On the difference between the pole and the $\overline{\mathrm{MS}}$ masses of the top quark at the electroweak scale

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- Motivation.
- The Concept of Quark Masses (QED/QCD)
- Standard Model and Masses
- Renormalization Group Equations (RGE) in Standard Model for the Masses.
- EW corrections to the ratio between pole and $\overline{\text{MS}}$ masses of top-quark.
- Summary

Motivation

November 2012,

according to CMS,

$$M_H = 125.8 \pm 0.4(\mathrm{stat}) \pm 0.5(\mathrm{syst})\mathrm{GeV} \ ,$$

according to ATLAS

$$M_H \ = \ 126.0(\mathrm{stat}) \pm 0.5(\mathrm{syst})\mathrm{GeV} \ ,$$

$$M_H \ = \ 125.2(\mathrm{stat}) \pm 0.5(\mathrm{syst})\mathrm{GeV} \ ,$$

Is there a new scale (physics) between the Fermi and Plank scales?

Self-consistency of the SM

The condition of stability of the SM vacuum (Bezrukov et al., May 13, 2012)

$$M_H \geq 129 \pm 6 \ \text{GeV} , \quad \text{Bezrukov et al.,}$$

$O(\alpha\alpha_s)$ matching (Bezrukov et al.,) 3-loop RGE (Chetyrkin-Zoller)

The leading contribution $O(M_t^4)$,$O(M_t^2 M_H^2)$ and $O(M_H^4)$, (Degrassi et al., May 29, 2012) with the result

$$M_H \geq 129.4 \pm 1.8 \ \text{GeV} , \quad \text{Degrassi et al.,}$$

Carefull analysis of uncertainties (Alekhin,Djouadi,Moch, July 17, 2012)

$$M_H \geq 129.4 \pm 5.6 \ \text{GeV} , \quad \text{Alekhin et al.,}$$

How to improve the stability analysis?

- The vacuum stability bound sensitively depends on the input parameters - the value of top-quark mass M_t.
- What mass ot top-quark is implemented in MC?
- To extract the value of top-quark from top-pair production, the QCD and EW parts should be joint together in one scheme, $\overline{\mathrm{MS}}$ or on-shell.

  Kühn, Scharf, Uwer (2007) Langenfeld, Moch, Uwer, (2009)
- Pole mass parametrization leads to artificially large perturbative corrections.
- $\overline{\mathrm{MS}}$ mass is preferable for precision fits.
- EW physics in $\overline{\mathrm{MS}}$ parametrization?

QCD

$$L_{\rm matter} = \sum_{\rm flavors \quad q} \bar{q} \left(i\hat{D} - m_q \right) q$$

Pole mass is defined as pole of renormalization propagator (pole scheme).
The position of the pole is gauge-invariant and infrared-finite quantity
Tarrach (1981); Kronfled (1998).
Pole mass suffers from renormalon contribution with uncertainty of order of
$\Lambda_{QCD} \sim 200\ MeV$ Bigi, Shifman,Uraltsev,Vainstein,Beneke,Braun (1994)
The $\overline{\rm MS}$ mass (running mass) is the renormalized quark mass in the MS-like scheme. Within dimensional regularization, the running mass is related with UV-counter-terms.
Appelquist-Carazzone decoupling theorem is valid.

In $\overline{\rm MS}$ -scheme the decoupling is performed "by hand".

Electroweak Part of Standard Model: $SU(2) \times U(1)$

Before Spontaneously Symmetry Breaking (SSB)

$$L_{\text{Yukawa}} = y_t \bar{q}_L \phi q_R + c.c.$$

where ϕ is a scalar field.

After SSB: $\phi = \frac{1}{\sqrt{2}} v + H$ and

$$L_{\text{fermion mass}} = \frac{y_t v}{\sqrt{2}} \bar{q}_L q_R + y_t H \bar{q}_L q_R + c.c.$$

Mass of quark is: $y_t \times v$.

Position of the pole is a gauge invariant and infrared finite quantity

Gambino & Grassi '00

Appelquist-Carazzone decoupling theorem is not valid:

ratio of two different masses is the ratio of coupling constants.

Renormalization Group Equation for $\overline{\rm MS}$ masses in SM

The RGE for the $\overline{\rm MS}$ masses in the broken phase of SM follows from the RG equations for massive parameter and massless coupling constants in unbroken phase of SM.

$$
\begin{aligned}
\mu^2 \frac{d}{d\mu^2} v^2(\mu^2) &= 4\,\mu^2 \frac{d}{d\mu^2}\left[\frac{m_W^2(\mu^2)}{g^2(\mu^2)}\right] = 4\,\mu^2 \frac{d}{d\mu^2}\left[\frac{m_Z^2(\mu^2) - m_W^2(\mu^2)}{g'^2(\mu^2)}\right] \\
&= 3\,\mu^2 \frac{d}{d\mu^2}\left[\frac{m_H^2(\mu^2)}{\lambda(\mu^2)}\right] = 2\,\mu^2 \frac{d}{d\mu^2}\left[\frac{m_f^2(\mu^2)}{y_f^2(\mu^2)}\right] \\
&= v^2(\mu^2)\left[\gamma_{m^2} - \frac{\beta_\lambda}{\lambda}\right],
\end{aligned}
$$

g', g are the $U(1)_Y$ and $SU(2)_L$ gauge couplings. m^2 and λ are the parameters of the scalar potential

$$
V = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4
$$

.

RGE for top-quark mass in SM

The full RGE for the $\overline{\mathrm{MS}}$ mass of top-quark can be written as

$$\mu^2 \frac{d}{d\mu^2} \ln m_t(\mu^2) = \gamma_t^{QCD} + \frac{1}{y_t}\left(\mu^2 \frac{d}{d\mu^2} y_t\right) + \frac{1}{2}\left(\mu^2 \frac{d}{d\mu^2} \ln m^2\right) - \frac{1}{2}\left(\frac{1}{\lambda}\mu^2 \frac{d}{d\mu^2}\lambda\right) ,$$

y_t is the Yukawa coupling of quark,
m^2 and λ are the parameters of the scalar potential

$$V = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$$

Behavior of RGE at high energies

For large values of μ^2:

$$\lim_{\mu\to M_{Plank}} \left| \frac{\beta_{y_t(\mu^2)}}{y_t(\mu^2)} \right| < \infty \,, \qquad \lim_{\mu\to M_{Plank}} m^2(\mu^2), \ln m^2(\mu^2) < \infty \,,$$

the values of running parameters $m^2(\mu^2)$ and $y_t(\mu^2)$ are bounded.

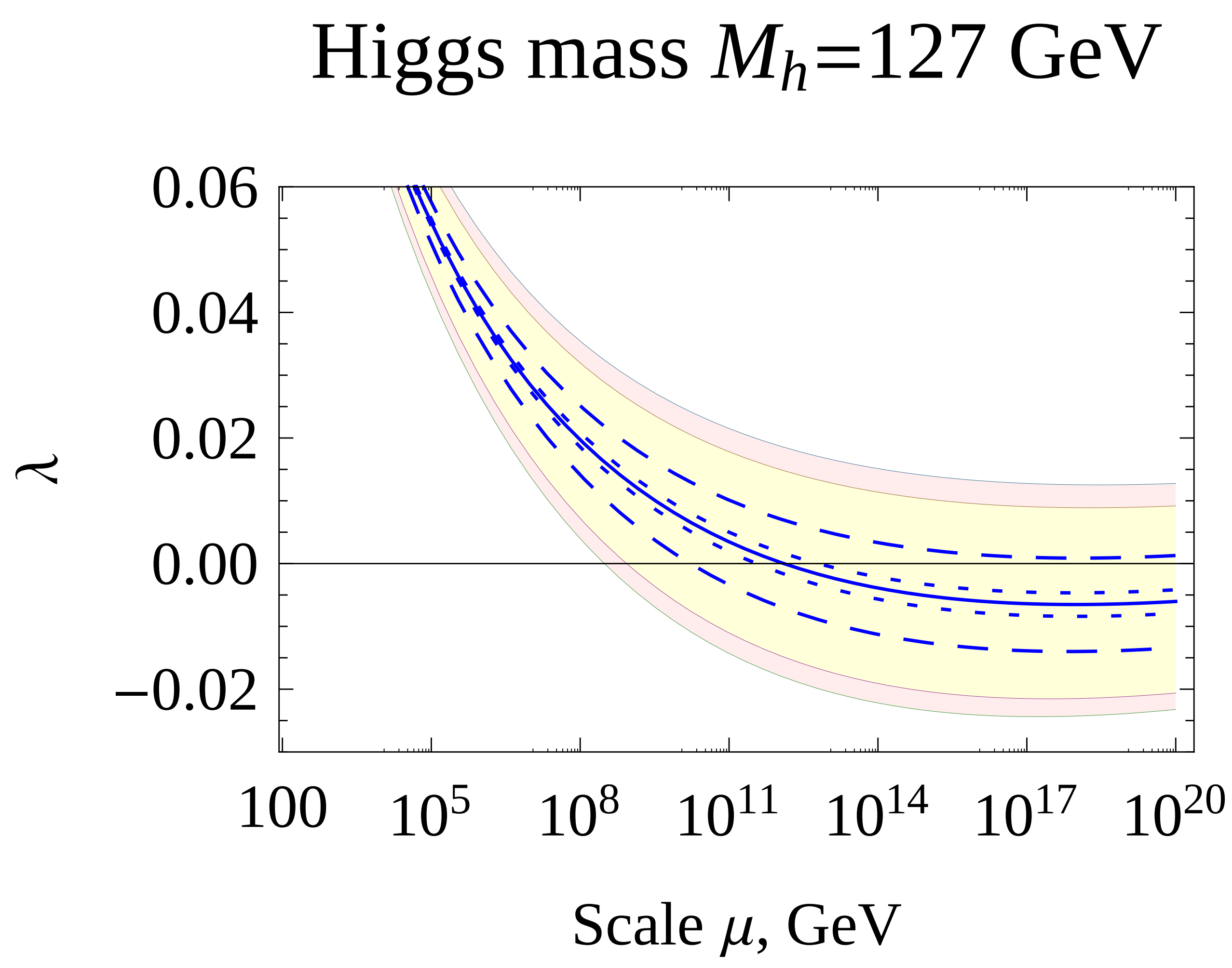
$$\lim_{\mu\to M_{Plank}} \beta_\lambda(\mu^2) \to 0 \,, \qquad \lim_{\mu\to M_{Plank}} \lambda(\mu^2) \to 0 \,.$$

The behavior of γ_t is defined by the Higgs self-coupling and the sign of its beta-function β_λ:

$$\lim_{\mu\to M_{Plank}} \mu^2 \frac{d}{d\mu^2} \ln m_t(\mu^2) \sim \frac{\beta_\lambda}{\lambda} \,, \quad \beta_\lambda < 0, \quad \lambda > 0$$

If vacuum is stable (SM is valid up to Plank scale), then

$$m_t(\mu^2)\Big|_{\mu\to\infty} \sim \left(\mu^2\right)^{\frac{\beta_\lambda(\mu^2)}{\lambda(\mu^2)}} \to 0.$$


Higgs mass M_h=127 GeV
0.06
0.04
0.02
0.00
−0.02
λ
100
10^5
10^8
10^{11}
10^{14}
10^{17}
10^{20}
Scale μ, GeV

Behavior of RGE at low energies

$$\mu^2 \frac{d}{d\mu^2} \ln m_t(\mu^2) = \gamma_t^{QCD}$$
$$+\frac{1}{y_t}\left(\mu^2 \frac{d}{d\mu^2} y_t\right) + \frac{1}{2}\left(\mu^2 \frac{d}{d\mu^2} \ln m^2\right) - \frac{1}{2}\left(\frac{1}{\lambda}\mu^2 \frac{d}{d\mu^2}\lambda\right) ,$$

The crucial question:

$$\mu^2 \frac{d}{d\mu^2} \ln \left(\frac{m^2}{\lambda(\mu^2)}\right)\Bigg|_{\mu \leq M_Z, 2M_W, M_H, M_t, 4M_t} = \text{constant or not?}$$

- If it is constant.

$$m_t(\mu^2) = 2^{-3/4} G_\mu^{-1/2} y_t(\mu^2) .$$

fermion masses and Yukawa couplings have equivalent RG evolution.

- It is non-constant.

$$m_t(\mu^2) = 2^{-3/4} G_\mu^{-1/2}(\mu^2) y_t(\mu^2) .$$

Decoupling at low scale in SM: is it exist?

In the weak sector of the SM, there is no decoupling because masses and couplings are interrelated by the Higgs mechanism.
So "decoupling by hand" as usually applied in QCD does not make sense in the weak sector.
The low-energy effective theory (obtained after elimination of the heavy state) is a non-renormalizable one exhibiting a completely wrong high-energy behavior.
The well-known non-decoupling effects:

$$\rho_{\rm eff}(0) = G_{\rm NC}/G_{\rm CC}(0) = 1 + \frac{N_c G_\mu}{8\pi^2\sqrt{2}} m_t^2$$

Input

Decoupling does not apply in the EW sector of SM, in particular not to the top quark mass effects.

For numerical evaluation we adopt the following values for the input parameters:

$$
\begin{aligned}
M_Z &= 91.1876(21)\ \text{GeV}, \quad M_W = 80.385(15)\ \text{GeV}, \\
M_t &= 173.5(1.0)\ \text{GeV}, \quad G_F = 1.16637 \times 10^{-5}\ \text{GeV}^{-2}, \\
\alpha^{-1} &= 137.035999, \quad \alpha^{-1}(M_Z^2) = 127.944, \\
\alpha_s^{(5)}(M_Z^2) &= 0.1184(7) \to \alpha_s^{(6)}(M_t^2) = 0.1079(6)
\end{aligned}
$$

Light quark masses M_f $(f \neq t)$ give negligible effects and do not play any role in our consideration.

$m_t - M_t$: QCD part

The QCD relation between the running and pole masses is

$$\left\{ m_t(M_t^2) - M_t \right\}_{\rm QCD} = M_t \left[-\frac{4}{3} \frac{\alpha_s^{(6)}(M_t^2)}{\pi} - 9.125 \left(\frac{\alpha_s^{(6)}(M_t^2)}{\pi} \right)^2 - 80.405 \left(\frac{\alpha_s^{(6)}(M_t^2)}{\pi} \right)^3 \right] .$$

Gray et al., (1990), Melnikov & Ritbergen; Chetyrkin & Steinhauser (2000).

$$\left\{ m_t(M_t^2) - M_t \right\}_{\rm QCD} = -7.95 \text{ GeV} - 1.87 \text{ GeV} - 0.57 \text{ GeV} = -10.38 \text{ GeV} .$$

A numerical estimation of the $O(\alpha_s^4)$ term is
A. Kataev & V. Kim (2010) ~ -0.02 GeV.

$m_t - M_t$; EW contribution

- $O(\alpha)$
  Fleischer,Jegerlehner (1981); Böhm et al.,(1986);
  Hempfling & Kniehl (1995);
- $O(\alpha\alpha_s)$ Jegerlehner,Kalmykov (2003)
  Numerical agreement with semi-numerical result of
  Eiras, Steinhauser (2006)
- $O(\alpha^2)$ The result is not available.
  In the approximation of vanishing electroweak gauge coupling
  Faisst et al., 2004; Martin, (2005).
  Different μ-dependence. In both papers, the result is of order $O(1)$ GeV.
- $O(\alpha^3), O(\alpha^2\alpha_s), O(\alpha\alpha_s^2)$ – Results are not available.

$m_t - M_t$: $O(\alpha^2)$ contribution: our estimation

At one-loop level, the largest contribution is coming from tadpole:

$$\left\{ \frac{m_t(\mu^2)}{M_t} \right\}^{O(\alpha)} \sim \left\{ \frac{m_t(\mu^2)}{M_t} \right\}^{O(\alpha)}_{\text{tadpole}} \sim \frac{3}{4} \frac{\alpha(\mu^2)}{\pi} \frac{M_t^4}{M_W^2 M_H^2} .$$

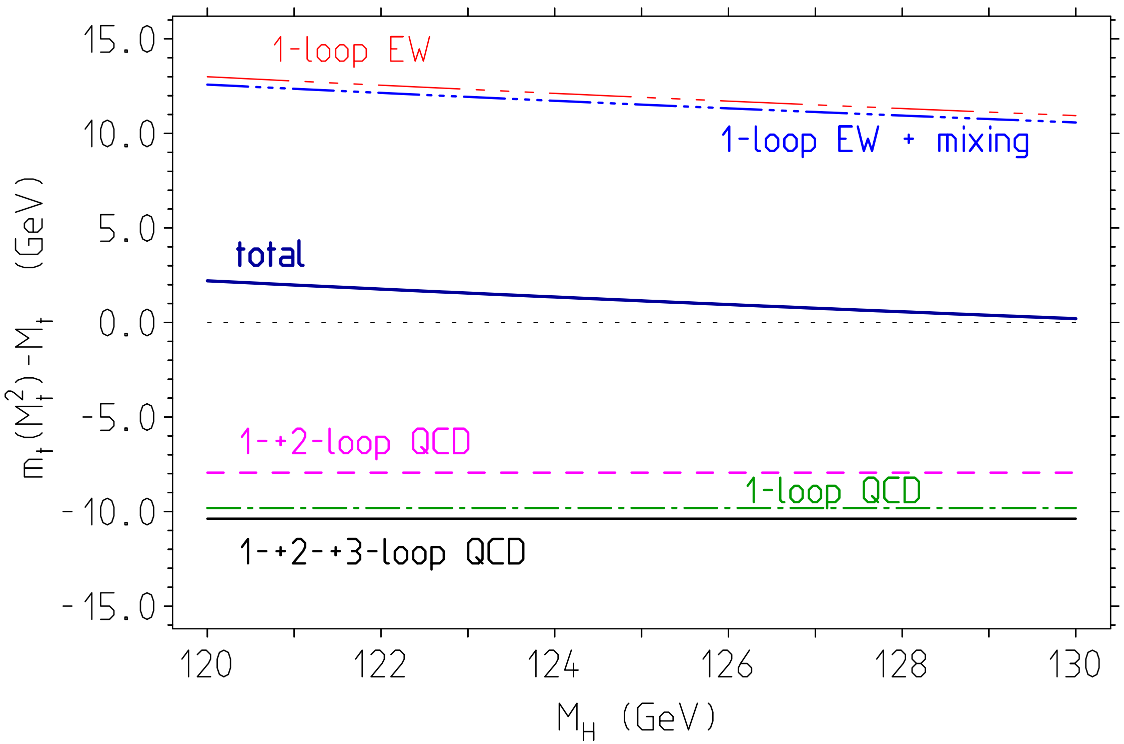
At 2-loop order $O(\alpha^2)$:

$$\left\{ \frac{m_t(\mu^2)}{M_t} \right\} \sim \left\{ \frac{m_t(\mu^2)}{M_t} \right\}_{\text{tadpole}} \sim \left\{ \sqrt{\frac{m_W^2(\mu^2)}{M_W^2}} \right\}_{\text{tadpole}} = \frac{1}{2} \left[\delta_{W,\alpha^2} - \frac{1}{4} \delta_{W,\alpha}^2 \right]$$

where

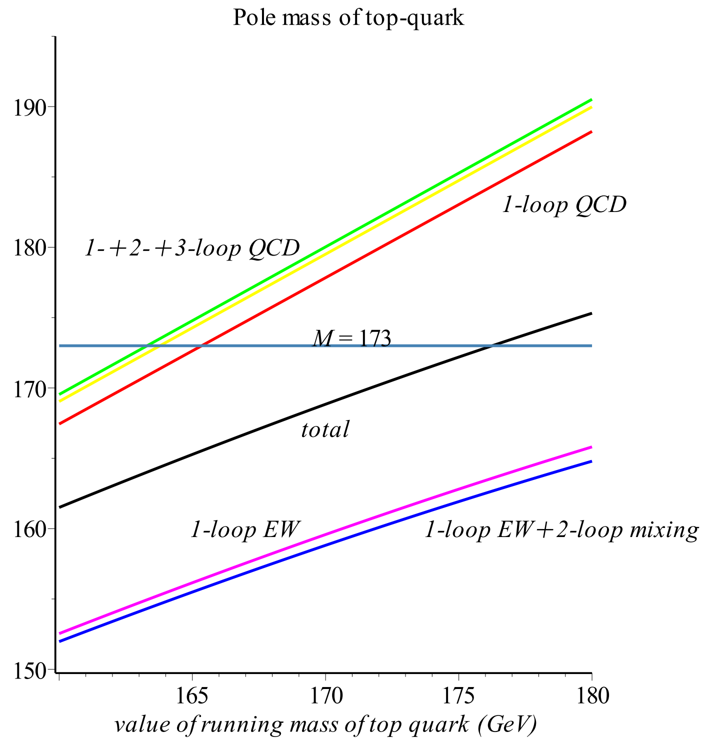
$$\frac{m_W^2(\mu^2)}{M_W^2} = 1 + \delta_{W,\alpha} + \delta_{W,\alpha^2} + \cdots$$ Jegerlehner,Kalmykov,Veretin(2001-2002)

Our estimation is $O(\alpha^2) \sim O(1)$ GeV



M_H [GeV]	$O(\alpha)$ [GeV]	$O(\alpha\alpha_s)$ [GeV]	$[m_t(M_t^2)-M_t]_{SM}$ [GeV]
124	12.11	-0.39	1.34
125	11.91	-0.39	1.14
126	11.71	-0.38	0.94

$$M_t - m_t(m_t)$$



Pole mass of top-quark
190
180
170
160
150
165
170
175
180
1-+2-+3-loop QCD
1-loop QCD
M = 173
total
1-loop EW
1-loop EW+2-loop mixing
value of running mass of top quark (GeV)

Conclusion

We have analysed the low/high scale behavior of $\overline{\text{MS}}$ mass of (top)-quark $m_t(\mu^2)$ defined via its propagator in the broken phase of SM.

For the large value of μ^2,

$$m_t(\mu^2)\Big|_{\mu\to\infty} \sim \left(\mu^2\right)^{\frac{\beta_\lambda(\mu^2)}{\lambda(\mu^2)}} \to 0.$$

For the low value of $\mu^2 \sim M_Z$, there is no decoupling or "decoupling by hand".

Result: the almost perfect cancellation between the QCD and EW effects for the Higgs boson mass $M_H \sim 125 - 126$ GeV.

$$M - m_t(m_t) \sim 1 \text{ GeV} \pm O(1) \text{ GeV}$$