Resummation of clustering logarithms for non-global QCD observables

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DIS 2013
Marseille, 25th April 2013

Jets shapes at hadron colliders

Jet shapes are of vital importance for LHC studies:

- exploting jet substructure for boosted heavy-particles studies
- testing and tunning Monte Carlos
- extracting and confirming QCD parameters (perturbative and non-perturbative)

Numerical MC estimates have been very handy, but analytical estimates also:

- predict dependence on jet algorithms (thus the concept of optimal jet algorithm and jet parameters)
- give confidence that higher-order effects are under control

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Analytical jet-shape distributions – the state of the art and problems

Event/Jet shapes received substantial progress:

- Perturbative aspects
  - up to N^{3}LL accuracy for resummation for global event shapes
  - matching up to to N^{3}LO fixed-order e.g. Abbate et al 2011, Chien et al 2010, Becher et al 2008,...
- Non-perturbative aspects see e.g. Dasgupta et al 2007, 2009
  - analytical computation of such effects
  - disentangling various components

but non-global jet shapes still suffer from problems:

- non-global logs only resummable numerically in the large-N_c limit Dasgupta and Salam 2002
- jet algorithms introduce "clustering logs"

We show here how to deal with clustering logs

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Clustering logs

Some history

- First calculation of non-global logs with jet clustering (k_t algorithm) in energy flow between gaps: effect of non-global logs reduced with k_t clustering — Appleby and Seymour, 2002
- However extra primary-emission (clustering) logs emerge when clustering is imposed. These were resummed numerically. Banfi and Dasgupta, 2005
- Analytical resummation of clustering logs for away from jet energy flow was performed. Non-global logs recalculated and found significantly reduced — Delenda, Appleby, Dasgupta and Banfi, 2006
- Recently a flurry of papers in SCET interested in resumming clustering logs — e.g. Kelley, Walsh and Zubrei, 2012

Clustering logs in energy flows vs. jet shapes

Why bother when resummation for E_t flow exists?

For energy flow into gaps between jets:

- collinear emissions to jets do not matter (to SL accuracy)
- thus no double logs, only single logs (SL)
- resummation resulted in a power series in R in the exponent: fast convergence of the series
- sufficient to compute the first few terms in the R-series

For jet shapes

- collinear emissions change the jet mass! double logs present
- may not have the R-series as in the E_t flow case (as we shall see)

How do different algorithms affect the jet mass distribution?

How does the dependence on the jet radius enter it?

Jet algorithms in the soft-gluon approximation

Sequential recombination algorithms

`Iterate until all objects are removed`

- `define` $d_{ij} = \min\left(k_{ti}^p, k_{tj}^p\right)\left(\delta\eta_{ij}^2 + \delta\phi_{ij}^2\right)$; $d_{iB} = k_{ti}^p R^2$.
- `search for smallest of all distances,` $d_{\min}$.
- `if` $d_{\min} = d_{iB}$, `object` i `is a jet and is removed.`
- `if` $d_{\min} = d_{ij}$, `objects` i `and` j `are merged.`

- $p = -2$ for anti-k_t algo $\Rightarrow$ clustering starts with hardest.
- $p = +2$ for k_t algo $\Rightarrow$ clustering starts with softest.
- $p = 0$ for Cambridge/Aachen algo $\Rightarrow$ clustering starts with geometrically closest.

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Jet algorithms in the soft-gluon approximation

SISCone algorithm

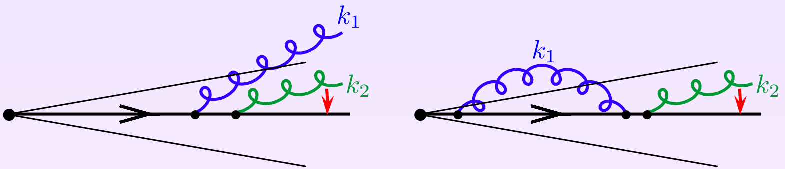
- Search for all stable cones of radius R (in a seedless way) [stable cone is one which points in same direction as 4-momentum of its contents].
- Resolve overlaps between jets with a split/merge procedure with overlap parameter f.

Clustering depends on split-merge proceedure.

How do jet algorithms affect the jet shape?

Primary emissions

C.f. the primary emission of two gluons off the hard (Q) initiating quark ($k_2 \ll k_1 \ll Q$) with $\theta_{12} < \theta_{2j} < R < \theta_{1j}$.



anti-k_t: complete real-virtual cancellation
no clustering logs

Clustering logs exponentiate just like Sudakov-type logs.

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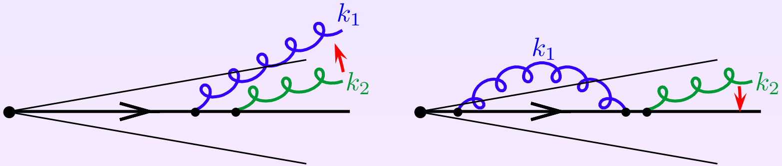

k_t, CA, SISCone algorithms: real-virtual mismatch
$\Rightarrow C_F^2 \alpha_s^2 L^2$ clustering logs

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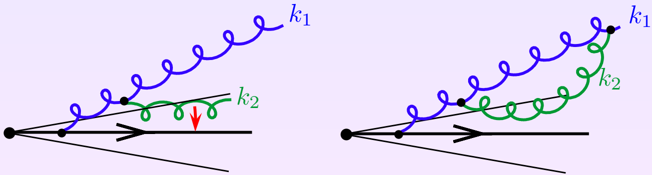
Secondary non-global emissions

C.f. the secondary emission of a softest gluon k_2 off another soft one k_1 ($k_2 \ll k_1 \ll Q$) with $\theta_{12} < \theta_{2j} < R < \theta_{1j}$



anti-k_t: real-virtual mismatch
maximum non-global contribution

Employing k_t, CA, SIScone algorithms reduces NG logs

but introduces new Abelian C_F^2 logs (relative to anti-k_t)

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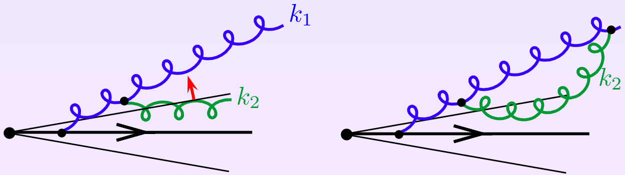

k_t, CA, SISCone: real-virtual cancellation
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How do jet algorithms affect the jet shape?

Optimal jet algorithms and jet radii

Current practice (in phenomenology community):

- avoid non-global logs by studying global event shapes
- use anti-k_t to get rid of clustering logs for non-global shapes
  e.g. Kang et al 2013, Chien et al 2012

however we disfavour use of anti-k_t algorithm: In the k_t algorithm:

- non-global logs have a small impact for large jet radii.
- watch for contamination with NP effects: favour small jet radii [underlying event $\sim R^2$, hadronisation $\sim 1/R$].
  Dasgupta Magnea and Salam, 2008
- hadronisation effects smaller for k_t than for anti-k_t algorithm.
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- must tune for optimal jet radii to minimise both perturbative and non-perturbative uncertainties [moderate jet radii?].

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We choose[1] the normalised invariant jet mass:

$$\rho = \left(\sum_{j\in\text{jet}} p_j\right)^2 / \left(\sum_i E_i\right)^2$$

j over particles in the measured jet; i over all particles.

We study the normalised single inclusive jet mass integrated distribution:

$$\Sigma(R^2/\rho) = \int_0^\rho \frac{1}{\sigma}\frac{d\sigma}{d\rho'}d\rho'$$

R: jet radius

For simplicity we consider $e^+e^- \to 2\,\text{jets}$. [work in progress for jet $+Z$ and 2 jets at hadron colliders].

[1] our work holds for other jet shapes

Jet mass distribution

Normalised invariant jet mass integrated distribution:

The NLL resummed integrated jet mass distribution is written as:

$$\Sigma^{\text{algo}} = \Sigma_{\text{glob}} \cdot \mathcal{S}_{\text{ng}}^{\text{algo}} \cdot \mathcal{C}_{\text{clus}}^{\text{algo}}$$

Σ_{glob} is the global part (algorithm-independent):

$$\Sigma_{\text{glob}} = \exp\left[Lg_1(\alpha_s L) + g_2(\alpha_s L)\right]$$

- $Lg_1(\alpha_s L)$ resums leading (double) logs $[L = \ln \frac{R^2}{\rho}]$.
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$\mathcal{S}_{\mathrm{ng}}^{\mathrm{algo}}$ is the non-global part:

- resums non-Abelian non-global single-logs numerically in the large-N_c limit.
- depends on the jet algorithm in use.
- effect is maximum for anti-k_t algorithm
- impact is significantly reduced for k_t, CA, SISCone algorithms [good!- less uncertainties due to large-N_c approx.]

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$\mathcal{C}_{\text{clus}}^{\text{algo}}$ is the clustering-induced part:

$$\mathcal{C}_{\text{clus}}^{\text{algo}} = \exp\left[\sum_{n\geq 2} \frac{1}{n!}\,\mathcal{F}_n^{\text{algo}}\left(-2\,C_F\,t\right)^n\right]$$

where $t = -\frac{1}{4\pi\beta_0}\ln(1-\alpha_s\beta_0 L)$, $L = \ln\frac{R^2}{\rho}$

- resums Abelian clustering-induced logs
- **In the anti-k_t algo:** no clustering logs: $\mathcal{C}_{\text{clus}}^{\text{anti}-k_t} = 1$

Clustering-logs coefficients

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The clustering-logs coefficients $\mathcal{F}_n^{\mathrm{algo}}$ are algorithm-dependent.
They are integrals over rapidity and azimuth with corresponding algorithm-dependent phase-space (Ξ^{algo}):

$$\mathcal{F}_n^{\mathrm{algo}}(R) = \frac{1}{\pi^n} \int \prod_i^n d\eta_i d\phi_i \frac{1}{\cosh^2 \eta_i - \cos^2 \phi_i} \Xi_n^{\mathrm{algo}}(k_1, k_2, \cdots, k_n)$$

R	0.1	0.4	0.7	1.0
$\mathcal{F}_2^{k_t,\mathrm{CA},\mathrm{SISCone}}$	0.183	0.184	0.188	0.208
$\mathcal{F}_3^{k_t}$	-0.052	-0.053	-0.055	-0.061
$\mathcal{F}_3^{\mathrm{CA}}$	-0.028	-0.029	-0.029	-0.030
$\mathcal{F}_3^{\mathrm{SISCone}}$ [preliminary]	0.033	0.034	0.037	0.060
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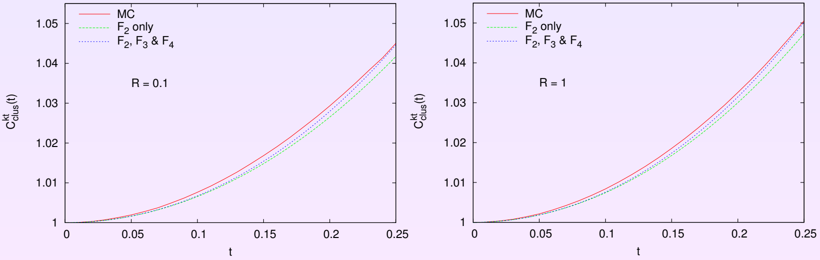
Clustering logs resummed: analytical vs. Monte Carlo

Comparison of the clustering part $\mathcal{C}^{k_t}_{\mathrm{clus}}$ in the k_t algorithm to MC

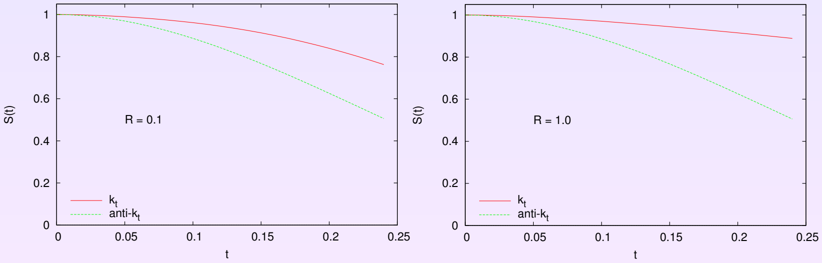
MC originally by Dasgupta and Salam, with modifications by Appleby, Seymour; and Banfi [private communication].



- small contribution of maximum order 5%
- largely dominated by $\mathcal{F}_2^{k_t}$ term
- uncalculated higher-order coeffs $\mathcal{F}_n^{k_t}$ have negligible impact

Non-global logs in k_t vs. anti-k_t

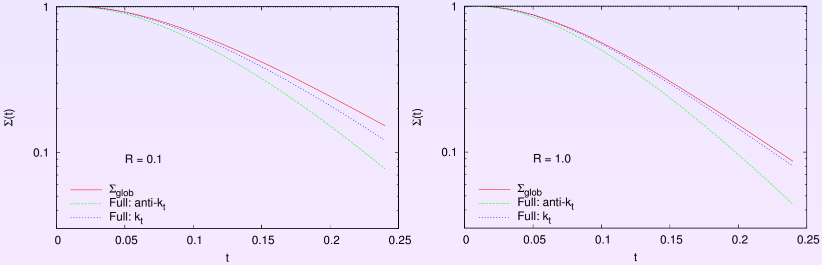
Comparison of the non-global components in the k_t and anti-k_t algorithms



- **anti-k_t algorithm**: coefficients of non-global logs are radius-independent – comparable to hemisphere mass in e^+e^- [untrue for hh collisions! – Dasgupta et al, 2012]
- **k_t algorithm**: non-global logs smaller in effect; and for $R \sim 1$ non-global logs significantly diminished.

Full distributions in MC

Comparison of the full distributions in the k_t and anti-k_t algorithms



Overall effect for $R = 1.0$:

- **k_t algorithm**: clustering and non-global logs (all together) modify the global part by a small $\mathcal{O}(7\%)$.
- **anti-k_t algorithm**: non-global logs modify the global part by a huge $\mathcal{O}(50\%)$.

Conclusions and Outlook

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- Non-global logs are a significant effect in the anti-k_t algorithm, but no clustering logs.
- Both non-global and clustering logs have a small contribution in k_t algorithm.
- Resummation of clustering logs available both for k_t, CA and SISCone algorithms.
- Impact of clustering logs: SISCone $<$ CA $<$ k_t.
- Work in progress for jet shapes at hadron colliders ($hh \to$ Z+jet and $\to$ 2 jets).
- Implementation of, at least, CA and maybe SISCone in Monte Carlo code is a future task.

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