Modeling the elastic differential cross-section at LHC

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Outline

1. Open problems in elastic scattering at high energies
2. Elastic scattering data *vs* models
3. BN model for total and inelastic cross-section
4. Parametrizing the elastic differential cross-sections
5. Predictions for future LHC data and asymptotia
6. Outlook and perspectives

Open problems in elastic scattering

- dynamics of large distance interaction $\leftrightarrow$ probing hadronic matter in the confinement region $\leftrightarrow$ energy dependence of $\sigma_{tot}(s)$

- unified description of hadronic interactions $\leftrightarrow$ link between 'soft' and 'hard' QCD sectors $\leftrightarrow$ Reggeon Field Theory ('soft' Pomeron dynamics/interactions) vs. pQCD approaches

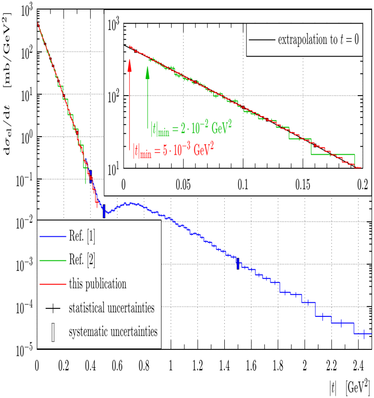
- need to model the differential elastic cross section $\leftrightarrow$ interplay between small and large $|t|$ phenomena $\leftrightarrow$ smooth transition from nPQCD to pQCD regime

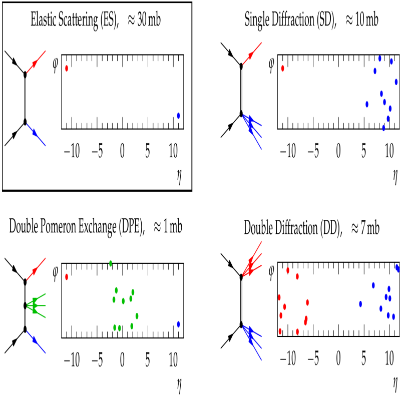
$$\downarrow$$

parametrizations as useful tool $\rightarrow$ model building

Measuring the elastic cross section
and topology of diffractive events

Large rapidity gaps (LRG) - devoid of produced hadrons[1]





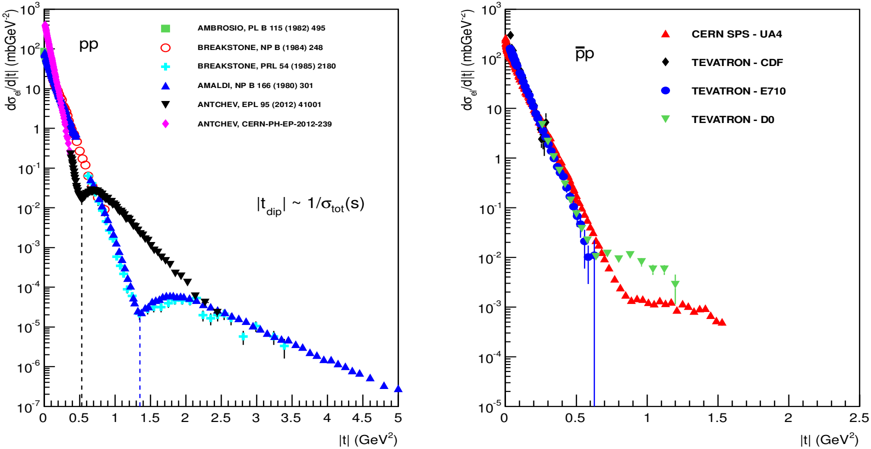
processes mediated by colorless exchange → quantum numbers preserved in the final state

[1] *Left*: G. Antchev, et al., Europhys.Lett. 101 (2013) 21002. *Right*: Jan Kaspar, Doctoral Thesis, CERN-Thesis-2011-214.

Elastic differential cross section

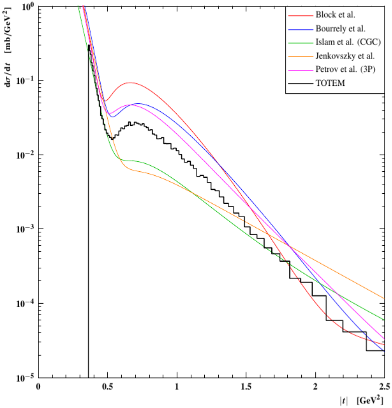
$pp/\bar{p}p$ data from ISR to LHC

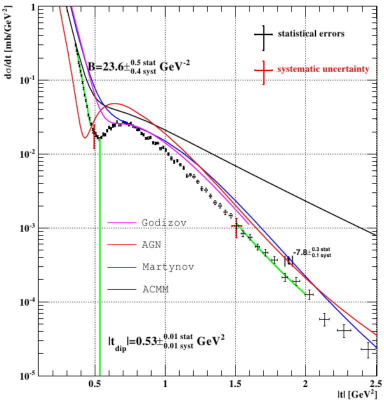
Diffractive pattern at high-energies



one 'dip' present in the *pp* channel and a 'shoulder' in $\bar{p}p$

LHC run at $\sqrt{s} = 7$ TeV, data from TOTEM[2]





none of the representative models fully agree with the data

[2] *Left*: G. Antchev *et al.*, Europhys.Lett. 95 (2011) 41001. *Right*: A.A. Godizov, PoS (IHEP-LHC-2011) 005.

Physics of the 'dip-shoulder' region

delicate cancelation between $C = \pm 1$ amplitudes

The 'dip'/'shoulder' occur through cancellations in elastic amplitude due to $t-$channel processes:

$$A^{pp,\bar{p}p}(s,t) = \frac{A^{+}(s,t) \pm A^{-}(s,t)}{2}$$

$A^{\pm}(s,t)$ are even/odd amplitudes corresponding to $C = \pm 1$ exchanges in the t-channel. In *Regge Phenomenology*, they are called "Pomeron" and "Odderon" terms, which can be translated into QCD (LO) language as $2g$-exchange[3] and $3g$-exchange[4]. Eventually, the nonleading contribution of secondary *Reggeons* and *Pomeron cuts* make their relative phase $\phi \neq \pi$.

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[3] F.E. Low, *Phys.Rev. D12 (1975) 163* and S. Nussinov, *Phys.Rev.Lett. 34 (1975) 1286*

[4] A. Donnachie and P.V. Landshoff, *Z.Phys. C2 (1979) 55*

General formalism

impact parameter picture and bounds

- Impact parameter framework

$$\begin{aligned}
A_{el}(s, t=-q^2) &= 2is\int d^2\mathbf{b}e^{i\mathbf{q}\cdot\mathbf{b}}A_{el}(s,b) \\
\frac{d\sigma_{el}}{dt} &= \frac{|A_{el}(s,t)|^2}{16\pi s} \to \boxed{\left.\frac{d\sigma_{el}}{dt}\right|_{t=0} = \frac{\sigma_{tot}^2(1+\varrho^2)}{16\pi}} \\
\sigma_{tot}(s) &= 2\int d^2\mathbf{b}\Re e A_{el}(s,b) \\
\sigma_{el}(s) &= \int d^2\mathbf{b}|A_{el}(s,b)|^2 \\
\sigma_{inel}(s) &= \int d^2\mathbf{b}[2\Re e A_{el}(s,b) - |A_{el}(s,b)|^2]
\end{aligned}$$

- $s-$channel unitarity: $\boxed{G_{inel}(s,b) = 2\Re e A_{el}(s,b) - |A_{el}(s,b)|^2}$

- Froissart-Martin-Lukaszuk bound: $\boxed{\sigma_{tot}(s) \leqslant \frac{1}{m_\pi^2}\ln^2(s/s_0)}$

- Pumplim Bound: $\boxed{\sigma_{el}(s) + \sigma_{diff}(s) \leqslant \sigma_{tot}(s)/2}$

In the eikonal formulation, multiple parton-parton independently distributed (Poisson) gives

$$\sigma_{inel}^{uncor.}(s) = \int d^2\mathbf{b}\,[1-e^{-\bar{n}(b,s)}]$$
$$\sigma_{tot}(s) = 2\int d^2\mathbf{b}\,[1-e^{-\bar{n}(b,s)/2}] = \sigma_{el}(s) + \sigma_{inel}^{diff}(s) + \sigma_{inel}^{uncor.}(s)$$

The average number of collisions, $\bar{n}(b,s)$, is modeled with two main ingredients:

- rise of σ_{tot} controlled by $\sigma_{jet} \sim s^{\epsilon}$ - $\epsilon = 0.3 - 0.4$
- parton $b-$distribution as the Fourier transform of soft gluon momentum transverse distribution:

$$A_{BN}(b,s) = N\int d^2\mathbf{K_t} e^{-i\mathbf{K_t}\cdot\mathbf{b}} \frac{d^2P(\mathbf{K_t})}{d^2\mathbf{K_t}}$$
$$d^2P(\mathbf{K_t}) = d^2\mathbf{K_t}\int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{K_t}\cdot\mathbf{b} - h(b,q_{max})}$$
$$h(b,q_{max}) = \int d^3\bar{n}_g(k_t)(1-e^{i\mathbf{k_t}\cdot\mathbf{b}}) \propto \int_{\mu\to 0}^{q_{max}} d^2\mathbf{k}_t \frac{\alpha_{eff}(k_t)}{k_t^2}(1-e^{-i\mathbf{k_t}\cdot\mathbf{b}}) \ln\frac{2q_{max}}{k_t}$$

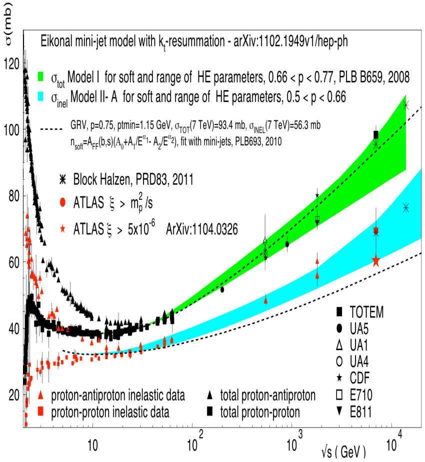
At large impact parameters ($k_t \to 0$)[5]:

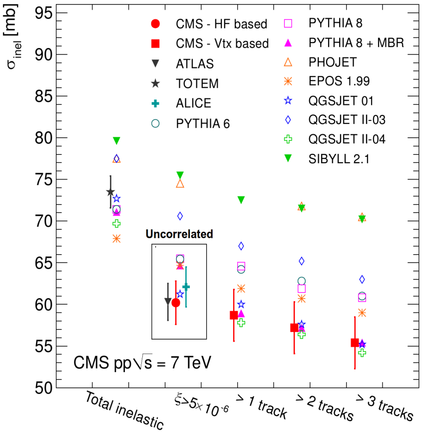
$$\alpha_{eff} \sim k_t^{-2p} \to A_{BN}(b,s) \sim e^{-(\bar{\Lambda}b)^{2p}} \to \sigma_{tot}(s) \sim [\epsilon \ln(s/s_0)]^{1/p} \quad (1/2 < p < 1)$$

[5] A. Grau, R.M. Godbole, G. Pancheri and Y.N. Srivastava, Phys.Lett. B682 (2009) 55.

Bloch-Nordsiek model for $\sigma_{tot}^{pp,\bar{p}p}$ and $\sigma_{inel}^{uncor.}$

a mini-jet model with IR soft gluon resummation





Inclusion of inelastic diffraction is essential to get the full inelastic cross-section → overestimate σ_{el}[6]

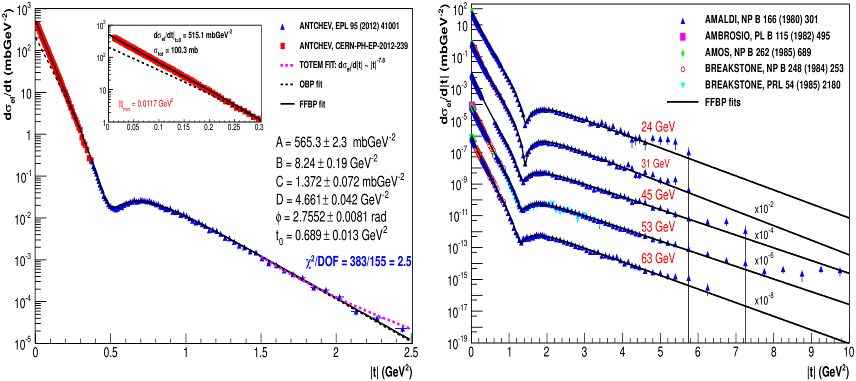
[6] *Left*: A. Achilli et al., Phys.Rev. D84 (2011) 094009. *Right*: CMS Collaboration, arXiv:1210.6718 [hep-ex].

Parametrizing the elastic differential cross-section

modified Barger-Phillips model

Our proposal (phenomenological): implementation of proton's FF at the BP amplitude[7]

$$\mathcal{A}_{el}^{pp}(s,t) = i[\sqrt{A(s)}e^{-B(s)|t|/2}\frac{1}{\left(1+\frac{|t|}{t_0}\right)^2} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]$$



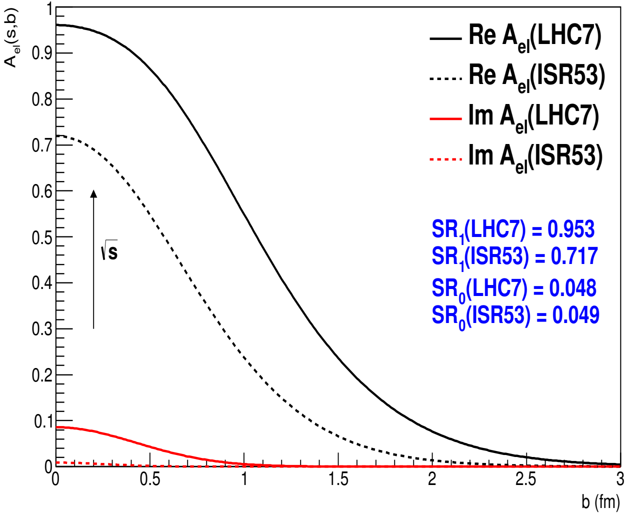
suggests the need to account for elastic rescatterings as $|t|$ increases ↔ the proton does not break up in the collision

[7] Phillips and Barger, *Phys.Lett. B46 (1973) 412* and Grau, Pacetti, Pancheri and Srivastava, *Phys.Lett. B714 (2012) 70*

Asymptotic sum rules

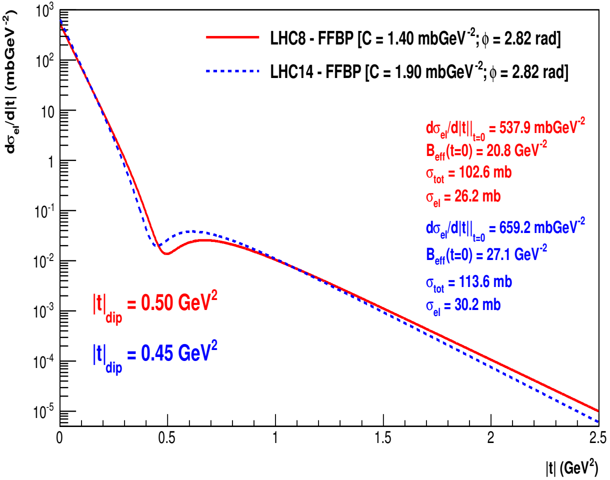
and amplitudes in $b-$space

$$SR_1 = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{0} dt \Im m \mathcal{A}_{el}(s,t) \to 1 \;;\; SR_0 = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{0} dt \Re e \mathcal{A}_{el}(s,t) \to 0$$



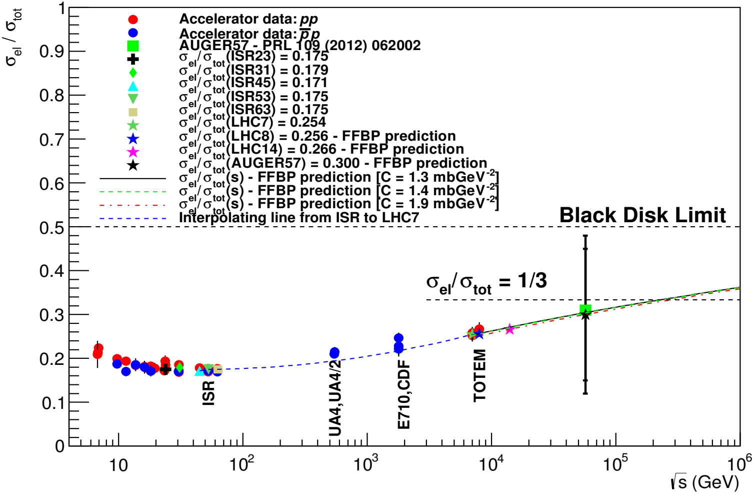
This model predictions
for LHC8 and LHC14

Using the sum rules, one determines (phenomenologically) the energy behaviour of fit parameters: $A(s) \sim ln^4 s$, $B(s) \sim ln^2 s$, $D(s) \sim lns$, $\phi \sim cte$, $t_0 \sim cte$ and $C(s) \sim cte$



This model predictions

from ISR to AUGER and asymptotia



$$\frac{\sigma_{el}}{\sigma_{tot}} \longrightarrow 1/2 \quad \text{at} \quad \sqrt{s} \simeq 10^{15}\text{GeV} \quad (E \sim 10^{30}\text{GeV})$$

Outlook and perspectives

What does this simple model teach us?

1. this modified version of the old Barger-Phillips model nicely reproduce data sets from ISR to LHC $\rightarrow$ good reproductions of $\sigma_{tot}(s), \sigma_{el}(s), \sigma_{inel}(s), B_{el}(s)$

2. the 'dip' structure arises from the interference of two exponentials with a relative phase (mixing $C = \pm 1$ processes)

3. from the sum rules we can guess the energy behaviour of fit parameters consistent with asymptotic theorems $\rightarrow$ predictions for LHC8 and LHC14

Still, many unanswered questions remain...

1. how to interpret this parametrization/formula in the framework QCD-unitarized models? Let us note that many eikonal/Pomeron schemes accomodate the diffractive cone, but not the dip-large $|t|$ region...

2. application of this model to other channels, like $\bar{p}p$?

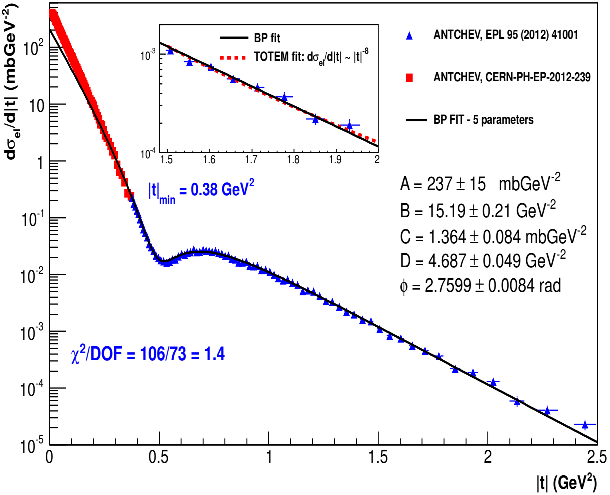
3. how to obtain the energy dependence of parameters in more fundamental grounds? For instance, can we understand their s-dependence in terms of soft gluon resummation model?

4. for the BN model, how to include inelastic diffraction?

Work is in progress to tackle these issues... THANK YOU!

Backup
the original Barger-Phillips amplitude

Suitably fit the data through the 'dip' and at large $|t|$

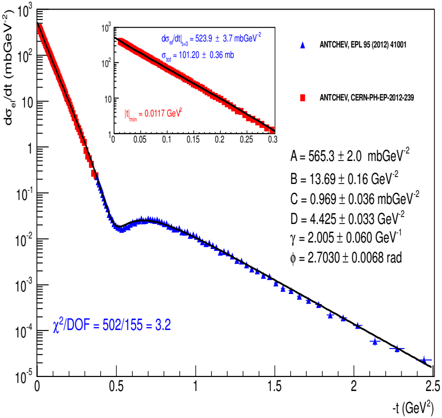


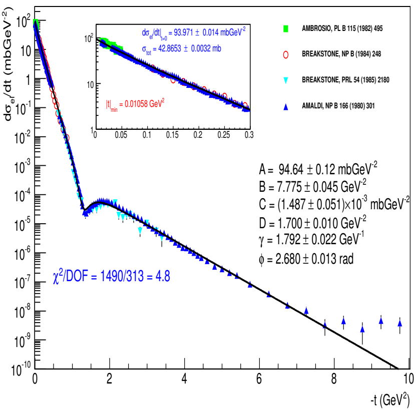
though misses the optical point and underestimate σ_{tot} by $\sim 30\%$ $\rightarrow$ problem of normalization at small $|t|$ region

Our first attempt - introduction of a square root threshold[8] at small $|t|$ (normalized):

$$\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{-B(s)|t|/2}e^{-\gamma(s)(\sqrt{4m_\pi^2+|t|}-2m_\pi)} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]$$

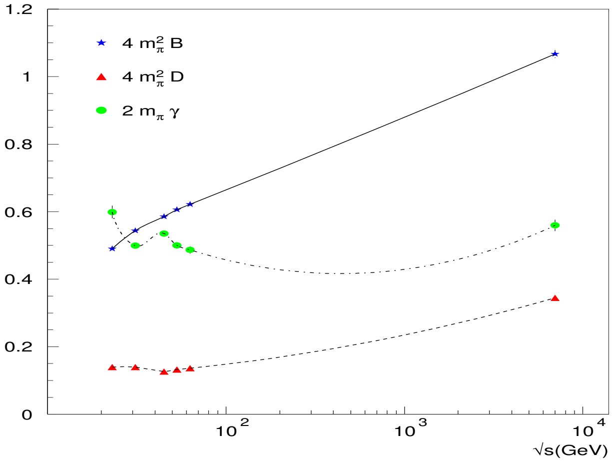
Provide a good fits to LHC7 and ISR53 (typical):





[8] motivated by the *two − pion loop* insertion in the Pomeron trajectory. See e.g. the recent review by Fiore et al. *Int.J.Mod.Phys. A24 (2009) 2551*

However, the new term do not behave as expected, with $\gamma(s) \sim \ln s$...



...instead it 'swings' with increasing c.m. energy → interpretation fails