

# Linearly Polarized Gluon Distributions at Small- $x$

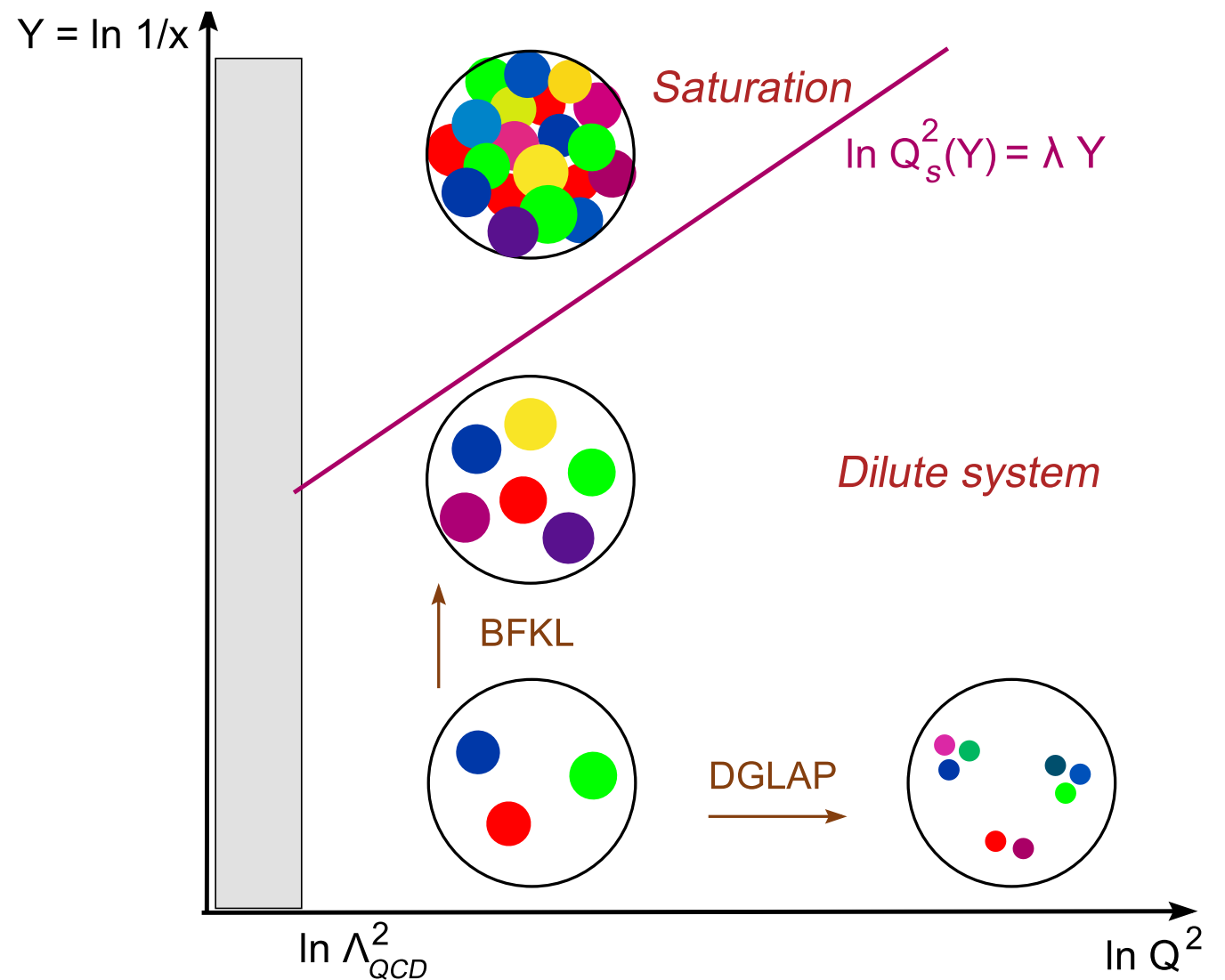
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In collaboration with J.-W. Qiu, B.-W. Xiao, and F. Yuan, Phys.Rev. D85 (2012) 045003

DIS Workshop  
Marseille, April 25, 2013

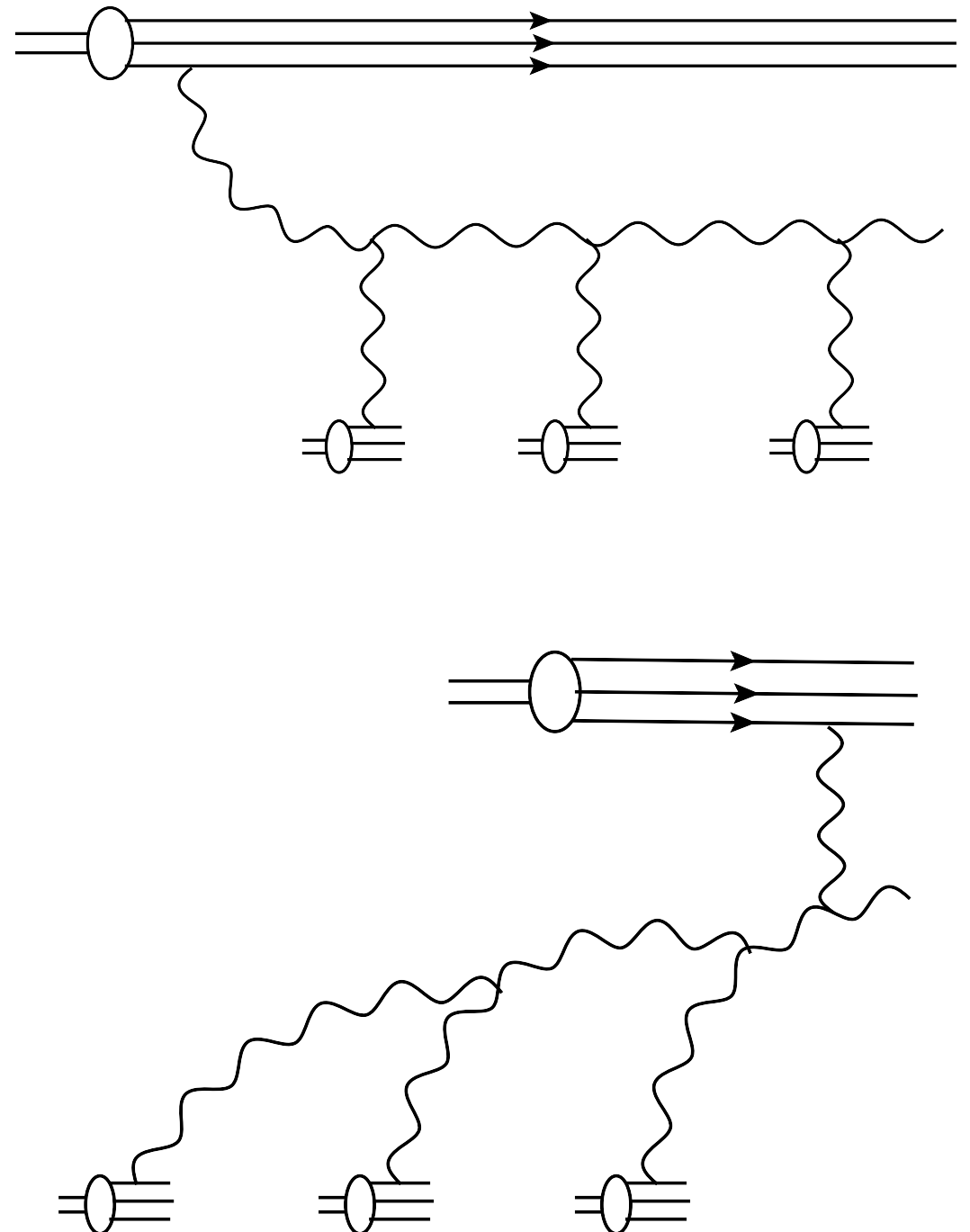
# High densities at small-x

- Separation between dense/dilute regimes given by saturation scale
- Non-linear evolution
- Collinear factorization does not apply for high-density regime
- Gluons are dominant



# Factorization at small- $x$

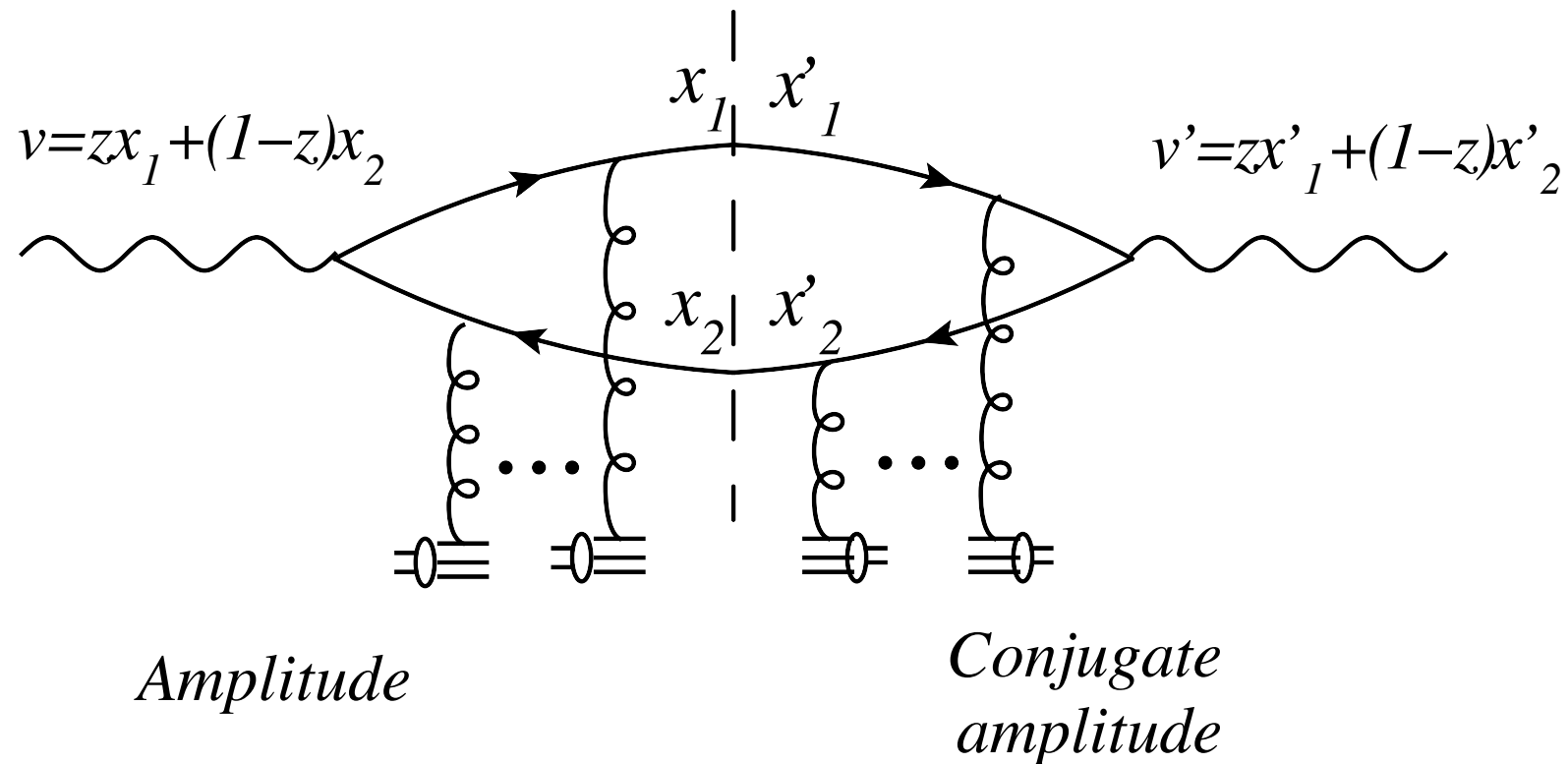
- It is necessary to include transverse momentum dependence
- Partonic interpretation is not straightforward and possibly gauge dependent (gluons)
- Additional semi-hard scale: saturation scale



# Multiple scatterings approach

- High energy probes propagating eikonally
- Target considered as a background field in a covariant gauge (appropriate for the small- $x$  degrees of freedom)
- Multiple scattering can be resummed in terms of Wilson lines

# Two-particle production in DIS

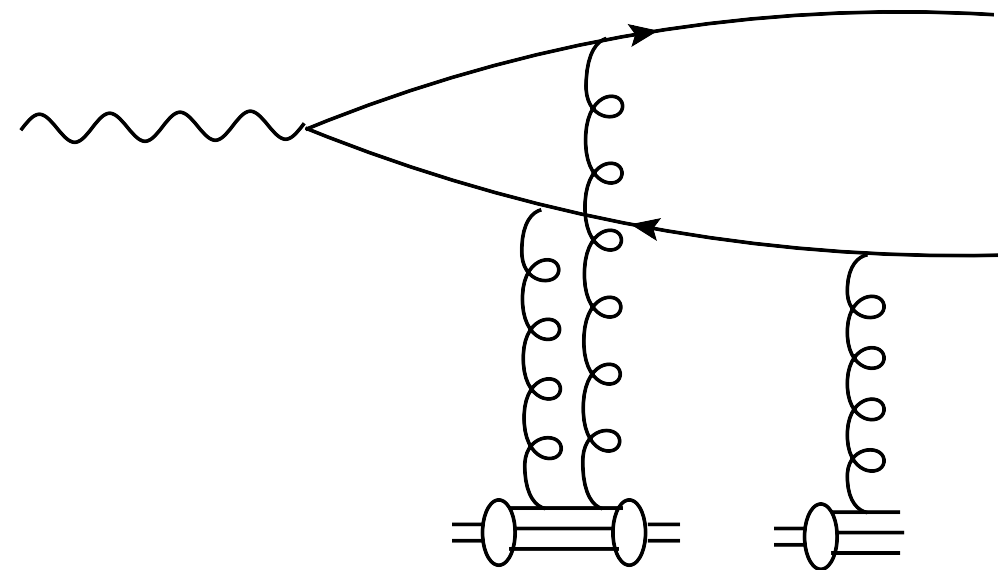


$$\begin{aligned}
 \frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} &= N_c \alpha_{em} e_q^2 \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2x_1}{(2\pi)^2} \frac{d^2x'_1}{(2\pi)^2} \frac{d^2x_2}{(2\pi)^2} \frac{d^2x'_2}{(2\pi)^2} \\
 &\times e^{-ik_{1\perp} \cdot (x_1 - x'_1)} e^{-ik_{2\perp} \cdot (x_2 - x'_2)} \sum \psi_T^*(x_1 - x_2) \psi_T(x'_1 - x'_2) \\
 &\times \left[ 1 + Q_{x_g}(x_1, x_2; x'_2, x'_1) - S_{x_g}^{(2)}(x_1, x_2) - S_{x_g}^{(2)}(x'_1, x'_2) \right]
 \end{aligned}$$

$$Q_{x_g}(x_1, x_2; x'_2, x'_1) = \frac{1}{N_c} \langle \text{Tr} U(x_1) U^\dagger(x'_1) U(x'_2) U^\dagger(x_2) \rangle_{x_g} \quad S_{x_g}^{(2)}(x_1, x_2) = \frac{1}{N_c} \langle \text{Tr} U(x_1) U^\dagger(x_2) \rangle_{x_g}$$

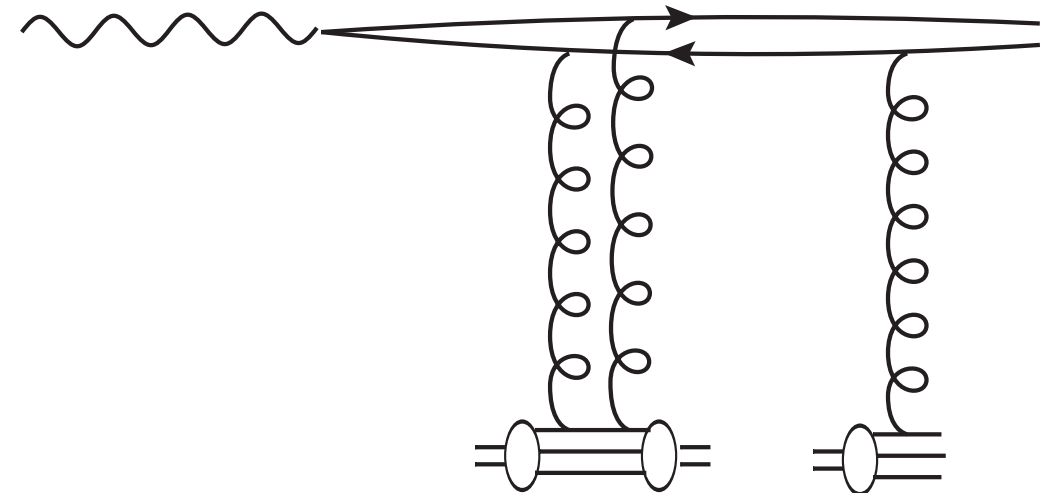
# Correlation limit

- Factorization requires a separation of scales
- Take momentum imbalance much smaller than individual transverse momenta
- In coordinate space this amounts to take a very small separation for the quark-antiquark pair



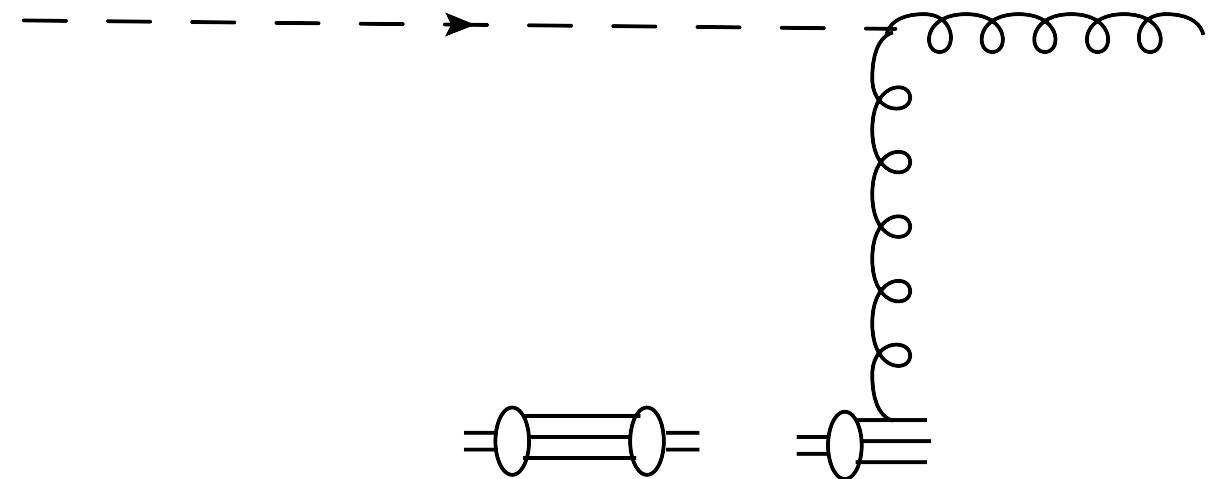
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Looks like colorless  
current liberating a gluon



# Factorized form

$$\begin{aligned}
 \frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}X}}{dy_1 dy_2 d^2 P_\perp d^2 q_\perp} = & \alpha_{em} e_q^2 \alpha_s \delta(x_{\gamma^*} - 1) z(1-z) (z^2 + (1-z)^2) \left[ \frac{\delta_{ij}}{(\tilde{P}_\perp^2 + \epsilon_f^2)^2} - \frac{4\epsilon_f^2 \tilde{P}_{\perp i} \tilde{P}_{\perp j}}{(\tilde{P}_\perp^2 + \epsilon_f^2)^4} \right] \\
 & \times (16\pi^3) \int \frac{d^3 v d^3 v'}{(2\pi)^6} e^{-iq_\perp \cdot (v-v')} 2 \left\langle \text{Tr} \left[ F^{i-}(v) \mathcal{U}^{[+]\dagger} F^{j-}(v') \mathcal{U}^{[+]} \right] \right\rangle_{x_g}
 \end{aligned}$$

# Factorized form

$$\frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}X}}{dy_1 dy_2 d^2 P_\perp d^2 q_\perp} = \alpha_{em} e_q^2 \alpha_s \delta(x_{\gamma^*} - 1) z(1-z) (z^2 + (1-z)^2) \left[ \frac{\delta_{ij}}{(\tilde{P}_\perp^2 + \epsilon_f^2)^2} - \frac{4\epsilon_f^2 \tilde{P}_{\perp i} \tilde{P}_{\perp j}}{(\tilde{P}_\perp^2 + \epsilon_f^2)^4} \right]$$

$$\times (16\pi^3) \int \frac{d^3 v d^3 v'}{(2\pi)^6} e^{-iq_\perp \cdot (v-v')} 2 \left\langle \text{Tr} \left[ F^{i-}(v) \mathcal{U}^{[+]\dagger} F^{j-}(v') \mathcal{U}^{[+]} \right] \right\rangle_{x_g}$$

$$\frac{1}{2} \delta^{ij} x G^{(1)}(x, q_\perp) + \frac{1}{2} \left( \frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij} \right) x h_\perp^{(1)}(x, q_\perp)$$

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WW Unpolarized distribution



# Factorized form

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WW Linearly polarized distribution

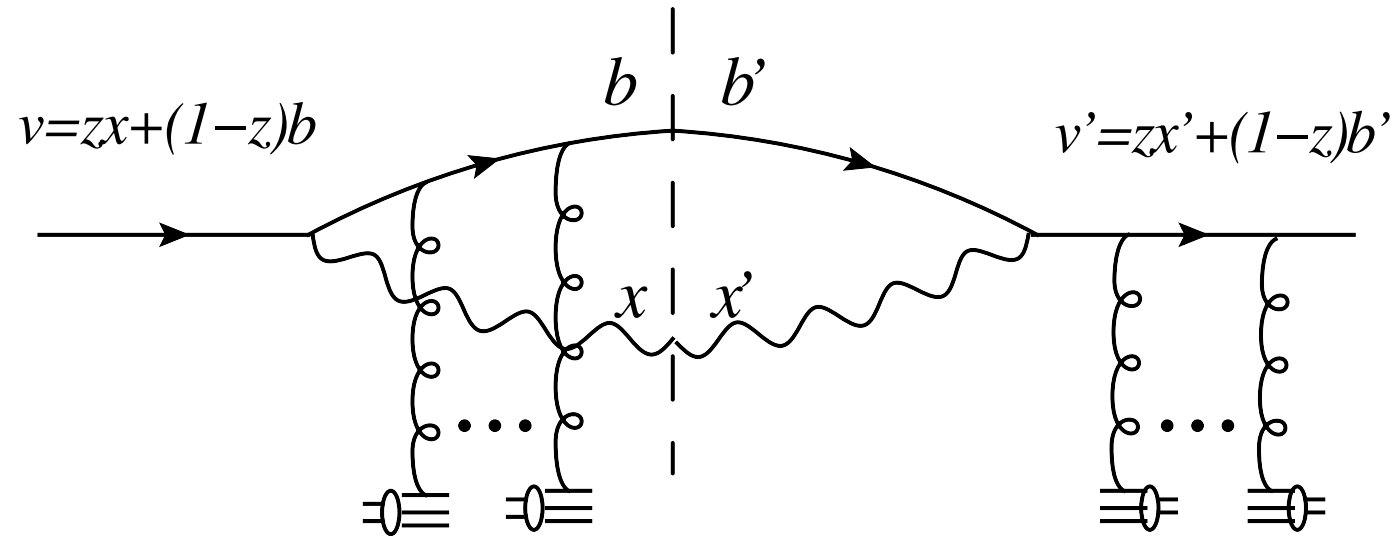
# Factorized form

$$\frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}X}}{dy_1 dy_2 d^2 P_\perp d^2 q_\perp} = \alpha_{em} e_q^2 \alpha_s \delta(x_{\gamma^*} - 1) z(1-z) (z^2 + (1-z)^2) \frac{\epsilon_f^4 + \tilde{P}_\perp^4}{(\tilde{P}_\perp^2 + \epsilon_f^2)^4} \\ \times \left[ xG^{(1)}(x, q_\perp) - \frac{2\epsilon_f^2 \tilde{P}_\perp^2}{\epsilon_f^4 + \tilde{P}_\perp^4} \cos(2\Delta\phi) xh_\perp^{(1)}(x, q_\perp) \right]$$

$$\Delta\phi = \phi_{\tilde{P}_\perp} - \phi_{q_\perp}$$

Sensitive to the linearly polarized gluon distribution through the azimuthal dependence

# Drell-Yan in pA collisions



*Amplitude*

*Conjugate  
amplitude*

$$\frac{d\sigma^{pA \rightarrow \gamma^* q X}}{dy_1 dy_2 d^2 k_{1\perp} d^2 k_{2\perp}} = \sum_f x_p q_f(x_p, \mu) \frac{\alpha_{em} e_f^2}{2\pi^2} (1-z) z^2 S_\perp F_{x_g}(q_\perp) \times \left\{ \left[ 1 + (1-z)^2 \right] \frac{q_\perp^2}{\left[ \tilde{P}_\perp^2 + \epsilon_M^2 \right] \left[ (\tilde{P}_\perp + zq_\perp)^2 + \epsilon_M^2 \right]} - \epsilon_M^2 \left[ \frac{1}{\tilde{P}_\perp^2 + \epsilon_M^2} - \frac{1}{(\tilde{P}_\perp + zq_\perp)^2 + \epsilon_M^2} \right]^2 \right\}$$

$$F_{x_g}(q_\perp) = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} \frac{1}{N_c} \langle \text{Tr} U(0) U^\dagger(r_\perp) \rangle_{x_g}$$

**Dipole amplitude**

# Factorized form

$$\frac{d\sigma^{pA \rightarrow \gamma^* q + X}}{dy_1 dy_2 d^2k_{1\perp} d^2k_{2\perp}} \Big|_{q_\perp \ll \tilde{P}_\perp} = \sum_f x_p q_f(x_p, \mu) x G^{(2)}(x_g, q_\perp) \left[ H_{qg \rightarrow q\gamma^*} - \cos(2\Delta\phi) H_{qg \rightarrow q\gamma^*}^\perp \right]$$

Linearly polarized distribution is not independent

$$H_{qg \rightarrow q\gamma^*} = \frac{\alpha_s \alpha_{e.m.} e_f^2 (1-z) z^2}{N_c} \left\{ \frac{1 + (1-z)^2}{[\tilde{P}_\perp^2 + \epsilon_M^2]^2} - \frac{2z^2 \epsilon_M^2 \tilde{P}_\perp^2}{[\tilde{P}_\perp^2 + \epsilon_M^2]^4} \right\}$$

$$H_{qg \rightarrow q\gamma^*}^\perp = \frac{\alpha_s \alpha_{e.m.} e_f^2 (1-z) z^2}{N_c} \frac{2z^2 \epsilon_M^2 \tilde{P}_\perp^2}{[\tilde{P}_\perp^2 + \epsilon_M^2]^4}$$

# Factorized form

Dipole type gluon distribution

$$\frac{d\sigma^{pA \rightarrow \gamma^* q + X}}{dy_1 dy_2 d^2k_{1\perp} d^2k_{2\perp}} \Big|_{q_\perp \ll \tilde{P}_\perp} = \sum_f x_p q_f(x_p, \mu) x G^{(2)}(x_g, q_\perp) \left[ H_{qg \rightarrow q\gamma^*} - \cos(2\Delta\phi) H_{qg \rightarrow q\gamma^*}^\perp \right]$$

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# Conclusions

- Correlation limit leads to TMD-factorized expressions
- Azimuthal dependence gives access to linearly polarized gluon distributions
- As for the unpolarized case, there are two types of linearly polarized gluon distributions at small- $x$