

# On the role of Sivers effect in $A_N$ for $pp \rightarrow h+X$



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In collaboration with

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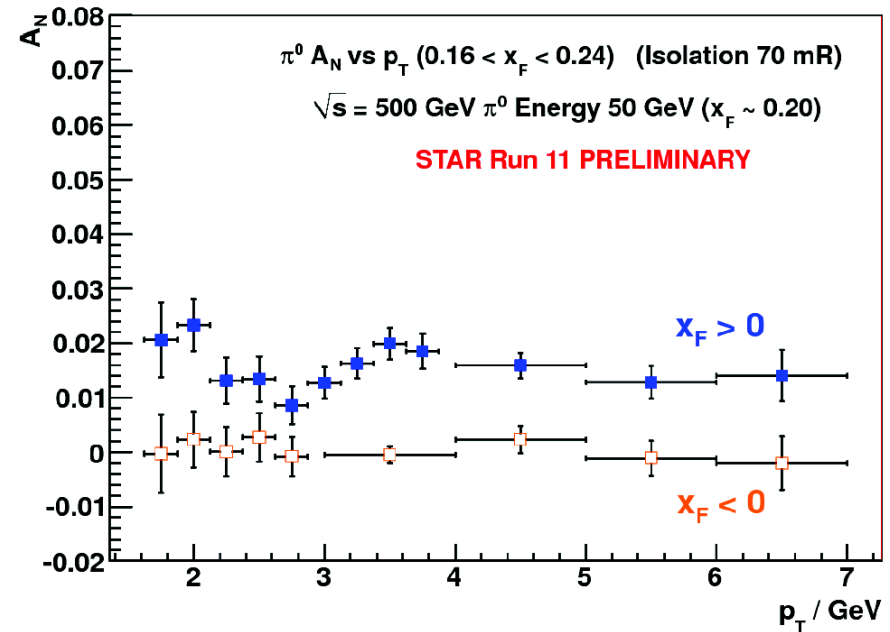
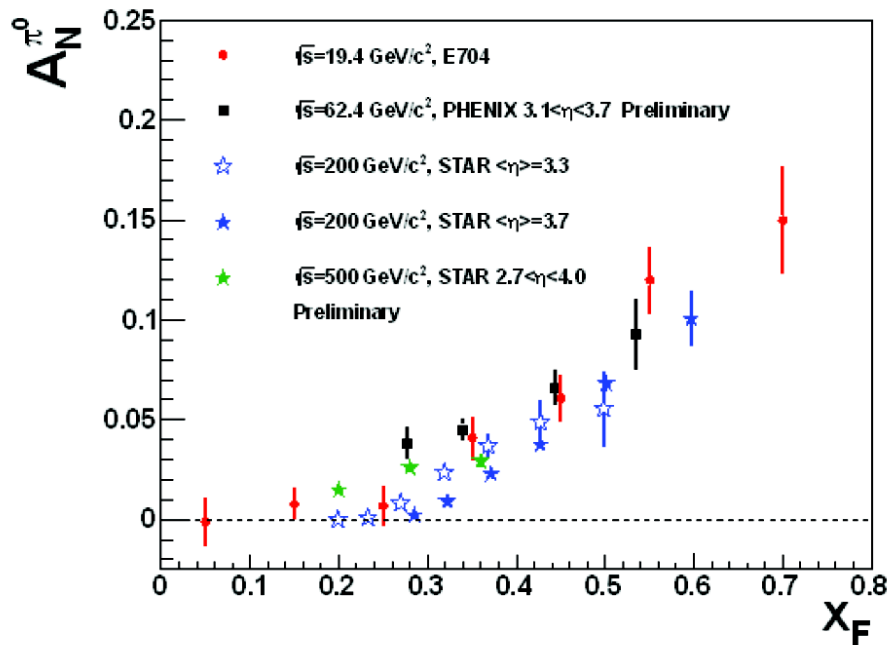
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# Outline

- Single Spin Asymmetries in inclusive hadron production
  - Generalized Parton Model (GPM)
  - Role of the Sivers effect in inclusive hadron production
  - Conclusions
-

# Single Spin Asymmetries in $pp^\uparrow \rightarrow h+X$

$$A_N = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\downarrow(\mathbf{P}_T)}{d\sigma^\uparrow(\mathbf{P}_T) + d\sigma^\downarrow(\mathbf{P}_T)} = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\uparrow(-\mathbf{P}_T)}{2 d\sigma^{\text{unp}}(P_T)}$$



➤ SSA  $A_N$  are large, and persist even at high energies and high  $P_T$ .

# Single Spin Asymmetries in $pp^\uparrow \rightarrow h+X$

- Puzzling in perturbative QCD at leading twist:

$$d\sigma^\uparrow - d\sigma^\downarrow = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{\Delta_T f_a}_{\text{transversity}} \otimes f_b \otimes \underbrace{[d\hat{\sigma}^\uparrow - d\hat{\sigma}^\downarrow]}_{\text{pQCD elementary SSA}} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

- The asymmetry can be generated only at partonic level:

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \hat{a}_N \propto \frac{m_q}{E_q} \alpha_s \sim 0$$

- Negligible at high energies, large transverse momenta

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# Single Spin Asymmetries in $pp^\uparrow \rightarrow h+X$

➤ Possible solutions:

- Going beyond twist 2:

collinear pQCD factorization at twist-3



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- Going beyond collinear approximation

Generalized Parton Model (TMD)

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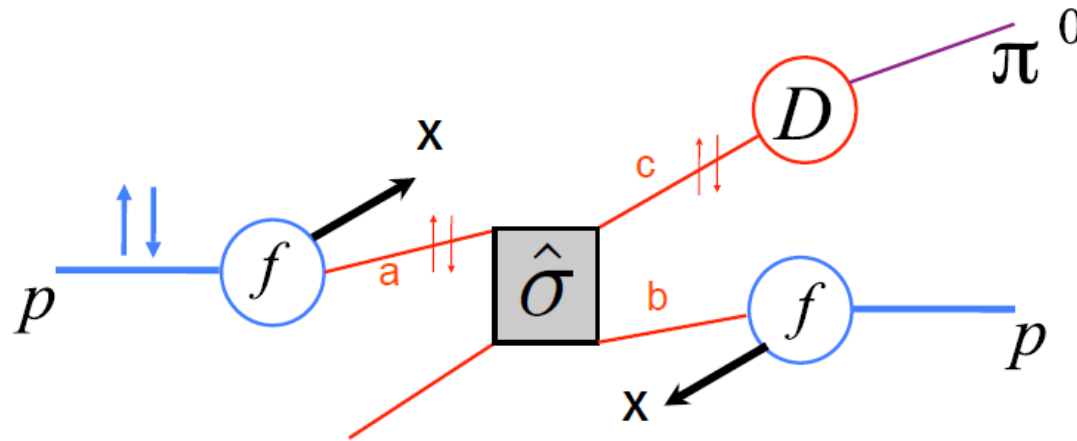
collinear pQCD factorization at twist-3

- Going beyond collinear approx.

Generalized Parton Model (TMD)

# Single Spin Asymmetries in $pp^\uparrow \rightarrow h+X$

- Going beyond collinear approx.: Generalized Parton Model (TMD)



$$d\sigma^\uparrow = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{single spin effects in TMDs}} \otimes \underbrace{f_{b/p}(x_b, \mathbf{k}_{\perp b})}_{\text{single spin effects in TMDs}} \otimes d\hat{\sigma}^{ab \rightarrow cd}(\mathbf{k}_{\perp a}, \mathbf{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \mathbf{p}_{\perp \pi})}_{\text{single spin effects in TMDs}}$$

➤ Warning: Factorization is assumed.



# Generalized Parton Model

$$d\sigma^\uparrow = \sum_{a,b,c=q,\bar{q},g} f_{a/p^\uparrow}(x_a, \mathbf{k}_\perp a) \otimes f_{b/p}(x_b, \mathbf{k}_\perp b) \otimes d\hat{\sigma} \otimes D_{h/c}(z, p_\perp h)$$

➤ Example: Sivers effect

$$\hat{f}_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)$$

Unpolarized TMD PDF



# Generalized Parton Model

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Sivers function

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^{\perp q}(x, k_\perp)$$

# Generalized Parton Model

$$d\sigma^\uparrow = \sum_{a,b,c=q,\bar{q},g} f_{a/p^\uparrow}(x_a, \mathbf{k}_\perp a) \otimes f_{b/p}(x_b, \mathbf{k}_\perp b) \otimes d\hat{\sigma} \otimes D_{h/c}(z, p_\perp h)$$

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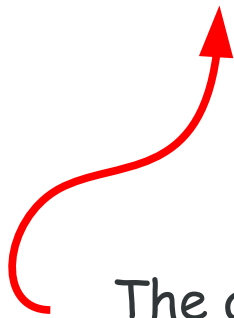
Vanishes in collinear approx.

# Generalized Parton Model

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➤ Example: Sivers effect

$$\begin{aligned} f_{q/p^\uparrow}(x, \mathbf{k}_\perp) - f_{q/p^\downarrow}(x, \mathbf{k}_\perp) &= \Delta^N f_{q/p^\uparrow}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp) \\ &= \Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi - \phi_S) \end{aligned}$$



The asymmetry is at level of PDFs and can be large

# Generalized Parton Model

- The asymmetry is given by the sum of different contributions:

$$d\sigma^\uparrow = \sum_{a,b,c=q,\bar{q},g} f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) \otimes f_{b/p}(x_b, \mathbf{k}_{\perp b}) \otimes d\hat{\sigma} \otimes D_{h/c}(z, p_{\perp h})$$

$$A_N = \Delta^N f_{q/p^\uparrow} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{h/c} \quad \text{Sivers effect}$$

+

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$$+ \Delta_{Tq} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes \Delta^N D_{h/q^\uparrow}$$

Collins effect

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Collins effect

$$+ \Delta_{Tq} \otimes \Delta f_{q^\uparrow/p} \otimes d\hat{\sigma} \otimes D_{h/c}$$

Boer-Mulders effect

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$$\begin{aligned} A_N = & \Delta^N f_{q/p^\uparrow} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{h/c} && \text{Sivers effect} \\ & + \Delta_{Tq} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes \Delta^N D_{h/q^\uparrow} && \text{Collins effect} \\ & + \Delta_{Tq} \otimes \Delta f_{q^\uparrow/p} \otimes d\hat{\sigma} \otimes D_{h/c} && \text{Boer-Mulders effect} \\ & + \text{Other quark TMD effects} + \text{Other gluon TMD effects} \end{aligned}$$



# Generalized Parton Model

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$$A_N \simeq \Delta^N f_{q/p^\uparrow} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{h/c}$$

Sivers effect

$$+ \Delta_{Tq} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes \Delta^N D_{h/q^\uparrow}$$

Collins effect

# Generalized Parton Model

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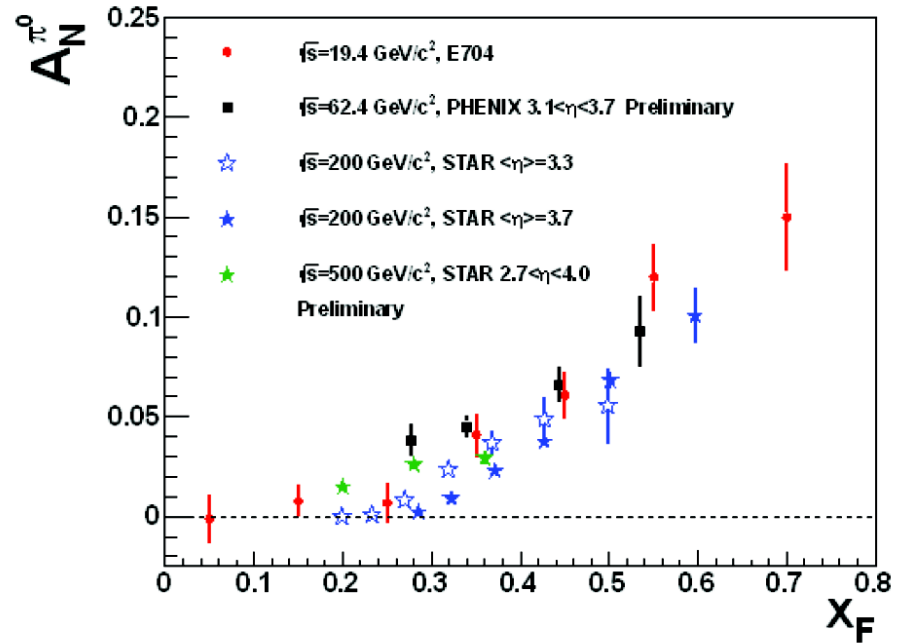
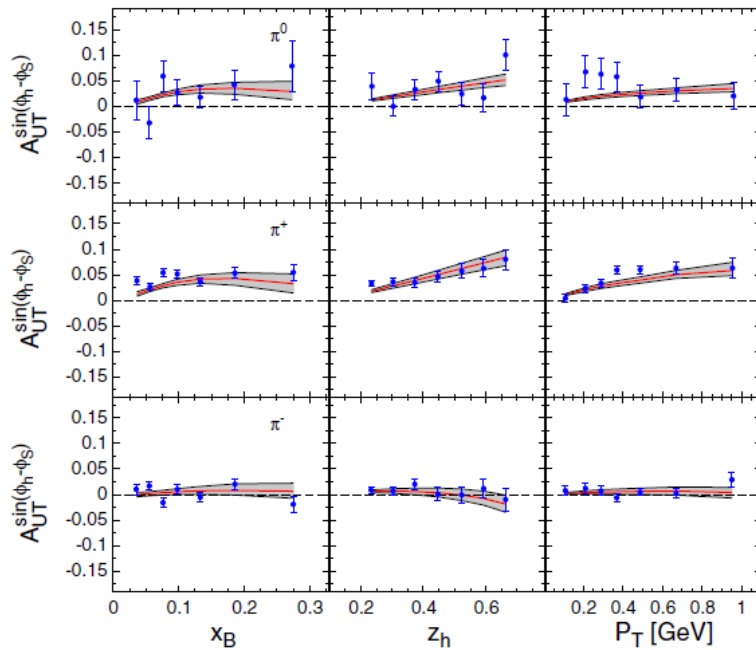
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$$+ \Delta_{Tq} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes \Delta^N D_{h/q^\uparrow} \quad \text{Collins effect}$$

- Current information on TMDs comes from the SIDIS experiment
- We can use this information to study TMD effects in inclusive hadron production
- Already done for Collins: Anselmino et al, PRD86,074032 (2012)

# Phenomenology



➤ However SIDIS data cover only a limited kinematical region  $x < 0.3$ , while in pp large  $x$  region are very important.

$$X_F < X < 1$$

# Parametrization of the Sivers function

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2 \mathcal{N}_q^S(x) f_{q/p}(x) h(k_\perp) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$h(k_\perp) = \sqrt{2} e \frac{k_\perp}{M} e^{-k_\perp^2 / M^2}$$

$$\mathcal{N}_q^S(x) = N_q^S x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

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Controls the small x behavior

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Controls the small x behavior

Controls the large x behavior

- The beta parameters are poorly known from SIDIS, but are crucial for pp asymmetries at high  $x_F$  strongly depends  $\beta_u / \beta_d$

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# Scan procedure

- I. We fit SIDIS data leaving  $\beta_u$  and  $\beta_d$  as free parameter.  
You get you best  $\chi^2_0$



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**II.** we consider several  $(\beta_u, \beta_d)$  pairs

$0 < \beta_u < 4$     $0 < \beta_d < 4$    At steps of 0.5

for each pair we perform a fit of SIDIS data  
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**III.** Is this new Sivers function able to describe nicely (statistically)  
the data? If so, let us take it!

$\chi^2 \leq \chi^2_0 + \Delta\chi^2$    Statistical criteria to accept the new function

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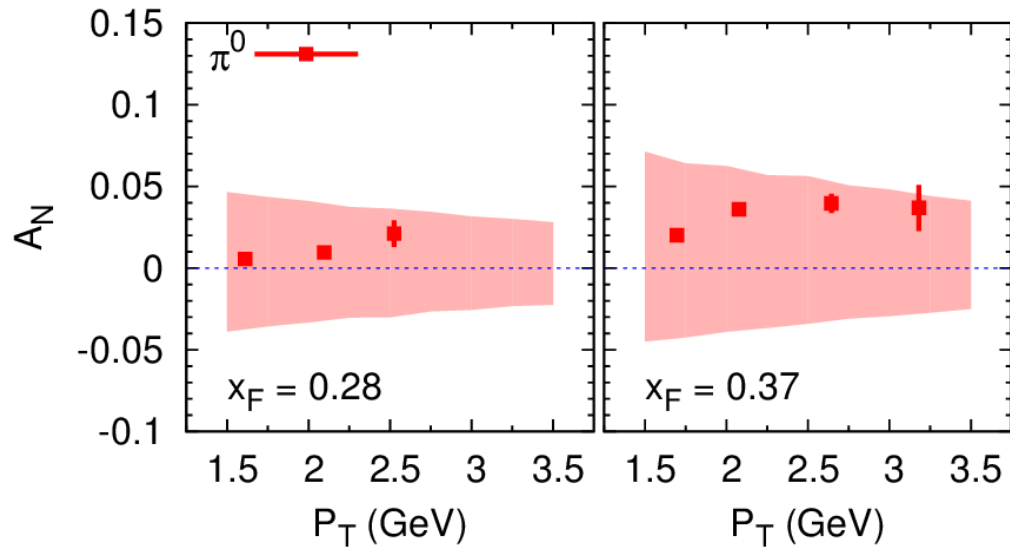
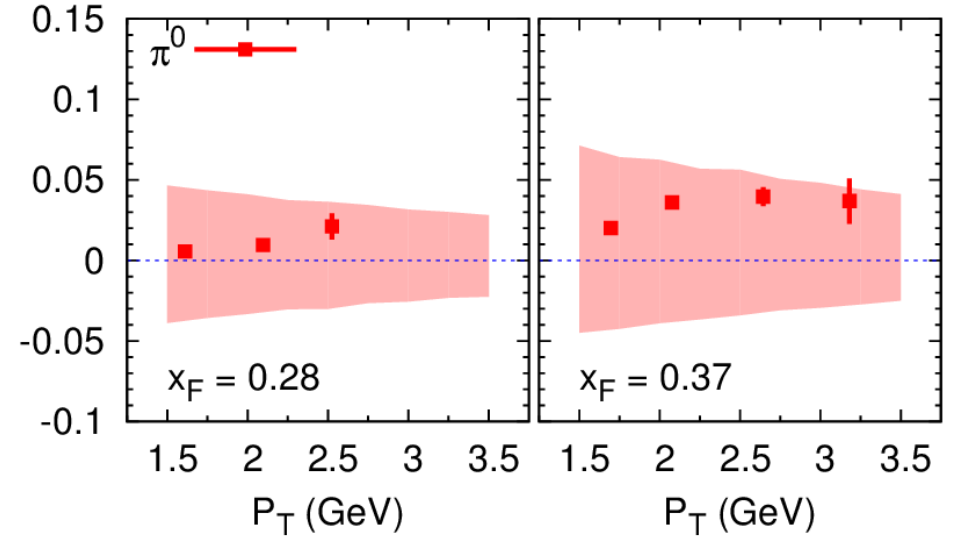
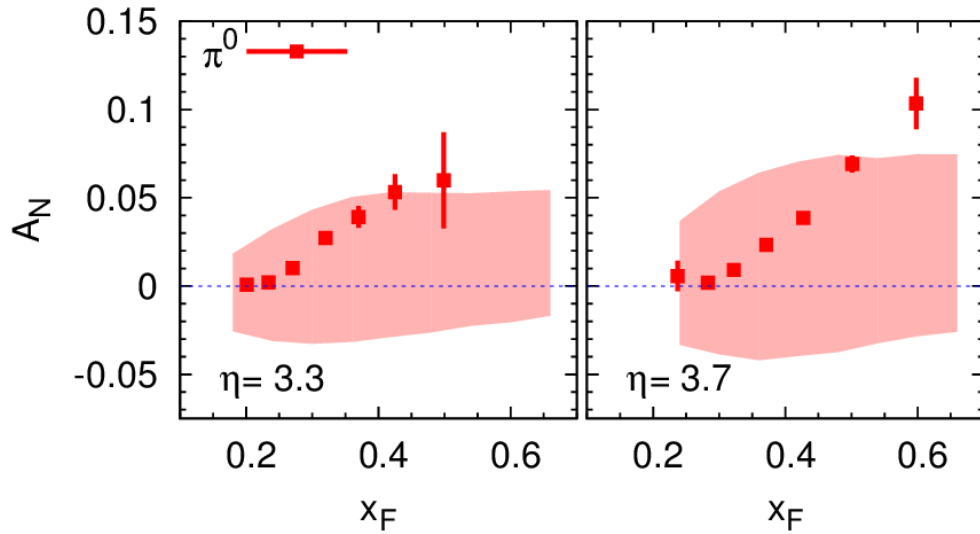
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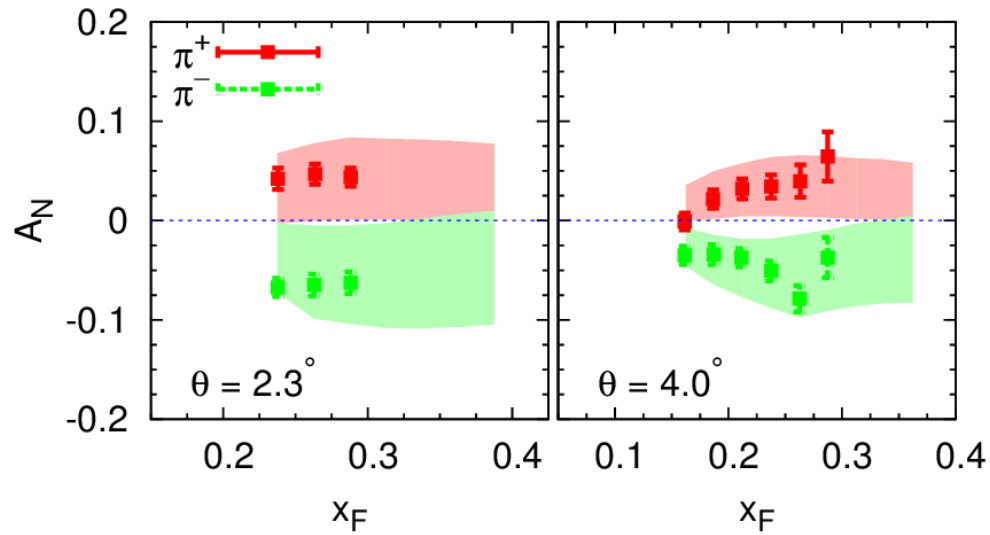
**IV.** Use the function able to describe the SIDIS data to calculate  
the Sivers effect in pp

# Results



 **STAR@RHIC**  
 $\sqrt{s} = 200$  GeV

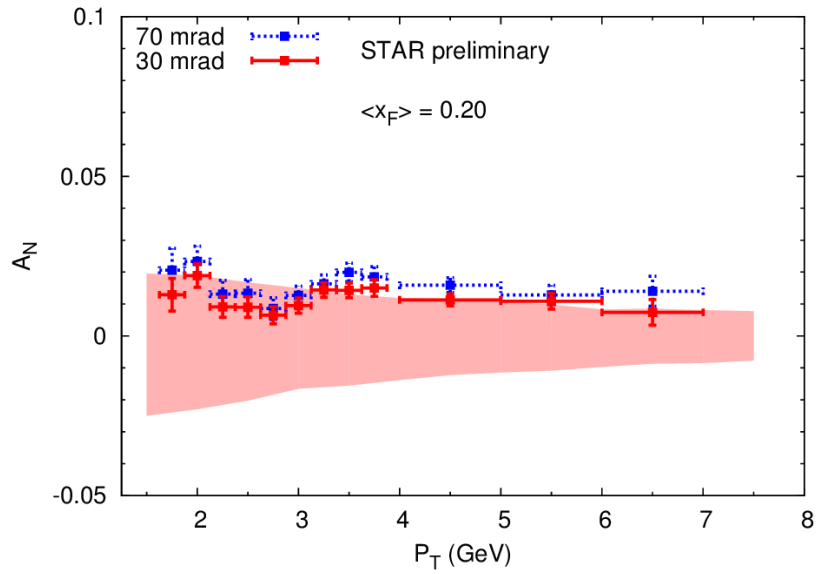
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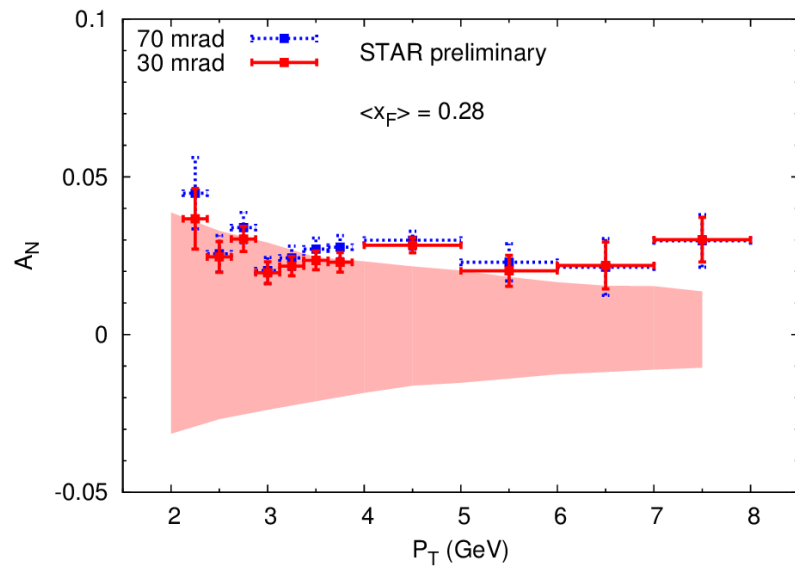
BRAHMS@RHIC

$\sqrt{s} = 200$  GeV

# Results



 STAR@RHIC  
 $\sqrt{s} = 500$  GeV,

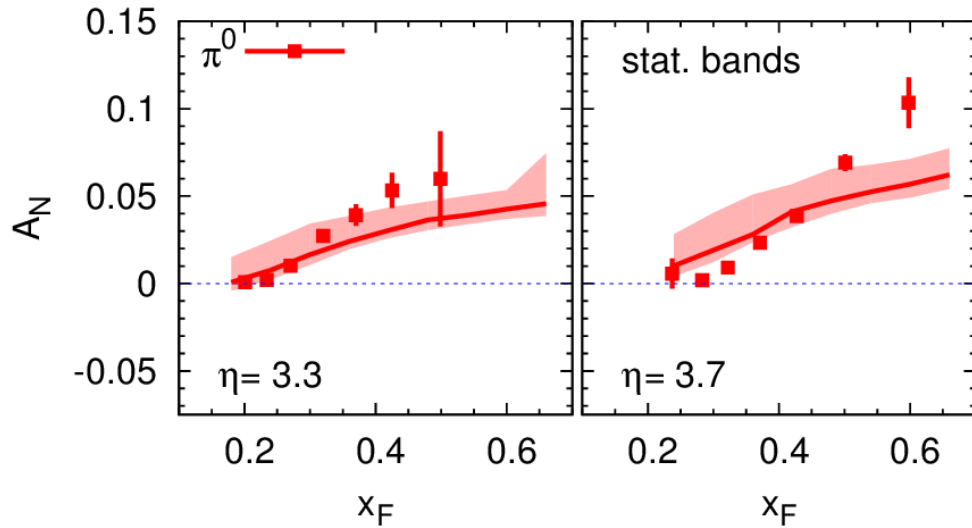



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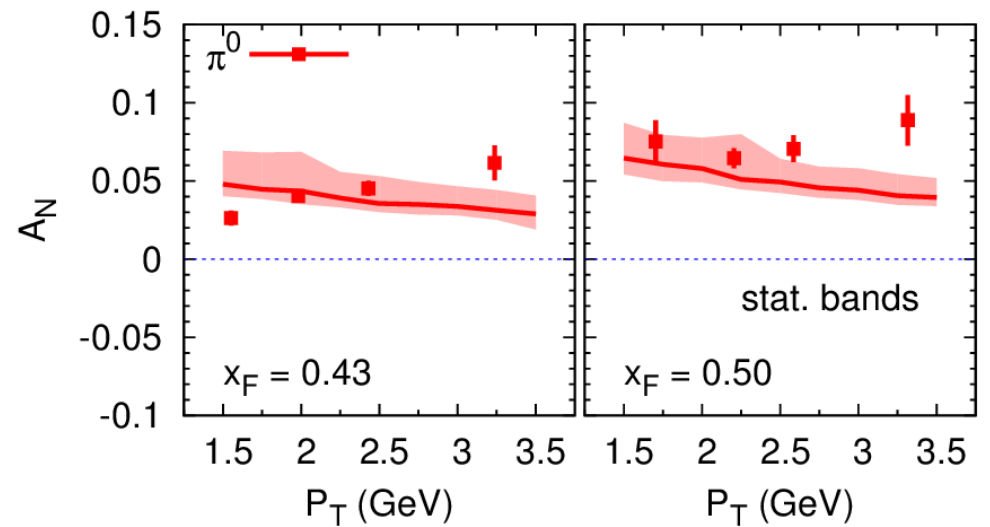
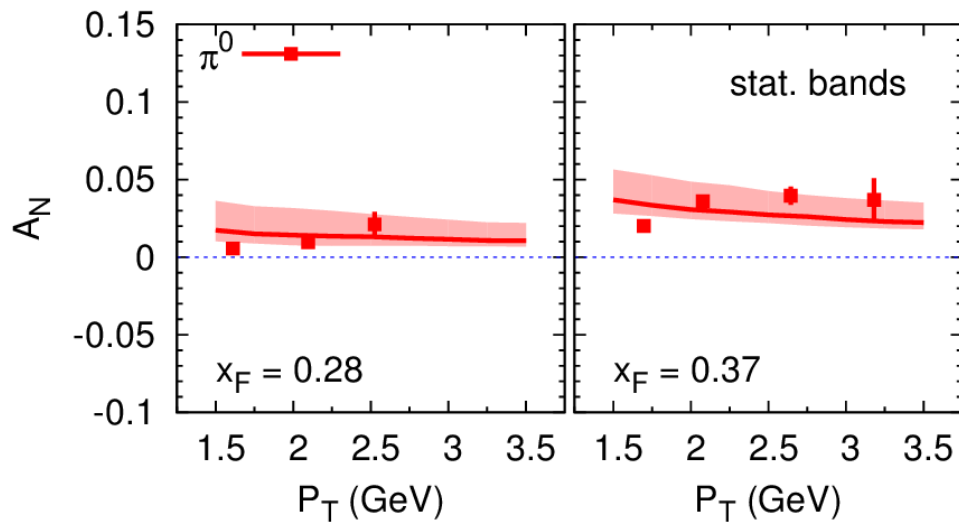
# Results

- Can we find a Sivers function able to explain qualitatively the data alone?

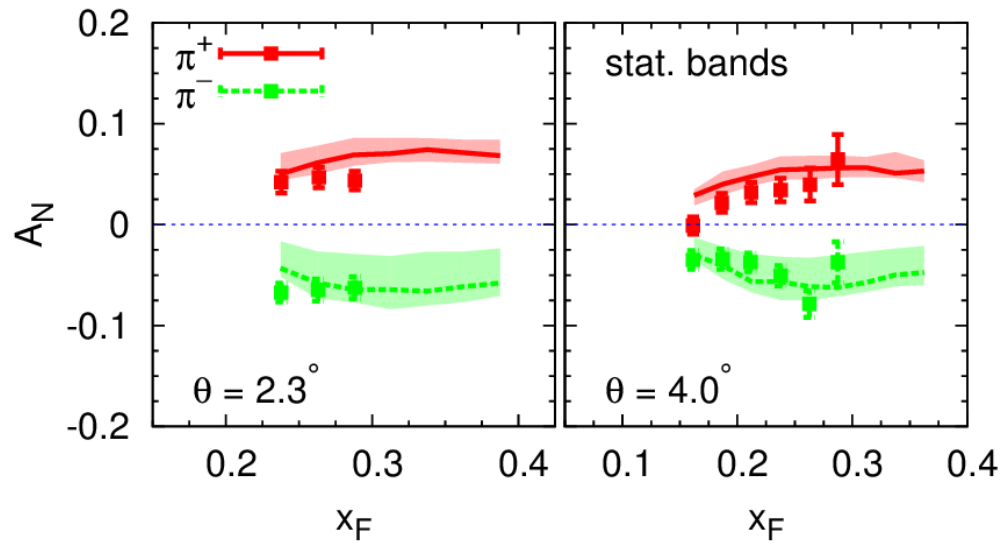
# Results



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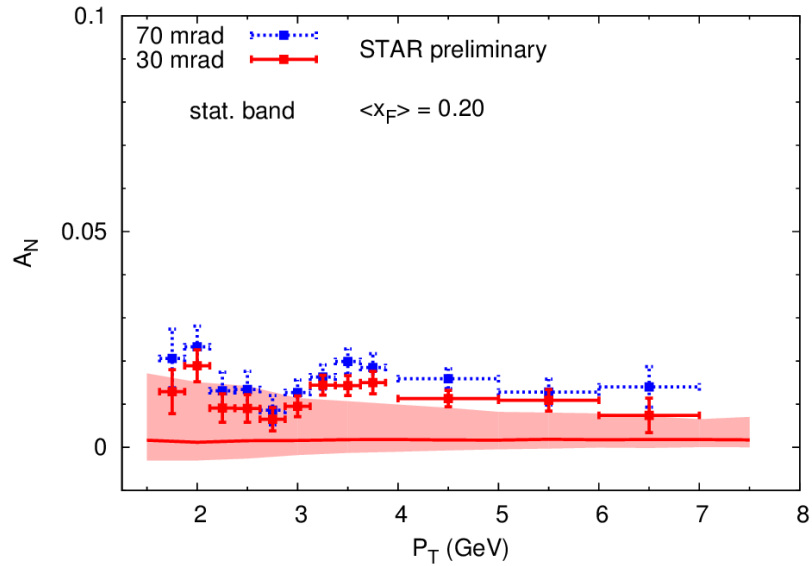


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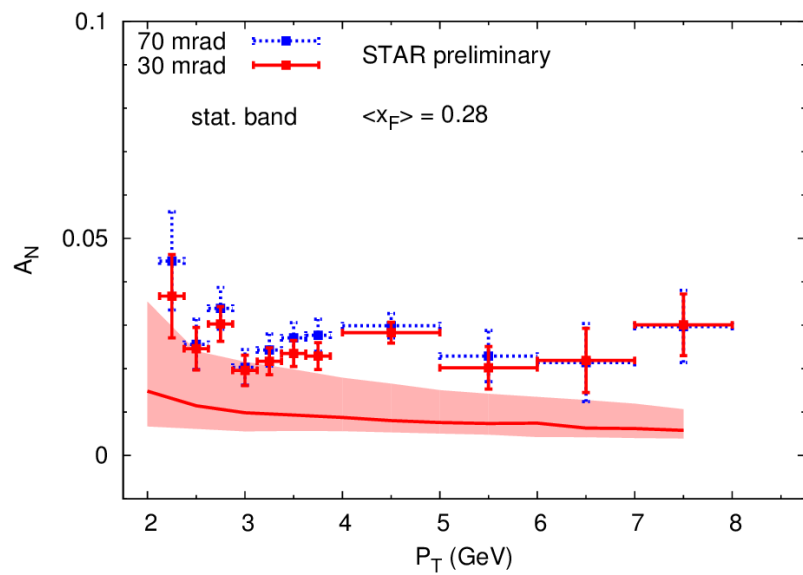
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# Results



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# Conclusions

- We used SIDIS data to estimate the Sivers effect in  $pp \rightarrow \text{pion} + X$
  - The Sivers effect can be large enough to explain  $A_N$  asymmetry
  - In principle we can find a Sivers function alone, to explain the asymmetry, but do not forget Collins effect!
  - Further necessary development: Factorization & evolution, data at large  $x$
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Back up slides

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# Scan procedure: some choices

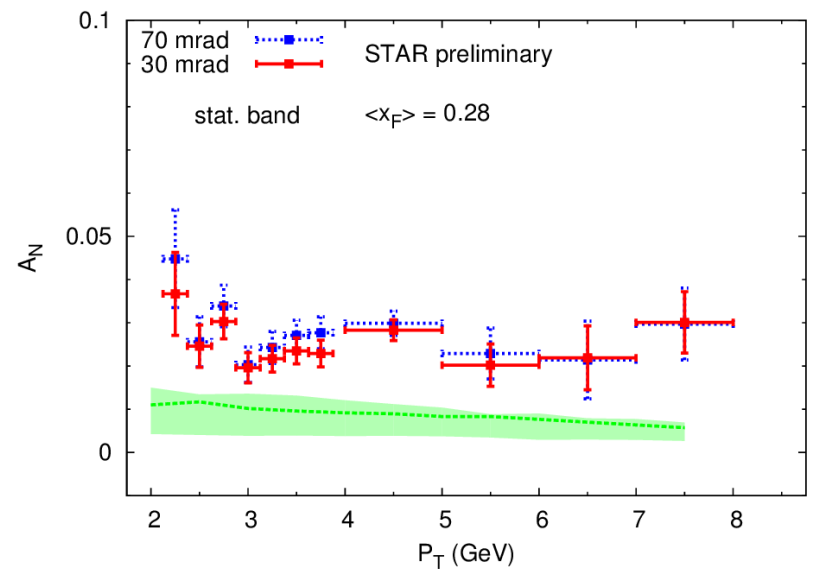
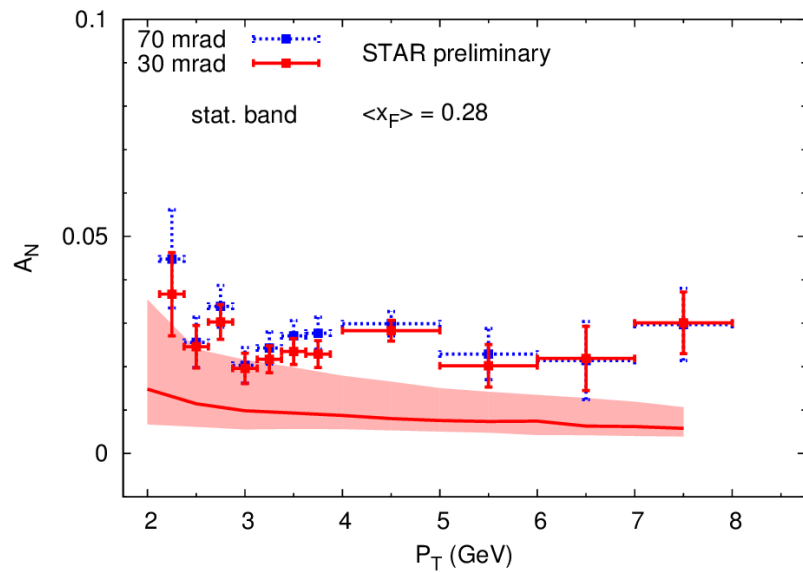
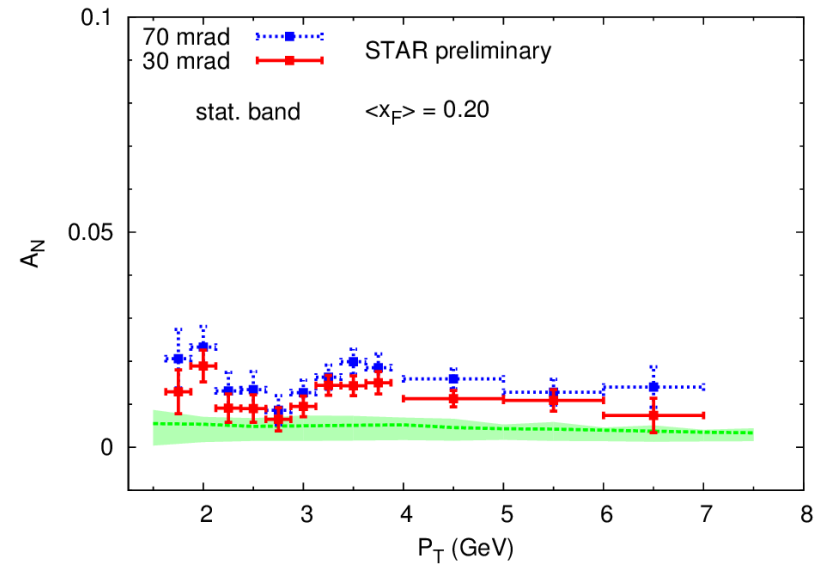
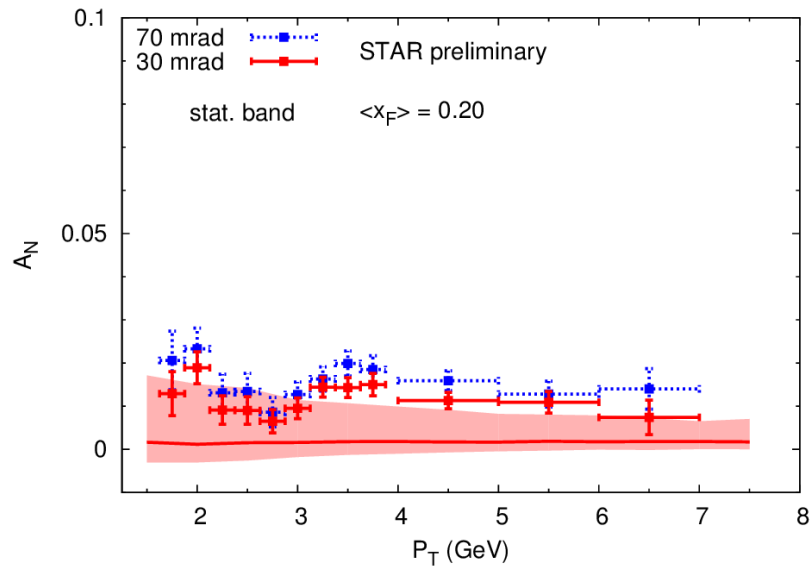
- Valence quark dominate pp. We neglect sea quarks in our procedure. This minimize the number of parameter:

$$\mathcal{N}_q^S(x) = N_q^S x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

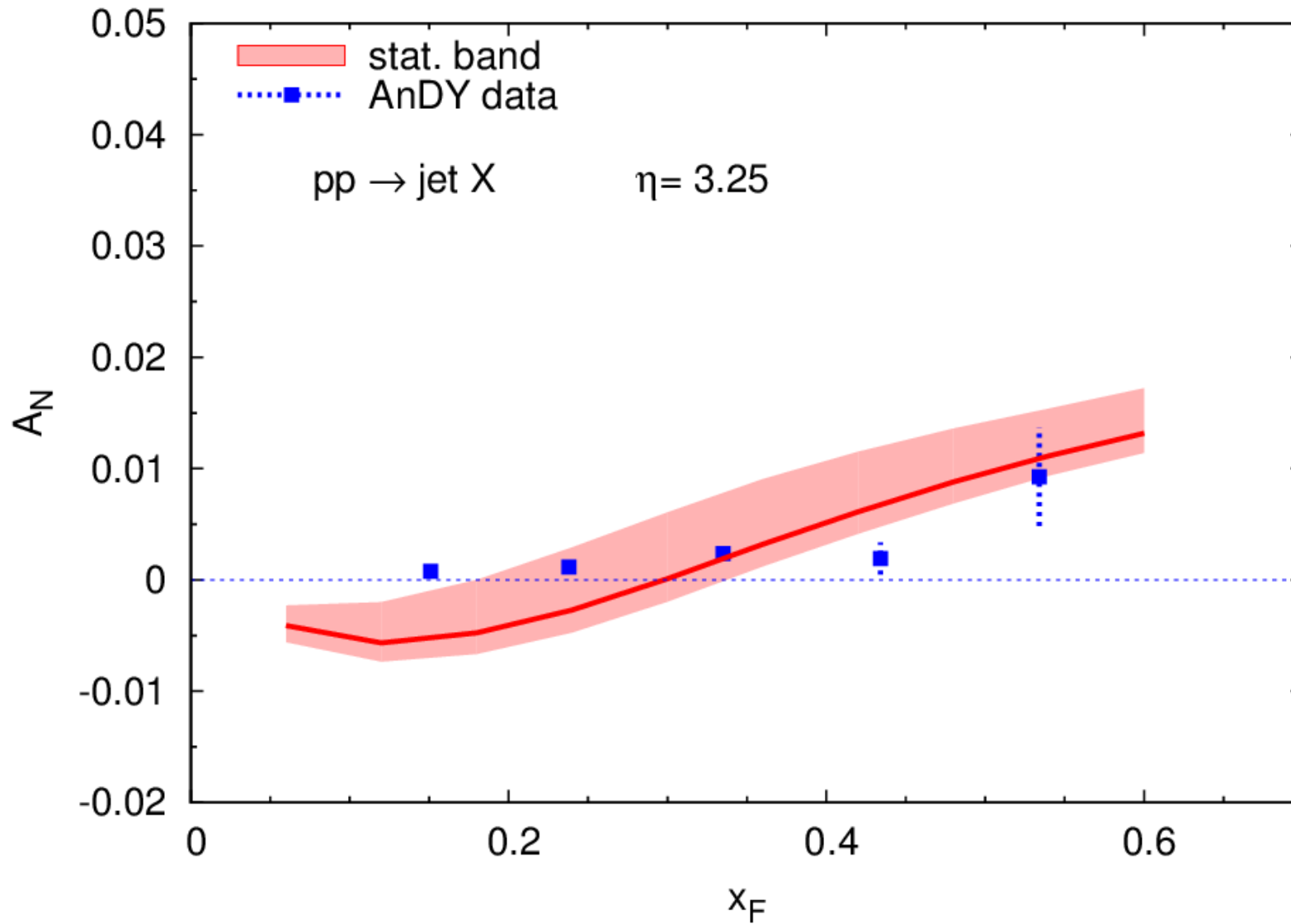
$$N_u, N_d, \alpha_u, \alpha_d, \beta_u, \beta_d, M$$

- We fit only HERMES data (in order to avoid problems related to evolution)
- GRV98 for PDFs and Kretzer FF

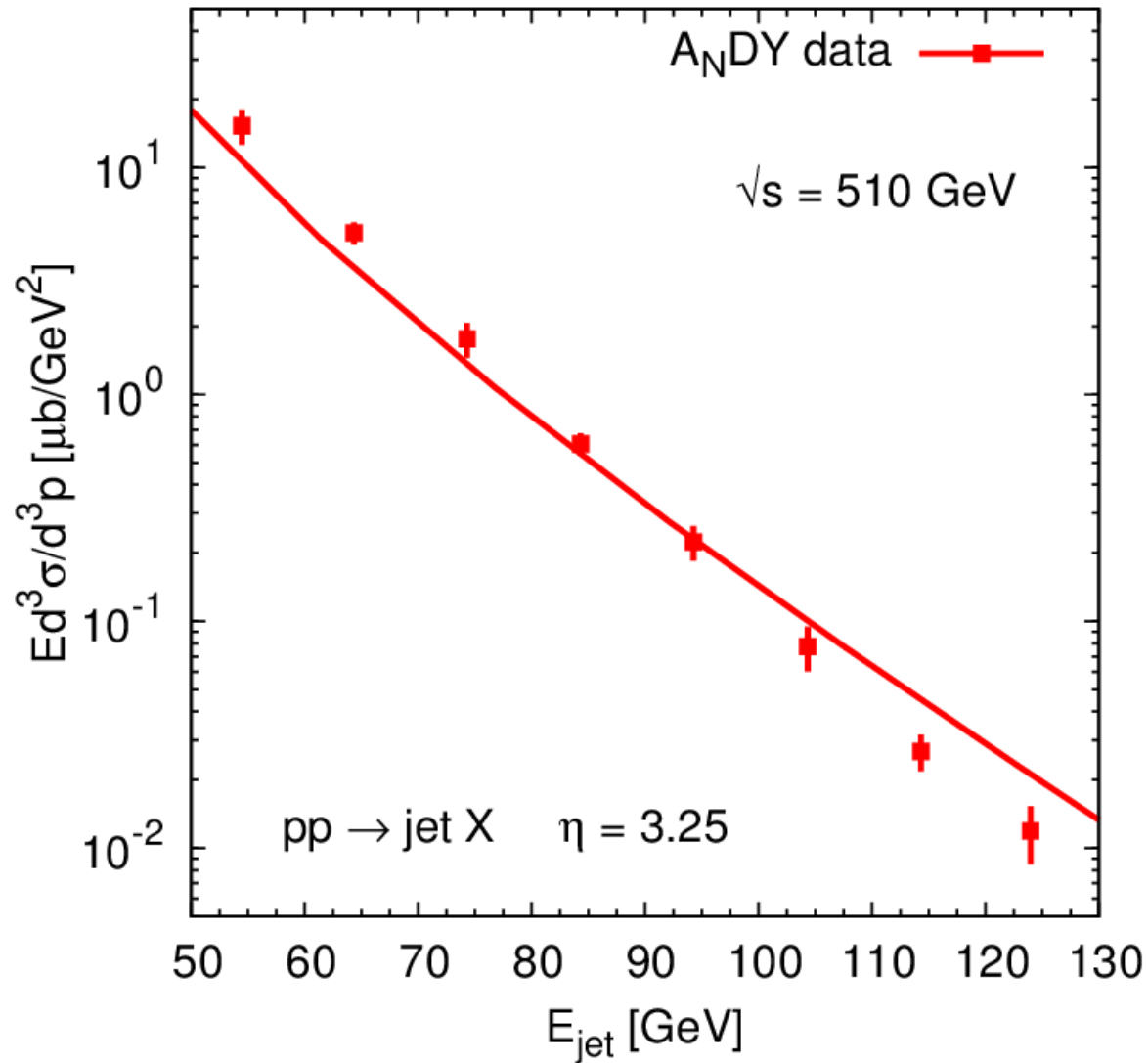
# Results



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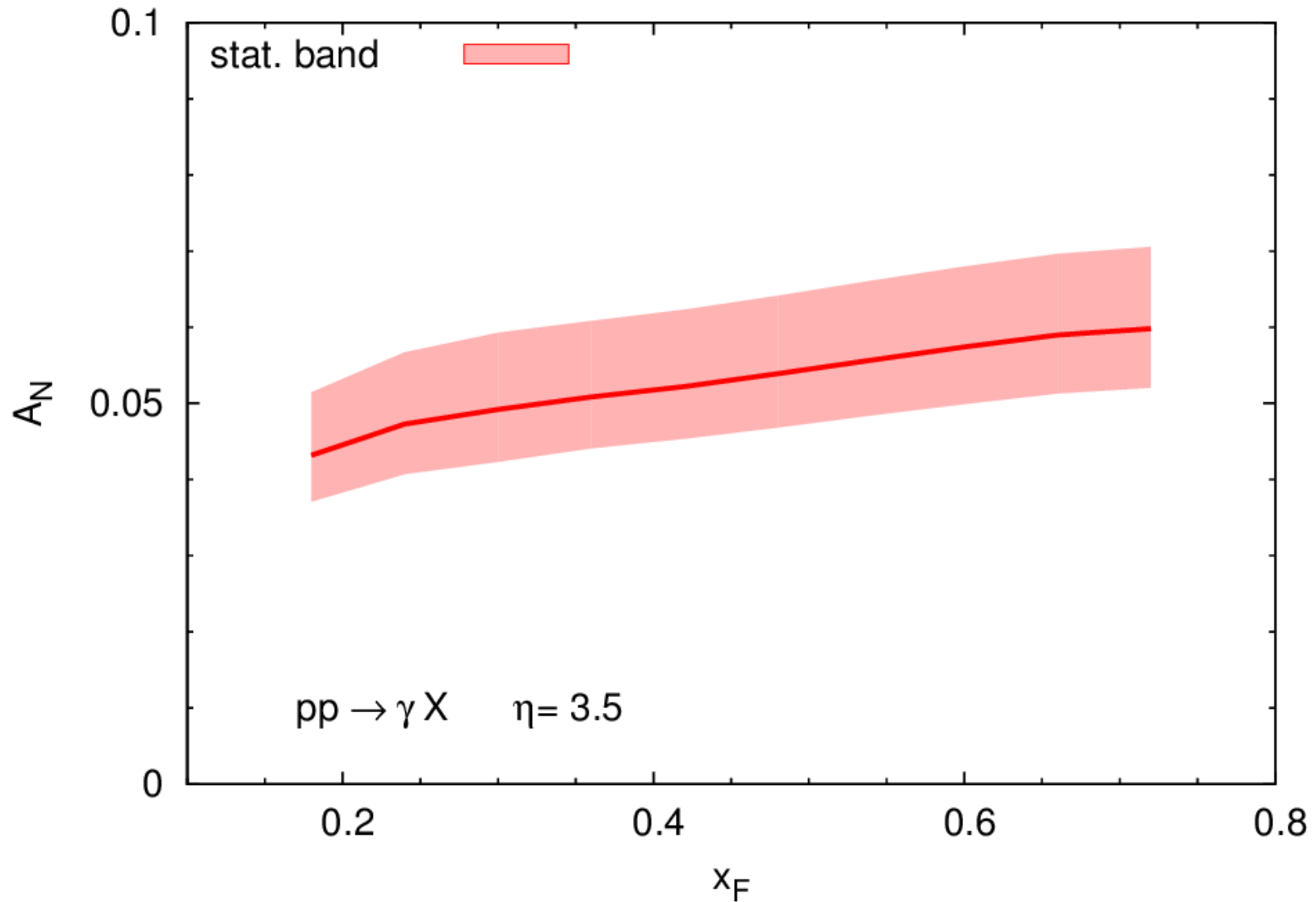


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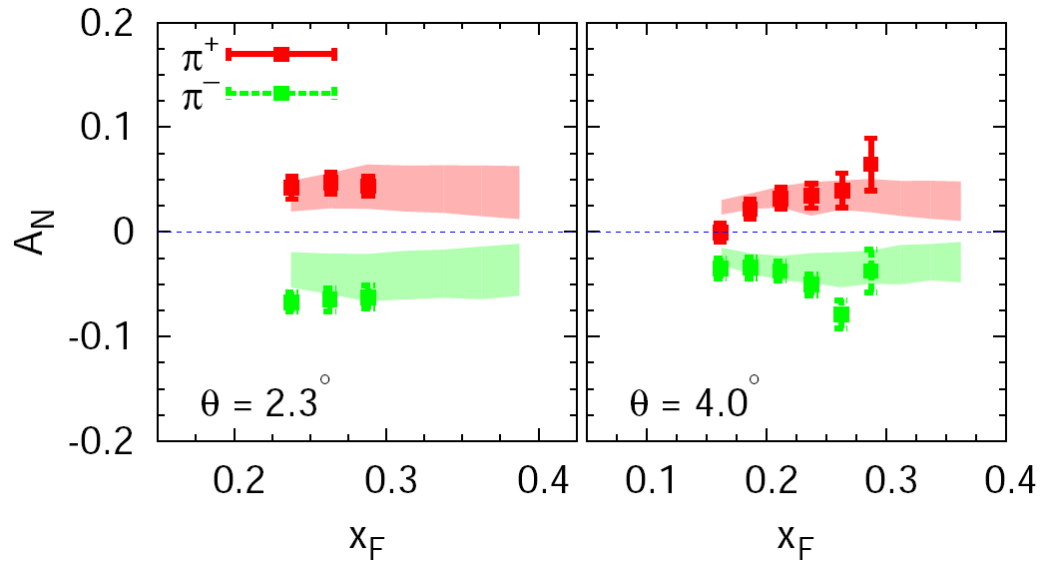




# Results

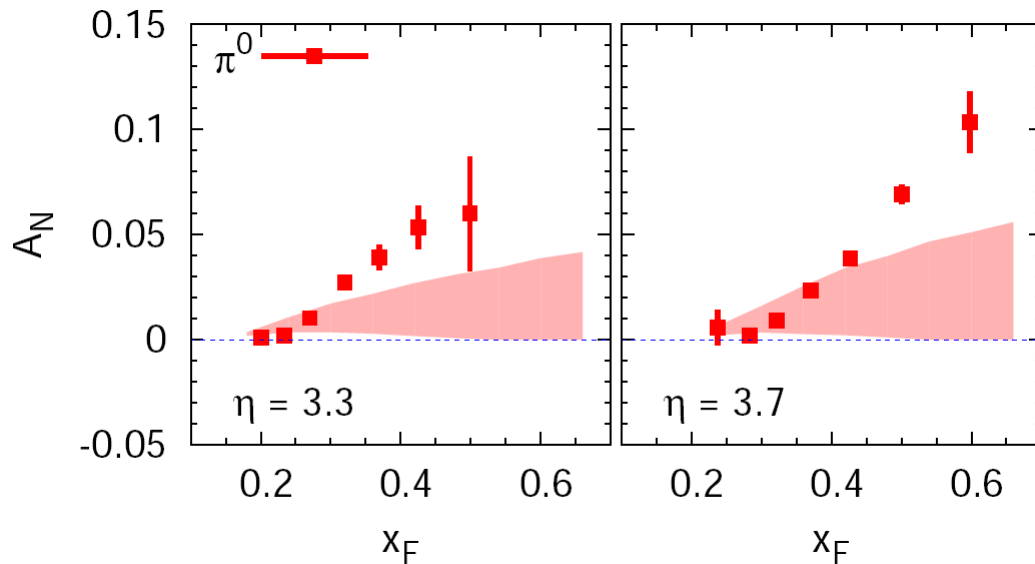


# Results: Collins



**BRAHMS@RHIC**

$\sqrt{s} = 200 \text{ GeV}$



**STAR@RHIC**

$\sqrt{s} = 200 \text{ GeV}$

# Single Spin Asymmetries in $pp^\uparrow \rightarrow h+X$

➤ Possible solution II:

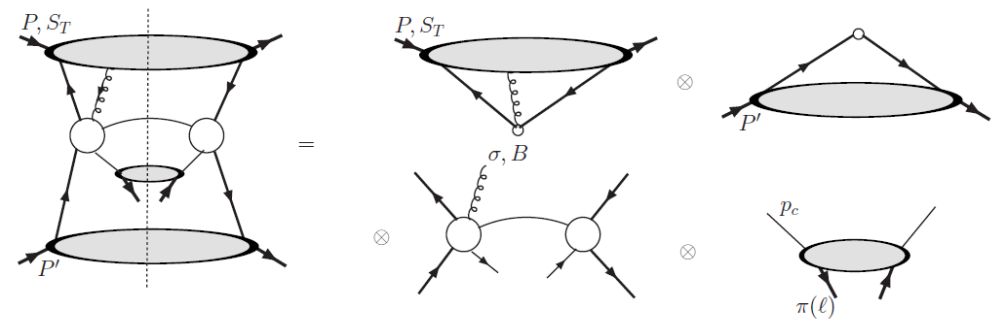
- Going beyond twist 2: collinear pQCD factorization at twist-3

$$A_N = \phi_{q/p^\uparrow}^{(3)} \otimes f_{q/p} \otimes D_{h/q} \quad \phi_{q/p^\uparrow}^{(3)} \rightarrow gT_q(x, x) \quad \text{ETQS Corr.func}$$

$$+ \Delta_{Tq} \otimes f_{q/p} \otimes D_{h/q}^{(3)}$$

$$+ \Delta_{Tq} \otimes \phi_{q^\uparrow/h} \otimes D_{h/q}$$

+ ...



Kouvaris, (2006), Kanazawa-Koike (2000), Kang-Yuan-Zhou (2010), Kang(2011)

# Relation between the Twist 3 and the TMD approaches

➤ Notice that in process like SIDIS and DY, where two factorization scale appears, we have:

$$q_T \ll Q$$

Asymmetry described by TMDs

$$q_T \gg Q$$

Asymmetry described by Twist3

➤ Twist3 functions and TMDs can be formally related to each other, for instance:

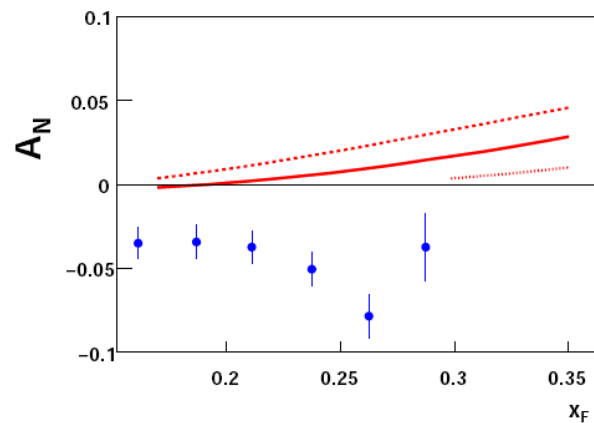
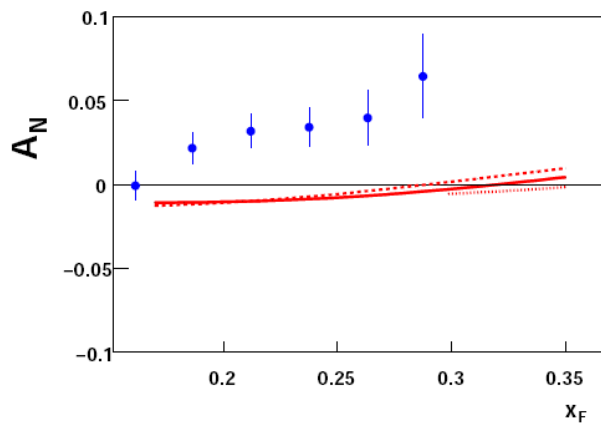
$$\int d^2 \mathbf{k}_\perp \left( \frac{\mathbf{k}_\perp^2}{M} \right) f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) |_{\text{SIDIS}} = -g T_q(x, x)$$

# Sign mismatch?

➤ Currently we do not have information on  $T_q(x,x)$  from SIDIS, but we can use this relation in order to give an estimation of  $A_N$  in inclusive production

$$\int d^2 \mathbf{k}_\perp \left( \frac{\mathbf{k}_\perp^2}{M} \right) f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) |_{\text{SIDIS}} = -g T_q(x, x)$$

➤ Brahms data: opposite sign.



Prokudin and Kang, PRD85,074 008 (2013)

# Single Spin Asymmetries in $pp^\uparrow \rightarrow h+X$

## ➤ Possible solutions:

- Going beyond twist 2: collinear pQCD factorization at twist-3
  - Factorization & universality proved
  - Not well known: requires  $P_T \gg Q$  in SIDIS, no data
  - Sign mismatch problem... (very indirect...)
- Going beyond collinear approx.: Generalized Parton Model (TMD)
  - Successfully applied in SIDIS lot of data
  - Factorization & universality are assumed for inclusive hadron production

# The Sivers function from SIDIS data

- We can build an azimuthal weighted asymmetry

$$A_{UT}^{\sin(\phi_h - \phi_S)} = 2 \frac{\int d\phi_S d\phi_h [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi_h - \phi_S)}{\int d\phi_S d\phi_h [d\sigma^\uparrow + d\sigma^\downarrow]}$$

- In details:

$$A_{UT}^{\sin(\phi_h - \phi_S)} = \frac{\sum_q \int d\phi_S d\phi_h d^2\mathbf{k}_\perp \Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi - \phi_S) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} D_q^h(z, p_\perp) \sin(\phi_h - \phi_S)}{\sum_q \int d\phi_S d\phi_h d^2\mathbf{k}_\perp f_{q/p}(x, k_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} D_q^h(z, p_\perp)}$$