The Full Two-loop Gluon Regge Trajectory in the Framework of Lipatov's Effective Action

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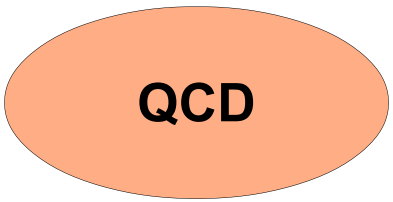
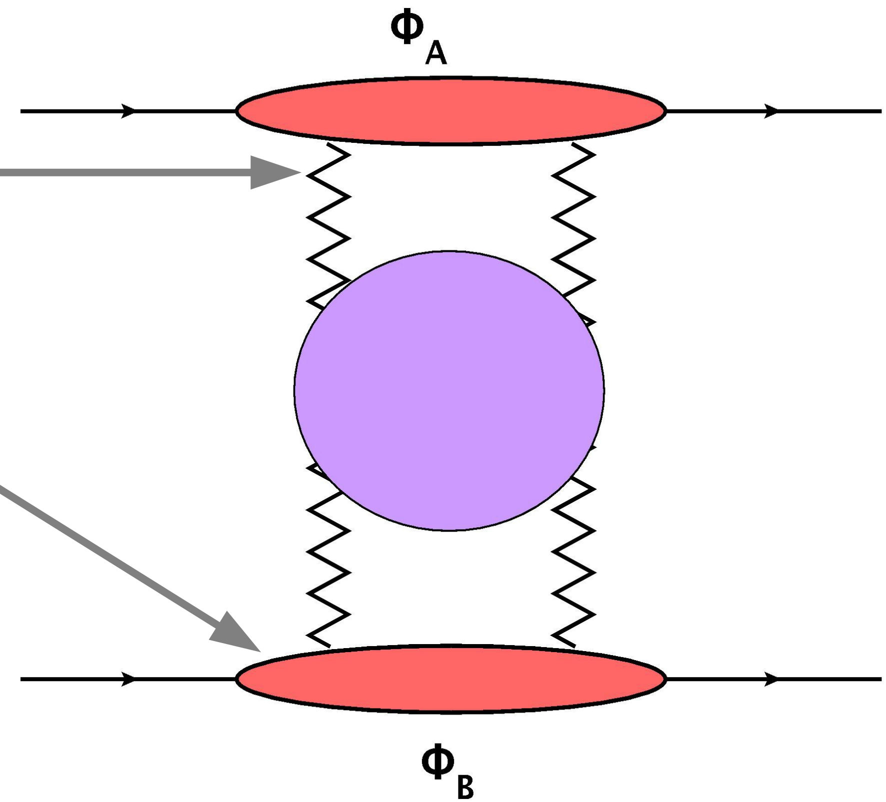
Strictly speaking, not an outline!

- Regge factorization
- BFKL dynamics in QCD
- Lipatov's high energy effective action framework
- Gluon Regge trajectory
- Techniques for loop Feynman diagrams within Lipatov's effective action
- Outlook

High energy (Regge) factorization





Hadronic processes
with a hard scale $Q^2 \gg \Lambda^2_{QCD}$

When the c.o.m energies $s \gg Q^2$
then resummation
is required for the terms: $\alpha_s(Q^2)\ln(s/Q^2) \approx 1$

BFKL dynamics

LL: Fadin, Kuraev, Lipatov (1977), Balitsky, Lipatov (1978)
NLL: Fadin, Lipatov (1998), Ciafaloni, Gamici (1998)

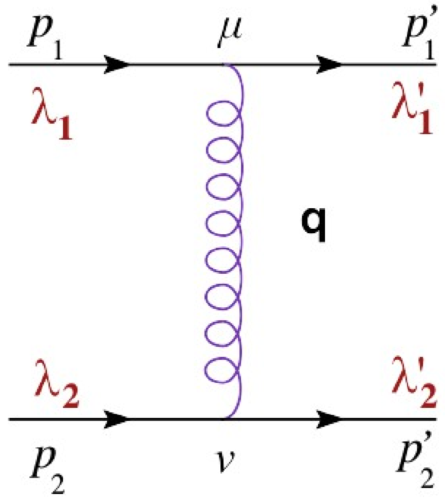
Within BFKL dynamics you need to calculate:

- Impact factors
- Reggeization of the gluons
- The Gluon Green's Function

And you would need to combine them all

The reggeized gluon

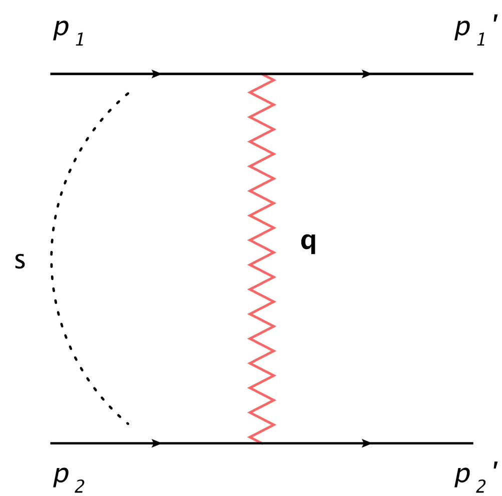
A normal gluon propagator: $D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}$



The reggeized gluon is a gluon with modified propagator:

$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\epsilon(q^2)}$$

(where $\mathbf{k}^2$ is a hard scale in the process at hand)



Gluon Regge Trajectory

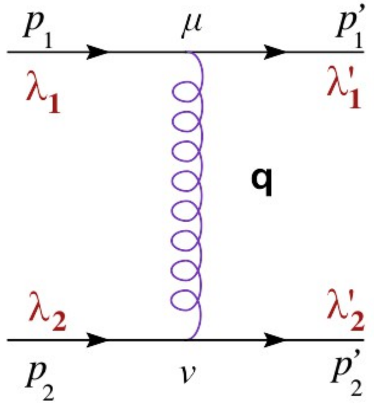
- The gluon Regge trajectory is the function: $1+\epsilon\left(q^2\right)$
- It is perturbatively calculable: $\epsilon\left(q^2\right)=\epsilon^{(1)}\left(q^2\right)+\epsilon^{(2)}\left(q^2\right)+\ldots$
- It encodes the virtual contributions to the BFKL equation
- It connects with the “cusp” anomalous dimension
- Known to NLO in QCD

Fadin, Fiore, Kotsky (1996)

- Known to all orders in N = 4 SYM

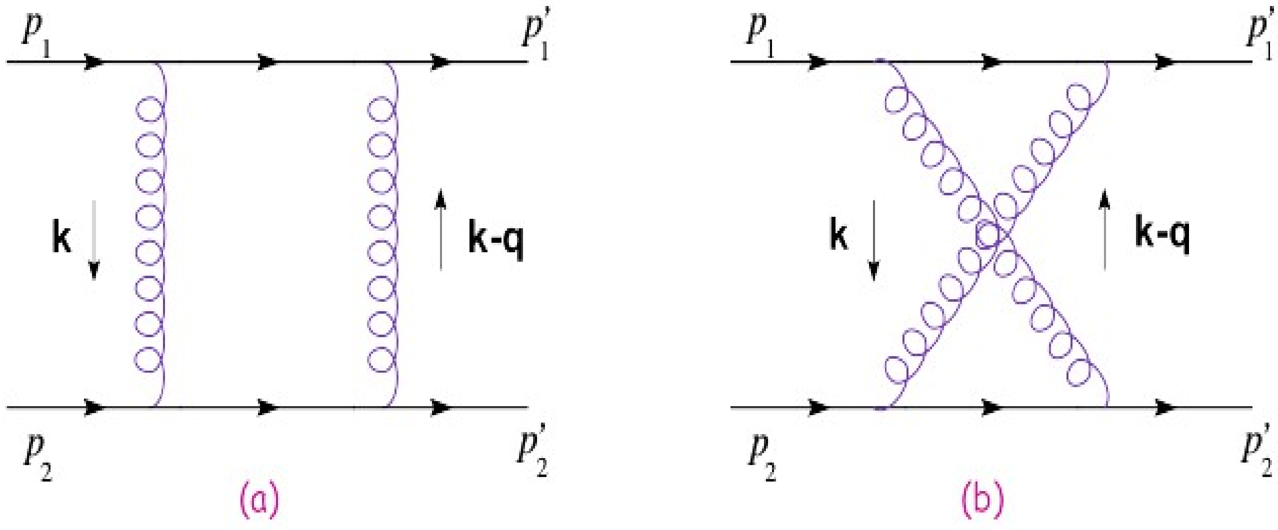
Kotikov, Lipatov (2000)
Beisert, Eden, Staudacher (2007)
Bartels, Lipatov, Sabio Vera (2009)

How to calculate in BFKL dynamics traditionally I:



Tree level

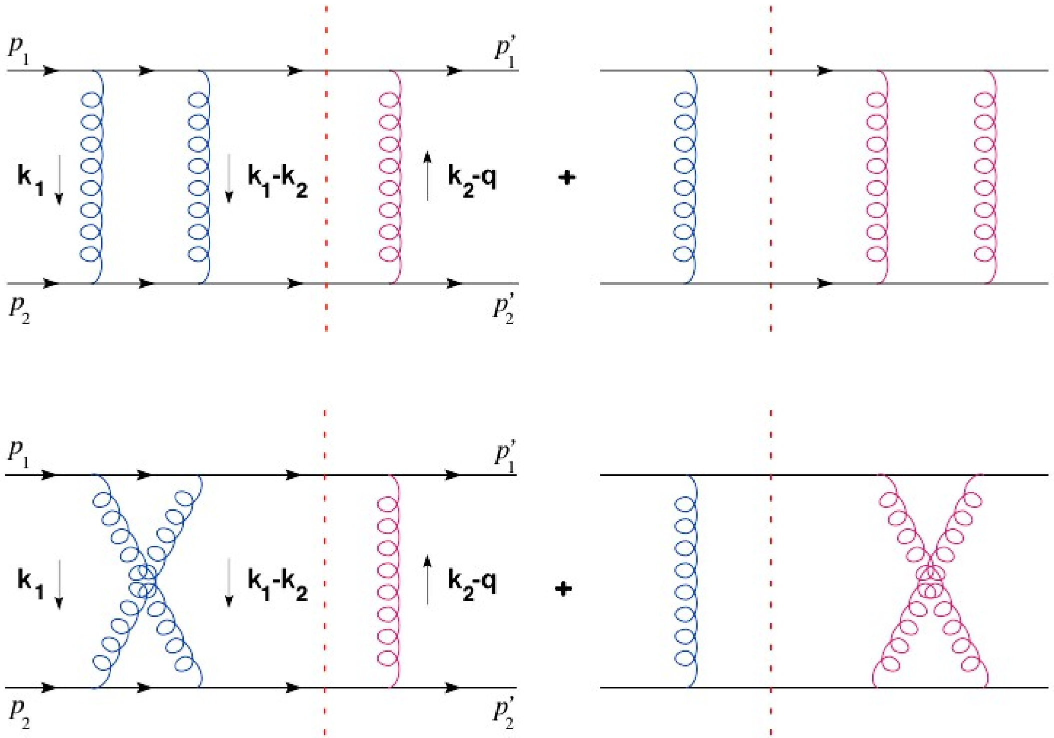
$$A^{(0)}(s,t) = 8\pi a_s t^{\alpha}_{ij} t^{\alpha}_{kl} \frac{s}{q^2} = 8\pi a_s t^{\alpha}_{ij} t^{\alpha}_{kl} \frac{s}{t}$$



One-loop

$$\epsilon(t) = \frac{N_c \alpha_s}{4\pi^2} \int -\mathbf{q}^2 \frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}, \quad A^{(1)}_8(s,t) = 8\pi a_s t^{\alpha}_{ij} t^{\alpha}_{kl} \frac{s}{t} \ln(\frac{s}{|t|})\, \epsilon(t) \;=\; A^{(0)} \ln(\frac{s}{|t|})\, \epsilon(t)$$

How to calculate in BFKL dynamics traditionally II:



Two-loop

$$A_8^{(2)}(s,t) = A^{(0)}(s,t)\,\frac{1}{2}\ln^2(\frac{s}{|t|})\,\epsilon^2(t)$$

$$A_8(s,t) = A^{(0)}(s,t)\,\left(1+\ln(\frac{s}{|t|})\,\epsilon(t)+\frac{1}{2}\ln^2(\frac{s}{|t|})\,\epsilon^2(t)+\ldots\right) \Rightarrow A_8(s,t) = A^{(0)}(s,t)\,\left(\frac{s}{|t|}\right)^{\epsilon(t)}$$

Lipatov's effective action

- Derived by integrating out heavy modes (LO)

Kirschner, Lipatov, Szymanowski (1994-95)

- Later, it was formulated in terms of gauge invariant interactions of reggeized gluons and QCD partons local in rapitidy

Lipatov (1995)

- Now available for calculations at loop level

Hentschinski, Sabio Vera (2011)

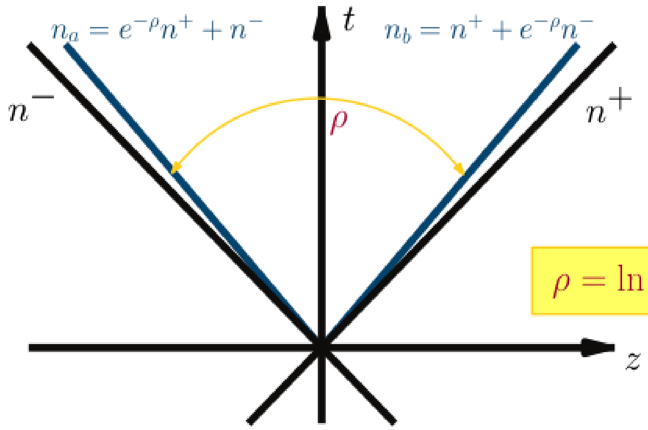
G.C., Hentschinski, Madrigal, Sabio Vera (2012)

- The main technical details were covered in Martin Hentschinski's talk just before

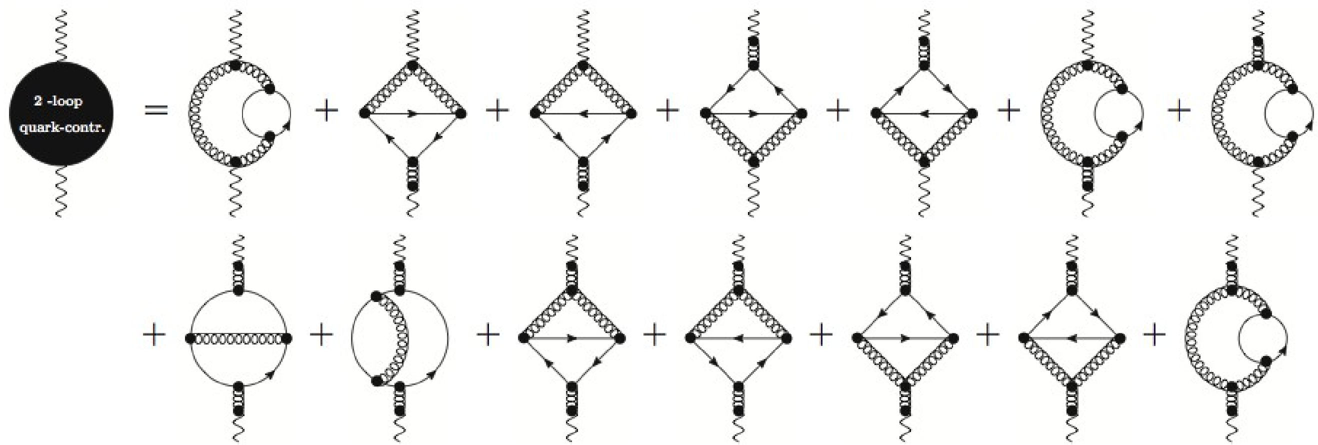
The (general) strategy: calculating in Lipatov's effective action

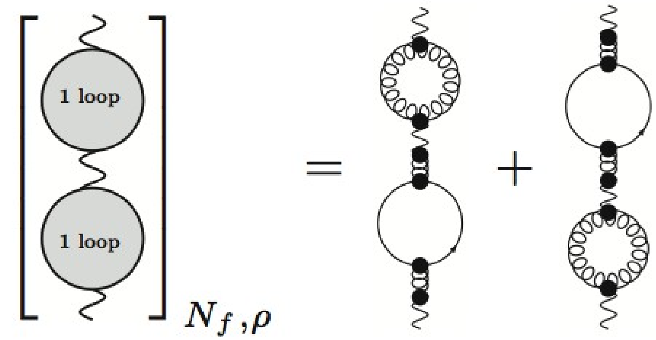
- Take a two-loop parton parton scattering amplitude in the high energy limit
- Subtract non-local contributions to avoid double counting
- Remove terms that are contributions of the 1-loop trajectory and 1-loop impact factors
- Keep terms that are

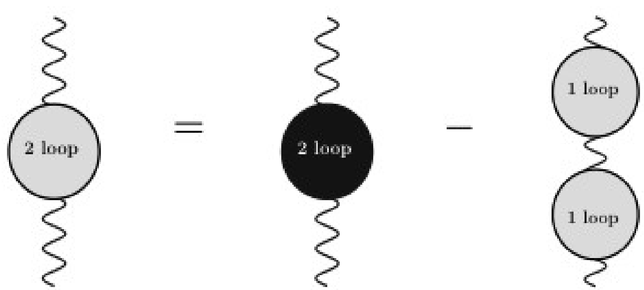
  ρ-enhanced (ρ = ln s)

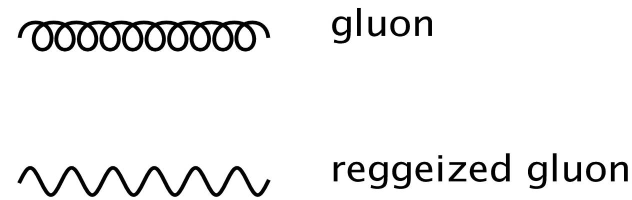


The topologies for the 2-loop quark contribution





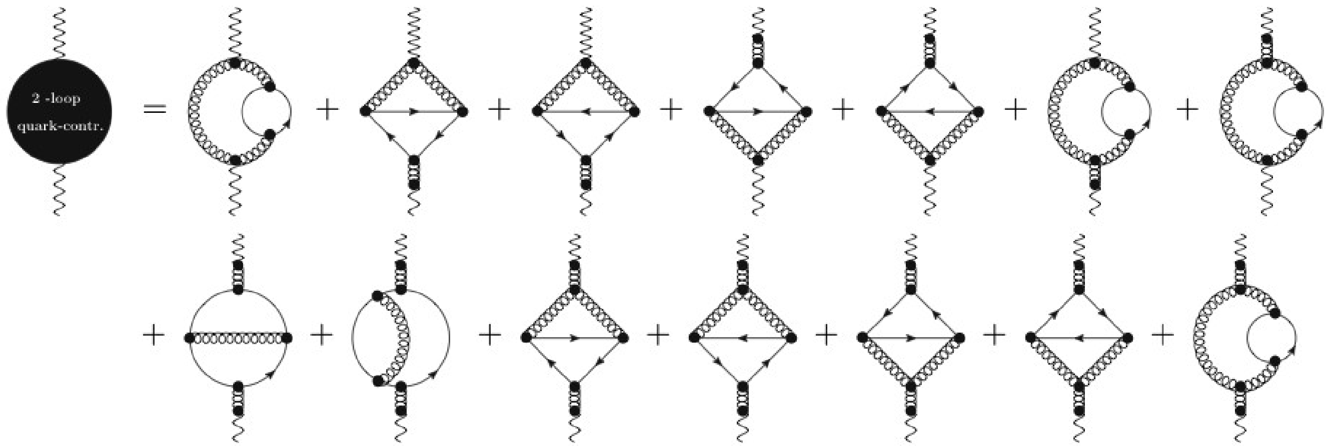


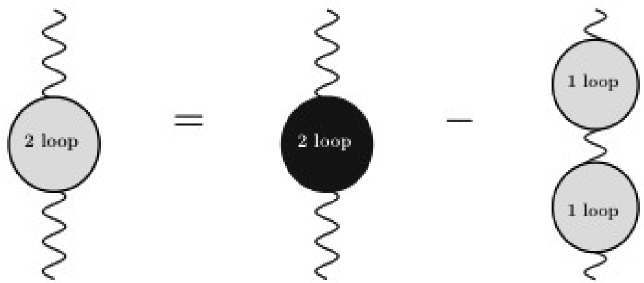


NOTE: for very good reasons, from now on the gluon Regge trajectory will be represented by

$$\omega\left(q^2\right)$$

The 2-loop result: quark contribution





G.C., Hentschinski, Madrigal, Sabio Vera
Nucl.Phys. B861 (2012) 133-144

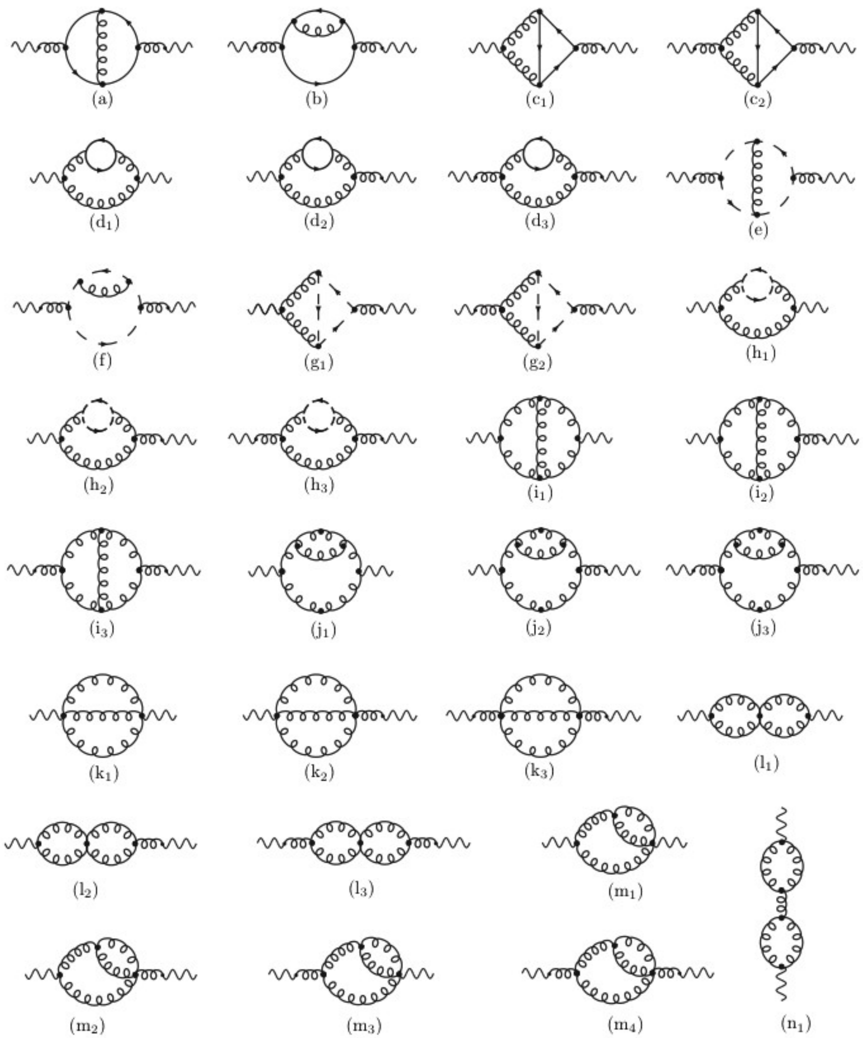
$$\omega^{(2)}_{n_f}\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = \bar{g}^4 \left(\frac{\boldsymbol{q}^2}{\mu^2}\right)^{2\epsilon} \frac{4n_f}{\epsilon N_c} \frac{\Gamma^2(2+\epsilon)}{\Gamma(4+2\epsilon)} \left[\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{2}{\epsilon} - \frac{3\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1+2\epsilon)}{\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)\epsilon}\right]$$

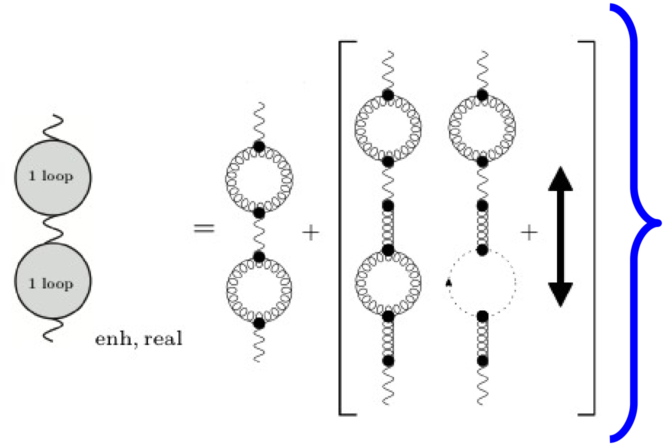
where $\bar{g}^2 = \frac{g^2 N_c \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}$

Full agreement with the literature →

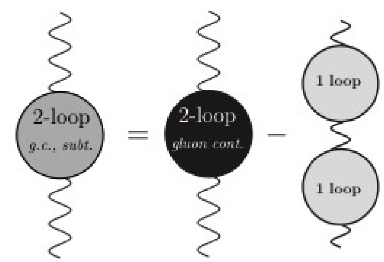
Fadin, Fiore, Kotsky (1996)
Fadin, Fiore, Quartarolo (1996)
Blumlein, Ravindran, van Neerven (1998)
Del Duca, Glover (2001)

All 2-loop topologies





Subtraction terms



From all the 2-loop topologies, pick the ones that belong to the gluon contribution and from these discard the ones which are not ρ-enhanced

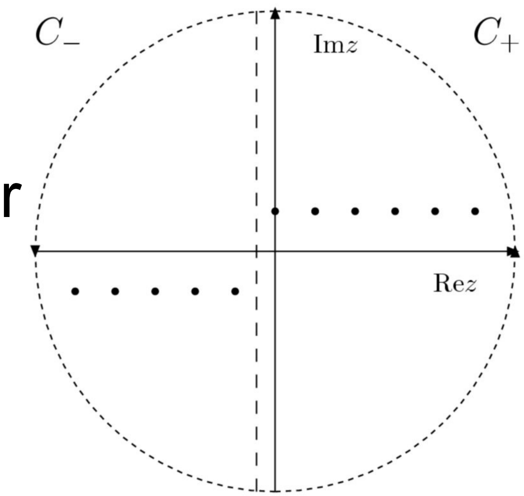
Laporta - Mellin-Barnes

- Use the Laporta algorithm for a reduction to master integrals Laporta (2000)
- The package **FIRE** was used for the reduction Smirnov (2008)
- Create the Mellin-Barnes (MB) representations for the master integrals
- Use the packages **MB.m** and **MBasymptotics.m** for further treatment of the MB integrals Czakon (2006)
- Also useful:

  - **Xsummer** Moch, Uwer (2002)

  - PSLQ algorithm Ferguson, Bailey, Arno (1999)

Mellin-Barnes representations



Under the assumption that n>0 and that the contour separates the poles of the Gamma functions

$$\frac{1}{(A+B)^n} = \frac{1}{2\pi i \Gamma(n)} \int_{-i\infty}^{i\infty} dz \frac{A^z}{B^{n+z}} \Gamma(-z)\Gamma(n+z)$$

Behaviour of the Gamma functions around non-positive arguments

$$z\Gamma(z) = \Gamma(1+z) \qquad \Rightarrow \qquad \Gamma(-n+z) = \frac{\Gamma(1+z)}{(-n+z)...(z)} \sim \frac{(-)^n}{n!}\frac{1}{z}$$

Take residues depending on the values of A and B

For $\quad A > B \quad \Rightarrow \quad z < 0 \quad \Rightarrow \quad z = -N-n, \ N = 0, 1, ...$

$$\sum_{N=0}^{\infty} \frac{(-)^N}{N!} \frac{\Gamma(N+n)}{\Gamma(n)} \frac{B^N}{A^{N+n}} = \frac{1}{A^n} \sum_{N=0}^{\infty} \frac{(N+n-1)!}{(n-1)!N!} \left(-\frac{B}{A}\right)^N = \frac{1}{A^n} \frac{1}{\left(1+\frac{B}{A}\right)^n} = \frac{1}{(A+B)^n}$$

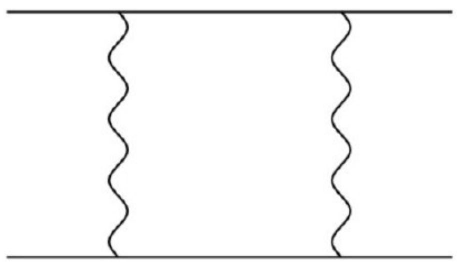
For $\quad A < B \quad \Rightarrow \quad z > 0 \quad \Rightarrow \quad z = N, \ N = 0, 1, ...$

$$\sum_{N=0}^{\infty} \frac{(-)^N}{N!} \frac{\Gamma(N+n)}{\Gamma(n)} \frac{A^N}{B^{N+n}} = \frac{1}{B^n} \sum_{N=0}^{\infty} \frac{(N+n-1)!}{(n-1)!N!} \left(-\frac{A}{B}\right)^N = \frac{1}{B^n} \frac{1}{\left(1+\frac{A}{B}\right)^n} = \frac{1}{(A+B)^n}$$

Mellin-Barnes: a simple example

$$\frac{1}{(A+B)^n} = \frac{1}{2\pi i\Gamma(n)} \int_{-i\infty}^{i\infty} dz \frac{A^z}{B^{n+z}} \Gamma(-z)\Gamma(n+z)$$

Massless one-loop box



$$e^{\epsilon\gamma}\Gamma(2+\epsilon) \int dx_1 ... dx_4 \delta(1-x_1-...-x_4) \frac{1}{(-sx_2x_3 - tx_1x_4)^{2+\epsilon}}$$

$$\frac{e^{\epsilon\gamma}}{2\pi i} \frac{1}{(-s)^{2+\epsilon}} \int_{-i\infty}^{i\infty} dz \left(\frac{t}{s}\right)^z \frac{\Gamma^2(-1-\epsilon-z)\Gamma(-z)\Gamma^2(1+z)\Gamma(2+\epsilon+z)}{\Gamma(-2\epsilon)}$$

$$\mathrm{Re}\,\epsilon = -\frac{1}{2}, \quad \mathrm{Re}\,z = -\frac{3}{4}$$

The generic 2-loop MB representation for the master integrals of the gluon contribution

$$\mathfrak{S} = \iint \frac{d^dk}{(2\pi)^d}\frac{d^dl}{(2\pi)^d}\frac{1}{(-k^2-i0)^A[-(k-q)^2-i0]^B(-l^2-i0)^C[-(l-q)^2-i0]^D[-(k-l)^2-i0]^E}$$

$$\times\frac{1}{(-\sigma_1 a\cdot k-i0)^{\lambda_1}(-\sigma_2 b\cdot k-i0)^{\lambda_2}(-\tau_1 a\cdot l-i0)^{\mu_1}(-\tau_2 b\cdot l-i0)^{\mu_2}}\quad(\sigma_i=\pm1,\tau_j=\pm1,\ a\cdot q=b\cdot q=0)$$

$$=\frac{-1}{2(4\pi)^d\Gamma(C)\Gamma(D)\Gamma(E)\Gamma(\mu_1)\Gamma(\mu_2)(\boldsymbol{q}^2)^{A+B+C+D+E+\frac{\lambda_1+\lambda_2+\mu_1+\mu_2}{2}-d}}$$

$$\times\int\cdots\int\frac{dz_1}{2\pi i}\cdots\frac{dz_6}{2\pi i}\frac{\Gamma(-z_1)\Gamma(-z_2)\Gamma(-z_3)\Gamma(-z_4)\Gamma(-z_5)\Gamma(-z_6)}{\Gamma(-2z_1)\Gamma(-2z_6)}\Gamma\left(z_1+z_2+z_3+z_4+C+D+E+\frac{\lambda_1+\lambda_2+\mu_1+\mu_2}{2}-\frac{d}{2}\right)$$

$$\times\Gamma\left(-z_1-z_2-z_3-z_4-z_5-C-D-E+\frac{\lambda_1-\lambda_2-\mu_1-\mu_2}{2}+\frac{d}{2}\right)\Gamma\left(-z_1+z_2+z_3+z_4+z_5+C+D+E-\frac{\lambda_1-\lambda_2-\mu_1-\mu_2}{2}-\frac{d}{2}\right)$$

$$\times\frac{\Gamma\left(-z_2-z_3-B-C-D-E-\frac{\lambda_1+\lambda_2+\mu_1+\mu_2}{2}+d\right)\Gamma\left(-z_2-z_4-A-C-D-E-\frac{\lambda_1+\lambda_2+\mu_1+\mu_2}{2}+d\right)}{\Gamma(-2z_2-z_3-z_4-A-B-2C-2D-2E-\lambda_1-\lambda_2-\mu_1-\mu_2+2d)\Gamma(2z_2+2z_3+2z_4+z_5+2C+2D+2E+\lambda_2+\mu_1+\mu_2-d)}$$

$$\times\Gamma(2z_2+2z_3+2z_4+z_5+2C+2D+2E+\mu_1+\mu_2-d)\Gamma(-z_2-z_3-z_4+z_6-C-D-E+d/2)\Gamma\left(z_2+A+B+C+D+E+\frac{\lambda_1+\lambda_2+\mu_1+\mu_2}{2}-d\right)$$

$$\times\Gamma(z_2+z_3+z_4+z_5-z_6+C+D+E+\mu_1-d/2)\Gamma(-z_2-z_3-z_4-z_5-z_6-C-D-E-\mu_1+d/2)$$

$$\times\frac{\Gamma(-2z_2-z_3-z_4-2C-2D-E-\mu_1-\mu_2+d)\Gamma(z_2+z_3+C)\Gamma(z_2+z_4+D)}{\Gamma(-z_3+A)\Gamma(-z_4+B)\Gamma(-z_5+\lambda_1)\Gamma(-C-D-E-\mu_1-\mu_2+d)}(e^{-\rho})^{z_1+z_6}(\sigma_1\sigma_2)^{-z_1-z_2-z_3-z_4-C-D-E-\frac{\lambda_1+\lambda_2+\mu_1+\mu_2}{2}+\frac{d}{2}}$$

$$\times\,\sigma_1^{-z_5}\sigma_2^{2z_2+2z_3+2z_4+z_5+2C+2D+2E+\mu_1+\mu_2-d}(\tau_1\tau_2)^{z_2+z_3+z_4-z_6+C+D+E-d/2}.$$

This particularly scary way to “represent” the MB representations is courtesy of Jose Daniel :)

The 2-loop gluon contribution result for the 2-loop gluon Regge trajectory

$$\omega^{(2)}(\boldsymbol{q}^2)|_{N_f=0} = \frac{(\omega^{(1)}(\boldsymbol{q}^2))^2}{4}\left[\frac{11}{3} + \left(\frac{\pi^2}{3} - \frac{67}{9}\right)\epsilon + \left(\frac{404}{27} - 2\zeta(3)\right)\epsilon^2\right]$$

Full agreement with the literature

Fadin, Fiore, Kotsky (1996)
Fadin, Fiore, Quartarolo (1996)
Blumlein, Ravindran, van Neerven (1998)
Del Duca, Glover (2001)

Outlook

- Use Lipatov's effective action for the study of the high energy factorization
- Use the effective action for the study of BFKL dynamics from a different perspective
- Develop consistently complementary tools
- Automate the process of calculating within the effective action

Back-up slides

Back-up slides

gluon propagator: $-i\delta^{ab}\,\frac{g_{\mu\nu}}{k^2}$,

Regge Ansatz

$$\mathcal{M}_{AB} = \frac{s}{t}\Gamma^{\alpha}_{A\to A'}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{\alpha}_{B\to B'}$$

quark-gluon vertex: $ig\bar{u}(p_1)\gamma^{\mu}T^{a}u(p_2)$,

quark propagator: $i\,\frac{\hat{p}+m}{p^2-m^2}$.

Reggeized gluon propagator: $-\frac{i}{2k^2}\delta^{ab}\left[(n^+)^{\mu}(n^-)^{\nu} + (n^+)^{\nu}(n^-)^{\mu}\right]$

$$\omega_B(q) = -\overline{g_{\mu}^2}\left(\frac{2}{\varepsilon} + 2\ln\frac{q^2}{\mu^2}\right), \quad K_r^B(\overrightarrow{q_1},\overrightarrow{q_2}) = \frac{4\,\overline{g_{\mu}^2}\,\mu^{-2\varepsilon}}{\pi^{1+\varepsilon}\Gamma(1-\varepsilon)}\,\frac{1}{(\overrightarrow{q_1}-\overrightarrow{q_2})^2}$$

$$\overline{g_{\mu}^2} = \frac{g_{\mu}^2\,N_c\,\Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}}$$