

Extraction of Transversity and Collins functions



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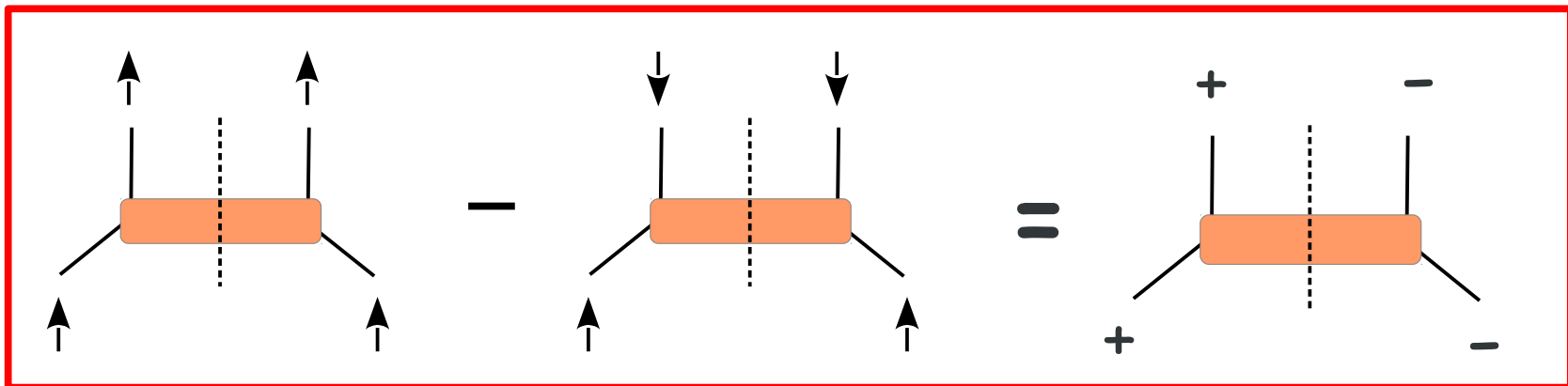
In collaboration with
M. Anselmino, M. Boglione, U. D'Alesio, F. Murgia and A. Prokudin

Outline

- New extraction (BELLE, HERMES and COMPASS D+P)
 - Polynomial parametrization of the Collins functions
 - Predictions for BaBar
 - Conclusions
-

The Transversity Function

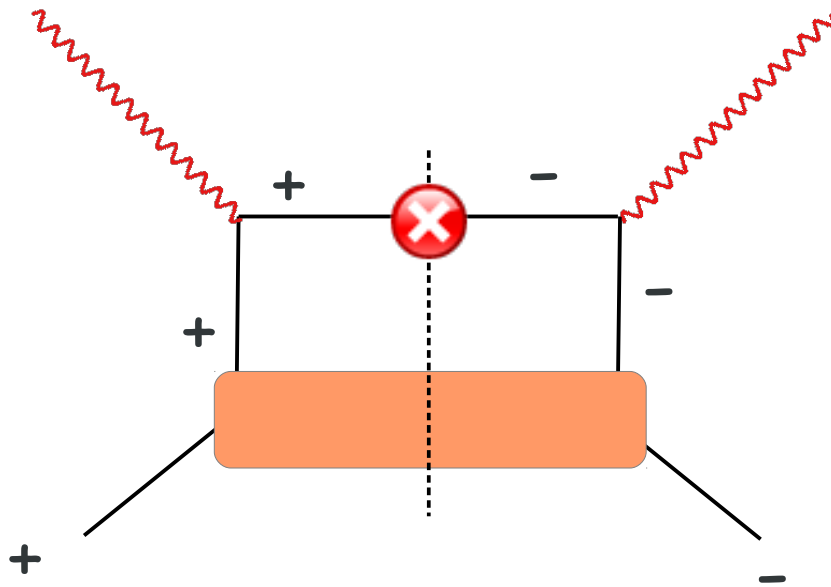
- The transversity is a twist two, collinear, distribution of transversely polarized quarks inside a transversely polarized hadron



- It is off diagonal in the helicity basis, i.e. it is a chiral odd function.

Chiral Odd nature of Transversity

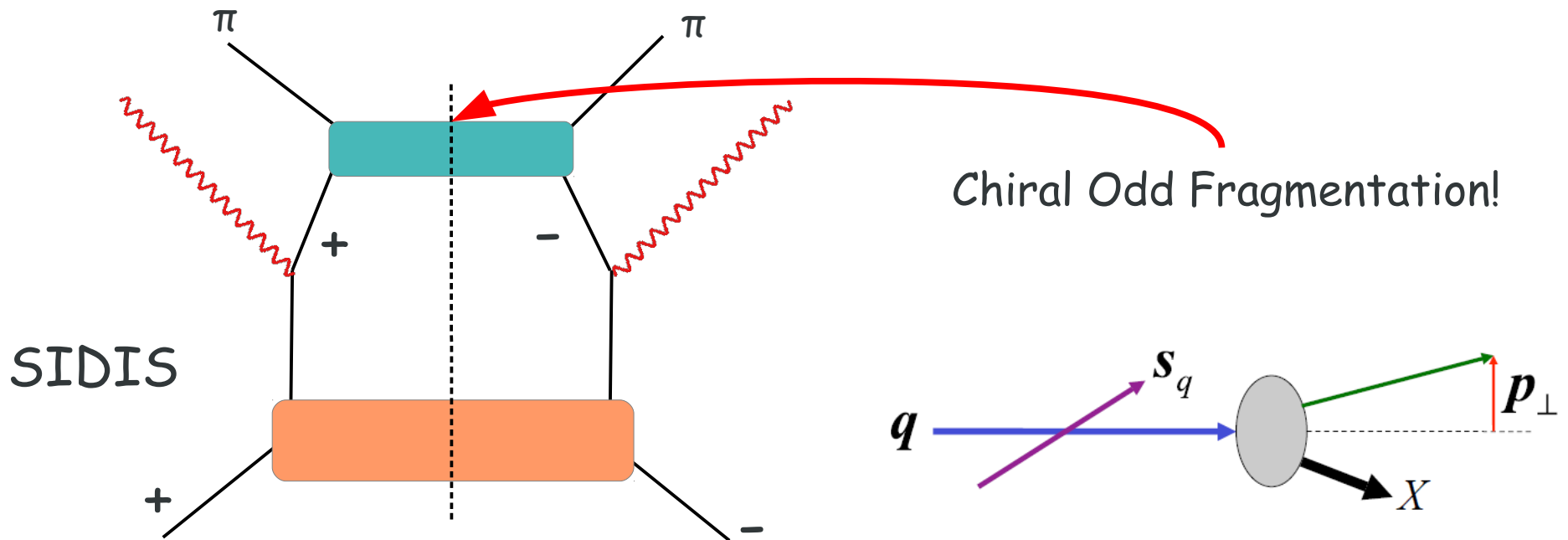
- Chiral Odd: It cannot be measured in DIS processes!



- It needs to be coupled with another chiral odd quantity...
- DIS is our main source of knowledge on the parton distribution.
- The transversity function has been unknown until 2007

Accessing the transversity

- In order to still have the power of DIS, let us consider the SIDIS instead of DIS



- Collins fragmentation function:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z, p_\perp) = D_{\pi/q^\uparrow}(z, p_\perp) - D_{\pi/q^\downarrow}(z, p_\perp)$$

Extraction of transversity & Collins functions

- Azimuthal asymmetry in polarized SIDIS

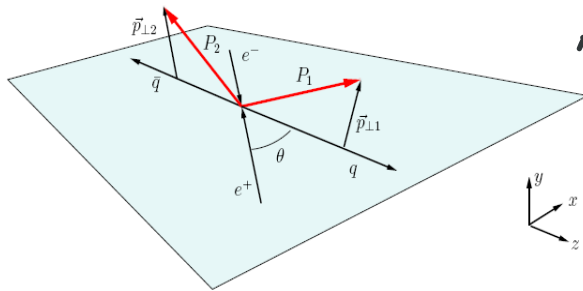
$$d\sigma^\uparrow - d\sigma^\downarrow = \sum_q h_{1q}(x, k_\perp) \otimes d\Delta\hat{\sigma}(y, \mathbf{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)$$

Transversity Collins function

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2 \frac{\int d\phi d\phi_S [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi + \phi_S)}{\int d\phi d\phi_S [d\sigma^\uparrow + d\sigma^\downarrow]}$$

Extraction of transversity & Collins functions

➤ $e^+e^- \rightarrow h_1 h_2$ X BELLE Data

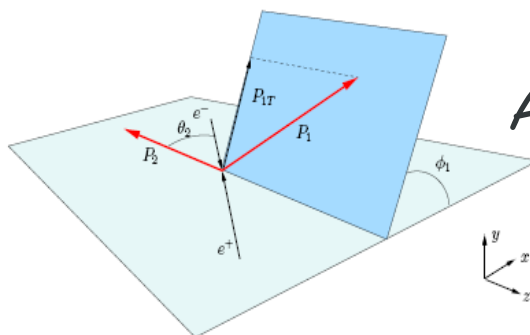


A_{12} asymmetry

Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d\cos\theta d(\varphi_1 + \varphi_2)}$$

$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$



A_0 asymmetry

Hadronic plane method

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \cos(2\phi_1) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

Polarized SIDIS & e^+e^- data: Extraction of Transversity

Parametrizations

➤ Gaussian parametrization of the unpolarized PDF & FF:

$$\bullet \quad f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

$$\bullet \quad D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle}$$

$$[*] \langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2 \quad \langle p_{\perp}^2 \rangle = 0.20 \text{ GeV}^2$$

Parametrizations

► Parametrization of Transversity function:

$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

Unpolarized PDF

Helicity PDF

$$\mathcal{N}_q^T(x) = N_q^T x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

N_q^T, α, β free parameters

Parametrizations

► Parametrization of the Collins function:

$$\Delta^N D_{\pi/q\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, p_\perp)$$

$$\bullet \mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta}$$

$$\bullet h(p_\perp) = \sqrt{2} e \frac{p_\perp}{M_h} e^{-p_\perp^2 / M_h^2}$$

Unpolarized FF

$N_q^C, \gamma, \delta, M_h$ free parameters

✓ Bound:

$$\Delta^N D_{\pi/q\uparrow}(z, p_\perp) \leq 2 D_{\pi/q}(z, k_\perp)$$

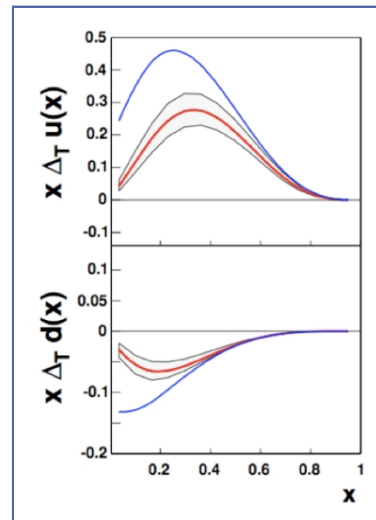
✓ Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q\uparrow}(z, p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z, p_\perp)$$

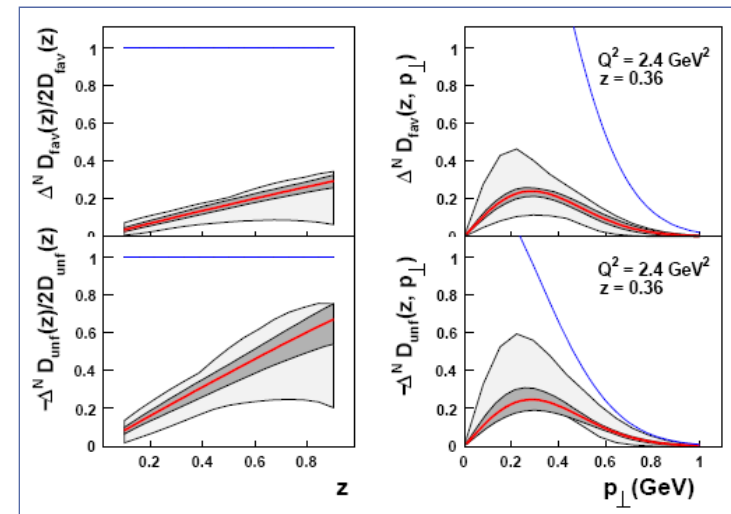
Extraction of transversity & Collins functions

➤ Simultaneous fit of HERMES, COMPASS D and BELLE (A_{12}^{UL}) data

$$\chi^2_{\text{dof}} = 1.3$$



Transversity



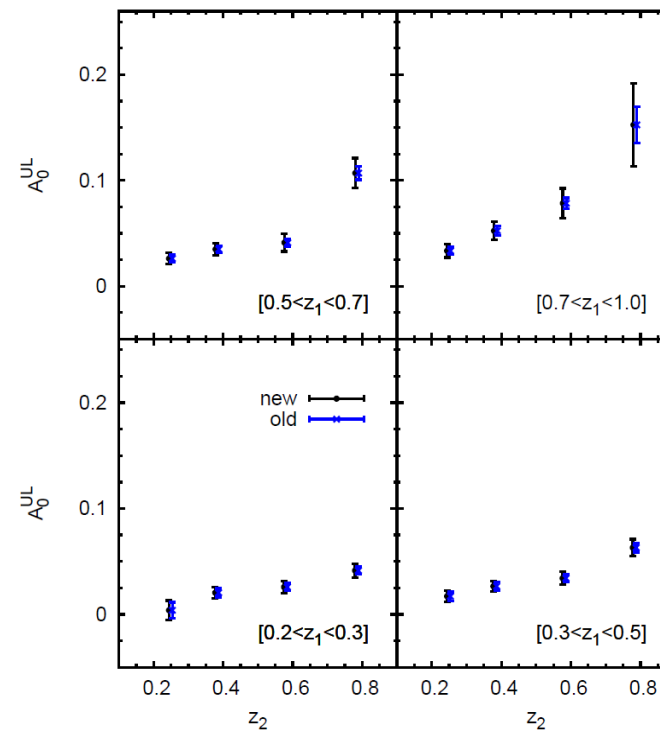
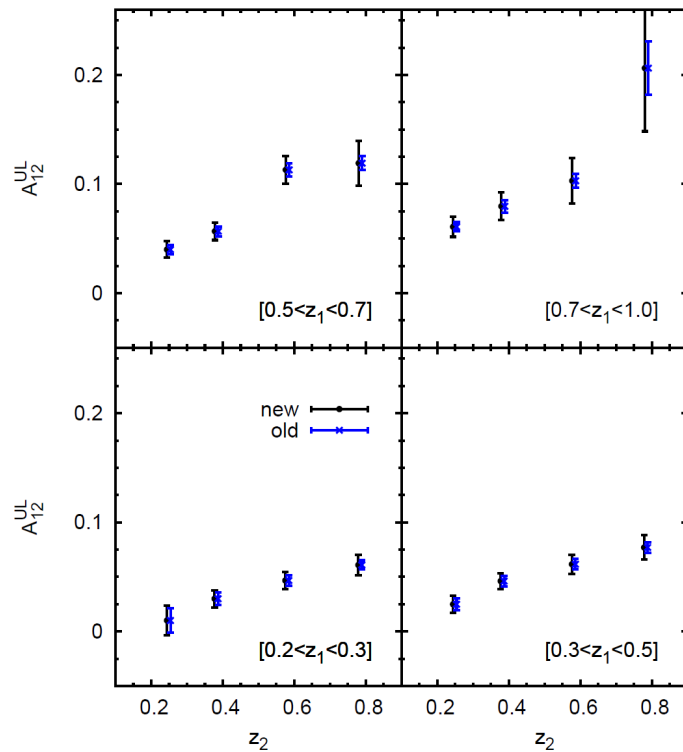
Collins functions

$N_u^T = 0.64 \pm 0.34$	$N_d^T = -1.00 \pm 0.02$
$\alpha = 0.73 \pm 0.51$	$\beta = 0.84 \pm 2.30$
$N_{fav}^C = 0.44 \pm 0.07$	$N_{unf}^C = -1.00 \pm 0.06$
$\gamma = 0.96 \pm 0.08$	$\delta = 0.01 \pm 0.05$
$M_h^2 = 0.91 \pm 0.52 \text{ GeV}^2$	

• Anselmino et. al arXiv: 0812.4366v1

BELLE Erratum

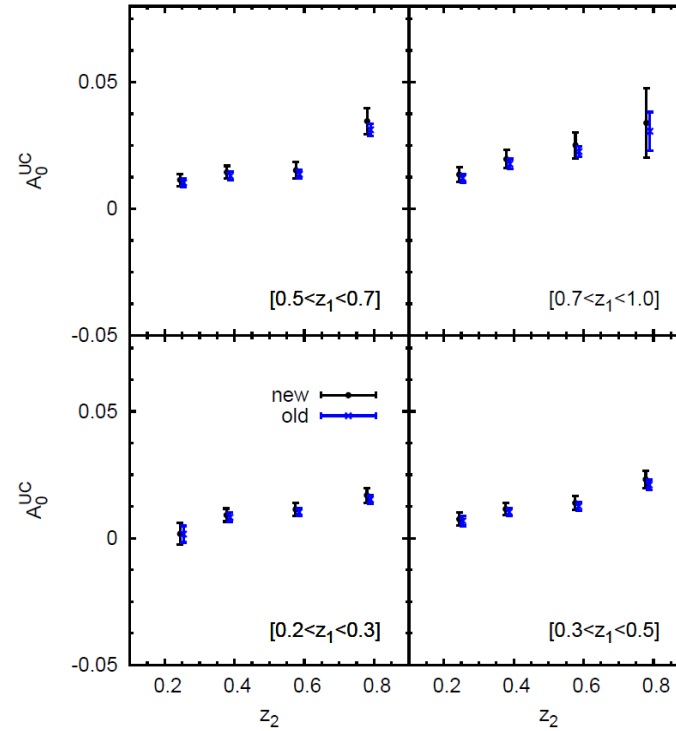
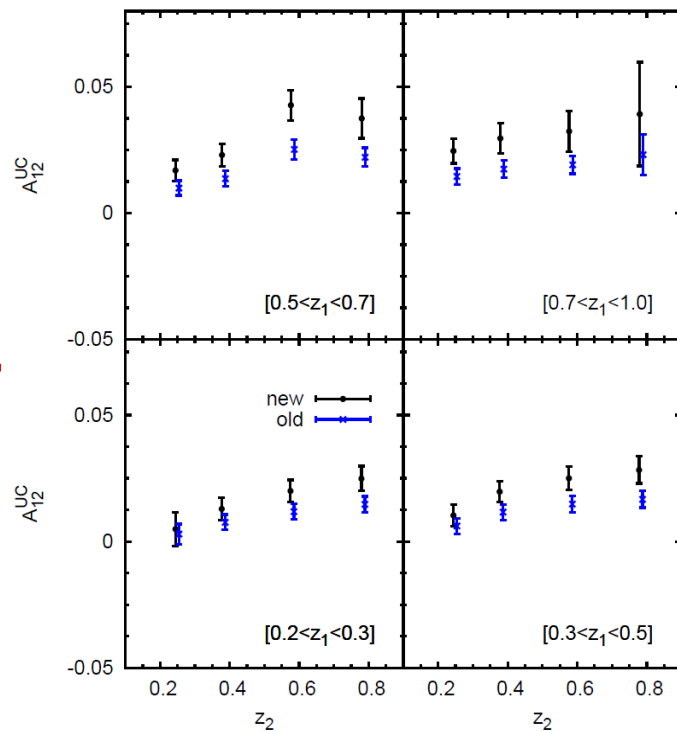
➤ BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



A^{UL} : Same central values, larger errors

BELLE Erratum

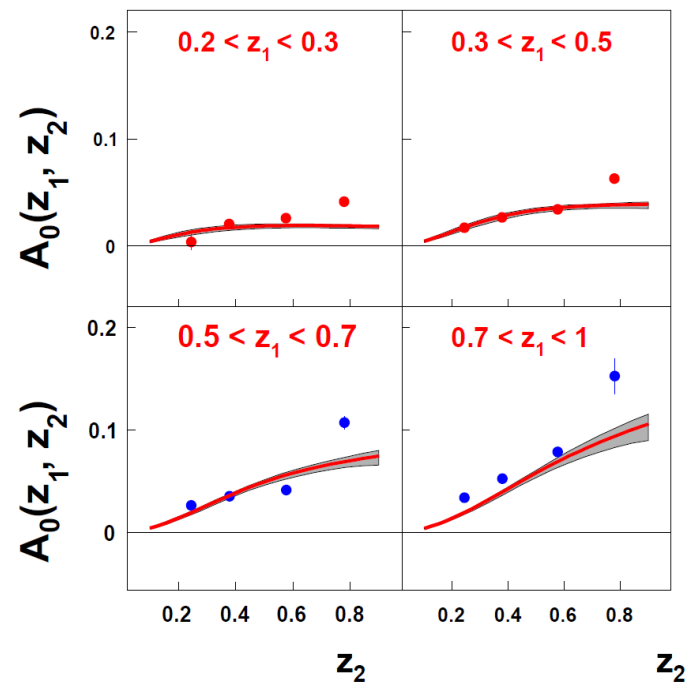
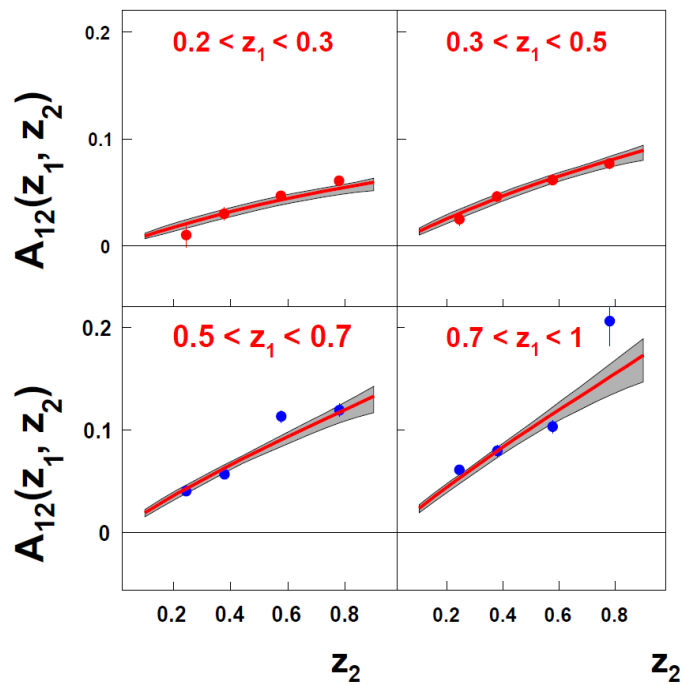
➤ BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



A^{UC} : Different normalization, larger errors

BELLE Erratum&2008 Extraction

➤ BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



◇ R. Seidl et al., Phys. Rev. D78

Good news! Previously partial incompatibility between the sets

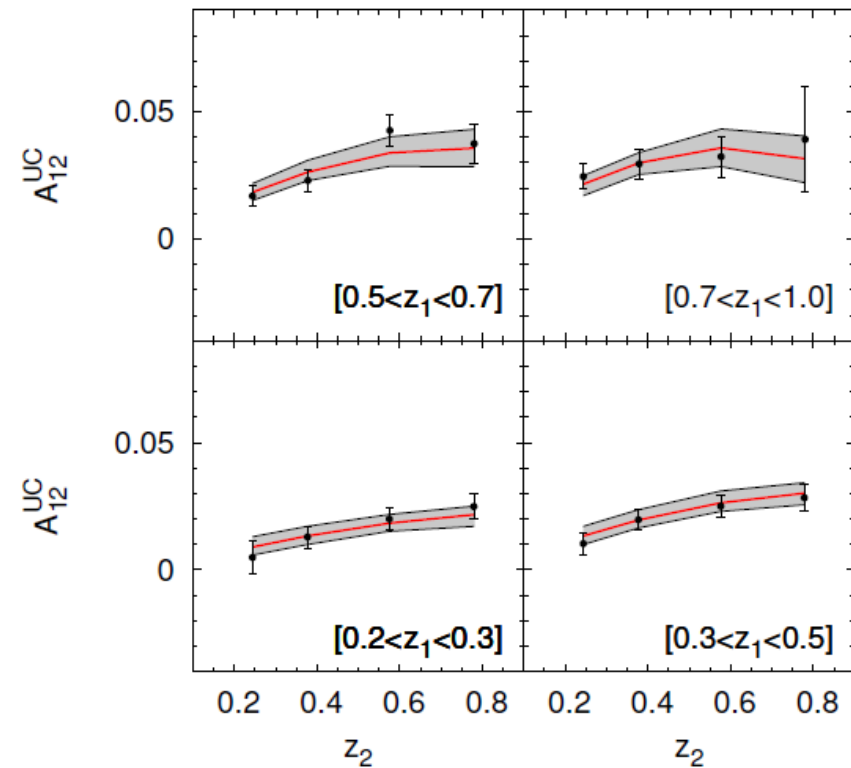
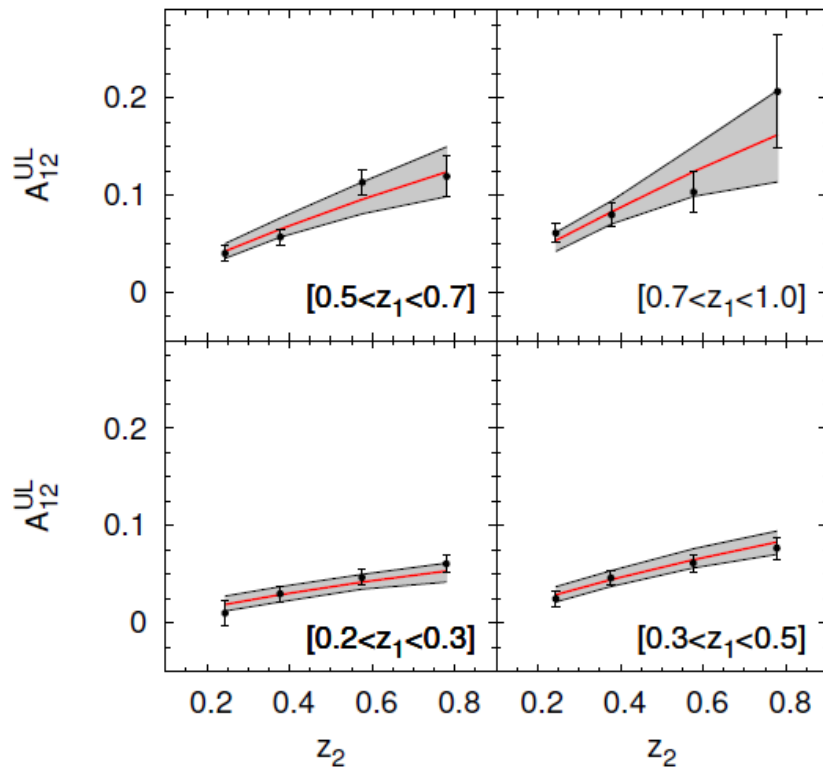
New analysis

➤ New analysis:

- HERMES (2009) $\pi^+ \pi^-$
- COMPASS Deuteron (2004) $\pi^+ \pi^-$
- COMPASS Proton (2013) $\pi^+ \pi^-$
- BELLE A_{12} or A_0

Extraction of transversity & Collins functions

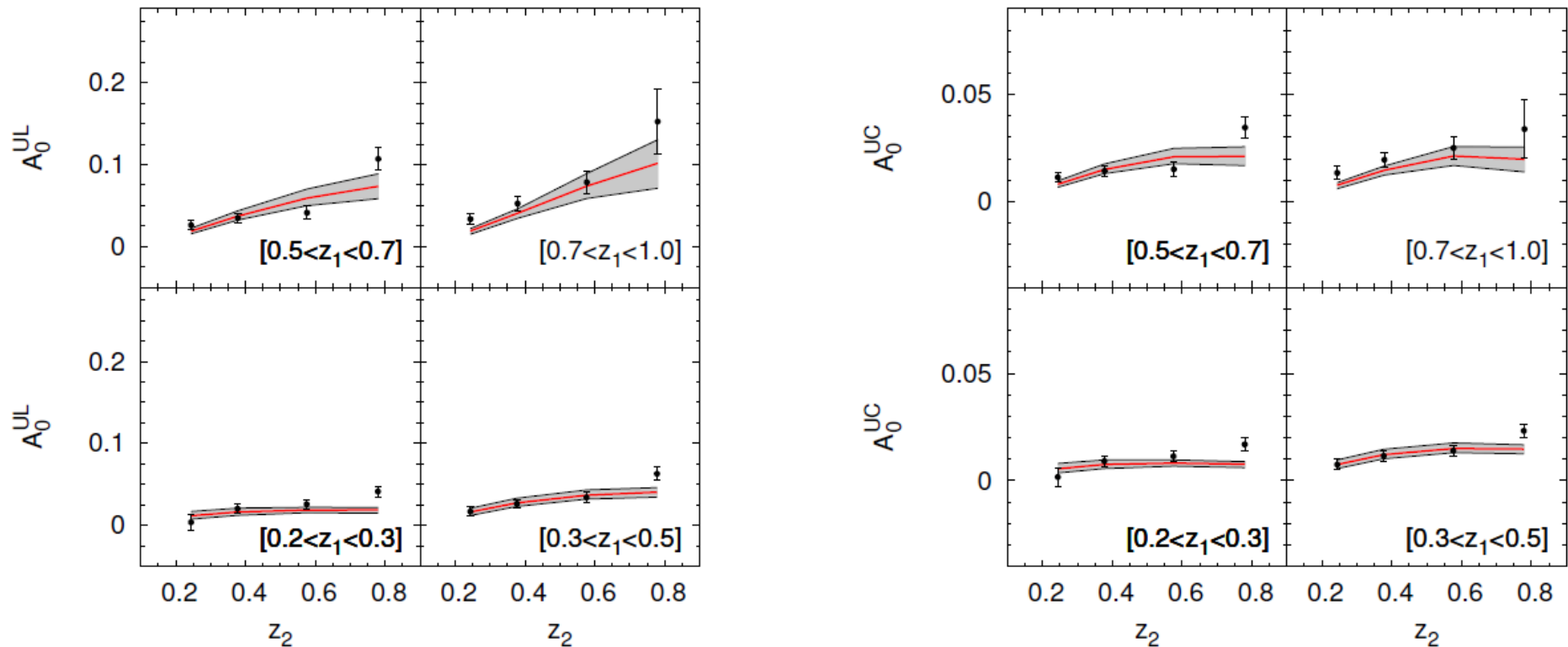
➤ FIT I: A_{12} BELLE data UL & UC + COMPASS+ HERMES



➔ Full compatibility between UL and UC

Extraction of transversity & Collins functions

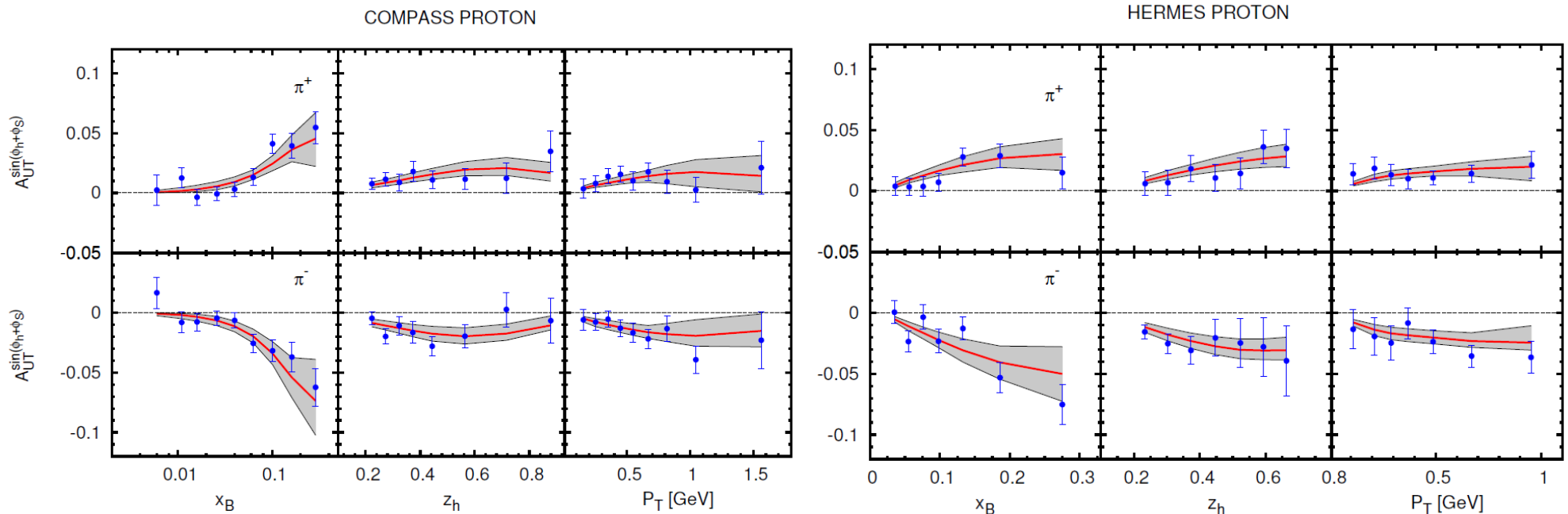
➤ FIT I: A_{12} BELLE data UL & UC + COMPASS+ HERMES



➔ Still tension between the two methods A_0 and A_{12}

Extraction of transversity & Collins functions

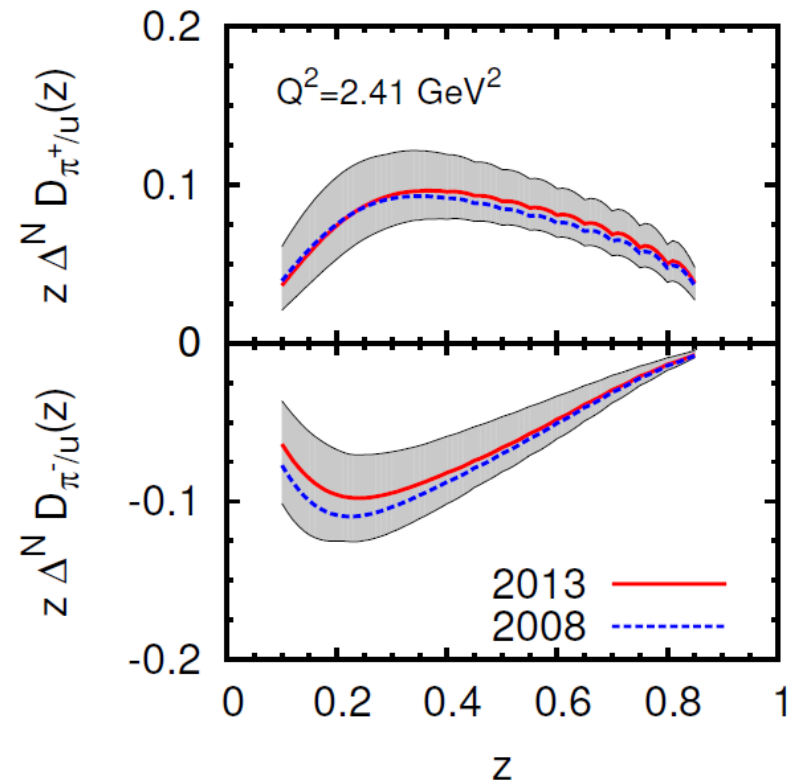
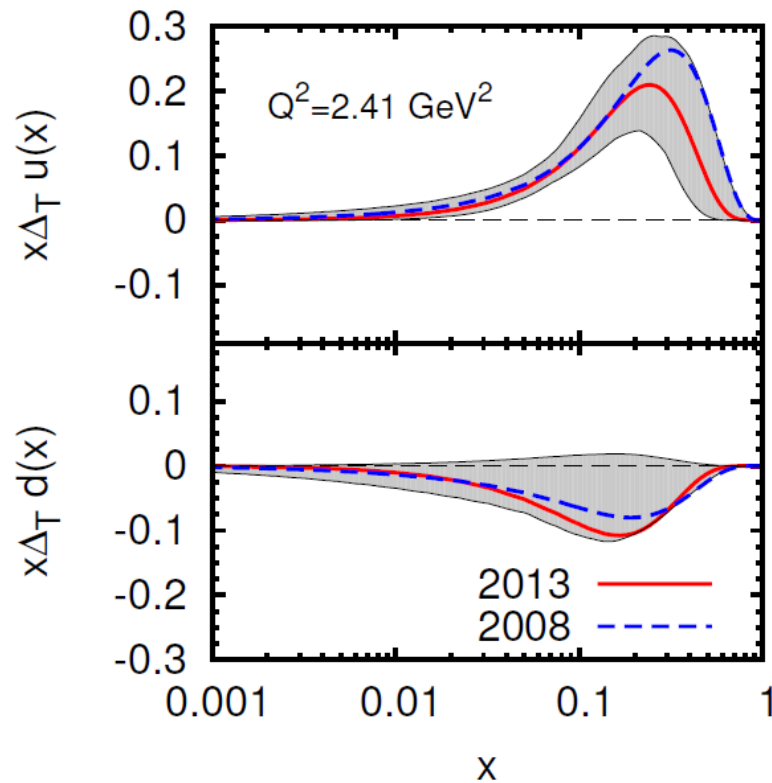
➤ FIT I: A_{12} BELLE data UL & UC + COMPASS+ HERMES



➔ Similar good description of HERMES and COMPASS

Extraction of transversity & Collins functions

➤ FIT I: A_{12} BELLE data UL & UC + COMPASS+ HERMES



➔ Similar results of 2008

Extraction of transversity & Collins functions

➤ FIT II: A_0 BELLE data UL & UC +COMPASS+ HERMES ???

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi_{d.o.f}^2 = 0.80$	$\chi_{tot}^2 = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi_{d.o.f}^2 = 1.12$	$\chi_{tot}^2 = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$

➔ A_0 data cannot be nicely described even if fitted...

Standard parametrization of the Collins function

- Parametrization of the z -dependent part of the Collins function:

$$\Delta^N D_{\pi/q\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, p_\perp)$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta}$$

Our standard parametrization

- It is equal to 0 at $z=0$ and $z=1$

New parametrization of the Collins function

- Let us try to change the parametrization of the z -dependent part of the Collins function:

$$\Delta^N D_{\pi/q\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, p_\perp)$$

$$\mathcal{N}_q^C(z) = N_q^C z [(1 - a - b) + az + bz^2]$$

NEW Polynomial parametrization

- It is equal to 0 at $z=0$ and equal to N_q at $z=1$

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
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Polynomial Parameterization $\chi_{d.o.f}^2 = 0.81$	$\chi_{tot}^2 = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi_{d.o.f}^2 = 1.01$	$\chi_{tot}^2 = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi_{d.o.f}^2 = 0.80$	$\chi_{tot}^2 = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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➔ If we fit A_{12} data we get the same description obtained with the std par.

➔ Almost identical Collins function, again the description of A_0 is not so good

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
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➔ If we fit A_0 data we can improve their description

➔ Still tension with A_{12}

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

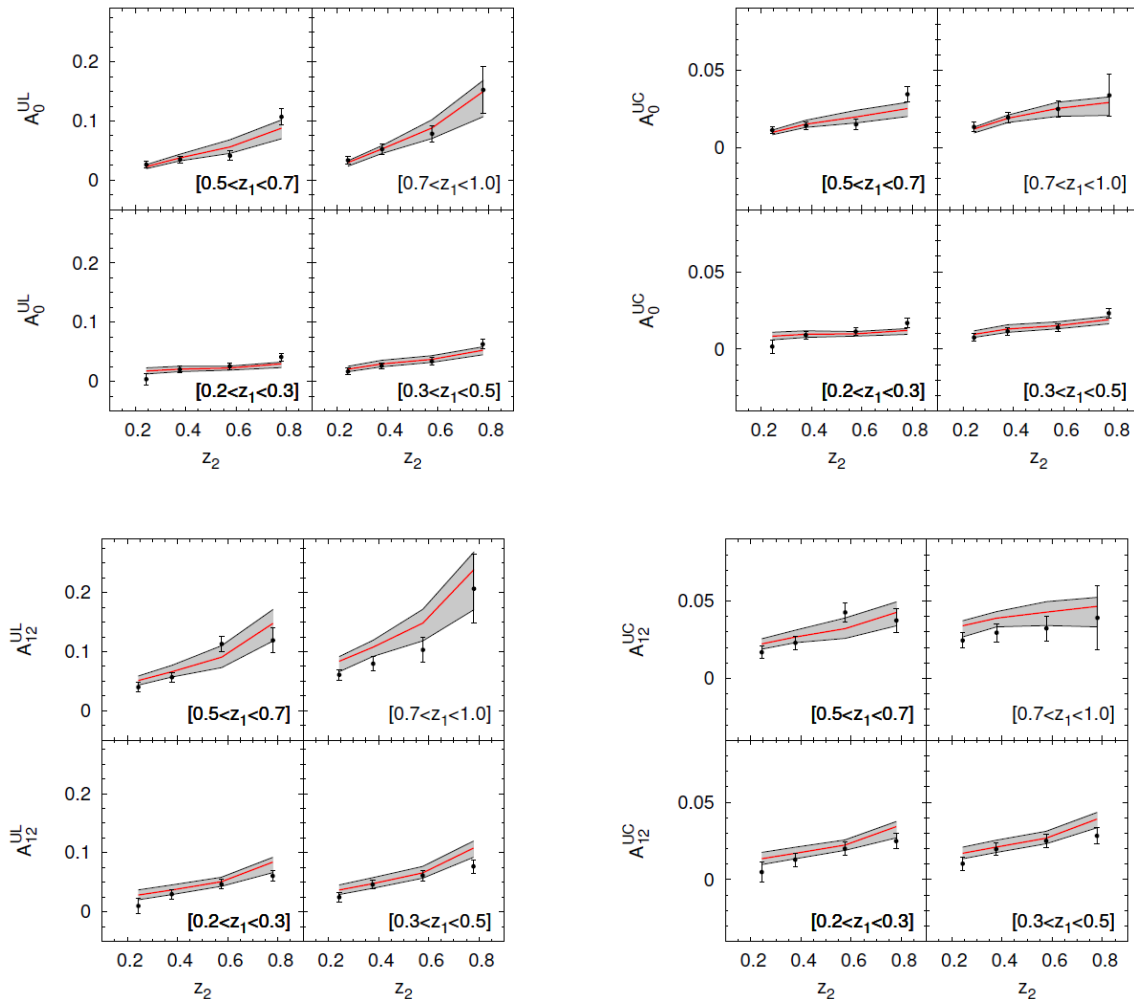
	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
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➔ If we fit A_0 data we can improve their description

➔ Still tension with A_{12}

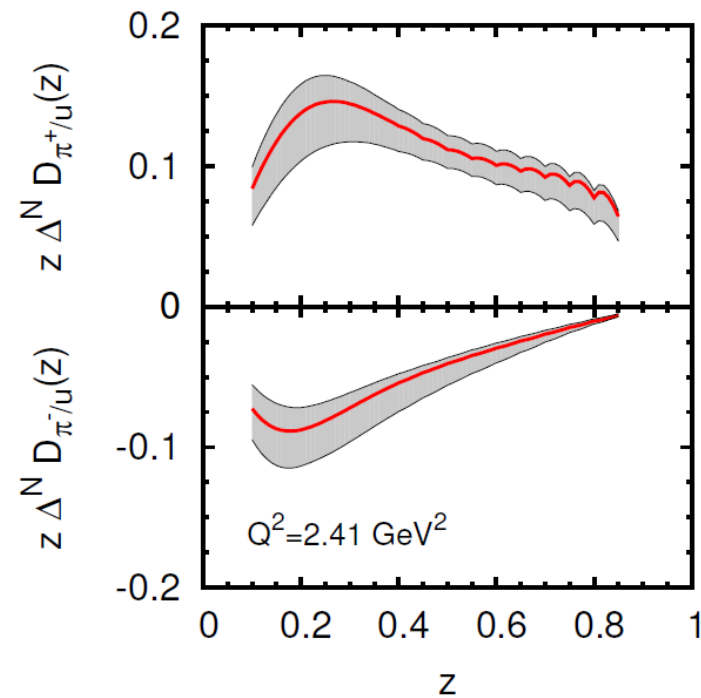
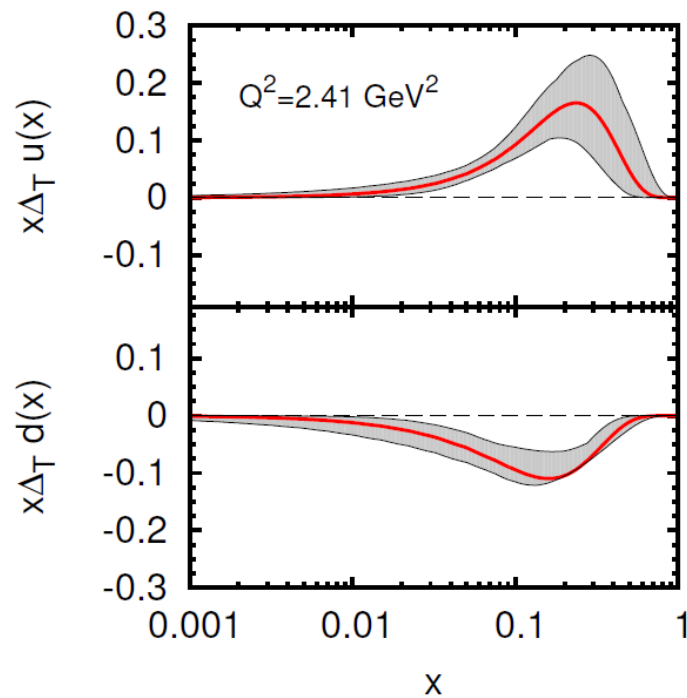
Extraction of transversity & Collins functions

➤ FIT IV: A_0 BELLE data UL & UC + COMPASS+ HERMES-POLYNOMIAL



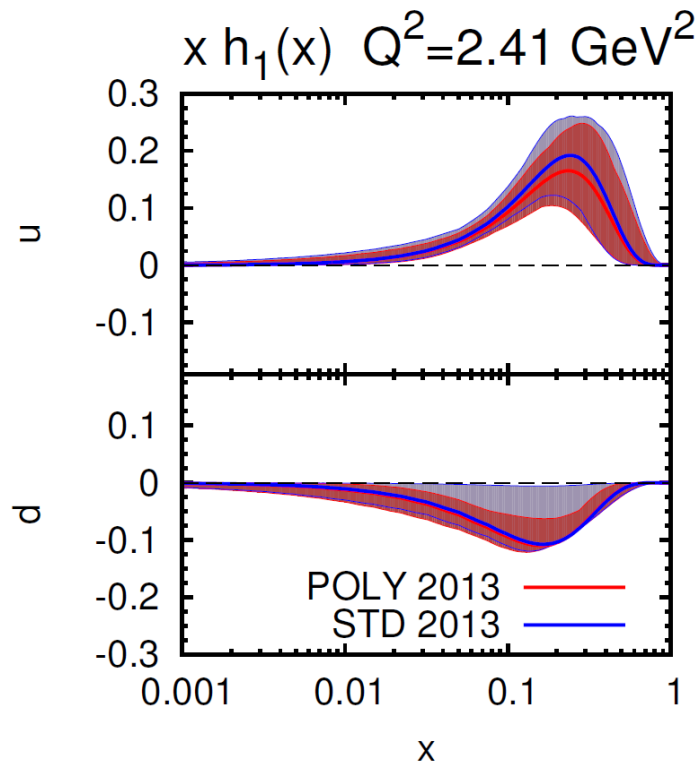
Extraction of transversity & Collins functions

- FIT IV: A_0 BELLE data UL & UC + COMPASS+ HERMES POLYNOMIAL

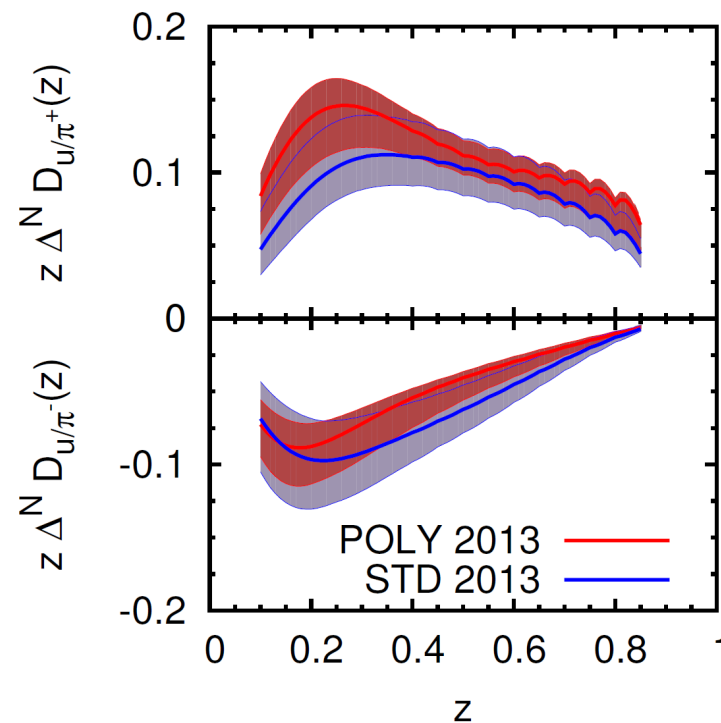


Extraction of transversity & Collins functions

➤ FIT II vs FIT IV (POLYNOMIAL vs STD; FITTED A_0)



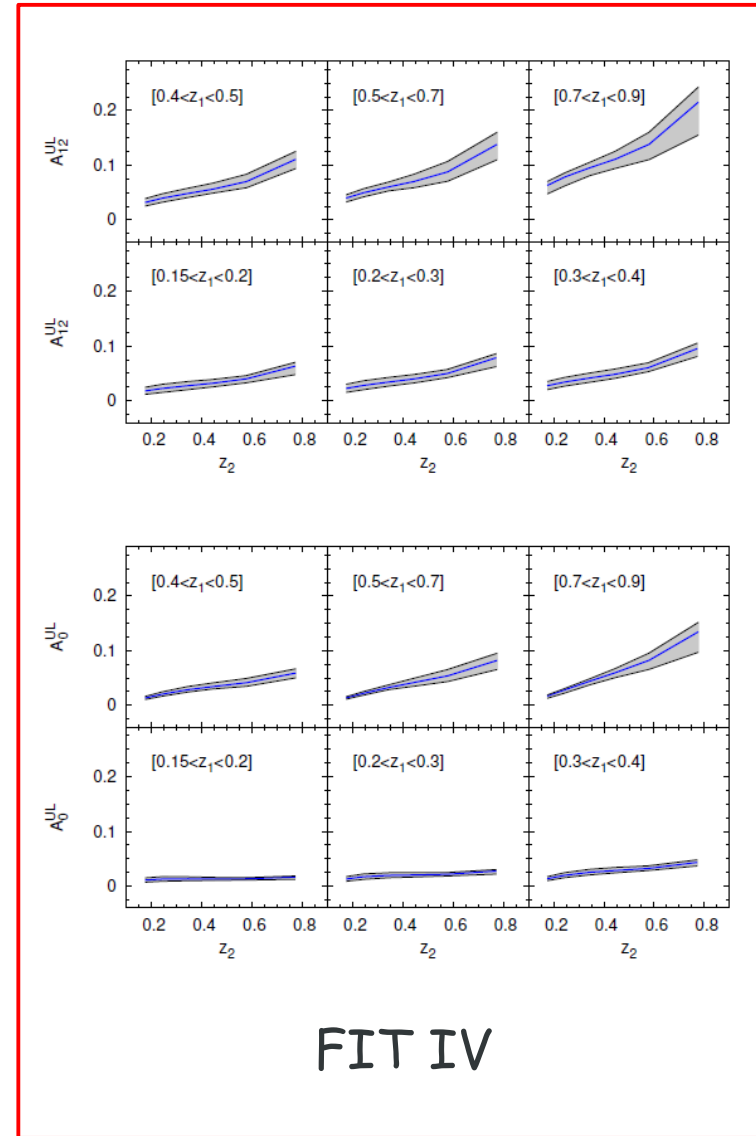
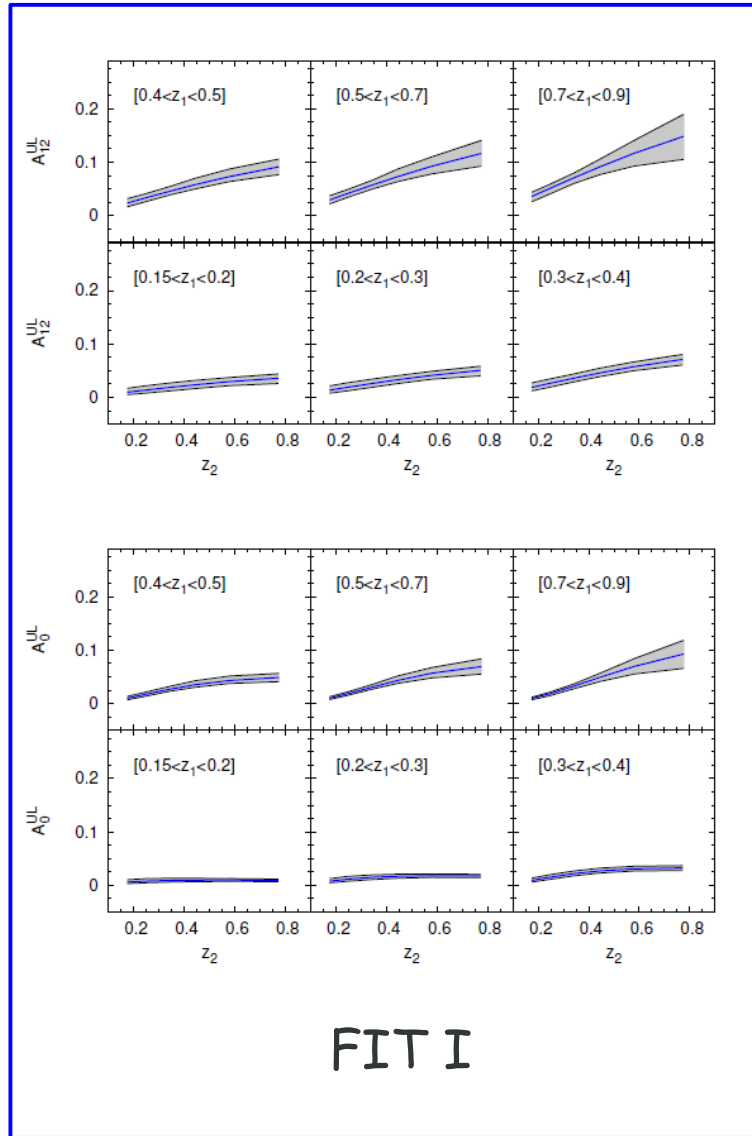
$$z \Delta^N D(z) = 4z H_1^{\perp(1/2)}(z) \quad Q^2 = 2.41 \text{ GeV}^2$$



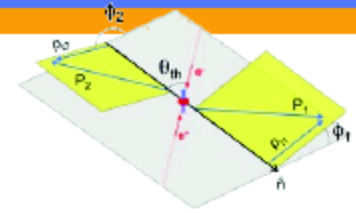
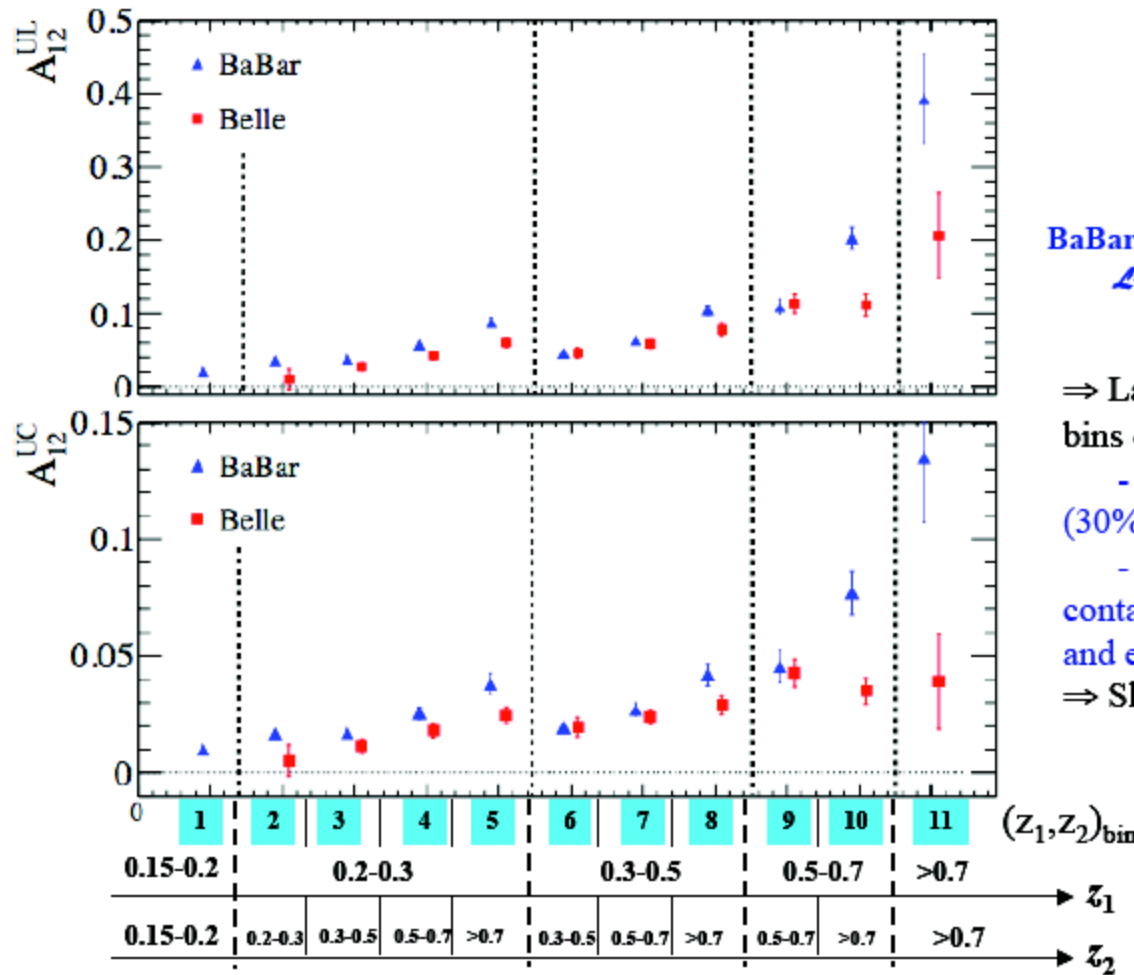
➔ Same transversity

➔ Different Collins functions (but not dramatically different)

BaBar Predictions



RF1 2: BaBar/Belle comparisons

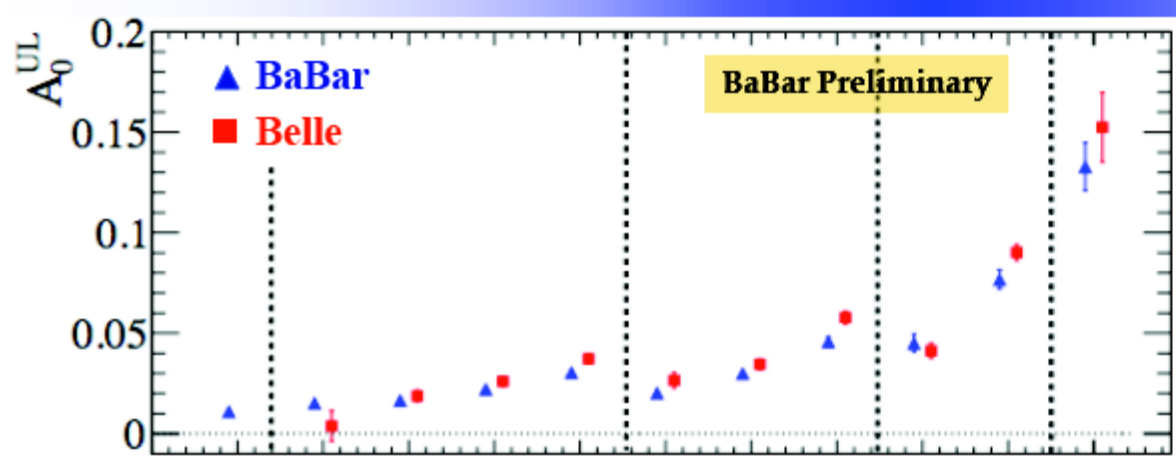
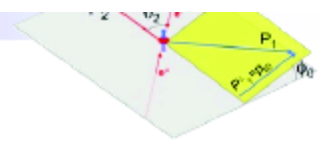


BaBar ($0.15 < z < 0.9$) $\mathcal{L} \sim 470 \text{ fb}^{-1}$
Belle ($0.2 < z < 1$) $\mathcal{L} \sim 547 \text{ fb}^{-1}$
 PRD 86, 039905(E) (2012)

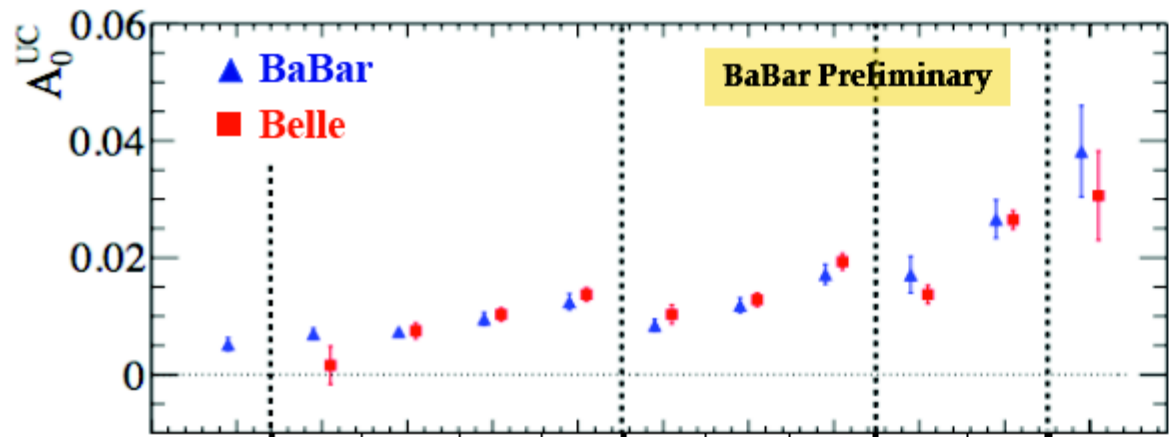
- ⇒ Large discrepancy in the last two bins of z :
 - bin-by-bin correction factors (30%)
 - $z < 0.9$ to remove the contamination from $\mu\mu\gamma$ background and exclusive events
- ⇒ Slightly higher at lower z

Conclusions

- New extraction of transversity and Collins functions
- UL and UC data fully compatible
- COMPASS & HERMES full compatible without TMD evolution
- A_{12} analysis in agreement with 2008 analysis
- A_0 is better described if we change parametrization
- Tension between A_{12} and A_0 sets
- BaBar predictions



BaBar ($0.15 < z < 0.9$) $\mathcal{L} \sim 470 \text{ fb}^{-1}$
Belle ($0.2 < z < 1$) $\mathcal{L} \sim 547 \text{ fb}^{-1}$
 PRD 78, 032011 (2008)

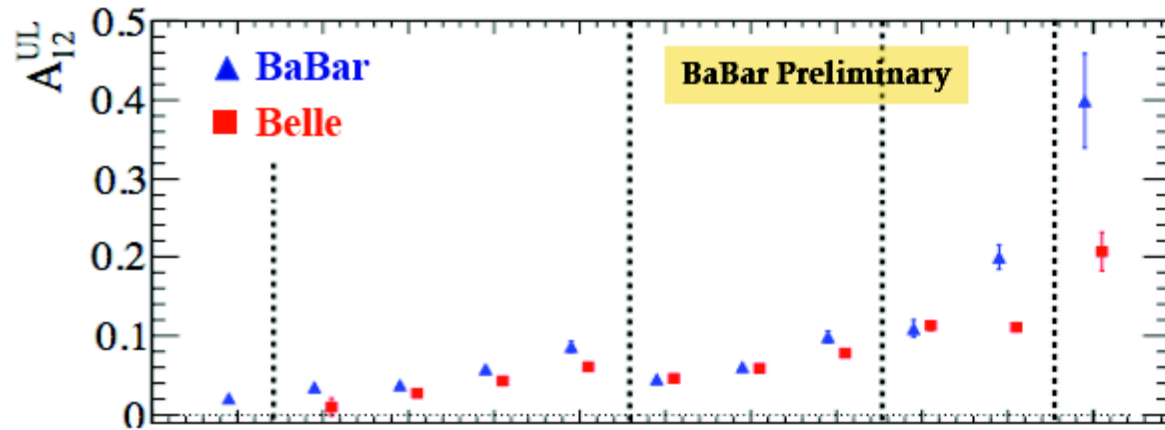
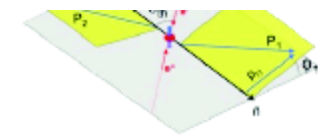


In order to perform this comparison, we used 10 (+1) symmetrized z -bin subdivisions, averaging the measured Belle and BaBar asymmetries which fell in the same symmetric bins

A_0^{UL} and A_0^{UC} : good agreement between the **BaBar asymmetries** and the **Belle results**.

1	2	3	4	5	6	7	8	9	10	11	$(z_1, z_2)_{\text{bin}}$
0.15-0.2	0.2-0.3	0.3-0.5	0.5-0.7	>0.7	0.3-0.5	0.5-0.7	>0.7	0.5-0.7	>0.7	>0.7	z_1
0.15-0.2	0.2-0.3	0.3-0.5	0.5-0.7	>0.7	0.3-0.5	0.5-0.7	>0.7	0.5-0.7	>0.7	>0.7	z_2

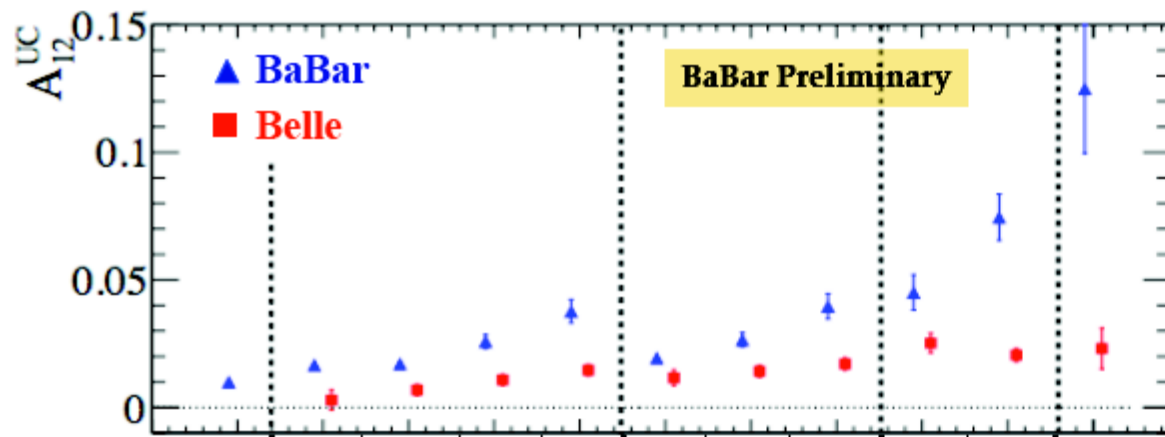




BaBar ($0.15 < z < 0.9$) $\mathcal{L} \sim 470 \text{ fb}^{-1}$
 Belle ($0.2 < z < 1$) $\mathcal{L} \sim 547 \text{ fb}^{-1}$
 PRD 78, 032011 (2008)

A_{12}^{UL} : large discrepancy in the last two bins of z

- bin-by-bin correction factors (30%)
- $z < 0.9$ to remove the contamination from $\mu\mu\gamma$ background and exclusive events



A_{12}^{UC} : BaBar asymmetry systematically above the Belle results for all z .

Belle analysts are investigating the source of discrepancies.

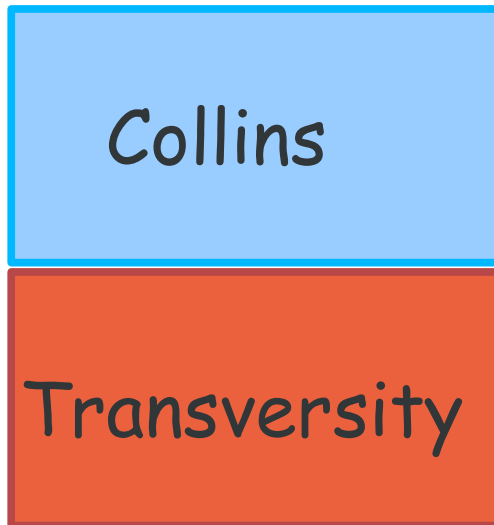
1	2	3	4	5	6	7	8	9	10	11	$(z_1, z_2)_{\text{bin}}$
0.15-0.2	0.2-0.3		0.3-0.5		0.5-0.7		>0.7				z_1
0.15-0.2	0.2-0.3	0.3-0.5	0.5-0.7	>0.7	0.3-0.5	0.5-0.7	>0.7	0.5-0.7	>0.7	>0.7	z_2



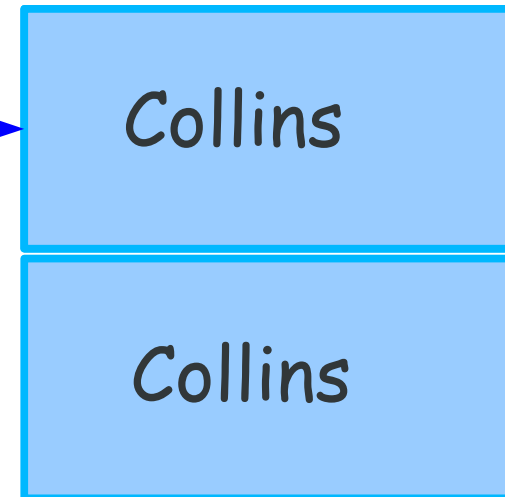
TMD evolution

- TMD evolution for the Collins function is still unknown.
- TMD evolution can suppress the Collins function at large Q^2 (Boer, 2001)

HERMES, COMPASS
 $Q^2=2.5-3.2 \text{ GeV}^2$



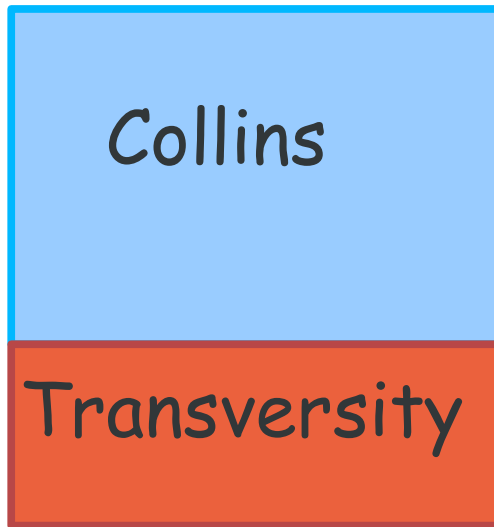
BELLE
 $Q^2=100 \text{ GeV}^2$



TMD evolution

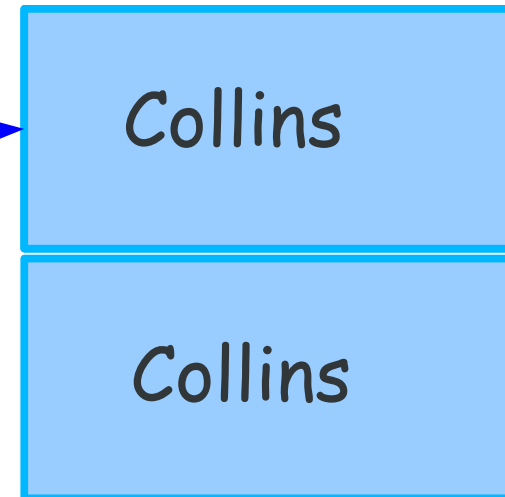
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- TMD evolution can suppress the Collins function at large Q^2
[D. Boer, Nucl. Phys. B603 (2001); Nucl. Phys. B806 (2009)]

HERMES, COMPASS
 $Q^2=2.5-3.2 \text{ GeV}^2$



TMD evolution??

BELLE
 $Q^2=100 \text{ GeV}^2$



The dihadron way: Pavia group extraction

➤ Comparison Pavia 2012-Torino 2008

