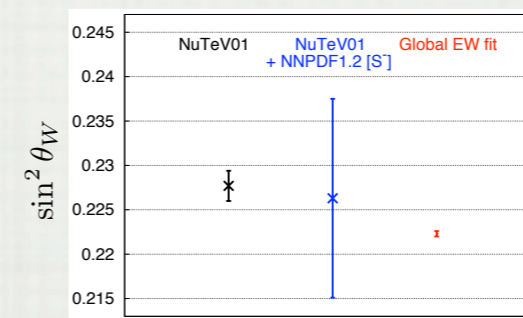


## 1. Strange quark content of the proton

(anti-)strange PDF from (anti-)neutrino DIS with heavy nuclei - nuclear effects important



*W-boson production @ LHC*



*weak mixing angle from NuTeV experiment*

## 2. Heavy ion collisions @ RHIC, LHC

lead & gold heavy nuclei - nuclear effects in gluon PDF substantial

# nPDF REVIEW

- Review of existing global analyses of nuclear PDF

1. Multiplicative nuclear correction factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

bound parton density free parton density

Hirai, Kumano, Nagai [[PRC76\(2007\)065207](#)] arXiv: 0709.0338

Eskola, Paukkunen, Salgado [[JHEP0904\(2009\)065](#)] arXiv: 0902.4154

de Florian, Sassot, Stratmann, Zurita [[PRD85\(2012\)074028](#)] arXiv: 1112.6324

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$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

*bound parton density*                      *free parton density*

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## 2. Native nuclear PDF

$$f_i^A(x_N, Q_0^2) = f_i(x_N, A, Q_0^2) \quad f_i(x_N, Q_0^2) = f_i(x_N, A = 1, Q_0^2)$$

*bound parton density*                      *free parton density*

nCTEQ [[PRD80\(2009\)094004](#)] arXiv: 0907.2357

# nPDF review

- Review of existing global analyses of nuclear PDF

## HIRAI, KUMANO, NAGAI'07 [PRC76(2007)065207] LO, NLO, ERROR PDFS

- uses multiplicative factor

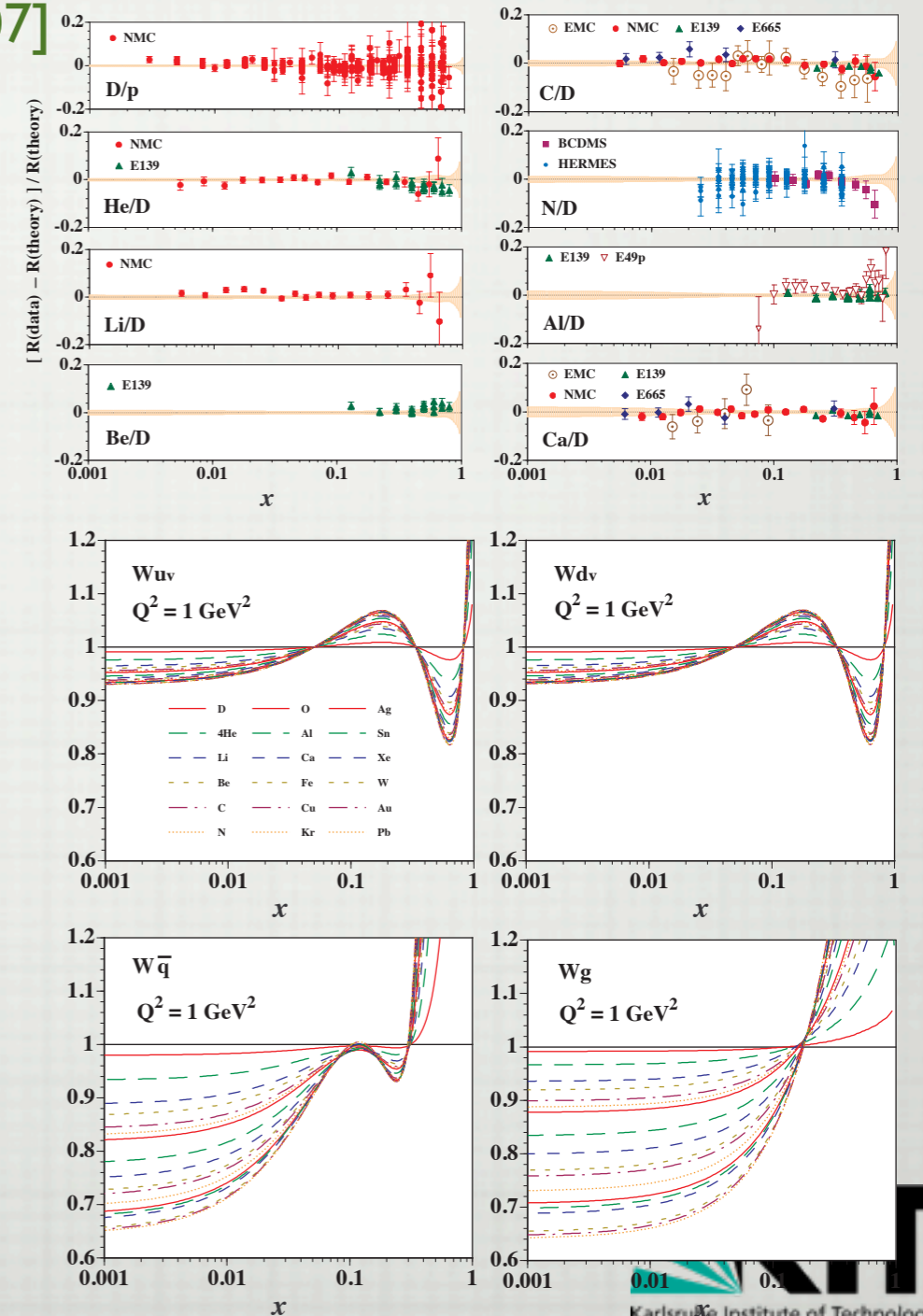
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in MRST 1998 and factor

$$R_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

- neglects region  $x > 1$
- includes all current DIS & DY data set (same as our analysis - discussed later)
- use Hessian method to produce error PDFs

$$\chi^2/\text{dof} = 1.2$$



# nPDF review

- Review of existing global analyses of nuclear PDF

**ESKOLA, PAUKKUNEN, SALGADO'09** [JHEP0904(2009)065]  
LO, NLO, ERROR PDFS

$\chi^2/\text{dof} = 0.8$

- uses multiplicative factor

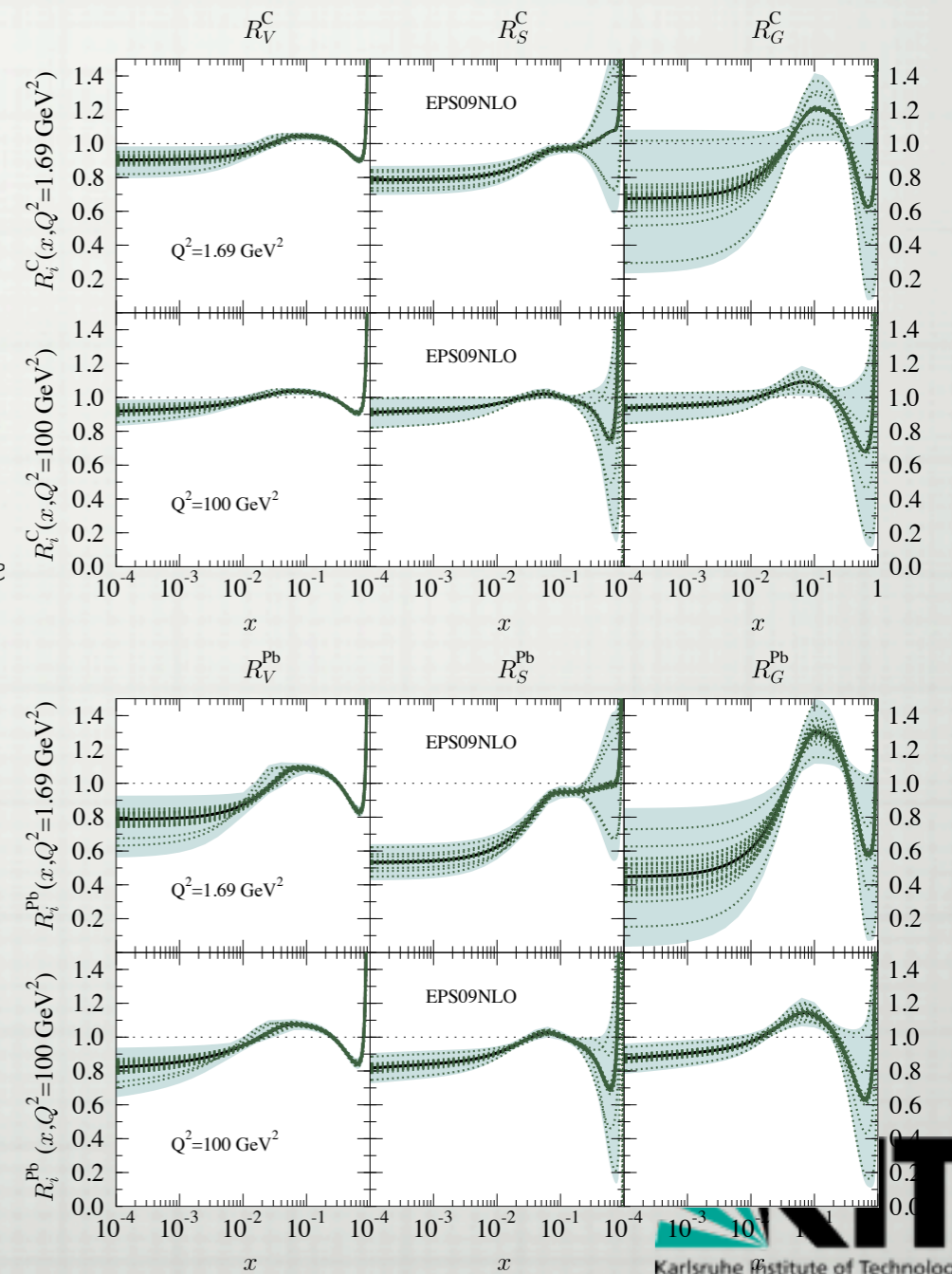
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in CTEQ6.1M and factor is a complicated piecewise defined function

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

with A-dependent parameters

- neglects region  $x > 1$
- includes all current DIS & DY data set &  $\pi^0$  RHIC data to constrain gluon
- use Hessian method to produce error PDFs



# nPDF review

- Review of existing global analyses of nuclear PDF

**DE FLORIAN, SASSOT, STRATMANN, ZURITA** [PRD85(2012)074028]  
 LO, NLO, ERROR PDFS

- uses multiplicative factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

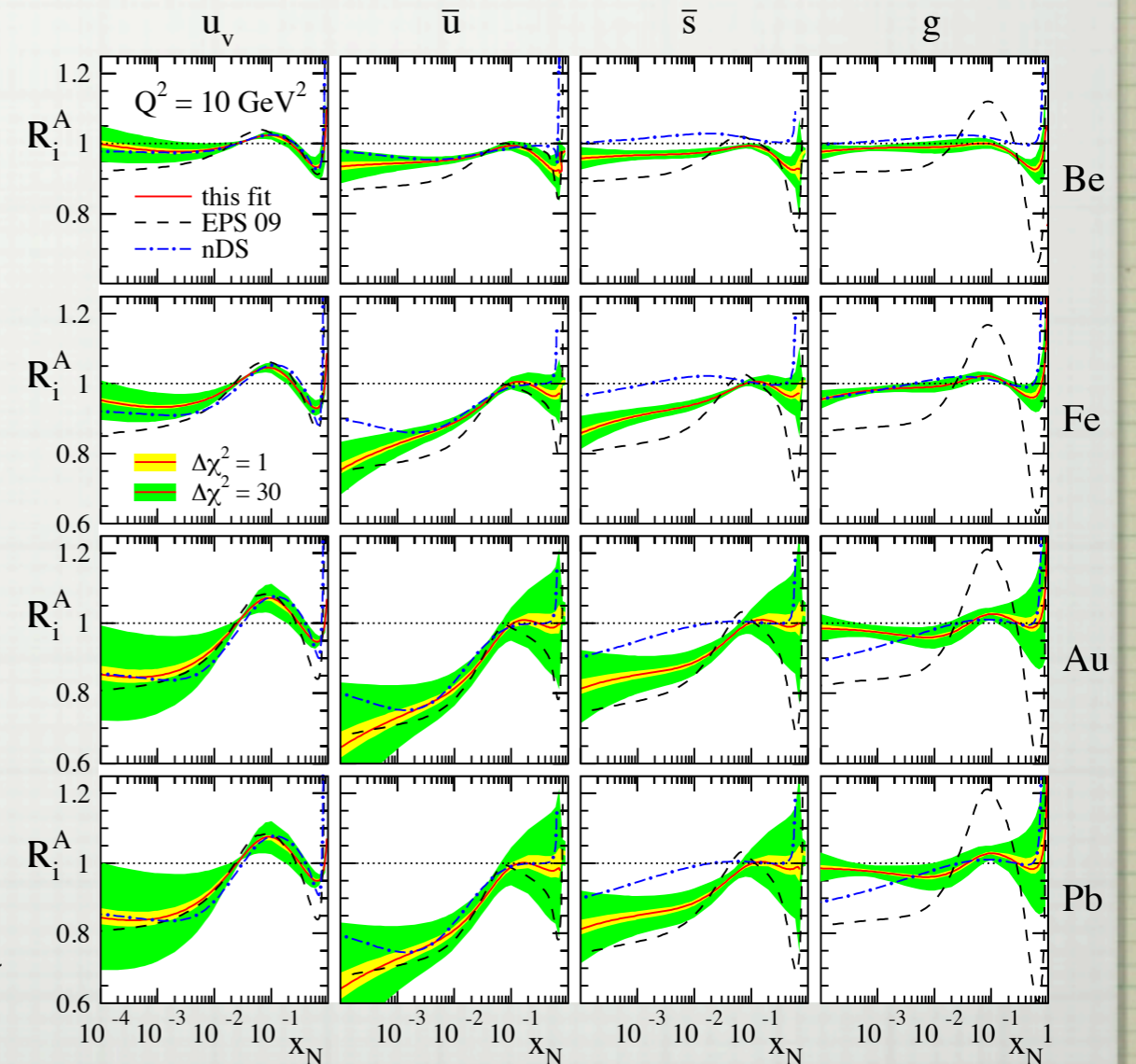
where proton PDF in MSTW08 and factor is a complicated function different for each flavour

$$R_v^A(x, Q_0^2) = \epsilon_1 x^{\alpha_v} (1-x)^{\beta_1} (1 + \epsilon_2 (1-x)^{\beta_2}) \times (1 + a_v (1-x)^{\beta_3})$$

$$R_s^A(x, Q_0^2) = R_v^A(x, Q_0^2) \frac{\epsilon_s}{\epsilon_1} \frac{1 + a_s x^{\alpha_s}}{a_s + 1}$$

$$R_g^A(x, Q_0^2) = R_v^A(x, Q_0^2) \frac{\epsilon_g}{\epsilon_1} \frac{1 + a_g x^{\alpha_g}}{a_g + 1}$$

- includes all current DIS & DY data set &  $\pi^0$  RHIC data and  $F_2^{\nu A}$  from neutrino data
- use Hessian method to produce error PDFs





- CTEQ framework for nuclear PDF - based on CTEQ6M proton fit

- functional form for bound protons same as for free proton PDF (restrict  $x$  to  $0 < x < 1$ )

$$x f_k(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0) / \bar{u}(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} + (1 + c_3 x)(1 - x)^{c_4}$$

- coefficients with A-dependance (reduces to proton for  $A=1$ )

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}), \quad k = \{1, \dots, 5\}$$

- proton coefficients  $c_{k,0}$  fixed to special CTEQ6M fit without much of nuclear data

- PDF for a nucleus with A-nucleons out of which Z-protons

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

- Input scale and other input parameters as in CTEQ6M proton analysis

$$Q_0 = m_c = 1.3 \text{ GeV} \quad m_b = 4.5 \text{ GeV} \quad \alpha_s(m_Z) = 0.118$$

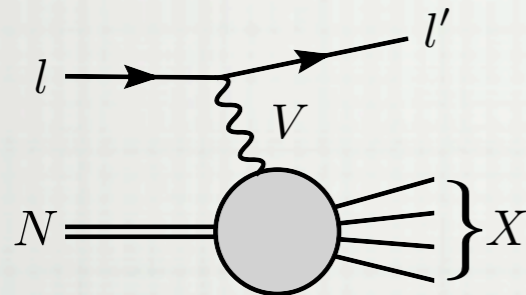
- Kinematic cuts on data

$$Q > 2 \text{ GeV} \quad W > 3.5 \text{ GeV}$$

- Experiments included in the analysis

## Charged lepton

Deep Inelastic Scattering



$$l + N \rightarrow l' + X$$

**CERN BCDMS & EMC & NMC**

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

**FNAL E-665**

$N = (\text{D, C, Ca, Pb, Xe})$

**DESY Hermes**

$N = (\text{D, He, N, Kr})$

**SLAC E-139 & E-049**

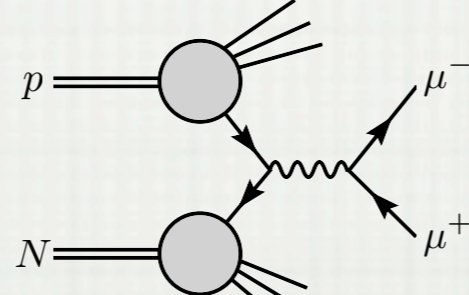
$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

**FNAL E-772 & E-886**

$N = (\text{D, C, Ca, Fe, W})$

*1233 data points (708 after cuts)*

Drell-Yan process

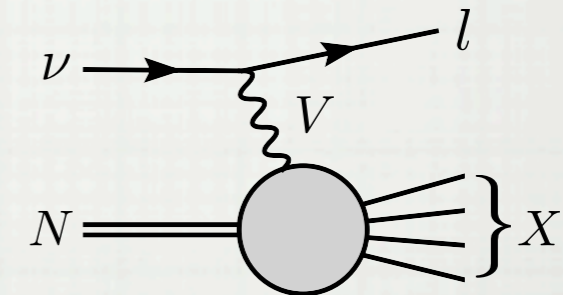


$$p + N \rightarrow \mu^+ \mu^- + X$$

## NOT (YET) INCLUDED

## Neutrino

Deep Inelastic Scattering



$$\nu(\bar{\nu}) + N \rightarrow l + X$$

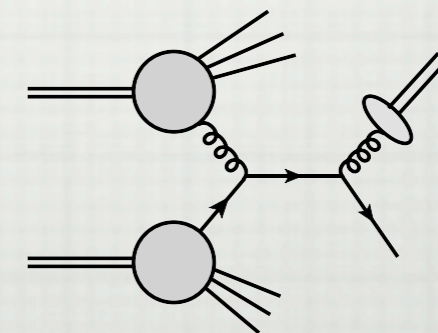
**CHORUS**

$N = \text{Pb}$

**CCFR & NuTeV**

$N = \text{Fe}$

## Single pion production



**RHIC - PHENIX & STAR**

$N = \text{Au}$

- NPDF fit properties:

- we fit nuclear data with NLO QCD predictions
- we include heavy quark effects (ACOT)
- applied standard CTEQ kinematical cuts  $Q > 2\text{GeV}$  &  $W > 3.5\text{GeV}$

- NPDF fit results:

- 708 (1233) data points after (before) cuts
- 17 free parameters - 691 degrees of freedom
- overall  $\chi^2/\text{dof} = 0.87$
- individually for different data subsets
  - for  $F_2^A / F_2^D$   $\chi^2/\text{pt} = 0.80$
  - for  $F_2^A / F_2^{A'}$   $\chi^2/\text{pt} = 0.51$
  - for  $\sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$   $\chi^2/\text{pt} = 0.85$

$F_2^A / F_2^D$ :			$F_2^A / F_2^{A'}$ :			
Observable	Experiment	# data	Observable	Experiment	# data	
D	NMC-97	275	Be/C	NMC-96	15	
He/D	SLAC-E139	18	Al/C	NMC-96	15	
	NMC-95, re	16	Ca/C	NMC-95	20	
	Hermes	92		NMC-96	15	
Li/D	NMC-95	15	Fe/C	NMC-95	15	
Be/D	SLAC-E139	17	Sn/C	NMC-96	144	
C/D	EMC-88	9	Pb/C	NMC-96	15	
	EMC-90	2	C/Li	NMC-95	20	
	SLAC-E139	7	Ca/Li	NMC-95	20	
	NMC-95, re	16	Total:		279	
	NMC-95	15				
	FNAL-E665-95	4				
	N/D	BCDMS-85	9			
		Hermes	92			
	Al/D	SLAC-E049	18			
		SLAC-E139	17			
Ca/D	EMC-90	2				
	SLAC-E139	7				
	NMC-95, re	15				
Fe/D	FNAL-E665-95	4				
	BCDMS-85	6				
	BCDMS-87	10				
	SLAC-E049	14				
	SLAC-E139	23				
	SLAC-E140	6				
Cu/D	EMC-88	9				
	EMC-93(addendum)	10				
	EMC-93(chariot)	9				
Kr/D	Hermes	84				
Ag/D	SLAC-E139	7				
Sn/D	EMC-88	8				
Xe/D	FNAL-E665-92	4				
Au/D	SLAC-E139	18				
Pb/D	FNAL-E665-95	4				
Total:		862	Total:		92	

$\sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$ :		
Observable	Experiment	# data
C/D	FNAL-E772-90	9
Ca/D	FNAL-E772-90	9
Fe/D	FNAL-E772-90	9
W/D	FNAL-E772-90	9
Fe/Be	FNAL-E866-99	28
W/Be	FNAL-E866-99	28
Total:		92

- NPDF Hessian analysis:

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0)(a_j - a_j^0) \quad H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- 17 free parameters - 7 gluon parameters
  - 8 valence parameters
  - 2 sea parameters

- Eigenvalues span 10 orders of magnitude  $\rightarrow$  numerical precision required

- Use improved derivatives - less sensitive to noise

$$\frac{\partial f}{\partial x} = \frac{f_1 - f_{-1}}{2h} \quad \begin{array}{l} \nearrow \\ \searrow \end{array} \quad \frac{f_1 - f_{-1} + 2(f_2 - f_{-2}) + 3(f_3 - f_{-3})}{28h}$$

central differences

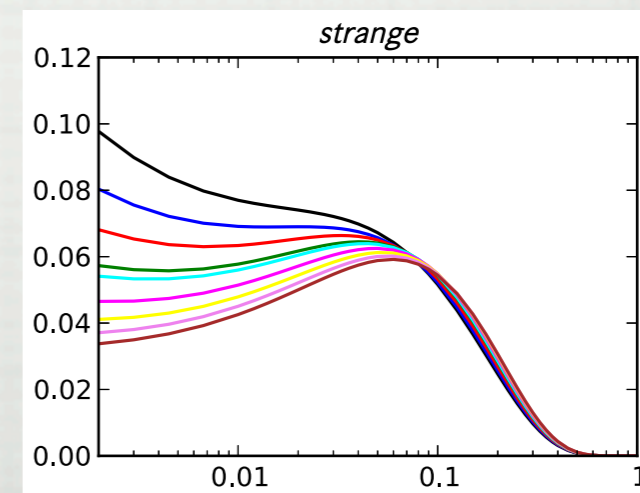
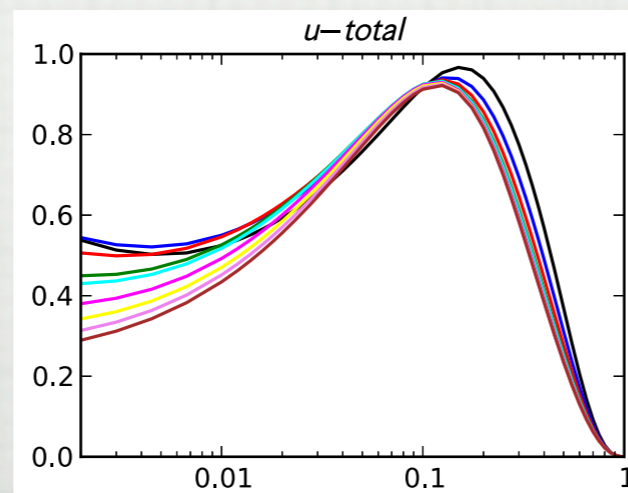
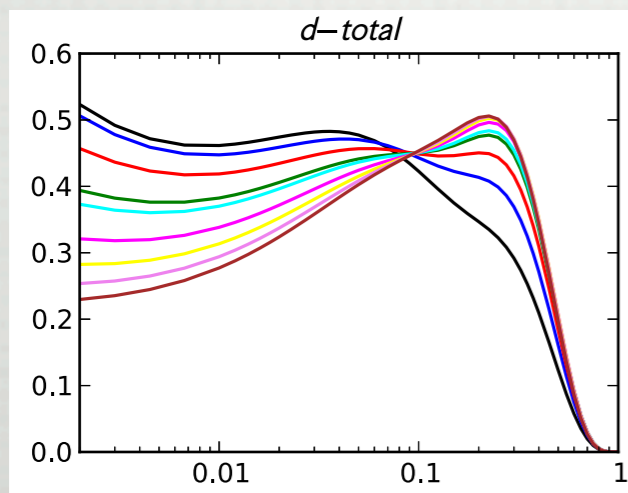
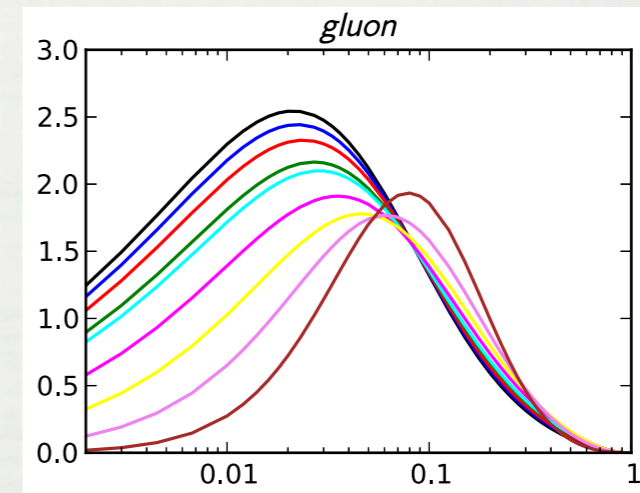
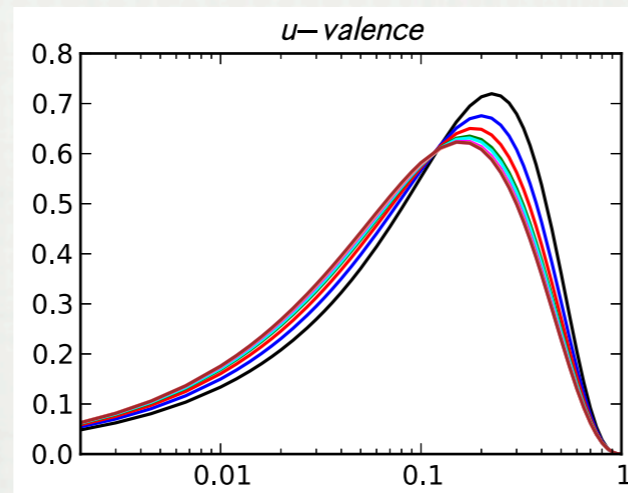
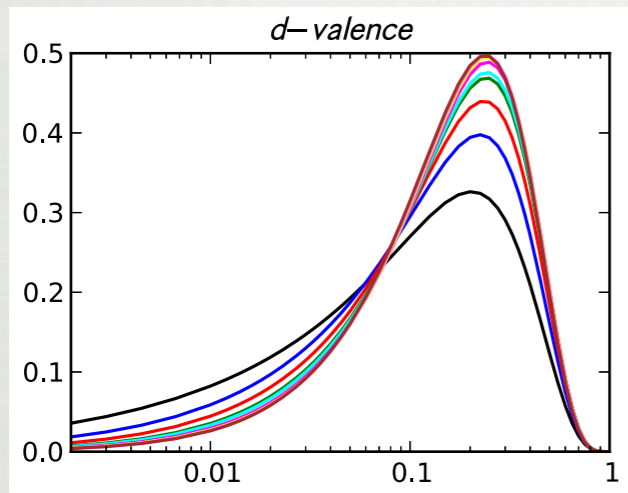
noise robust Lanczos 3, 5-point derivative

$$\frac{f_1 - f_{-1} + 2(f_2 - f_{-2}) + 3(f_3 - f_{-3}) + 4(f_4 - f_{-4}) + 5(f_5 - f_{-5})}{110h}$$

- $\Delta\chi^2 = 35$  determined so that every nuclear target is described within 90% C.L.

- Parton density functions for bound partons as a function of  $x$

black      yellow      brown  
 $x f_k^A(x, Q)$  for  $A = (1, 2, 4, 9, 12, 27, 56, 108, 207)$   
red      purple

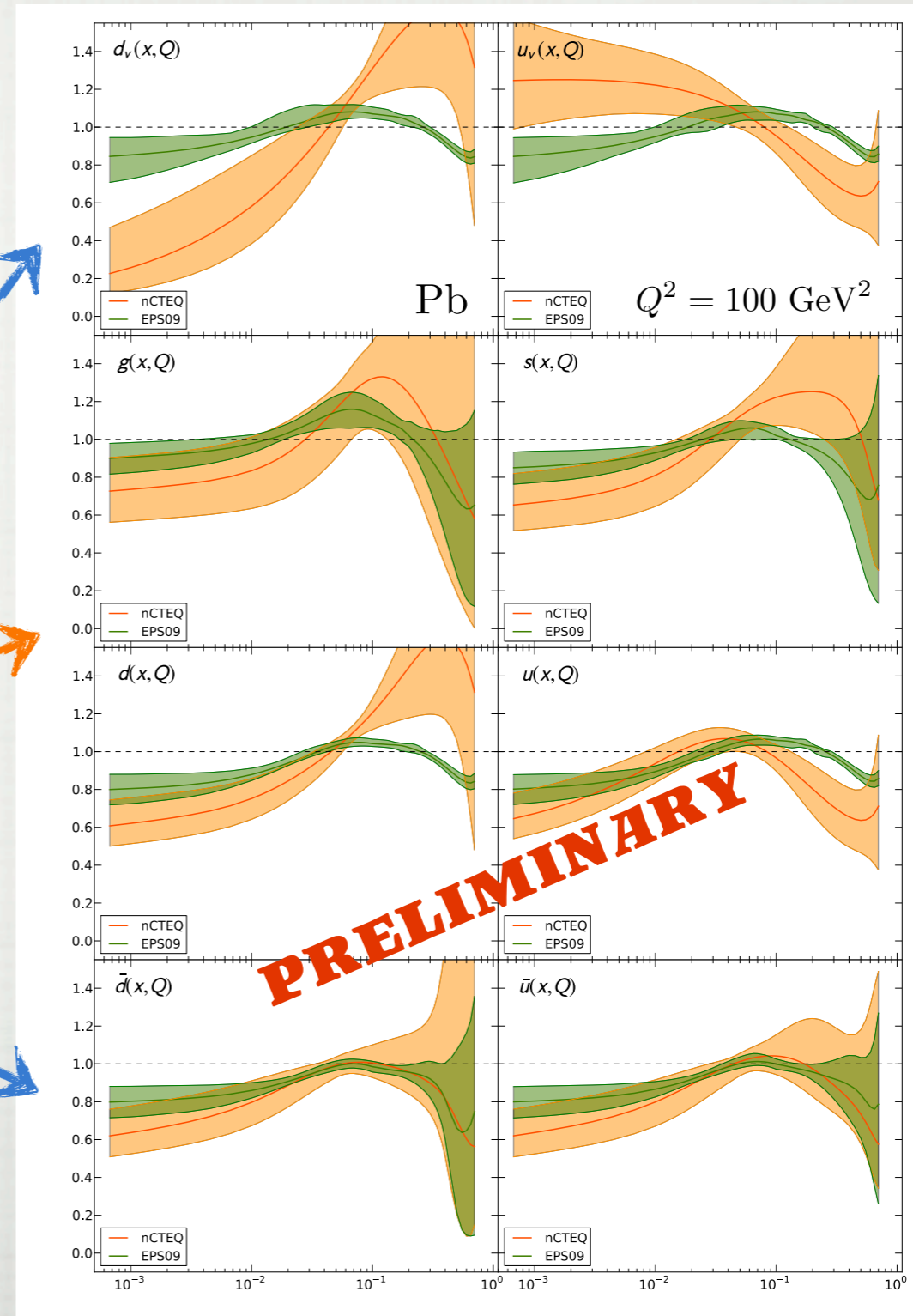


# nCTEQ

- nCTEQ nuclear correction factors with uncertainties

$$R_i(\text{Pb}) = \frac{f_i^{\text{Pb}}(x, Q)}{f_i^{\text{p}}(x, Q)} \quad @ \quad Q^2 = 100 \text{ GeV}^2$$

- different solution for d-valence & u-valence compared to EPS09
- larger uncertainty @ gluon nuclear correction factor & bigger low-x suppression
- sea quark nuclear correction factors similar to EPS09
- nuclear correction factors depend largely on underlying proton baseline

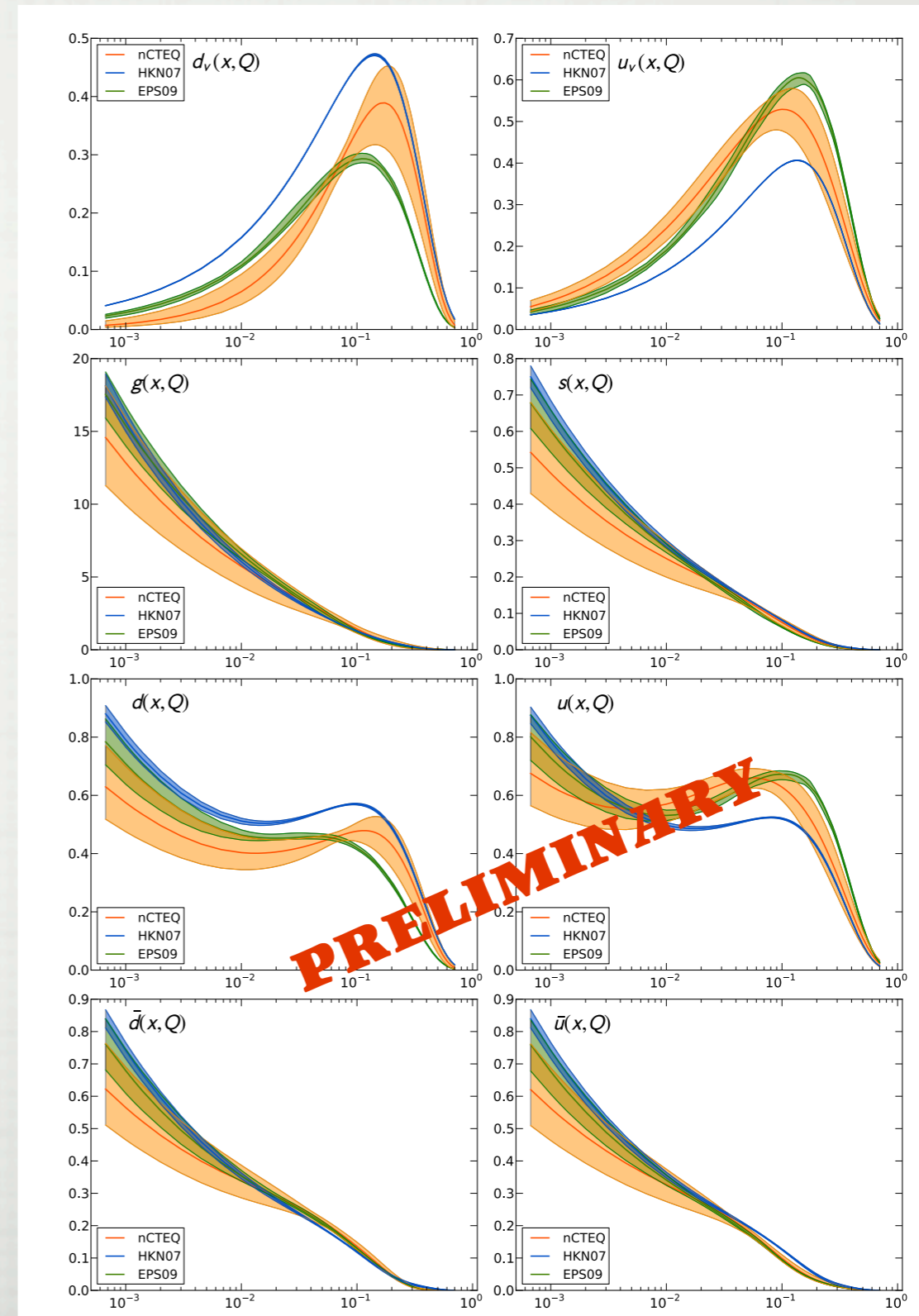


# nCTEQ

- nCTEQ nuclear PDFs with uncertainties

$$x f_i^{\text{Pb}}(x, Q) \quad @ \quad Q^2 = 100 \text{ GeV}^2$$

- nCTEQ d-valence & u-valence solution between HKN07 & EPS09
- nCTEQ nuclear uncertainties larger than previous nPDF analyses
- nPDFs not dependant on proton baseline - better agreement between different nPDFs
- Results still very preliminary

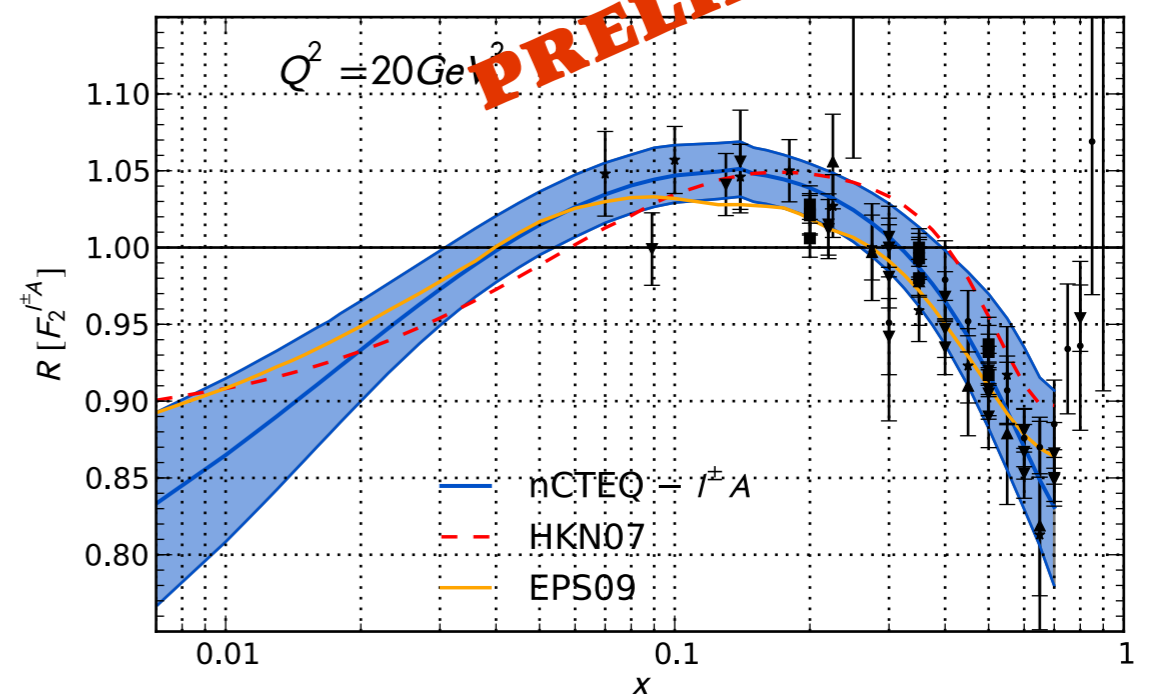
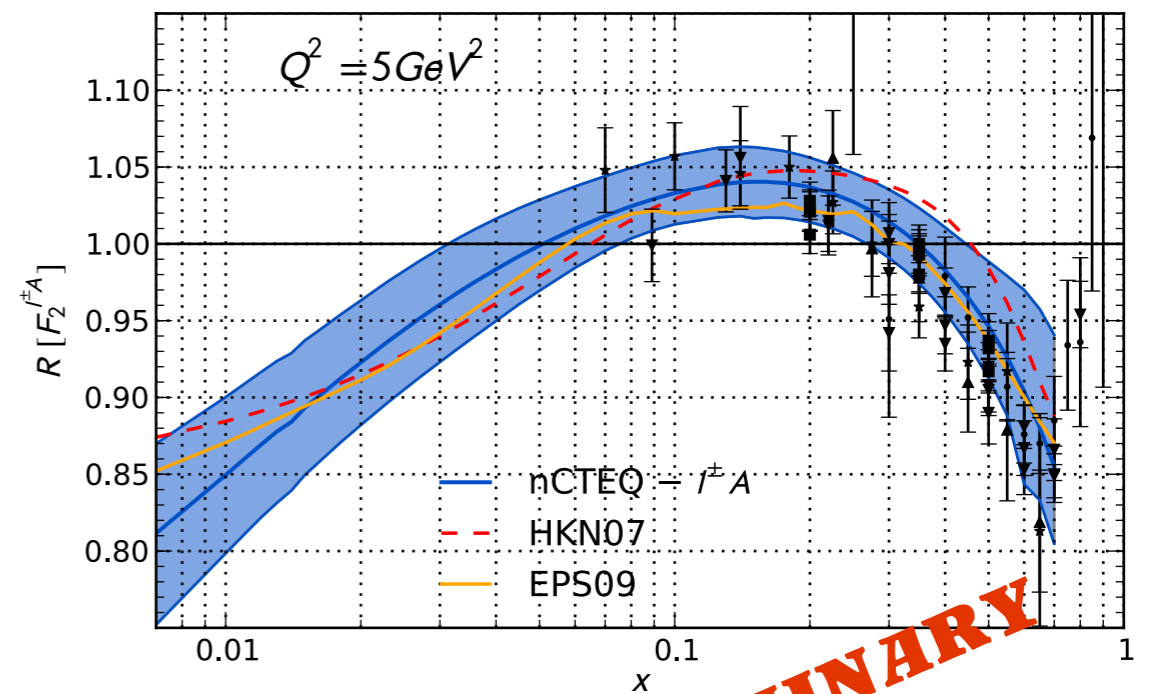


# nCTEQ

- nCTEQ structure function ratios with uncertainties

$$R = \frac{F_2^{Fe}(x, Q)}{F_2^D(x, Q)}$$

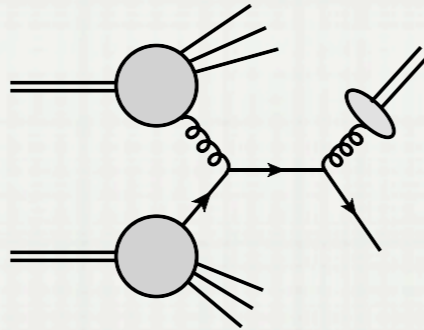
- Structure function ratios are fitted observables
- Despite different d-valence & u-valence solutions - ratio of structure functions remain very similar
- Good description of data & differences between nCTEQ and other nPDFs appear at low-x where there's no data





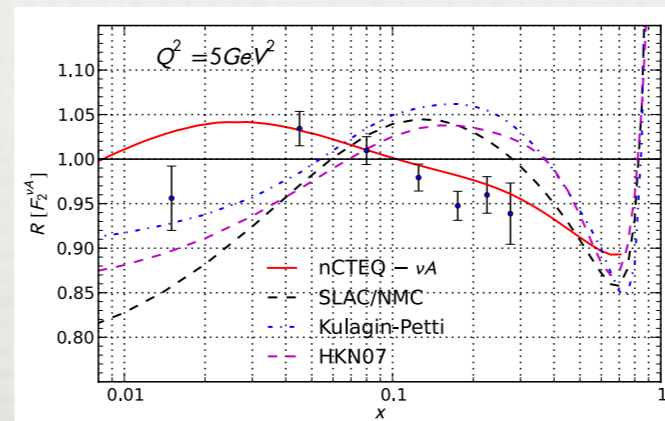
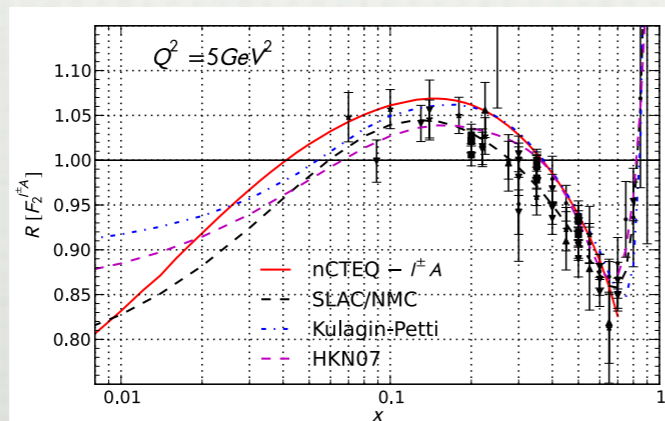
- Why is the nCTEQ analysis still **PRELIMINARY?**

- On-going work - inclusion of single inclusive pion production data from d-Au from RHIC



- more realistic estimate of the nuclear gluon correction factor @ intermediate & high- $x$

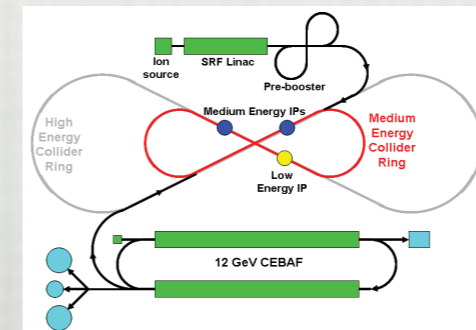
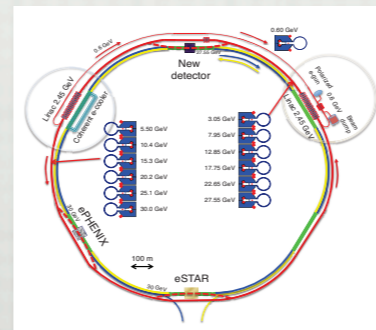
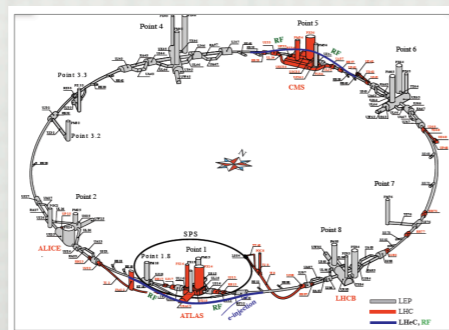
- In discussion - inclusion of neutrino DIS data (inconsistencies within NuTeV data)



- better flavour separation for nuclear effects

# Conclusions & Outlook

- nCTEQ analysis still preliminary - RHIC data being included & analysed at the moment
- nCTEQ has larger uncertainties & larger nuclear suppression for gluon @ low-x
- LHC pPb data have a large potential to constrain nPDF
  - need baseline w/o LHC data first
- Next-generation colliders LHeC or EIC would be a game-changer for nPDFs

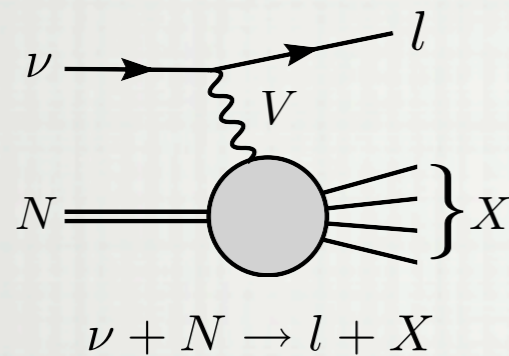


**THANK YOU**

**BACKUP SLIDES**

# NEUTRINO DIS

- Neutrino DIS cross-section data



NuTeV & di-muon  
 $N = \text{Fe}$   
 CHORUS  
 $N = \text{Pb}$

→ 2310 data points  
 → 824 data points

All charged lepton  
 DIS & Drell-Yan data  
 → 708 data points

- Challenges in combining the neutrino & charged lepton data

- deal with the disparity of number of data points - assigning weights to neutrino data
- neutrino DIS data only with 2 heavy nuclei - insufficient to get a reliable A-dependance
- do all neutrino data show the different behavior or only NuTeV ?

- Different neutrino observables

$$\frac{d\sigma^{\nu A}}{dx dQ^2} \quad \& \quad \frac{d\sigma^{\bar{\nu} A}}{dx dQ^2} \quad \text{vs.} \quad F_2^{\nu+\bar{\nu}}(x, Q^2) \quad \& \quad xF_3^{\nu+\bar{\nu}}(x, Q^2)$$

↗  
*needs theory assumptions to extract*

- Nuclear correction factors

- we show correction factors defined e.g. as  $R[F_2^{\nu}] = F_2^{\nu A} / F_2^{\nu A, \text{free}}$

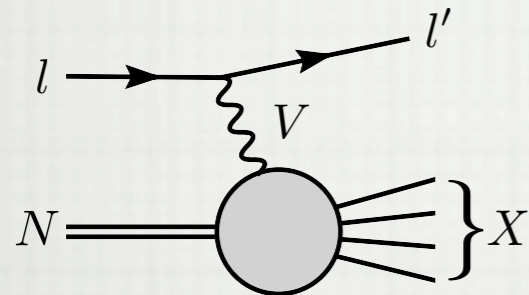
↖  
*from free proton PDF*

# NEUTRINO DIS

- Experiments included in the analysis

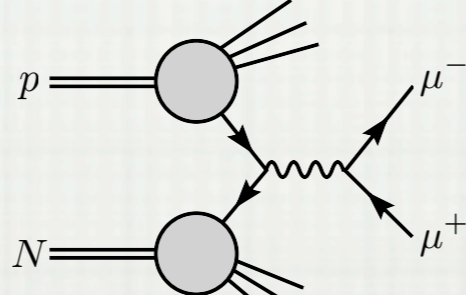
## Charged lepton

Deep Inelastic Scattering



$$l + N \rightarrow l' + X$$

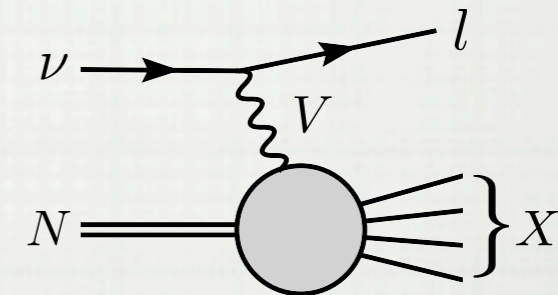
Drell-Yan process



$$p + N \rightarrow \mu^+ \mu^- + X$$

## Neutrino

Deep Inelastic Scattering



$$\nu(\bar{\nu}) + N \rightarrow l + X$$

### CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

### FNAL E-665

$N = (\text{D, C, Ca, Pb, Xe})$

### DESY Hermes

$N = (\text{D, He, N, Kr})$

### SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

### FNAL E-772 & E-886

$N = (\text{D, C, Ca, Fe, W})$

### CHORUS

$N = \text{Pb}$

### CCFR & NuTeV

$N = \text{Fe}$

1233 data points (708 after cuts)

3832 data points (3134 after cuts)

# NEUTRINO DIS

- Comparison of charged lepton and neutrino fits

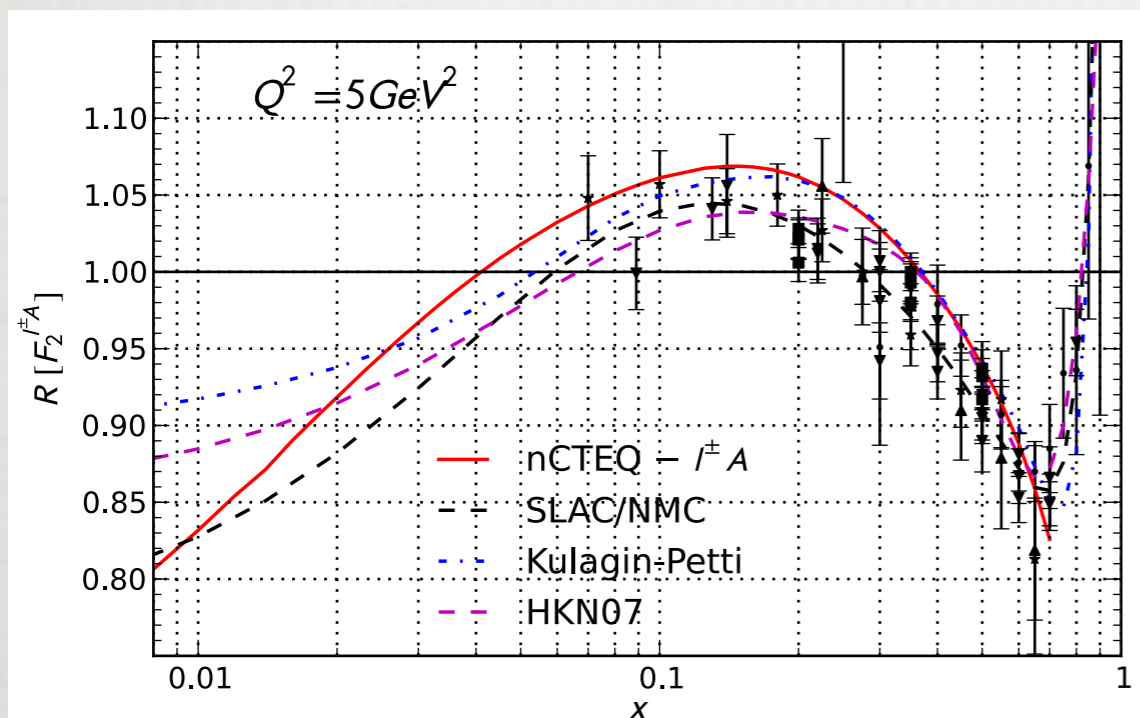
KK et al.

[Phys.Rev.Lett. 106(2011) 122301] arXiv: 1012.1178

*Fit to charged lepton data*

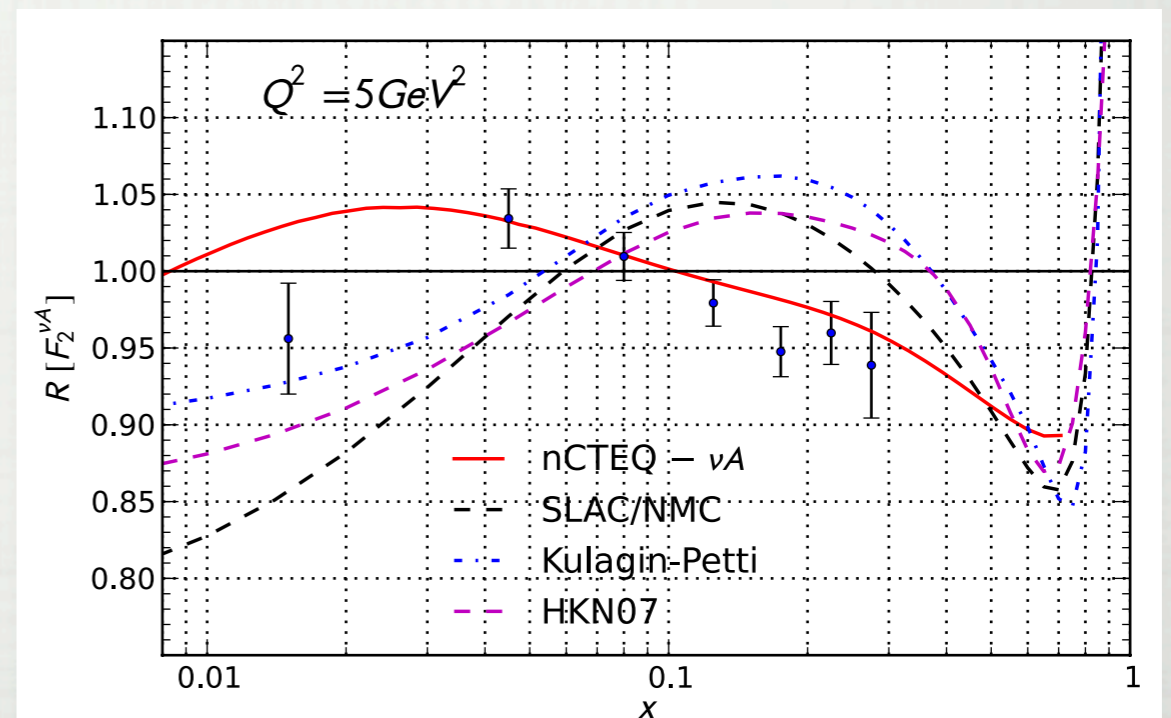
*DIS & DY*

$$\chi^2/\text{d.o.f} = 0.89$$



*Fit to only neutrino DIS*

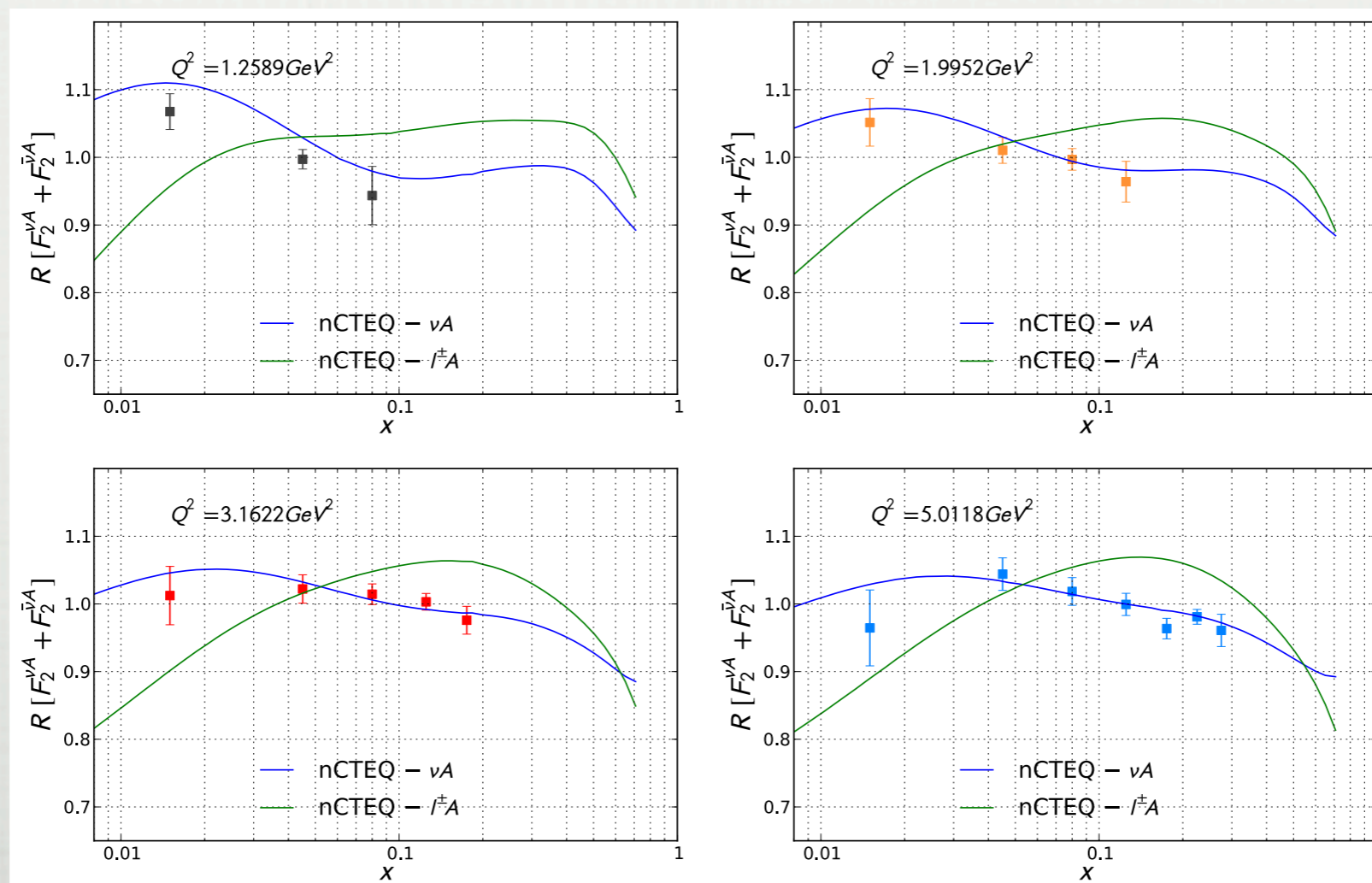
$$\chi^2/\text{d.o.f} = 1.33$$



- can we explain the difference and fit all data together in a global fit ?

# NEUTRINO DIS

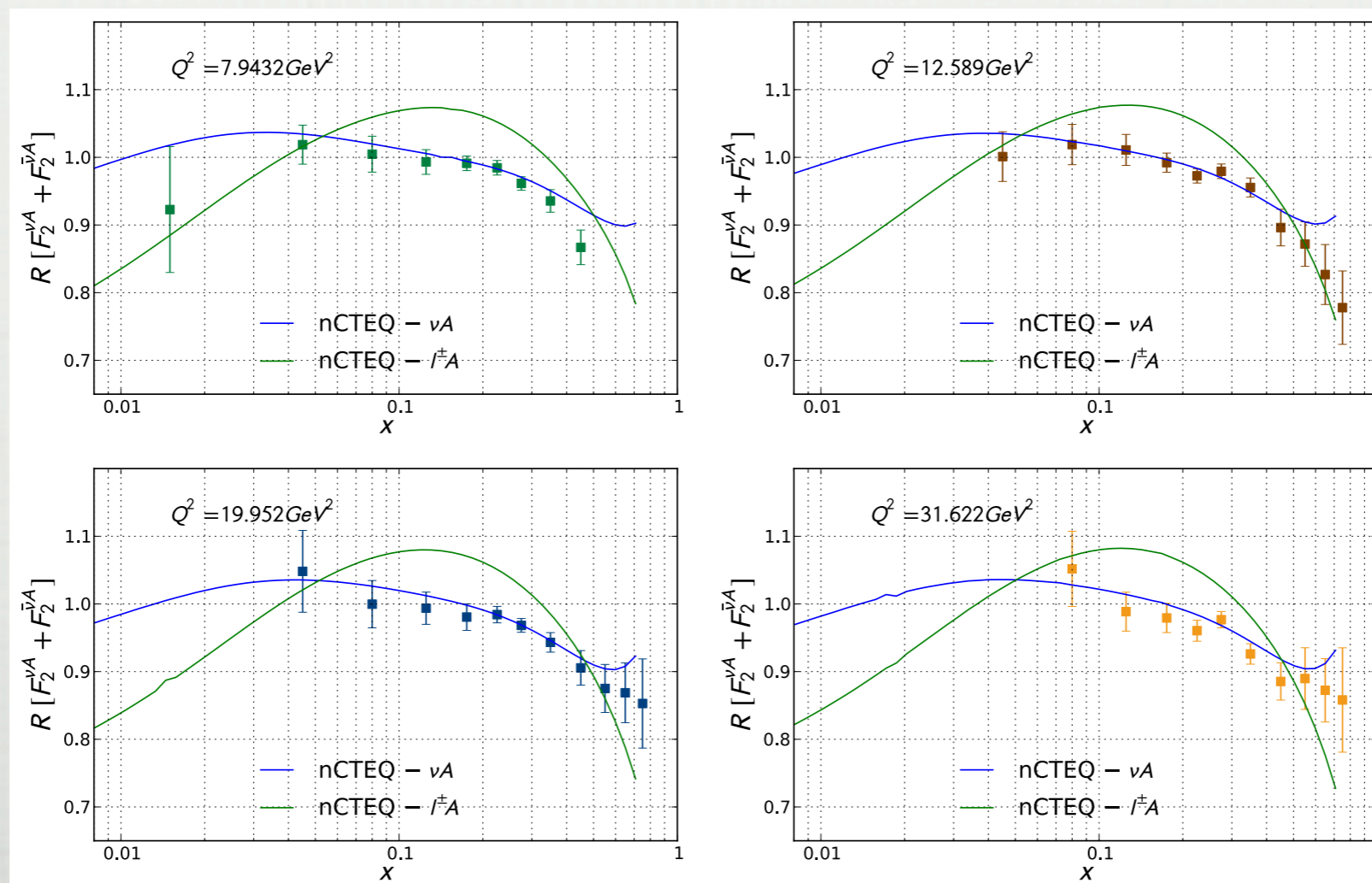
- NLO QCD calculation of  $\frac{F_2^{\nu A} + F_2^{\bar{\nu} A}}{2}$  in the ACOT-VFN scheme
  - comparison of nCTEQ - only neutrino fit against extracted NuTeV data at different  $Q^2$
  - charge lepton fit undershoots low-x data & overshoots mid-x data
  - low- $Q^2$  and small-x data cause tension with the shadowing observed in charged lepton data





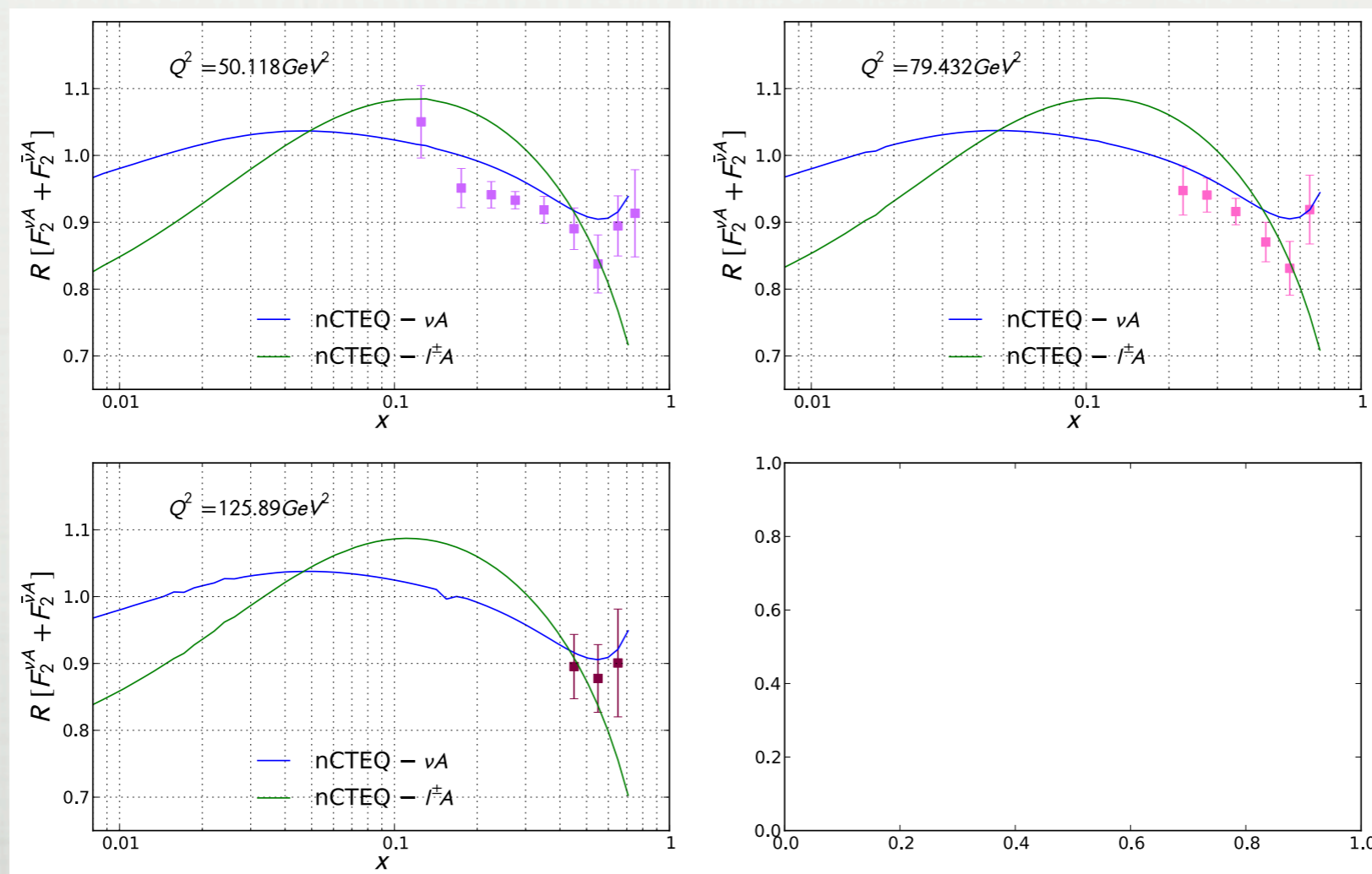
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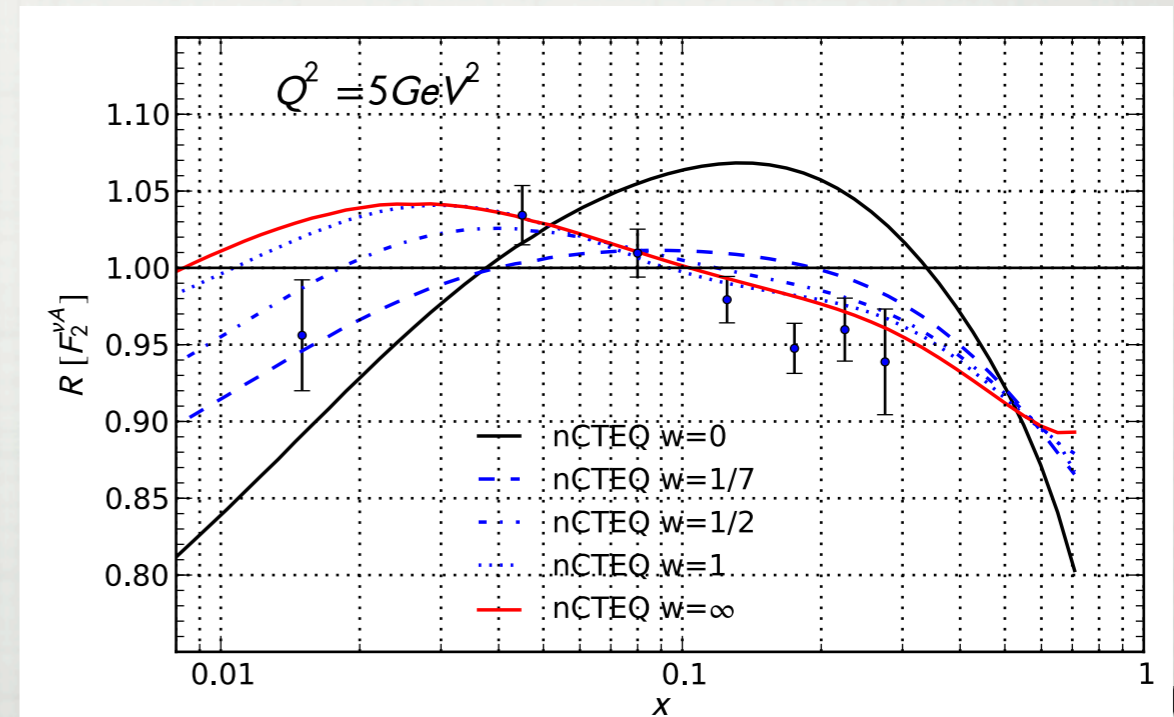
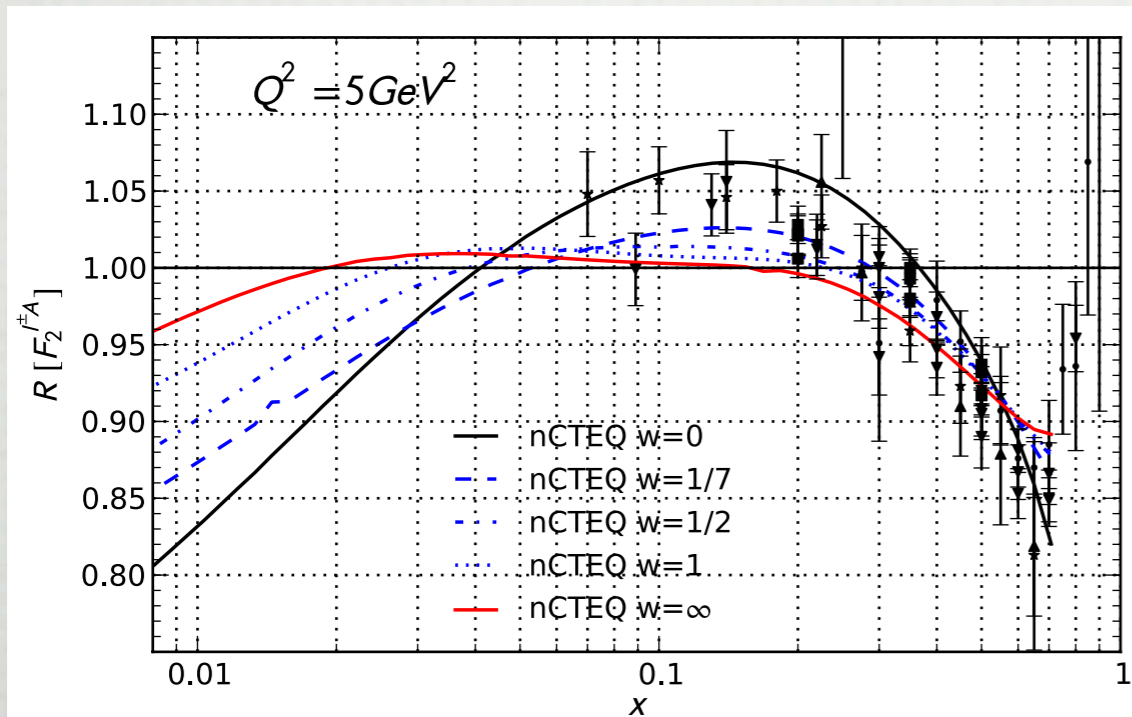


# NEUTRINO DIS

- Analysis of fits with different weights of neutrino DIS (correlated errors)

$w$	$l^\pm A$	$\chi^2$ (/pt)	$\nu A$	$\chi^2$ (/pt)	total $\chi^2$ (/pt)
0	708	630 (0.89)	-	-	$630 \pm 58$
1/7	708	645 (0.91)	3134	4681 (1.50)	$5326 \pm 203$
1/2	708	680 (0.96)	3134	4375 (1.40)	$5055 \pm 192$
1	708	736 (1.04)	3134	4246 (1.36)	$4983 \pm 190$
$\infty$	-	-	3134	4167 (1.33)	$4167 \pm 176$

$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

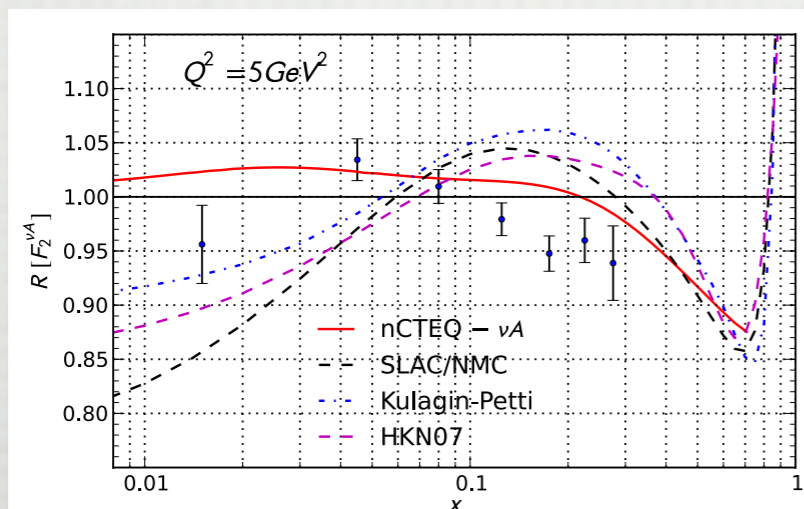
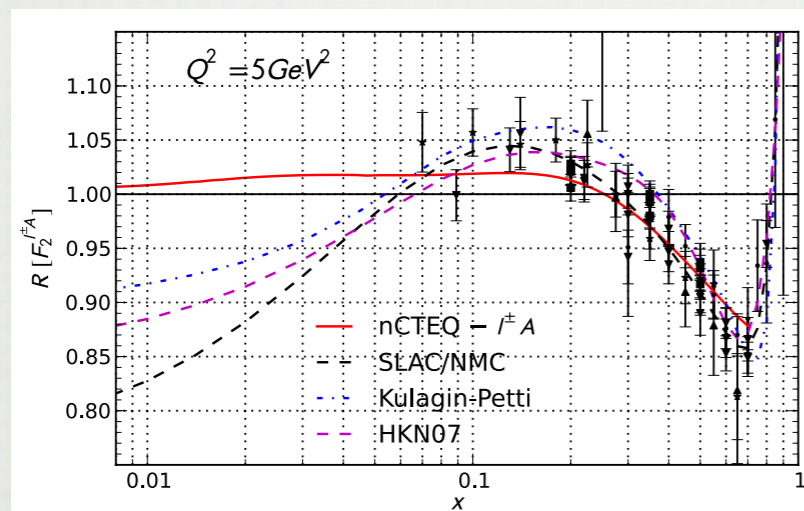


# NEUTRINO DIS

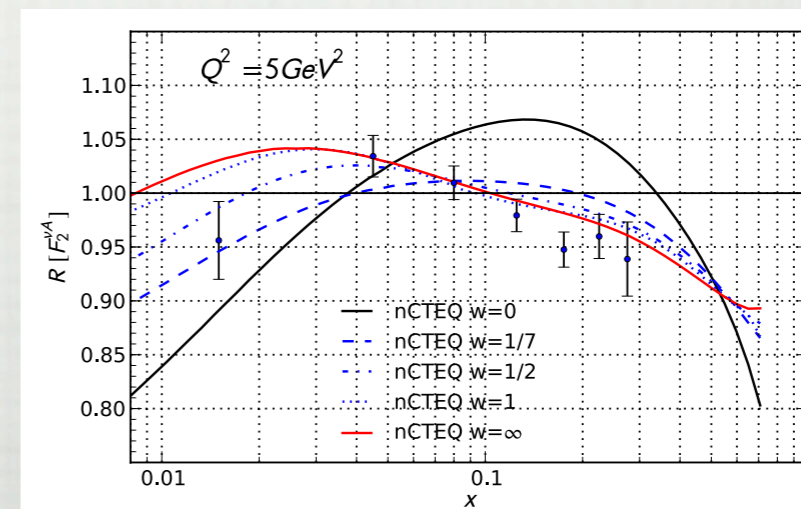
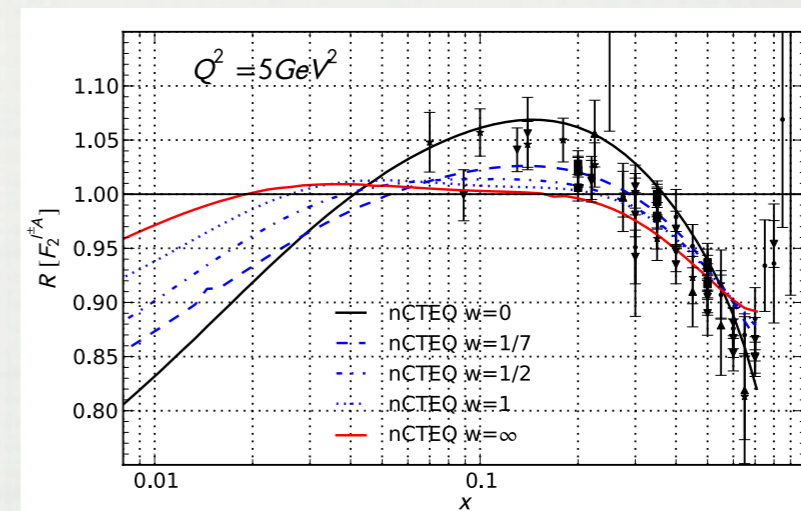
- Analysis of fits with neutrino DIS (uncorrelated errors)

$w$	$l^\pm A$	$\chi^2$ (/pt)	$\nu A$	$\chi^2$ (/pt)	total $\chi^2$ (/pt)
1-corr	708	736 (1.04)	3134	4246 (1.36)	4983 (1.30)
1-uncorr	708	809 (1.14)	3110	3115 (1.00)	3924 (1.02)

uncorrelated errors



correlated errors



# NEUTRINO DIS

- Properties of neutrino fits

- CHORUS data are in good agreement with the charged lepton data

combined:  $\chi^2/pt=1.03$

- NuTeV data (with correlated errors) difficult to fit alone or with the charged lepton data

alone:  $\chi^2/pt=1.35$

combined:  $\chi^2/pt=1.33$

- Neutrino data dominate the combined fit without re-weighting - final result depends from the weight chosen

