Inclusion of open-charm and $W^{\pm}$ production data in a polarized PDF extraction via Bayesian reweighting

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   - The proton spin content and the Bjorken sum rule
3. Inclusion of new datasets via Bayesian reweighting
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   - $W^{\pm}$ boson production at RHIC
4. Conclusions
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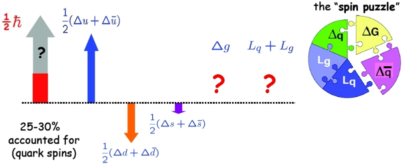
1. Motivation

Why are we interested in helicity-dependent PDFs?

1. The "spin puzzle" or how quarks and gluons carry proton's spin

$$\langle S_z \rangle = \frac{1}{2}\langle \Delta\Sigma \rangle + \langle \Delta g \rangle + (L_q + L_g) \sim \frac{1}{2}$$

$$\Delta\Sigma(x) = \sum_{i=u,d,s} (\Delta q_i + \Delta\bar{q}_i)$$



→ what about the $\Delta q + \Delta\bar{q}$ and Δg contributions to the proton spin?

→ how much are they uncertain?

→ what are the antiquark distributions (*i.e.* Δq and $\Delta\bar{q}$ separately)?

2. Explore QCD beyond helicity-averaged case

→ could determination of α_s from polarized data be competitive?

3. Test for physics beyond SM

→ possible polarization upgrades of the Tevatron or the LHC

→ polarized hadron collisions will require statistically sound knowledge of polarized PDFs

Issues in standard PDF determination

- Extraction of a set of functions Δf with error bands from a set of data points
- We need an error band, i.e. a probability density $\mathcal{P}[\Delta f(x)]$ in the space of PDFs

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Delta f \, \mathcal{P}[\Delta f] \mathcal{O}[\Delta f]$$

$$\sigma^2_{\mathcal{O}} = \int \mathcal{D}\Delta f \, \mathcal{P}[\Delta f] (\mathcal{O}[\Delta f] - \langle \mathcal{O} \rangle)^2$$

Standard approach

1. Choose a fixed functional form like

   $$\Delta f_i(x, Q_0^2) = A_i x^{b_i} (1-x)^{c_i} (1+\dots)$$

2. Determine best-fit parameters
3. Errors determined via Gaussian linear error propagation

But...

1. Is the parametrization flexible enough?
2. What is the error associated to any particular choice?
3. Need to rely on linear error propagation

The NNPDF methodology in a nutshell

1. Monte Carlo sampling of experimental data
   - generate experimental data replicas assuming multi-Gaussian probability distribution
   - validate against experimental data to determine the sample size ($N_{\rm rep} \sim 100$)

   $\Rightarrow$ **no need to rely on linear error propagation, no tolerance needed**
2. Fit a set of PDFs parametrizing each replica with Neural Networks
   - redundant and flexible parametrization, $\mathcal{O}(200)$ parameters
   - fit to each data replica by optimizing χ^2 (genetic algorithm + cross-validation)

   $\Rightarrow$ **reduce the theoretical bias due to the parametrization**

Treat resulting PDF replicas as equally probable members of a statistical ensemble
Expectation values for observables (and errors, etc.) are Monte Carlo integrals

$$\langle \mathcal{O}[\Delta f] \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{O}[\Delta f_k]$$

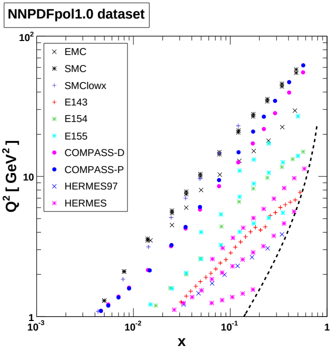
The NNPDF methodology provides unbiased and statistically meaningful PDF ensemble which samples the probability density in the space of PDFs

Some optimized/new features used in polarized NNPDF for the first time

2. The NNPDFpol1.0 parton set

[arXiv:1303.7236]

Experimental data and QCD analysis features

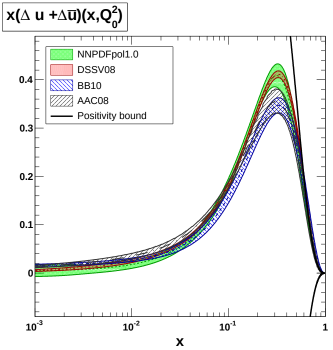


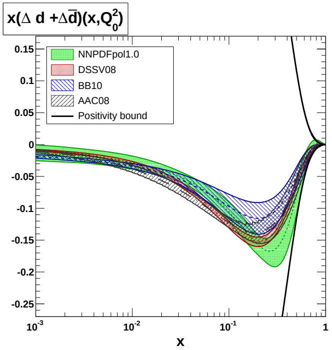
- Global fit to inclusive DIS world data on $g_1^{p,d,n}$ (proton, deuteron and neutron targets)
- Kinematical cuts
  $Q^2 \geq 1$ GeV^2 (pure perturbative QCD analysis)
  $W^2 = Q^2(1-x)/x \geq 6.25$ GeV^2 [arXiv:0807.1501]
  (remove sensitivity to dynamical higher-twists)
  Higher twist terms added to observables and fitted to data become compatible with zero
- Initial scale $Q_0^2 = 1$ GeV^2 + FastKernel evolution method [arXiv:1002.4407]

Inclusive (NC) DIS does not allow us to disentangle the contributions from q and $\bar{q}$
Choose a basis of four polarized PDFs (gluon + linear combinations of light quarks)
e.g. $\{\Delta\Sigma; \Delta T_3; \Delta T_8; \Delta g\}$ or $\{\Delta u + \Delta\bar{u}; \Delta d + \Delta\bar{d}; \Delta s + \Delta\bar{s}; \Delta g\}$

- Require integrability and positivity of Δg and $\Delta q + \Delta\bar{q}$ combinations

The NNPDFpol1.0 parton set at $Q_0^2 = 1\ \mathrm{GeV}^2$





DSSV08

[arXiv:0904.3821]

DIS+SIDIS+pp (π^0/jet)

BB10

[arXiv:1005.3113]

DIS only

AAC08

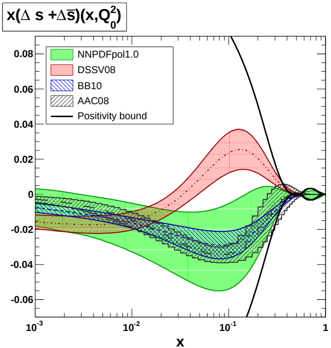
[arXiv:0808.0413]

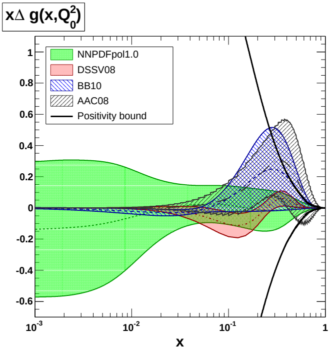
DIS+pp (π^0)

$\Delta u + \Delta\bar{u}$ and $\Delta d + \Delta\bar{d}$

- Central values in reasonable agreement with those of other parton sets (best with DSSV08, slightly worse with AAC08, worst with BB10)
- Uncertainties slightly larger for NNPDF than for other sets, especially DSSV08 (notice that DSSV08 fit is based on a much wider dataset)
- Where no data or theoretical constraints are available, uncertainties are larger (flexibility of the Neural Network)

The NNPDFpol1.0 parton set at $Q_0^2 = 1$ GeV^2





DSSV08
[arXiv:0904.3821]
DIS+SIDIS+pp (π^0/jet)

BB10
[arXiv:1005.3113]
DIS only

AAC08
[arXiv:0808.0413]
DIS+pp (π^0)

$\Delta s + \Delta \bar{s}$

- Good agreement with BB10 and AAC08, but larger uncertainty for almost all x
- Inconsistency at 2σ with DSSV08 in the medium-small x region (SIDIS data)

Δg

- Central value compatible with zero
- Uncertainty much larger than any other set, especially in the low-x region

The proton spin content

$$[\Delta\Sigma] \equiv \int_0^1 \Delta\Sigma(x, Q_0^2)dx \qquad [\Delta g] \equiv \int_0^1 \Delta g(x, Q_0^2)dx$$

Singlet and Gluon first moments in $\overline{\text{MS}}$ scheme at $Q_0^2 = 1$ GeV^2

	NNPDFpol1.0	DSSV08	AAC08 (positive Δg)	ABFR98
$[\Delta\Sigma]$	0.22 ± 0.20	$0.26 \pm 0.02 \pm 0.13$	0.26 ± 0.06	$0.12 \pm 0.05^{+0.19}_{-0.13}$
$[\Delta g]$	-1.2 ± 4.2	$-0.12 \pm 0.12 \pm 0.06$	0.40 ± 0.28	$1.6 \pm 0.4 \pm 0.8$

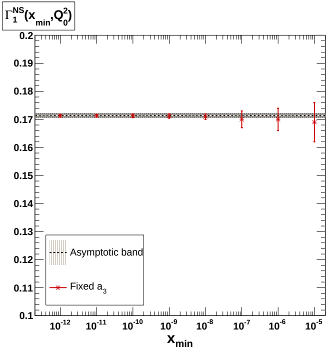
$$\int_{10^{-3}}^1 \Delta g(x, Q^2 = 1\text{GeV}^2) = -0.12 \pm 1.21$$

- Singlet
  → good agreement with determinations from other analyses
  → comparable uncertainty, if somewhat larger, to that estimated including extrapolation
- Gluon
  → compatible with zero, uncertainty larger than any other set even in the measured region (about three times in the measured region, up to ten times within extrapolation)

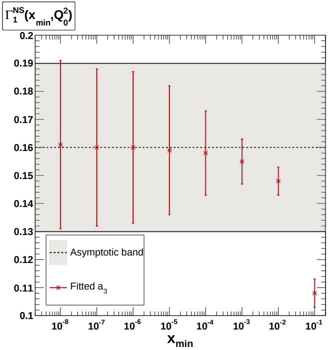
The experimental status of the gluon first momentum is still completely uncertain
Uncertainty larger than any previous estimate even in the measured region

The Bjorken sum rule

$$\Gamma_1^{\rm NS}(x_{\rm min}, Q^2) \equiv \int_{x_{\rm min}}^{1} dx[g_1^p(x,Q^2) - g_1^n(x,Q^2)] \xrightarrow{x_{\rm min}=0} \frac{1}{6} a_3 \Delta C_{\rm NS}[\alpha_s(Q^2)]$$



fixed $a_3 = 1.2701 \pm 0.0025$



fitted $a_3 = 1.19 \pm 0.22$

Determination of α_s from Bjorken sum rule is presently not competitive

3. Inclusion of new dataset

Bayesian reweighting

1. Take a prior ensemble of PDF replicas $\{f_k\}$, with $k = 1, \ldots, N_{\rm rep}$
2. Take a new dataset $\{y\} = \{y_1, \ldots, y_n\}$ and its covariance matrix σ_{ij} (if available)
3. Compute the prediction $y_i[f_k]$ for the experimental point i for the k-th replica
4. Compute the χ^2_k and the relative weight w_k of each replica on the new dataset y

$$\chi^2_k = \sum_{i,j}^{n} \{y_i - y_i[f_k]\}\, \sigma_{ij}\, \{y_j - y_j[f_k]\} \quad w_k \propto (\chi^2_k)^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k} \quad \text{with} \quad N_{\rm rep} = \sum_{k=1}^{N_{\rm rep}} w_k$$

5. Replicas are no longer equally probable. Expectation values are given by

$$\langle \mathcal{O} \rangle_{\rm new} = \sum_{i=1}^{N_{\rm rep}} w_k \mathcal{O}[f_k]$$

6. Loss of efficiency: $N_{\rm eff} \equiv \exp\left[-\sum_{k=1}^{N_{\rm rep}} p_k \log p_k\right]$ with $p_k = w_k / N_{\rm rep}$
   $0 < N_{\rm eff} < N_{\rm rep}$; $N_{\rm eff}$ must not be too low $\Rightarrow$ increase the number of replicas in prior

Reweighting allows to incorporate new datasets without need of refitting provided a prior PDF ensemble

Reweighting the gluon with COMPASS open-charm data

New dataset on virtual photon cross section asymmetry from open-charm muoproduction

$$A^{\gamma N \to D^0 X} = \frac{\Delta g \otimes d\Delta\hat{\sigma}_{\gamma g} \otimes D_c^H}{g \otimes d\hat{\sigma}_{\gamma g} \otimes D_c^H}$$

FEATURES

- directly sensitive to Δg which is probed through photon-gluon fusion process (in DIS Δg is probed through scaling violations instead)

EXPERIMENTAL MEASUREMENT

- COMPASS (2002-2007) [arXiv:1211.6849] (talk by L. Silva)
  ($\Delta g(x, Q^2)$ is probed at $0.06 \lesssim x \lesssim 0.22$ and $Q^2 = 4(m_c^2 + p_T^2) \sim 13$ GeV^2)

THEORETICAL PREDICTION

- the asymmetry is computed at LO for the time being
  (actually NLO contributions do not cancel in the asymmetry [arXiv:1212.1319])
- use `NNPDFpol1.0` for Δg, `NNPDF2.3` for g (both at NLO) and Peterson D_c^H
  (charm quarks lose little fraction z of their momentum in the hadronization and D_c^H is peaked at fairly large values of z; hence different choices of D_c^H slightly affect $A^{\gamma N \to D^0 X}$)

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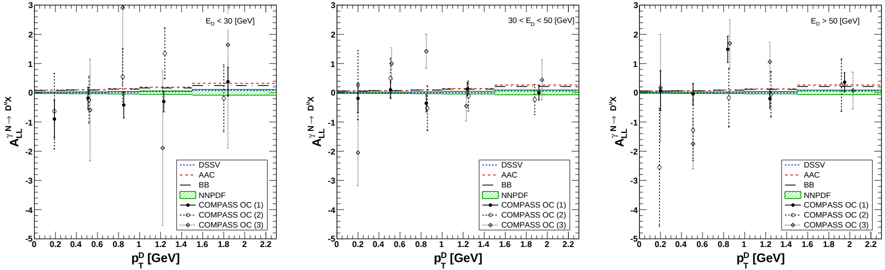
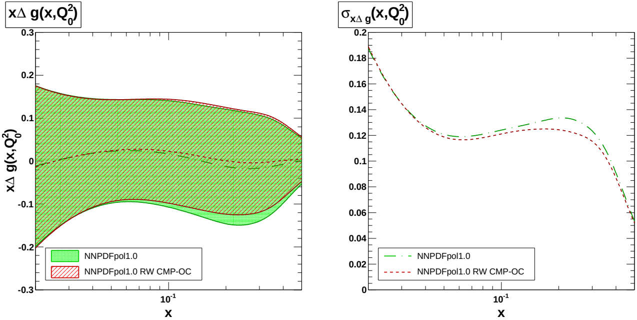


Reweighting the gluon with COMPASS open-charm data



- The new dataset has little impact on the polarized gluon
  → at $Q^2 = 1$ GeV^2 only a slight reduction in the polarized gluon uncertainty is observed
  → data description does not improve, as the value of χ^2 per data point is unchanged after reweighting: $\chi^2/N_{\rm dat} = 1.23$

Little impact on the reweighted polarized gluon due to large experimental uncertainties
More experimental precision should be achieved to better constrain Δg
with open-charm data in the kinematic region probed by COMPASS

Reweighting with $W^{\pm}$ production at RHIC

New dataset on longitudinal single-spin asymmetry for $W^{\pm}$ boson production

$$A_L^{W^+} \sim \frac{-\Delta u(x_1)\bar{d}(x_2) + \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)} \qquad A_L^{W^-} \sim \frac{-\Delta d(x_1)\bar{u}(x_2) + \Delta\bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)}$$

FEATURES

- sensitive to individual quark and antiquark flavours ($\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}$)
  (purely weak process coupling q_L with $\bar{q}_R$ at partonic level, $u_L\bar{d}_R \to W^+$ or $d_L\bar{u}_R \to W^-$)
- no need of fragmentation functions (instead of SIDIS)

EXPERIMENTAL MEASUREMENT

- STAR and PHENIX at RHIC (talks by B. Surrow and S. Park)
  (only preliminary measurements from STAR (2012) [arXiv:1302.6639] will be considered here)

THEORETICAL PREDICTION

- `NNPDFpol1.0` itself is not good since q and $\bar{q}$ contributions are not disentangled
  → use instead `NNPDFpol1.0` + reasonable assumption on q and $\bar{q}$ separation
  → if new data bring in sufficient new information, results will be independent from prior
- the asymmetry is computed at NLO using CHE code [arXiv:1003.4533]

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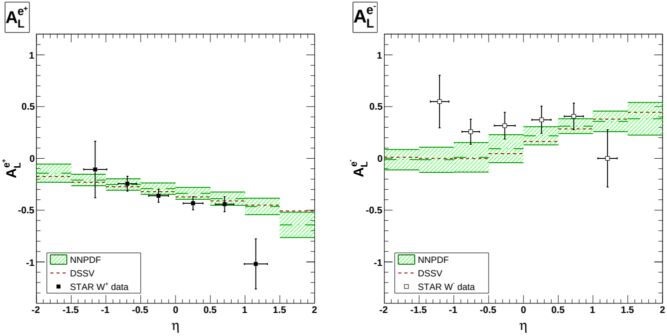
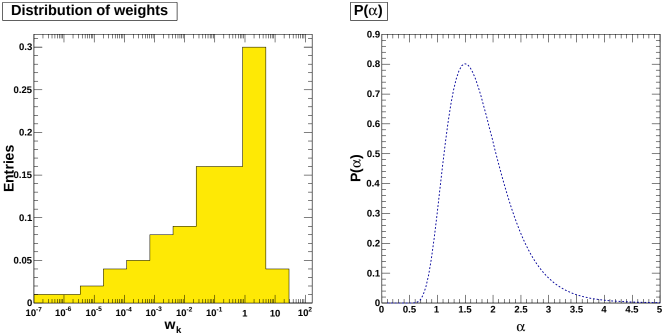


PRIOR: `NNPDFpol1.0` supplemented with a NN fit to $\Delta\bar{u}$ and $\Delta\bar{d}$ $(cv + 2\sigma)$ from DSSV08 with $N_{\rm rep} = 500$ replicas

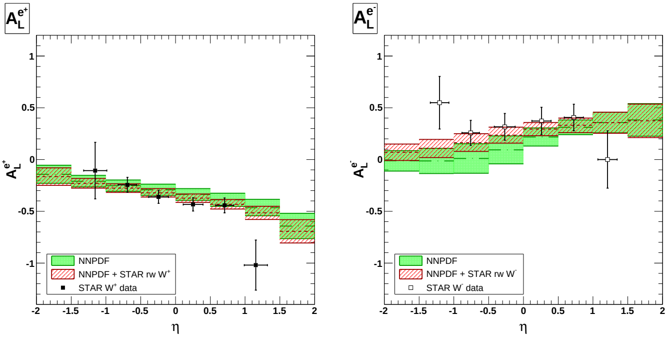
Simultaneous reweighting of $W^{\pm}$ data



- The distribution of weights shows a long tail to small values (about 30% replicas with $w_k < 10^{-2}$)
- Data are fairly consistent with the experimental information in the prior (the $\mathcal{P}(\alpha)$ distribution is peaked just above one)

$\mathcal{P}(\alpha)$ is the probability density of the rescaling parameter α, $\mathcal{P}(\alpha) \propto \frac{1}{\alpha}\sum_{k=1}^{N_{\rm rep}} w_k(\alpha)$,
where $w_k(\alpha)$ are evaluated by rescaling $\chi^2 \to \chi^2/\alpha$
If $\mathcal{P}(\alpha)$ peaks close to one, the new datasets are consistent with the old

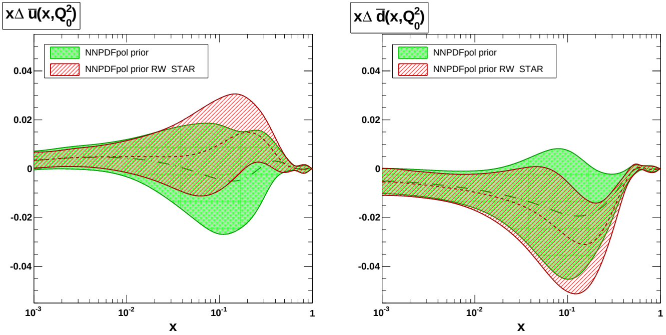
Simultaneous reweighting of $W^{\pm}$ data



Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$	$\chi^2_{\rm rw}/N_{\rm dat}$	$N_{\rm eff}/N_{\rm rep}$
STAR		12	2.10	1.12	0.38
	STAR W^+	6	1.58	1.04	0.60
	STAR W^-	6	2.62	1.20	0.30

After reweighting, χ^2 per data point $\chi^2_{\rm rw}/N_{\rm dat} \sim 1$
as wished for a good description of data

Simultaneous reweighting of $W^{\pm}$ data



- Significant modification of the shape of $\Delta\bar{u}$ and $\Delta\bar{d}$ from prior
- All other PDFs (including strangeness) are almost unaffected (and not shown here)
- Reweighting with separate W^+ and W^- datasets shows that $\Delta\bar{u}$ and $\Delta\bar{d}$ behaviour is driven by experimental information from W^- and W^+ respectively. Results shown above include both these effects
- Full check of independence from prior in progress

$W^{\pm}$ production data provide pivotal experimental information on $\Delta\bar{u}$ and $\Delta\bar{d}$!

4. Conclusions

Summary

1. Fit to polarized inclusive DIS world data based on the NNPDF methodology

$\Delta q + \Delta\bar{q}$ $\rightarrow$ good agreement with other available global analysis

$\rightarrow$ uncertainties slightly large especially in the low-x extrapolation region

Δg $\rightarrow$ the experimental status of the gluon momentum is completely uncertain

The `NNPDFpol1.0` ensemble with $N_{\rm rep} = 100$ replicas can be downloaded from

`https://nnpdf.hepforge.org/`

2. Inclusion of new datasets via Bayesian reweighting
   - COMPASS open-charm muoproduction data

     $\rightarrow$ more experimental precision is needed for constraining the polarized gluon
   - STAR $W^{\pm}$ production data

     $\rightarrow$ relevant impact of data in disentangling Δu (Δd) and $\Delta\bar{u}$ ($\Delta\bar{d}$)

     $\rightarrow$ expecting new experimental information from STAR/PHENIX ongoing analyses

3. Starting point for a new polarized release including this experimental information

Summary

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Thank you for your attention!

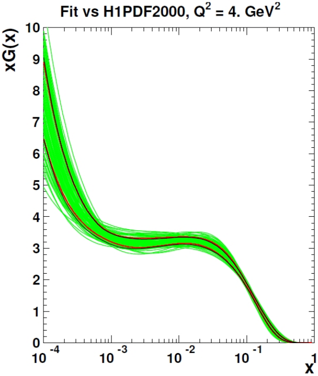
5. Backup

PDF fitting: state of the art

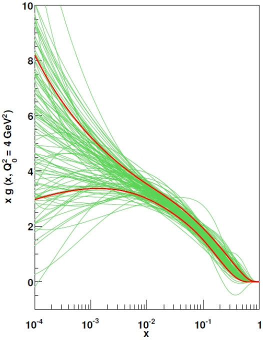
1. First stage: first moments of polarized PDFs and polarized sum rules (last 25 years)
   $\longrightarrow$ "historical" experimental collaborations (at CERN, SLAC, DESY, JLAB): EMC, SMC, E142, E143, E154, E155, COMPASS, HERMES, CLAS, ...
2. Second stage: polarized PDF fits from global NLO QCD analysis (last $\sim$15 years)
   $\longrightarrow$ different choice of datasets, parton parametrization, treatement of higher twists, ...
   ABFR (arXiv:hep-ph/9803237, 1998), BB (arXiv:1005.3113, 2010) (DIS only);
   AAC (arXiv:0808.0413, 2008), LSS (arXiv:1010.0574, 2010) (DIS+SIDIS);
   DSSV (arXiv:0904.3821, 2009) (DIS+SIDIS+pp)
3. Third stage: provide uncertainties on polarized PDFs (last $\sim$10 years)
   $\longrightarrow$ Gaussian error propagation, Lagrange multiplier + Hessian method; fit with orthogonal polynomials (arXiv:1011.4873, 2010)

Simple functional forms vs Neural Networks

HERA-LHC 2009 PDF benchmarks



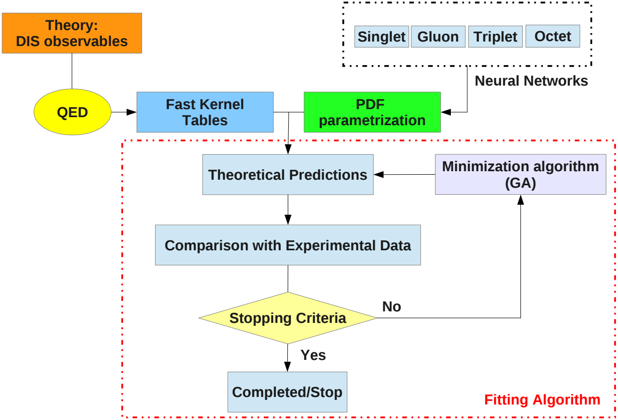
simple functional forms



Neural Networks

- Simple functional forms $\Delta q(x) = Ax^b(1-x)^c P(x)$
  $\longrightarrow$ systematic underestimation of uncertainties $\Rightarrow$ tolerance
- Artificial Neural Networks as universal interpolants
  $\longrightarrow$ reduce theoretical bias from choice of PDF functional form

A general overview on the methodology



Ingredients:
Monte Carlo sampling and Neural Networks

Ingredient 1: Monte Carlo sampling of experimental data

MONTE CARLO SAMPLING

- Sample the probability density $\mathcal{P}[\Delta q]$ in the space of functions assuming multi-Gaussian data probability distribution

$$g_{1,p}^{(\mathrm{art}),(k)}(x,Q^2) = \left[1 + \sum_c r_{c,p}^{(k)}\sigma_{c,p} + r_{s,p}^{(k)}\sigma_{s,p}\right] g_{1,p}^{(\mathrm{exp})}(x,Q^2)$$

$\sigma_{c,p}$: correlated systematics $\sigma_{s,p}$: statistical errors (also uncorrelated systematics)

$r_{c,p}^{(k)}$, $r_{s,p}^{(k)}$: Gaussian random numbers

- Generate MC ensemble of N_{rep} replicas with the data probability distribution

MAIN FEATURES

- Expectation values for observables are Monte Carlo integrals

$$\langle \mathcal{O}[\Delta q] \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \mathcal{O}[\Delta q_k]$$

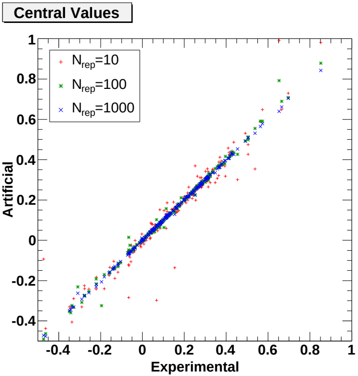
...and the same is true for errors, correlations etc.

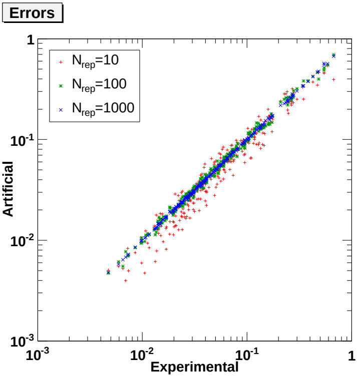
- No need to rely on linear propagation of errors
- Possibility to test for non-Gaussian behaviour in fitted PDFs

Ingredient 1: Monte Carlo sampling of experimental data

DETERMINING SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy





Accuracy of few % requires $\sim$ 100 replicas

DETERMINING SAMPLE SIZE

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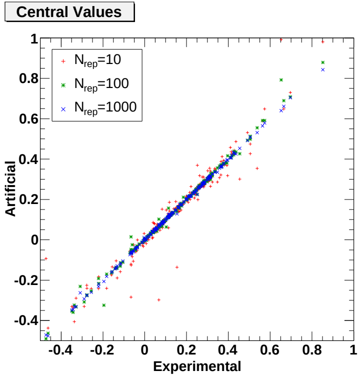

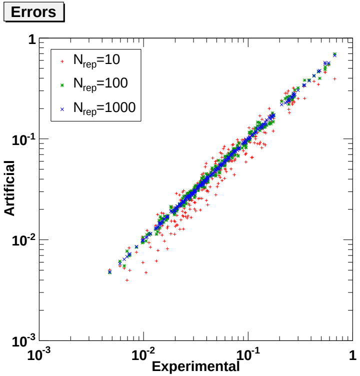

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Quantitative approach: devise proper statistical estimators

	$\left\langle \mathrm{PE}\left[\langle g_1^{(\mathrm{art})}\rangle\right]\right\rangle$ [%]			$r\left[g_1^{(\mathrm{art})}\right]$		
N_{rep}	10	100	1000	10	100	1000
EMC	23.7	3.5	2.9	.76037	.99547	.99712
SMC	19.4	5.6	1.2	.94789	.99908	.99993
...	...	...	...	...	...	...

$$\left\langle PE\left[\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right]\right\rangle_{\mathrm{dat}} = \frac{1}{N_{\mathrm{dat}}}\sum_{i=1}^{N_{\mathrm{dat}}}\left|\frac{\langle F_i^{(\mathrm{art})}\rangle_{\mathrm{rep}} - F_i^{(\mathrm{exp})}}{F_i^{(\mathrm{exp})}}\right|$$

Percentage Error

$$r\left[F^{(\mathrm{art})}\right] = \frac{\left\langle F^{(\mathrm{exp})}\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}} - \langle F^{(\mathrm{exp})}\rangle_{\mathrm{dat}}\left\langle\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}}}{\sigma_s^{(\mathrm{exp})}\sigma_s^{(\mathrm{art})}}$$

Scatter Correlation

Ingredient 1: Monte Carlo sampling of experimental data

DETERMINING SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Quantitative approach: devise proper statistical estimators

	$\left\langle PE\left[\langle \delta g_1^{(art)}\rangle\right]\right\rangle$ [%]			$r\left[\delta g_1^{(art)}\right]$		
N_{rep}	10	100	1000	10	100	1000
EMC	12.8	4.9	2.0	.97397	.99521	.99876
SMC	22.4	5.4	1.7	.96585	.99489	.99980
...	...	...	...	...	...	...

$$\left\langle PE\left[\langle F^{(art)}\rangle_{rep}\right]\right\rangle_{dat} = \frac{1}{N_{dat}}\sum_{i=1}^{N_{dat}}\left|\frac{\langle F_i^{(art)}\rangle_{rep} - F_i^{(exp)}}{F_i^{(exp)}}\right|$$

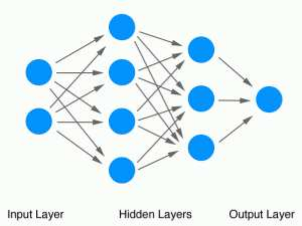
Percentage Error

$$r\left[F^{(art)}\right] = \frac{\left\langle F^{(exp)}\langle F^{(art)}\rangle_{rep}\right\rangle_{dat} - \langle F^{(exp)}\rangle_{dat}\left\langle\langle F^{(art)}\rangle_{rep}\right\rangle_{dat}}{\sigma_s^{(exp)}\sigma_s^{(art)}}$$

Scatter Correlation

Ingredient 2: Neural Networks

A convenient functional form
providing redundant and flexible parametrization
used as a generator of random functions in the PDF space

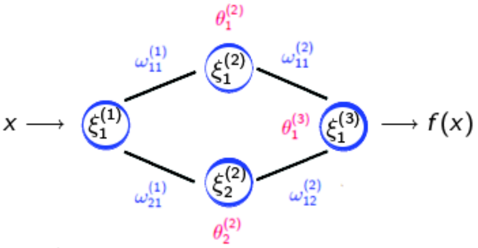


$$\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(x) = \frac{1}{1+e^{-x}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in preceding layer (feed-forward NN)
- activation determined by parameters (weights and thresholds)
- activation determined according to a non-linear function (except the last layer)

EXAMPLE: THE SIMPLEST 1-2-1 NN



$$f(x) \equiv \xi_1^{(3)} = \left\{ 1 + \exp\left[\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}} \right] \right\}^{-1}$$

Recall: $\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)} \right)$; $g(x) = \frac{1}{1 + e^{-x}}$

Ingredient 2: Neural Networks

NEURAL NETWORKS

- Parametrize each polarized PDF replica with flexible Neural Network

DSSV, AAC, LSS, BB	NNPDFpol
$\mathcal{O}(10-20)$ parameters	$\mathcal{O}(200)$ parameters

- Train NN to determine the best fit for each replica
- Compute an ensemble of observables and compare to experimental data

MAIN FEATURES

- Only require smoothness of the fitted function
- Do not require any other prejudice on *a priori* functional form
- Reduce the bias associated to the choice of some functional form

One more ingredient: minimization and stopping

GENETIC ALGORITHM

Standard minimization unefficient owing to the large parameter space
and non-local x-dependence of the observables
Genetic algorithm provides better exploration of the whole parameter space

- Set Neural Network parameters randomly
- Make clones of the parameter vector and mutate them
- Define a figure of merit or error function for the k-th replica

$$E^{(k)} = \frac{1}{N_{\rm rep}} \sum_{i,j=1}^{N_{\rm rep}} \left(g_{1,i}^{({\rm art})(k)} - g_{1,i}^{({\rm net})(k)} \right) \left(({\rm cov})^{-1} \right)_{ij} \left(g_{1,j}^{({\rm art})(k)} - g_{1,j}^{({\rm net})(k)} \right)$$

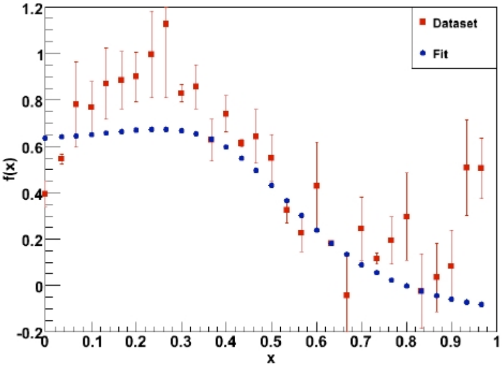
$g_{1,i}^{({\rm art})(k)}$: generated from Monte Carlo sampling
$g_{1,i}^{({\rm net})(k)}$: computed from Neural Network PDFs

- Select the best set of parameters and perform other manipulations (crossing, mutating, ...) until stability is reached.

DRAWBACK

- NN can learn fluctuations owing to their flexibility

UNDERLEARNING

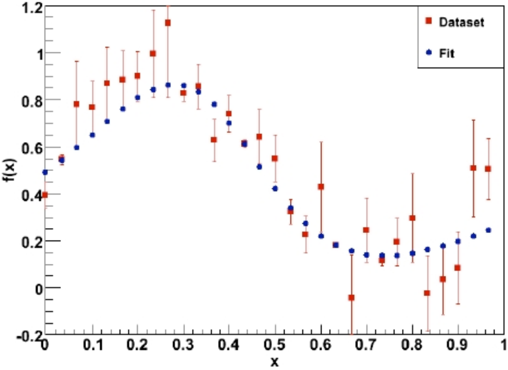


One more ingredient: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

PROPER LEARNING

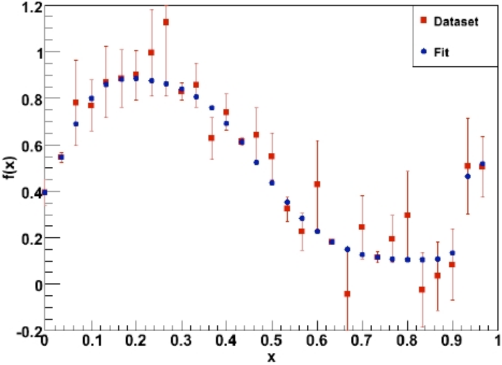


One more ingredient: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

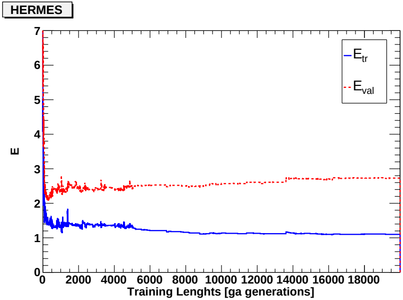
OVERLEARNING



One more ingredient: minimization and stopping

CROSS-VALIDATION METHOD

- divide data into two subsets (training & validation)
- train the NN on training subset and compute χ^2 for each subset
- stop when χ^2 of validation subset no longer decreases
  (NN are learning fluctuations!)



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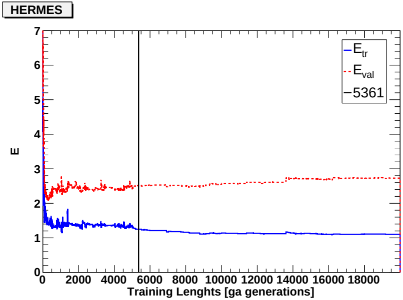

The best fit does not coincide with the χ^2 absolute minimum

Polarized PDF evolution

In Mellin space the DGLAP equations

$$\begin{aligned}
\mu^2 \frac{\partial}{\partial \mu^2} \Delta q_{NS}^{\pm,\nu}(N,\mu^2) &= \Delta\gamma_{NS}^{\pm,\nu} q_{NS}^{\pm,\nu}(N,\mu^2) \\
\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} (N,\mu^2) &= \begin{pmatrix} \Delta\gamma_{qq}(N,\alpha_s(Q^2)) & \Delta\gamma_{qg}(N,\alpha_s(Q^2)) \\ \Delta\gamma_{gq}(N,\alpha_s(Q^2)) & \Delta\gamma_{gg}(N,\alpha_s(Q^2)) \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}
\end{aligned}$$

can be solved analitically

$$\Delta q_{NS}^{\pm,\nu}(N,Q^2) = \Gamma_{NS}^{\pm,\nu}(N,a_s,a_0)\Delta q_{NS}^{\pm,\nu}(N,Q_0^2) \ , a_s \equiv \alpha_s/2\pi$$

where, at NLO,

$$\Gamma_{NS,NLO}^{\pm,\nu}(N,a_s,a_0) = \exp\left\{ \frac{U_1^{\pm,\nu}}{b_1} \ln\left(\frac{1+b_1 a_s}{1+b_1 a_0} \right) \right\} \left(\frac{a_s}{a_0} \right)^{-R_0^{NS}}$$

Polarized PDF evolution

NNPDF NLO polarized PDF evolution (**Fast Kernel method**) benchmarked with the Les Houches PDF benchmarks (G. Salam and a. Vogt, hep-ph/0511119)

x	$\epsilon_{\rm rel}\,(\Delta u_V)$	$\epsilon_{\rm rel}\,(\Delta d_V)$	$\epsilon_{\rm rel}\,(\Delta\Sigma)$	$\epsilon_{\rm rel}\,(\Delta g)$
10^{-3}	$1.1\,10^{-4}$	$9.2\,10^{-5}$	$9.9\,10^{-5}$	$1.1\,10^{-4}$
10^{-2}	$1.4\,10^{-4}$	$1.9\,10^{-4}$	$3.5\,10^{-4}$	$9.3\,10^{-5}$
0.1	$1.2\,10^{-4}$	$1.6\,10^{-4}$	$5.4\,10^{-6}$	$1.7\,10^{-4}$
0.3	$2.3\,10^{-6}$	$1.1\,10^{-5}$	$7.5\,10^{-6}$	$1.7\,10^{-5}$
0.5	$5.6\,10^{-6}$	$9.6\,10^{-6}$	$1.6\,10^{-5}$	$2.5\,10^{-5}$
0.7	$1.2\,10^{-4}$	$9.2\,10^{-7}$	$1.6\,10^{-4}$	$7.8\,10^{-5}$
0.9	$3.5\,10^{-3}$	$1.1\,10^{-2}$	$4.1\,10^{-3}$	$7.8\,10^{-3}$

Very accurate evolution!

Input polarized PDF basis

Four polarized PDFs (gluon + linear combinations of light quarks)

1. singlet $\Delta\Sigma(x) \equiv \Sigma_{i=1}^{n_f} \Delta q_i(x)$
2. gluon $\Delta g(x)$
3. triplet $\Delta T_3(x) \equiv \Delta u(x) - \Delta d(x)$
4. octet $\Delta T_8(x) \equiv \Delta u(x) + \Delta d(x) - 2\Delta s(x)$

$$\Delta q_i(x, Q^2) = q_i^{\uparrow\uparrow}(x, Q^2) + \bar{q}_i^{\uparrow\uparrow}(x, Q^2) - q_i^{\uparrow\downarrow}(x, Q^2) + \bar{q}_i^{\uparrow\downarrow}(x, Q^2)$$

$$\Delta g(x, Q^2) \equiv g^{\uparrow\uparrow}(x, Q^2) - g^{\uparrow\downarrow}(x, Q^2)$$

Inclusive neutral-current DIS data do not allow disentangling the contributions from q and $\overline{q}$.
In our notation, Δq takes into account flavor plus anti-flavor contributions.

- At initial scale $Q_0^2 = 1 GeV^2$
- Assume all heavy quarks are generated radiatively
- Adopt $\alpha_s(M_Z^2) = 0.119$, $m_c = 1.4$ GeV, $m_b = 4.75$ GeV

PDF Parametrization

$$\Delta\Sigma(x, Q_0^2) = (1-x)^{m_{\Delta\Sigma}} x^{-n_{\Delta\Sigma}} NN_{\Delta\Sigma}(x)$$

$$\Delta g(x, Q_0^2) = (1-x)^{m_{\Delta g}} x^{-n_{\Delta g}} NN_{\Delta g}(x)$$

$$\Delta T_3(x, Q_0^2) = A_{\Delta T_3}(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)$$

$$\Delta T_8(X, Q_0^2) = A_{\Delta T_8}(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)$$

1. Each polarized PDF parametrized with a multi-layer feed-forward NN (2-5-3-1)
2. Parametrization supplemented with a preprocessing polynomial:
   → exponents m and n randomly chosen in fixed intervals;
   → intervals must be sufficient large not to introduce a bias on the fit
   → check *a posteriori* by studying asymptotic exponents
3. Overall normalization constant factored out for triplet and octet.

$$A_{\Delta T_3} = \frac{a_3}{\int_0^1 dx[(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)]}$$

$$A_{\Delta T_8} = \frac{a_8}{\int_0^1 dx[(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)]}$$

PDF Parametrization

$$\Delta\Sigma(x, Q_0^2) = (1-x)^{m_{\Delta\Sigma}} x^{-n_{\Delta\Sigma}} NN_{\Delta\Sigma}(x)$$

$$\Delta g(x, Q_0^2) = (1-x)^{m_{\Delta g}} x^{-n_{\Delta g}} NN_{\Delta g}(x)$$

$$\Delta T_3(x, Q_0^2) = A_{\Delta T_3}(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)$$

$$\Delta T_8(X, Q_0^2) = A_{\Delta T_8}(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)$$

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PDF Parametrization

$$\Delta\Sigma(x, Q_0^2) = (1-x)^{m_{\Delta\Sigma}} x^{-n_{\Delta\Sigma}} NN_{\Delta\Sigma}(x)$$

$$\Delta g(x, Q_0^2) = (1-x)^{m_{\Delta g}} x^{-n_{\Delta g}} NN_{\Delta g}(x)$$

$$\Delta T_3(x, Q_0^2) = A_{\Delta T_3}(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)$$

$$\Delta T_8(X, Q_0^2) = A_{\Delta T_8}(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)$$

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Observables

The g_1 structure function

1. Longitudinal and transverse asymmetries

$$A_{\parallel} = \frac{d\sigma^{\rightarrow\Rightarrow} - d\sigma^{\rightarrow\Leftarrow}}{d\sigma^{\rightarrow\Rightarrow} + d\sigma^{\rightarrow\Leftarrow}} \quad A_{\perp} = \frac{d\sigma^{\rightarrow\Uparrow} - d\sigma^{\rightarrow\Downarrow}}{d\sigma^{\rightarrow\Uparrow} + d\sigma^{\rightarrow\Downarrow}}$$

2. Polarized structure functions

$$g_1(x,Q^2) = \frac{F_1(x,Q^2)}{(1+\gamma^2)(1+\eta\zeta)}\left[(1+\gamma\zeta)\frac{A_{\parallel}}{D} - (\eta-\gamma)\frac{A_{\perp}}{d}\right]$$

$$g_2(x,Q^2) = \frac{F_1(x,Q^2)}{(1+\gamma^2)(1+\eta\zeta)}\left[\left(\frac{\zeta}{\gamma}-1\right)\frac{A_{\parallel}}{D} + \left(\eta+\frac{1}{\gamma}\right)\frac{A_{\perp}}{d}\right]$$

3. What do experiments effectively measure (and publish)?

$$A_{\parallel}: \quad g_1(x,Q^2) = \frac{F_1(x,Q^2)}{1+\gamma\eta}\frac{A_{\parallel}}{D} + \frac{\gamma(\gamma-\eta)}{\gamma\eta+1}g_2(x,Q^2)$$

$$A_1: \quad g_1(x,Q^2) = A_1(x,Q^2)F_1(x,Q^2) + \gamma^2 g_2(x,Q^2)$$

4. Two possible assumptions on g_2
   - $g_2(x,Q^2) = 0$
   - $g_2(x,Q^2) \approx g_2^{t_2}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2) \equiv g_2^{WW}(x,Q^2)$

Target mass corrections

- Extracting both structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ from data requires measuring longitudinal and transverse spin asymmetries $A_\parallel$ and $A_\perp$
- Experimental information on $A_\perp$ is rather poor (in most cases only $A_\parallel$ is measured), thus $g_1(x, Q^2)$ and $g_2(x, Q^2)$ are related

$$g_1(x, Q^2) = \frac{F_1(x, Q^2)}{1+\gamma\eta}\frac{A_\parallel}{D} + \frac{\gamma(\gamma-\eta)}{\gamma\eta+1} g_2(x, Q^2)$$

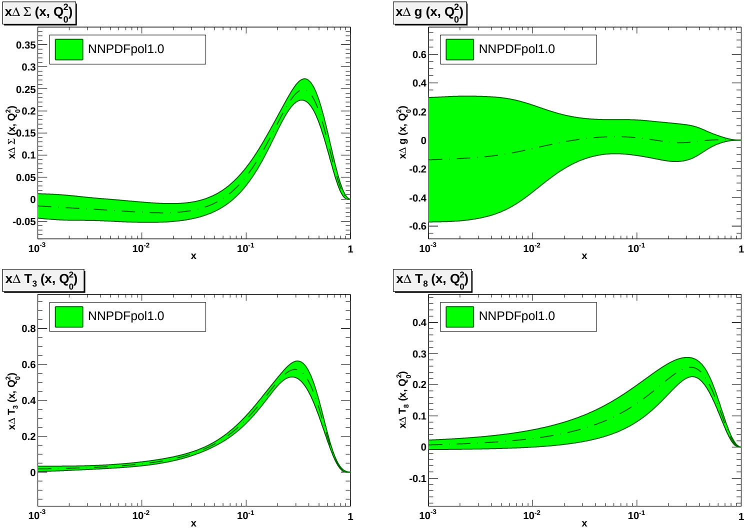
$$\gamma = \frac{2m_N x}{Q} \; ; \quad \eta = \frac{\epsilon\gamma y}{1-\epsilon(1-y)} \; ; \quad D = \frac{1-(1-y)\epsilon}{1+\epsilon R(x, Q^2)} \; ; \quad \epsilon = \frac{4(1-y)-\gamma^2 y^2}{2y^2+4(1-y)+\gamma^2 y^2} \; ; \quad y = 1 - \frac{E'}{E}$$

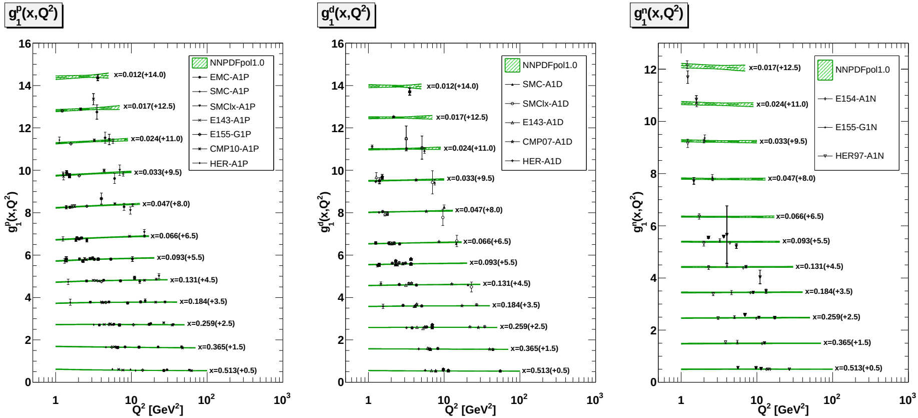
MUST MAKE SOME ASSUMPTION ON $g_2(x, Q^2)$

$g_2 = 0$ OR $g_2 = g_2^{WW}$ (relate g_2 to g_1 by means of the Wandzura-Wilczek relation)

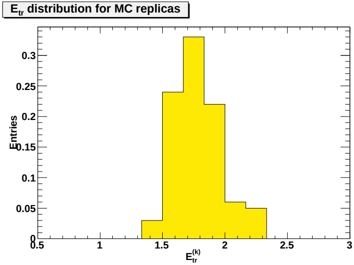
- Target mass corrections implemented iteratively during the minimization procedure

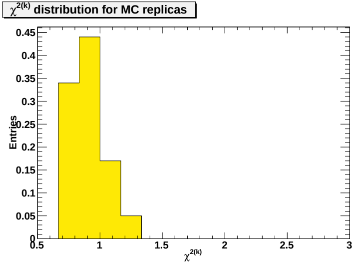
The NNPDFpol1.0 parton set: parametrization basis

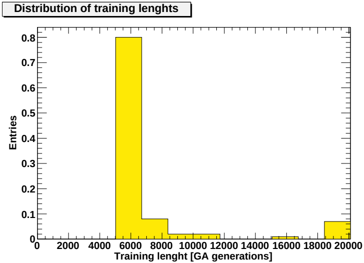



$g_1^p(x,Q^2)$
NNPDFpol1.0
EMC-A1P
SMC-A1P
SMClx-A1P
E143-A1P
E155-G1P
CMP10-A1P
HER-A1P
x=0.012(+14.0)
x=0.017(+12.5)
x=0.024(+11.0)
x=0.033(+9.5)
x=0.047(+8.0)
x=0.066(+6.5)
x=0.093(+5.5)
x=0.131(+4.5)
x=0.184(+3.5)
x=0.259(+2.5)
x=0.365(+1.5)
x=0.513(+0.5)
Q^2 [GeV^2]
$g_1^d(x,Q^2)$
NNPDFpol1.0
SMC-A1D
SMClx-A1D
E143-A1D
CMP07-A1D
HER-A1D
$g_1^n(x,Q^2)$
NNPDFpol1.0
E154-A1N
E155-G1N
HER97-A1N

NNPDFpol1.0: global χ^2

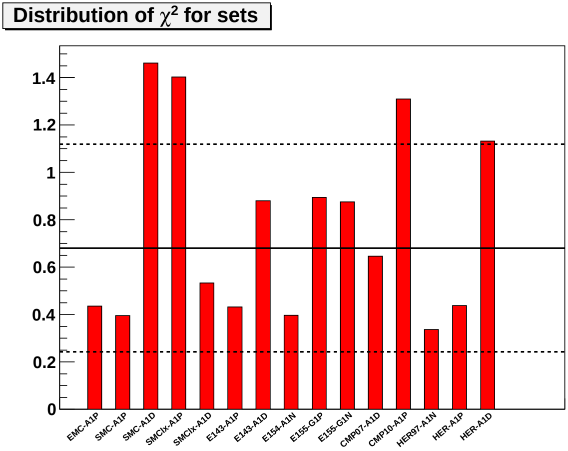






χ^2_{tot}	0.77
$\langle E \rangle \pm \sigma_E$	1.82 $\pm$ 0.18
$\langle E_{\mathrm{tr}} \rangle \pm \sigma_{E_{\mathrm{tr}}}$	1.66 $\pm$ 0.49
$\langle E_{\mathrm{val}} \rangle \pm \sigma_{E_{\mathrm{val}}}$	1.88 $\pm$ 0.67
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	0.91 $\pm$ 0.12
$\langle TL \rangle \pm \sigma_{TL}$	6927 $\pm$ 3839

NNPDFpol1.0: individual experiments χ^2



$$\chi^{2(k)} = \frac{1}{N_{\rm dat}} \sum_{i,j=1}^{N_{\rm dat}} \left(g_{1,i}^{(\rm exp)} - g_{1,i}^{(\rm net)(k)} \right) \left((\rm cov)^{-1} \right)_{ij} \left(g_{1,j}^{(\rm exp)} - g_{1,j}^{(\rm net)(k)} \right)$$

Comparing different NNPDF fits: stability of results

Compare two sets of $N_{\text{rep}}^{(1)}$ and $N_{\text{rep}}^{(2)}$ replicas coming from different fits
Do they have belong to the same underlying probability distribution?

1. Directly compare the parton set plots
2. Look at the statistical estimators for the fit
3. Define the distance between central values of different fits

$$d^2\left(\langle q^{(k)}\rangle_{(1)}, \langle q^{(k)}\rangle_{(2)}\right) = \frac{\left(\langle q^{(k)}\rangle_{(1)} - \langle q^{(k)}\rangle_{(2)}\right)^2}{\sigma^2\left[\langle q^{(k)}\rangle_{(1)}\right] + \sigma^2\left[\langle q^{(k)}\rangle_{(2)}\right]}$$

and similarly for standard deviations

- Distances have a χ^2 probability distribution with one degree of freedom
- Statistically equivalent fits have $d \sim 1$

Distances

Compare two sets of $N_{\rm rep}^{(1)}$ and $N_{\rm rep}^{(1)}$ replicas coming from different fits
Do they have belong to the same underlying probability distribution?

MEAN VALUE

$$d^2\left(\langle q^{(k)}\rangle_{(1)},\langle q^{(k)}\rangle_{(2)}\right)=\frac{\left(\langle q^{(k)}\rangle_{(1)}-\langle q^{(k)}\rangle_{(2)}\right)^2}{\sigma^2\left[\langle q^{(k)}\rangle_{(1)}\right]+\sigma^2\left[\langle q^{(k)}\rangle_{(2)}\right]}$$

$$\langle q^{(k)}\rangle_{(i)}=\frac{1}{N_{{\rm rep}(i)}}\sum_{l=1}^{N_{{\rm rep}(i)}}q_l^{(k)} \qquad \sigma^2\left[\langle q^{(k)}\rangle_{(i)}\right]=\frac{1}{N_{{\rm rep}(i)}}\sigma^2\left[q_{(i)}^{(k)}\right]=\frac{1}{N_{{\rm rep}(i)}-1}\sum_{l=1}^{N_{{\rm rep}(i)}}\left(q_{l,(i)}-\langle q\rangle_{(i)}\right)^2$$

UNCERTAINTY

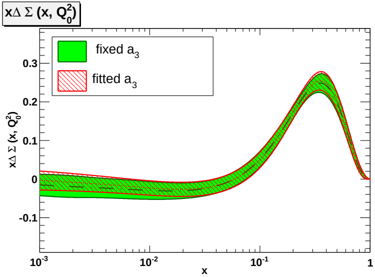
$$d^2\left(\sigma^2_{(1)},\sigma^2_{(2)}\right)=\frac{\left(\bar\sigma^2_{(1)}-\bar\sigma^2_{(2)}\right)}{\sigma^2\left[\sigma^2_{(1)}\right]+\sigma^2\left[\sigma^2_{(2)}\right]}$$

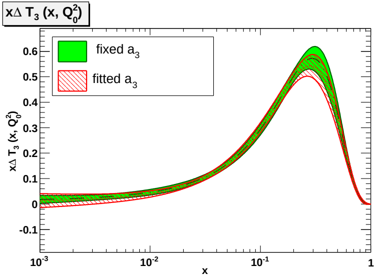
$$\bar\sigma^2_{(i)}\equiv\sigma^2\left[q_{(i)}^{(k)}\right] \qquad \sigma^2\left[\sigma^2_{(i)}\right]=\frac{1}{N_{{\rm rep}(i)}}\left[\frac{1}{N_{{\rm rep}(i)}}\sum_{l=1}^{N_{{\rm rep}(i)}}\left(q_{l,(i)}-\langle q\rangle_{(i)}\right)^4-\frac{N_{{\rm rep}(i)}-3}{N_{{\rm rep}(i)}-1}\left(\bar\sigma^2_{(i)}\right)^2\right]$$

By definition, the distances have a χ^2 probability distribution with one degree of freedom
mean $\langle d\rangle = 1$ and $d \lesssim 2.3$ at 90% confidence level

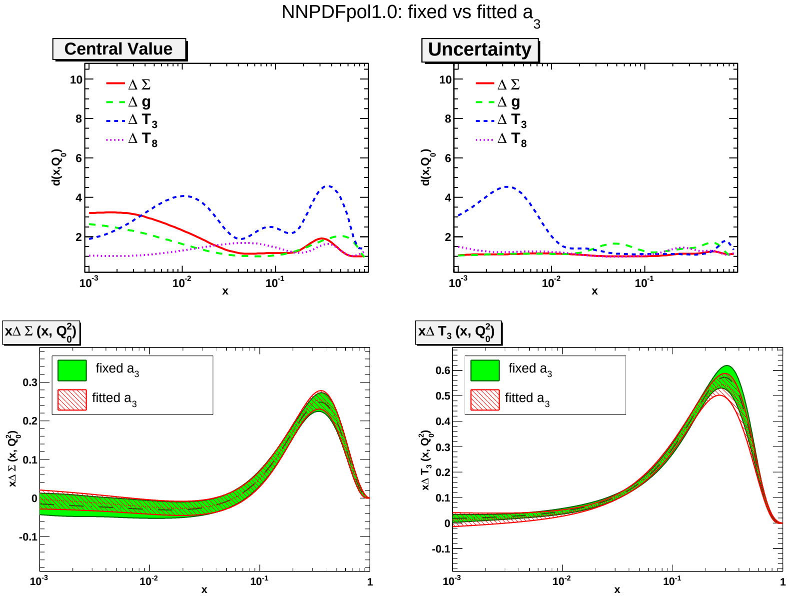
Impact of sum rules: fixed vs fitted a_3

Fit	a_3 free	reference
$\chi^2_{\rm tot}$	0.79	0.77
$\langle E \rangle \pm \sigma_E$	1.84 ± 0.19	1.82 ± 0.18
$\langle E_{\rm tr} \rangle \pm \sigma_{E_{\rm tr}}$	1.73 ± 0.41	1.66 ± 0.49
$\langle E_{\rm val} \rangle \pm \sigma_{E_{\rm val}}$	1.93 ± 0.58	1.88 ± 0.67
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	0.93 ± 0.12	0.91 ± 0.12

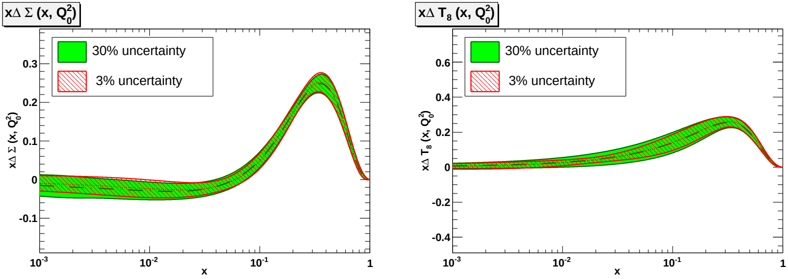




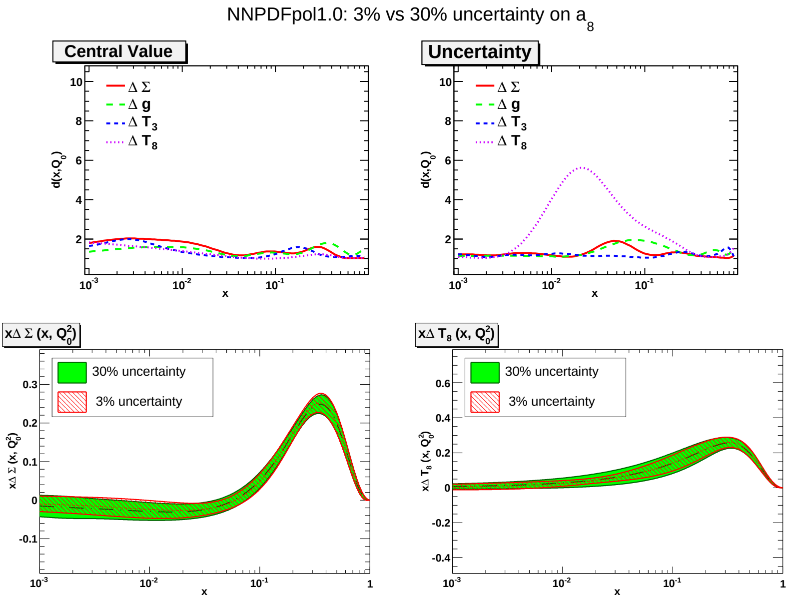
Impact of sum rules: fixed vs fitted a_3



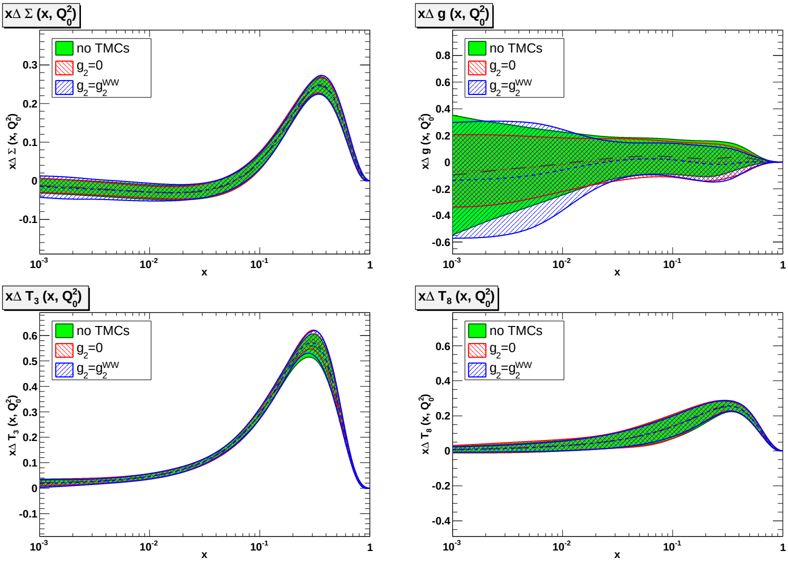
Fit	a_8 large	reference
$\chi^2_{\rm tot}$	0.77	0.77
$\langle E \rangle \pm \sigma_E$	1.86 ± 0.19	1.82 ± 0.18
$\langle E_{\rm tr} \rangle \pm \sigma_{E_{\rm tr}}$	1.66 ± 0.53	1.66 ± 0.49
$\langle E_{\rm val} \rangle \pm \sigma_{E_{\rm val}}$	1.87 ± 0.71	1.88 ± 0.67
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	0.92 ± 0.15	0.91 ± 0.12



Impact of sum rules: a_8 uncertainty



Impact of Target Mass Corrections (TMCs)

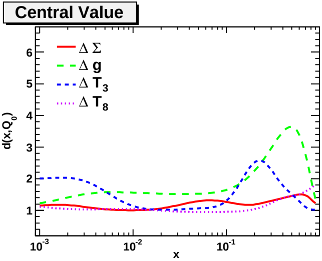


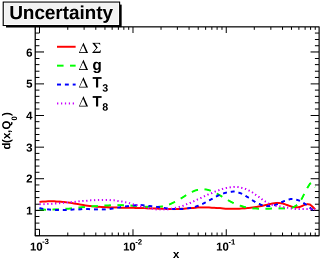
Impact of Target Mass Corrections (TMCs)

Fit	noTMCs	$g_2 = 0$	$g_2 = g_2^{WW}$
χ^2_{tot}	0.78	0.75	0.77
$\langle E \rangle \pm \sigma_E$	1.81 $\pm$ 0.16	1.83 $\pm$ 0.15	1.82 $\pm$ 0.18
$\langle E_{\mathrm{tr}} \rangle \pm \sigma_{E_{\mathrm{tr}}}$	1.62 $\pm$ 0.50	1.70 $\pm$ 0.38	1.66 $\pm$ 0.49
$\langle E_{\mathrm{val}} \rangle \pm \sigma_{E_{\mathrm{val}}}$	1.84 $\pm$ 0.70	1.96 $\pm$ 0.56	1.88 $\pm$ 0.67
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	0.90 $\pm$ 0.09	0.86 $\pm$ 0.09	0.91 $\pm$ 0.12

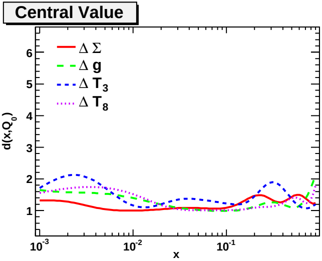
Impact of Target Mass Corrections (TMCs)

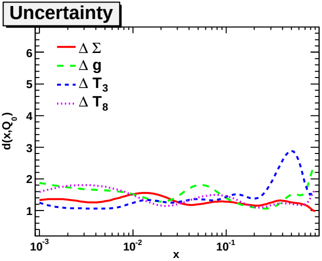
NNPDFpol1.0: no TMCs vs $g_2 = g_2^{WW}$





NNPDFpol1.0: $g_2 = 0$ vs $g_2 = g_2^{WW}$





Theoretical constraints: positivity

$$A_1 \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{g_1(x,Q^2)}{F_1(x,Q^2)} \Rightarrow |g_1(x,Q^2)| \leq F_1(x,Q^2)$$

- This relation must hold for scattering off any target.
- It can be imposed on structure functions $g_1^{(i)}$ and $F_1^{(i)}$ by separately including only the contribution from the i-th flavor ($i = u, d, s$).
- We get three positivity constraints on the combination $\Delta q_i + \Delta\bar{q}_i$.
- At LO we get

$$|g_1^{(i)}(x,Q^2)| \leq F_1^{(i)}(x,Q^2) \xrightarrow{LO} |\Delta q_i(x,Q^2)| \leq q_i(x,Q^2)$$

- A similar equation relation can be obtained for the gluon by imposing the condition $|\Delta\sigma(x,m_h^2)| \leq \sigma(x,m_h^2)$ on the polarized and unpolarized cross-sections for inclusive Higgs production in gluon-proton scattering

  $g + p \rightarrow H + X \xrightarrow{LO} g + g \rightarrow H$

$$|\Delta g(x,Q^2)| \leq g(x,Q^2)$$

Positivity: technical insight

$$|\Delta f_i(x,Q^2)| \leq f_i(x,Q^2) + \sigma_i(x,Q^2)$$

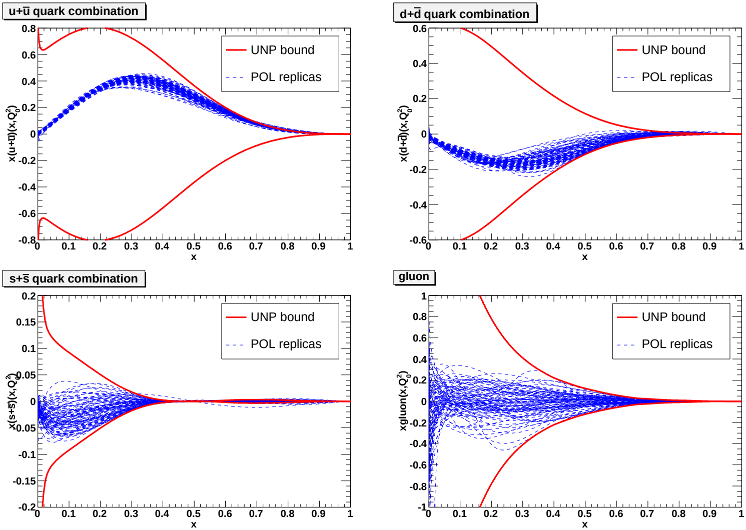
Modify $E^{(k)}$ by a Lagrange multiplier $\lambda_{\rm pos}$, $N_{dat,pos}$=20, $x_p \in [10^{-5}, 0.9]$ linearly spaced

$$E^{(k)} \longrightarrow E^{(k)} - \lambda_{\rm pos} \sum_{p=1}^{N_{dat,pos}} \left\{ \sum_{j=u+\bar{u},d+\bar{d},s+\bar{s},g} \Theta \left[\left(f_j + \sigma_j\right)(x_p, Q_0^2) - \left| \Delta f_j^{(net)(k)}(x_p, Q_0^2) \right| \right] \cdot \left[\left(f_j + \sigma_j\right)(x_p, Q_0^2) - \left| \Delta f_j^{(net)(k)}(x_p, Q_0^2) \right| \right] \right\}$$

Replicas not fulfilling positivity bound are penalized in the fitting procedure.
The unpolarized PDF and its error are computed as MV and the SD from the NNPDF2.1_NNLO fit.

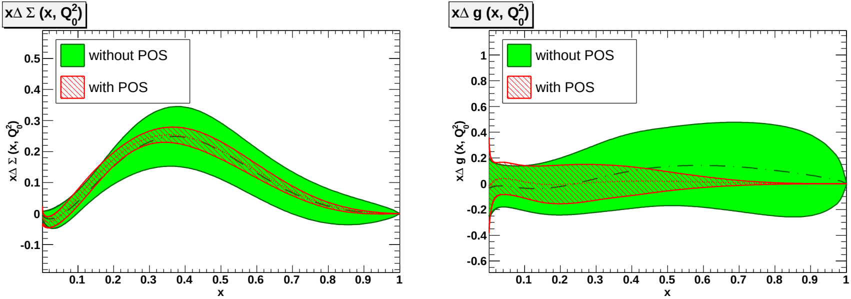
$$\begin{cases} \lambda_{\rm pos} = \lambda_{\rm max}^{(N_{\rm gen}-1)/(N_{\lambda_{\rm max}}-1)} & N_{\rm gen} < N_{\lambda_{\rm max}} \\ \lambda_{\rm pos} = \lambda_{\rm max} & N_{\rm gen} \geq N_{\lambda_{\rm max}} \end{cases} \qquad \lambda_{\rm max} = 10 \qquad N_{\lambda_{\rm max}} = 2000$$

Positivity: technical insight



Theoretical constraints: positivity

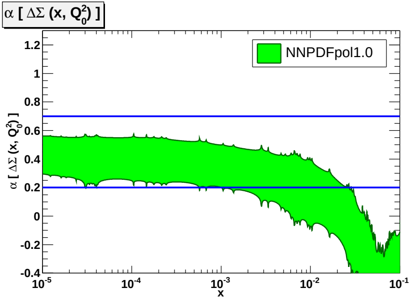
Polarized PDFs are only loosely constrained by the data.
At high-x positivity constraints are as important as data!

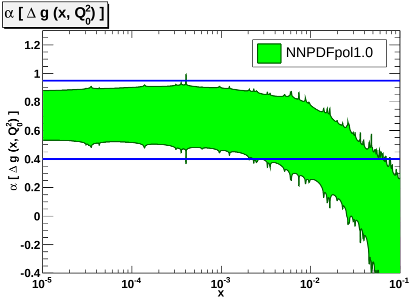


$\chi^2_{\text{tot}} = 0.72$ (no positivity constraints)
$\chi^2_{\text{tot}} = 0.75$ (with positivity constraints)

Preprocessing: effective asymptotic exponents

$0.0 \leq n_{\Delta\Sigma} \leq 0.6$ $\quad$ $0.0 \leq n_{\Delta g} \leq 0.6$



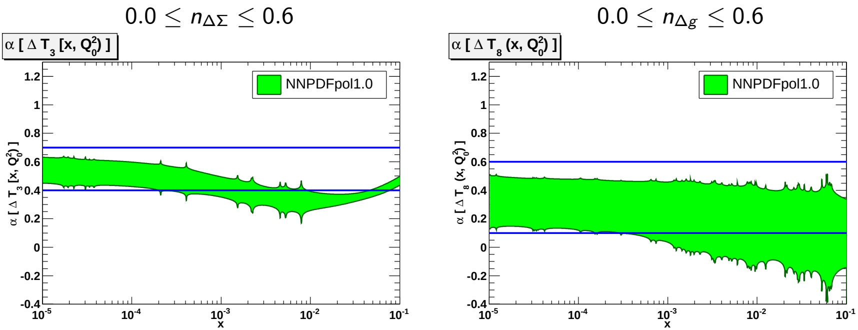


$$\alpha_{\rm eff}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1/x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

Effective exponents always contained in the preprocessing exponents range

The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

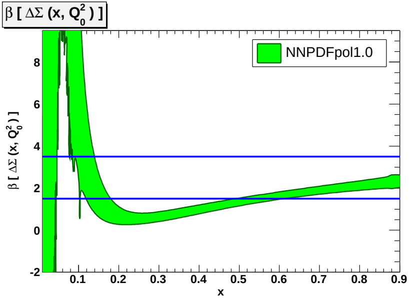


$$\alpha_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1/x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

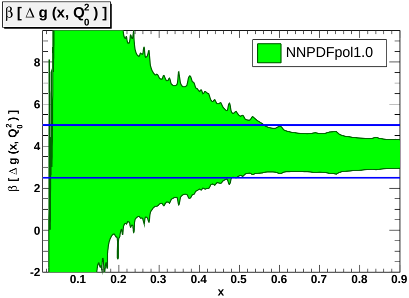
Effective exponents always contained in the preprocessing exponents range
The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

$0.5 \leq m_{\Delta\Sigma} \leq 5.0$



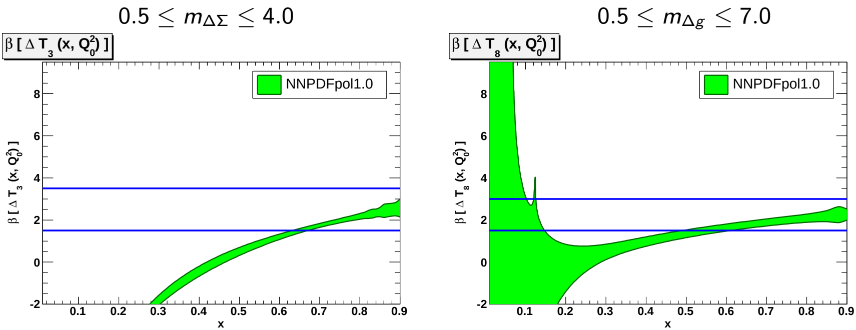
$0.5 \leq m_{\Delta g} \leq 4.0$



$$\beta_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2)) / \ln(1 - x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

Effective exponents always contained in the preprocessing exponents range
The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

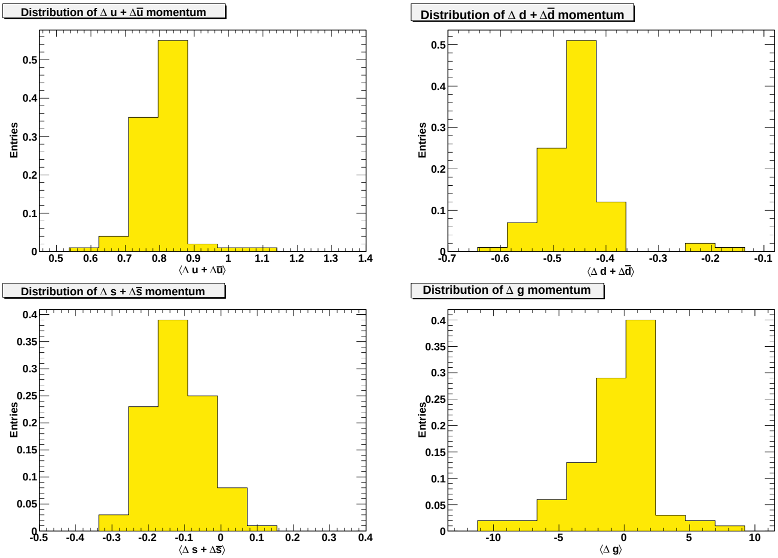


$$\beta_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1-x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

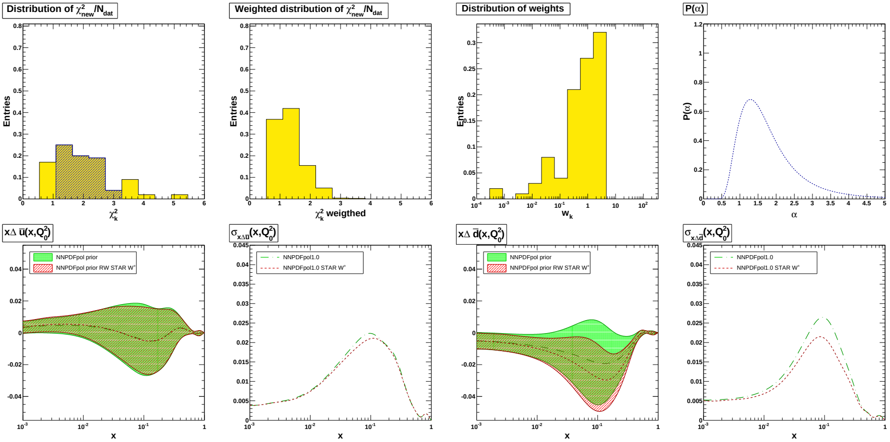
Effective exponents always contained in the preprocessing exponents range

The polarized PDF is driven only by experimental data

Momentum distributions



Reweighting with W^+ dataset



Reweighting with W^- dataset