Charm quark mass dependence in CTEQ global QCD analysis.

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We report on the study of the effect of the charm quark mass in the CTEQ global analysis of PDFs of the proton.
J.Gao, Pavel Nadolsky and M.G. arXiv:1304.3494[hep-ph]

We discuss physics assumptions behind S-ACOT-χ and differences compared to other schemes used in recent extractions of m_c:
see H1 and ZEUS Coll. EPJC (2013); Martin, Stirling, Thorne, Watt (2010); Alekhin, Bluemlein, Daum, Lipka, Moch (2013); Alekhin, Daum, Lipka, Moch (2012)

This is a part of the CTEQ-TEA collaboration efforts towards the CT1X NNLO global fit of PDFs of the proton.
(Nadolsky's talk in tuesday's parallel session)

also

(Joey Huston's talk PDF4LHC on Wed., April 17, CERN)
https://indico.cern.ch/conferenceDisplay.py?confId=244990

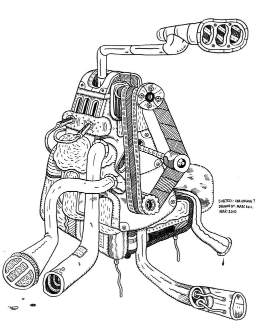
Why is m_c important?

- Global analysis of PDFs of the proton: sensitive to the method by which the heavy-quark masses are included in experiments, especially at $Q \approx m_c$.
- DIS experiments have the best potential to constrain m_c.
- Impact on the extracted PDFs non-negligible: modifications due to heavy-quark treatment have phenomenological consequences for EW precision measurements at the LHC.
- Motivated by H1-ZEUS measurements (2012), of comb. cross sections on incl. and semi-incl. DIS charm production at HERA, we examined constraints on the $\overline{\mathrm{MS}}$ mass of the charm quark in the S-ACOT-χ heavy-quark factorization scheme at NNLO.
- The value of m_c from the hadronic scattering data in the CT10 NNLO fit, including these new data, is found to agree, within the uncertainties, with the world average $\overline{\mathrm{MS}}$ value.

Which charm mass is constrained by DIS data?

$\overline{\mathrm{MS}}$ $m_c(m_c)$ or pole mass m_c?

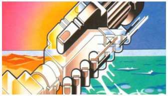
Depends on
heavy-quark scheme,
N_f in $\alpha_s(M_Z)$,
QCD order,
QCD scales,
switching point energies
$\Longrightarrow$



$\Longrightarrow$ m_c **?**

Which parameter dominates the most?

"Welcome to the **fitting** machine"



"$\Longrightarrow$ determine $m_c(m_c)$ including all uncertainties"

Can a VFN scheme be used to extract m_c?

In many previous PDF analyses, the heavy-quark masses have been treated as effective parameters rather than fundamental constants. They were anticipated to deviate from the $\overline{\mathrm{MS}}$ masses or even be fully independent.

Several heavy-quark factorization schemes
FFN, ACOT, BMSN, CSN, FONLL, TR'...

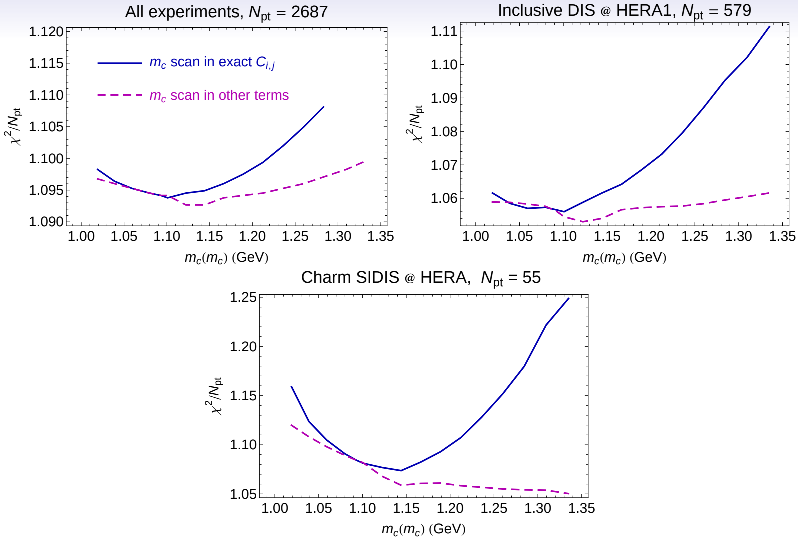
Different approximations in DIS calculations

- *reshuffling of m_c^2/Q^2 between coeff. for charm and coeff. for gluon*
- *differences in what it is called LO and NLO $\rightarrow$ relevant at low Q^2*
- *method of truncating the expansions is different*

Approximations affect extraction of the mass values from the hadronic data, but their effect is of a higher order in α_s according to the QCD factorization theorem. Thus, m_c extracted in all schemes converges to the m_c of the SM Lagrangian at high enough order.

Energy scales of order m_c in heavy-quark schemes

$\mathcal{O}(m_c)$ scales appearing FFN or VFN		
	is in FFN ?	is in VFN ?
M_c or $m_c(m_c)$ in exact $\gamma^* g \to c\bar{c}$ in NC DIS	$\surd$ **dominates**	$\surd$ dominates
switching scales in $\alpha_s(\mu)^{n_f} \to \alpha_s(\mu)^{n_f+1}$	$\surd$	$\surd$
switching scales in PDFs evolution	$\times$	$\surd$
kinem. approx. in FE coeff. func.	$\times$	$\surd$
scales in quark-fragmentation into hadrons	$\surd$	$\surd$



Dependence of χ^2/N_{pt} as a function of $m_c(m_c)$ in the exact flavor-creation coefficient functions (solid blue lines) and auxiliary energy scales listed in the text (dashed magenta lines).

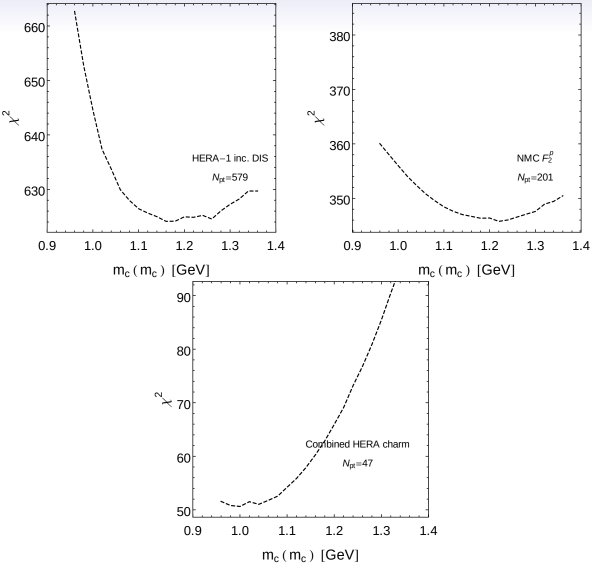
Details of the implementation

- $\overline{\mathrm{MS}}$ masses as the input in the whole calculation.
- OME $A_{ab}^{(k)}$: pole masses, we convert $\overline{\mathrm{MS}}$ by using 2-loop relation Chetyrkin et al. (2000)
- S-ACOT-χ GMVFN at 2-loop accuracy NNLO $\rightarrow$ exact flavor-creation terms; approx flavor-excitation terms

Advantages of the S-ACOT-χ scheme

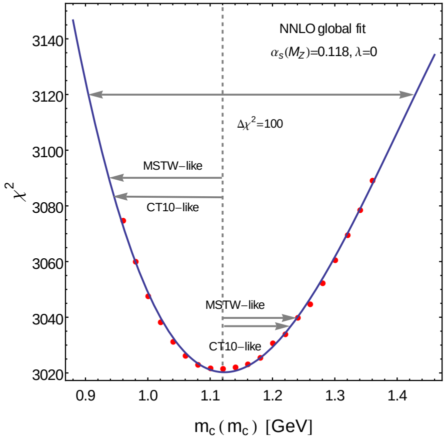
- It is proved to all orders by the QCD factorization theorem for DIS *(Collins, 1998); (M.G., Nadolsky, Lai, Yuan, 2011)*
- **Universal** PDFs
- One value of N_f (and one PDF set) in each Q range
- matching to FFN is controlled by a single parameter λ in our current implementation. The χ scaling (Tung, Kretzer, Schmidt, 2001; kinematically motivated) or no scaling (disfavoured) is obtained with $\lambda = 0$ and $\gg 1$, respectively.

Tightest constraints on m_c from data



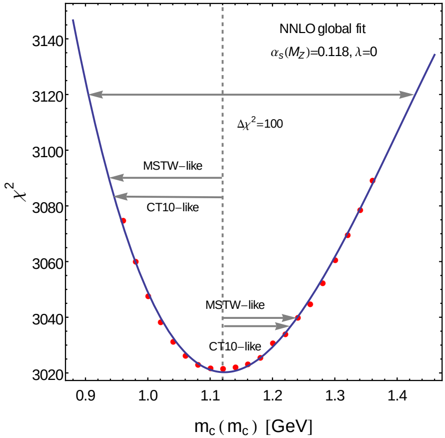
Comparable constraints are imposed by HERA incl. DIS data and combined HERA charm production data, weaker constraints from fixed-target DIS (NMC F_2^p,...)

Results



Global χ^2 of the S-ACOT-χ NNLO fit as a function of the $\overline{\mathrm{MS}}$ charm mass. Lines with left/right arrows indicate 90% C.L. intervals obtained with different tolerance criteria. Best-fit $m_c = 1.12$ GeV, depends on the parametric form for the small-x gluon PDF

Results



The PDF uncertainty (for fixed other assumptions) is of order $^{+0.11}_{-0.17}$ GeV according to either CT10 or MSTW-like convention. Larger than the total uncertainty of 0.05 GeV (PDF + ...) in recent extractions by Alekhin et al.

Estimate of PDF uncertainties

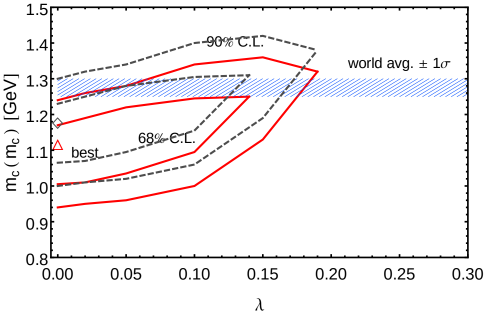
PDF uncertainty δm_c [GeV] (90% C.L.)	
$\Delta\chi^2 \leq 100$	$\delta m_c =^{+0.30}_{-0.22}$
CT10-like	$\delta m_c =^{+0.11}_{-0.17}$
MSTW-like	$\delta m_c =^{+0.12}_{-0.18}$

90% C.L. on $m_c(m_c)$ is defined by comparing 3 different criteria for defining the PDF uncertainty:

1. Uniform χ^2 tolerance: 90% C.L. to a $\Delta\chi^2 \leq 100$ variation as in the CTEQ6 PDF analysis.
2. CT10-like criterion, which supplements the uniform χ^2 tolerance condition by additional χ^2 penalties to prevent strong disagreements with individual experiments *on average*.
3. MSTW-like criterion: it does not introduce the uniform tolerance, but requires the χ^2 value for *every* individual experiment to lie within the specified confidence interval.

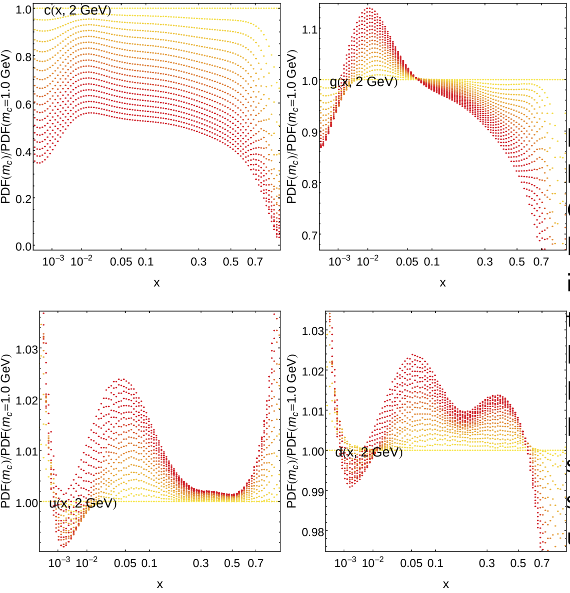
Results

Theor. sys. uncer.	DIS scale	$\alpha_s(M_Z)$	λ	χ^2 def.
Parameter range	$[Q/2,\ 2Q]$	$[0.116,\ 0.120]$	$[0,\ 0.2]$	–
$\delta m_c(m_c)$ (GeV)	$^{+0.02}_{-0.02}$	$^{+0.01}_{-0.01}$	$^{+0.14}_{-0}$	$^{+0.06}_{-0}$



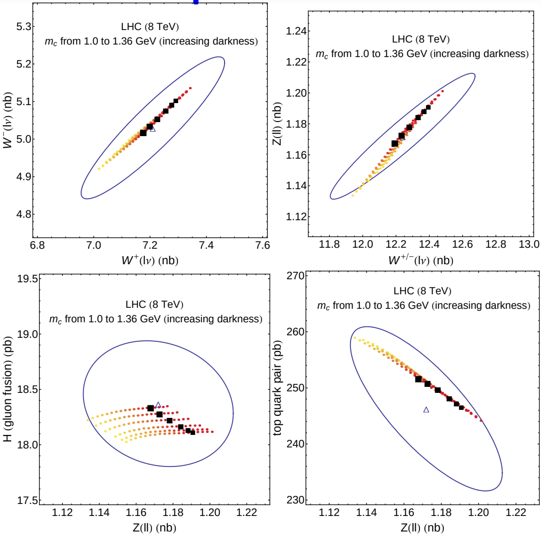
Preferred regions for $m_c(m_c)$ vs. the rescaling parameter λ. The best-fit values and confidence intervals are shown for two alternative methods for implementation of correlated systematic errors.

m_c dependence of PDFs



Relative changes in select PDFs $f_{a/p}(x, Q)$ at $Q = 2$ GeV obtained in a series of PDF fits with $m_c(m_c)$ ranging from 1 to 1.36 GeV, plotted as ratios to the respective PDFs for $m_c(m_c) = 1$ GeV. Default rescaling ($\lambda = 0$) in DIS coefficient functions is assumed. Darker colors correspond to larger $m_c(m_c)$ values.

Impact on LHC@8 TeV cross sections



LHC@14 TeV in the backup

Plot of NNLO cross sections for $W^{\pm}$, Z^0, Higgs boson production through gluon fusion, and top quark pair production at the LHC (8 TeV) for charm quark mass $m_c(m_c)$ ranging from 1 to 1.36 GeV and $\lambda = \{0, 0.02, 0.05, 0.1, 0.15, 0.2\}$. Darker color corresponds to larger mass values.

Black squares represent the cross sections evaluated by using $m_c(m_c) = 1.28$ GeV $\Longrightarrow$ **smaller uncertainty**.

The triangle and ellipse indicate the central predictions and 90% C.L. interval based on CT10NNLO.

Conclusions and main messages

- We tried to address the most important sources of uncertainties in the extraction of $m_c(m_c)$ from a global fit in the NNLO S-ACOT-χ GMVFN scheme .
- The best-fit $\overline{\mathrm{MS}}$ charm mass value and its 90% C.L. PDF uncertainty (defined according to the CT10 criterion) are $m_c(m_c) = 1.12^{+0.11}_{-0.17}$ GeV.
- Other significant sources of uncertainty exist and are given explicitly. The quadrature sum of their estimates gives: $^{+0.16}_{-0.02}$ GeV.
- $m_c(m_c)$ in the S-ACOT-χ scheme ($\lambda \approx 0$) is compatible with the world-average value $m_c(m_c) = 1.275$ GeV within the uncertainties. (Larger values of λ are disfavoured.)

BACKUP

Differences among GM schemes

- Main differences among GMVFN schemes: $(m_c^2/Q^2)^p$, $p > 0$ powerlike contributions near the threshold in approx FE terms.
- FE terms: constructed from the respective ZM expressions evaluated at the rescaling mom. fraction $\chi(M_f)$.
- demonstrated M.G., Nadolsky, Lai, Yuan (2011) to be compliant with the QCD factorization theorem for DIS cross sections to all orders Collins (1998). $x = \zeta \left(1 + \zeta^{\lambda} M_f^2/Q^2\right)^{-1}$
- The CTEQ global fit is sensitive to λ. Changes in the preferred m_c provide an estimate of the uncertainty due to the powerlike corrections.

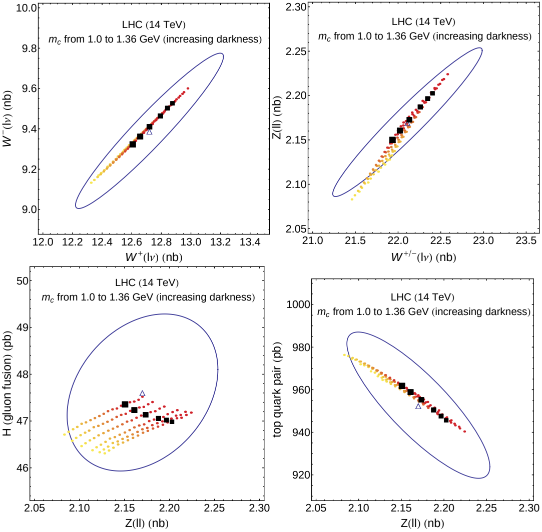
$\rightarrow$ **This gives the bulk of the differences between schemes in the m_c extraction.**

$\overline{\mathrm{MS}}$ mass and Pole mass

- Pole mass M_c: is defined as the position of the pole in the renormalized quark propagator;
  - ☺ it is infrared-safe, gauge-invariant, and is derived in the on-shell renormalization scheme.
  - ☹ m_c close to nonperturbative region, the accuracy of in mass determination is limited by renormalons contrib. (not better than a few hundred MeV.)
- $\overline{\mathrm{MS}}$ mass $m_c(\mu)$: is the renormalized quark mass in the modified-minimal-subtraction scheme.
  - ☺ it is defined as a short-distance mass, not affected by nonperturbative ambiguities. Precise determinations of $m_c(m_c)$ can be achieved (uncertainty of order 30 MeV or less.)
  - ☹ it is evaluated at a momentum scale μ of the hard process, often equal to m_c itself (You have to run the RGE's).

The $\overline{\mathrm{MS}}$ mass starts to differ from the pole mass beginning at order $\mathcal{O}(\alpha_s)$. The conversion between the $\overline{\mathrm{MS}}$ mass to the pole mass may be required because the experimental mass definition is often closer to the on-shell mass. Gray et al. (1990), Chetyrkin et al. (1999), Melnikov et al. (2000), Marquard et al. (2007)

Impact on LHC@14 TeV cross sections



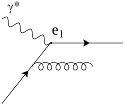
Plot of NNLO cross sections for $W^{\pm}$, Z^0, Higgs boson production through gluon fusion, and top quark pair production at the LHC (14 TeV) for charm quark mass $m_c(m_c)$ ranging from 1 to 1.36 GeV and $\lambda = \{0, 0.02, 0.05, 0.1, 0.15, 0.2\}$. Darker color corresponds to larger mass values. The empty diamonds represent the cross sections evaluated by using $m_c(m_c) = 1.28$ GeV. The triangle and ellipse indicate the central predictions and 90% C.L. interval based on CT10NNLO.

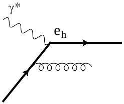
Components of inclusive $F_{2,L}(x, Q)$.

Components of inclusive $F_{2,L}(x, Q^2)$ are classified according to the quark couplings to the photon

$$F = \sum_{l=1}^{N_l} F_l + F_h \quad (1)$$

$$F_l = e_l^2 \sum_a \left[C_{l,a} \otimes f_{a/p} \right](x, Q), \quad F_h = e_h^2 \sum_a \left[C_{h,a} \otimes f_{a/p} \right](x, Q). \quad (2)$$





At $\mathcal{O}(\alpha_s^2)$:

$$F_h^{(2)} = e_h^2 \left\{ c_{h,h}^{NS,(2)} \otimes (f_{h/p} + f_{\bar{h}/p}) + C_{h,l}^{(2)} \otimes \Sigma + C_{h,g}^{(2)} \otimes f_{g/p} \right\}$$

$$F_l^{(2)} = e_l^2 \left\{ C_{l,l}^{NS,(2)} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c_{l,g}^{(2)} \otimes f_{g/p} \right\} \quad (3)$$

Rescaling variable

- In the default implementation: the rescaling variable $\chi = x(1 + 4m_c^2/Q^2)$, corresponding to $M_f = 2m_c$ for the lightest final state ($c\bar{c}$).
- The rescaling ratio χ/x is thus independent of x. Generally it may be expected that the threshold suppression is less pronounced at $W^2 = Q^2(1/x - 1) \to \infty$ for fixed Q, corresponding to $x \to 0$. In this limit, it may be desirable to reduce or even eliminate the rescaling altogether, as quasi-collinear production of heavy quarks becomes more feasible.
- To allow for this possibility, a generalized rescaling variable ζ can be implicitely defined by Nadolsky, Tung (2009)

$$x = \zeta \left(1 + \zeta^{\lambda} M_f^2/Q^2\right)^{-1}, \tag{4}$$

where λ is a positive parameter, typically $0 \leq \lambda \lesssim 1$. The S-ACOT-$\chi$ scheme is reproduced with $\lambda = 0$, and the rescaling is fully turned off for $\lambda \gg 1$.

Data sets

Besides the combined HERA data on inclusive and semiinclusive charm production, we include experimental data from DIS measurements by

- BCDMS, NMC, CDHSW, and CCFR;
- NuTeV and CCFR dimuon production;
- F_{2c} measurements at HERA that are not included in the combined set;
- fixed-target Drell-Yan process;
- vector boson and inclusive jet production at the Tevatron
- inclusive jet production at the LHC, $\rightarrow$ slightly reduces the uncertainty in the gluon PDF.

In the several series of fits that we have carried out, we varied the assumptions and inputs to test their impact on the optimal charm mass.

The results that we will present are common to all these fits, hence are robust against the variations in the assumptions.