

Initial conditions for evolution of double parton distributions

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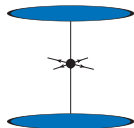
in collaboration with Krzysztof Golec-Biernat

- ▶ Double parton distribution functions (DPDF) are used in the description of double hard scattering.
- ▶ Their QCD evolution equations are known in the leading logarithmic approximation (LLA).
- ▶ The DPDF obey nontrivial **sum rules** which are conserved by the evolution equations.
- ▶ Problem of this talk: **how to specify initial conditions for the evolution equations which obey these sum rules?**

- ▶ Parton distribution functions
- ▶ Evolution equations
- ▶ Sum rules
- ▶ Initial conditions
- ▶ Summary

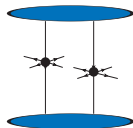
Parton distribution functions

- ▶ Single parton scattering



PDF: $D_f(x, Q)$

- ▶ Double parton scattering



DPDF: $D_{f_1 f_2}(x_1, x_2, Q_1, Q_2)$

- ▶ Two hard scales: Q_1, Q_2 and two flavours: f_1, f_2 (including gluon).
- ▶ Sum of partons' momenta cannot exceed total nucleon momentum

$$x_1 + x_2 \leq 1$$

QCD evolution equations for single PDF

- ▶ General form of evolution equations for single PDF ($t = \ln Q^2$)

$$\partial_t D_f(x, t) = \sum_{f'} \int_0^1 du \mathcal{K}_{ff'}(x, u, t) D_{f'}(u, t)$$

- ▶ The integral kernels describe **real** and **virtual** parton emission



$$\mathcal{K}_{ff'}(x, u, t) = \mathcal{K}_{ff'}^R(x, u, t) - \delta(u - x) \delta_{ff'} \mathcal{K}_f^V(x, t)$$

Evolution equations (cont.)

- ▶ The real emission kernels

$$\mathcal{K}_{ff'}^R(x, u, t) = \frac{1}{u} P_{ff'}\left(\frac{x}{u}, t\right) \theta(u - x)$$

- ▶ Splitting functions

$$P_{ff'}(z, t) = \frac{\alpha_s(t)}{2\pi} P_{ff'}^{(0)}(z) + \frac{\alpha_s^2(t)}{(2\pi)^2} P_{ff'}^{(1)}(z) + \dots$$

- ▶ Well known DGLAP evolution equations for single PDF

$$\partial_t D_f(x, t) = \sum_{f'} \int_x^1 \frac{dz}{z} P_{ff'}(z, t) D_{f'}\left(\frac{x}{z}, t\right) - D_f(x, t) \sum_{f'} \int_0^1 dz z P_{f'f}(z, t)$$

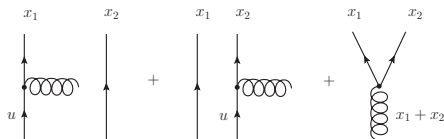
Evolution equations for DPDF

- ▶ Evolution of DPDF with equal hard scales $Q_1 = Q_2 \equiv Q$:

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \rightarrow D_{f_1 f_2}(x_1, x_2, Q, Q) \equiv D_{f_1 f_2}(x_1, x_2, t = \ln Q^2)$$

- ▶ Evolution equations with three terms:

$$\begin{aligned} \partial_t D_{f_1 f_2}(x_1, x_2, t) &= \sum_{f'} \int_0^{1-x_2} du \mathcal{K}_{f_1 f'}(x_1, u, t) D_{f' f_2}(u, x_2, t) \\ &+ \sum_{f'} \int_0^{1-x_1} du \mathcal{K}_{f_2 f'}(x_2, u, t) D_{f_1 f'}(x_1, u, t) \\ &+ \sum_{f'} \mathcal{K}_{f' \rightarrow f_1 f_2}^R(x_1, x_1 + x_2, t) D_{f'}(x_1 + x_2, t) \end{aligned}$$



- ▶ The third splitting term contains single PDF

$$\frac{\alpha_s(Q)}{2\pi} \sum_{f'} \frac{1}{x_1 + x_2} P_{f' \rightarrow f_1 f_2} \left(\frac{x_1}{x_1 + x_2} \right) D_{f'}(x_1 + x_2, Q)$$

- ▶ The DPDF evolution equations need to be solved together with the DGLAP equations for single PDF.
- ▶ Initial conditions for both DPDF and PDF have to be specified:

$$D_{f_1 f_2}(x_1, x_2, Q_0)$$

$$D_f(x, Q_0)$$

Momentum sum rule

- ▶ Momentum sum rule for PDF (conserved by DGLAP eqs.)

$$\sum_f \int_0^1 dx x D_f(x, Q) = 1$$

- ▶ By analogy: momentum sum rule for DPDF

$$\sum_{f_1} \int_0^{1-x_2} dx_1 x_1 \frac{D_{f_1 f_2}(x_1, x_2, Q)}{D_{f_2}(x_2, Q)} = (1 - x_2)$$

- ▶ The ratio in red is a conditional probability to find a parton with the momentum fraction x_1 while the second parton fraction x_2 is fixed.
- ▶ Momentum sum rule is a relation between DPDF and PDF

$$\sum_{f_1} \int_0^{1-x_2} dx_1 x_1 D_{f_1 f_2}(x_1, x_2, Q) = (1 - x_2) D_{f_2}(x_2, Q) \quad (1)$$

Valence number sum rule

- ▶ Valence number sum rule for PDF (N_i = no. of valence quarks)

$$\int_0^1 dx \{D_{q_i}(x, Q) - D_{\bar{q}_i}(x, Q)\} = N_i$$

- ▶ Valence number sum rule for DPDF

$$I_{q_i f_2} = \int_0^{1-x_2} dx_1 \{D_{q_i f_2}(x_1, x_2, Q) - D_{\bar{q}_i f_2}(x_1, x_2, Q)\}$$
$$= \begin{cases} N_i D_{f_2}(x_2, Q) & \text{for } f_2 \neq q_i, \bar{q}_i \\ (N_i - 1) D_{f_2}(x_2, Q) & \text{for } f_2 = q_i \\ (N_i + 1) D_{f_2}(x_2, Q) & \text{for } f_2 = \bar{q}_i \end{cases} \quad (2)$$

- ▶ Relations (1) and (2) are conserved by the evolution equations **once imposed at some initial scale** Q_0 .

- ▶ In practice: initial DPDF are built out of the **existing** single PDFs, e.g. (Gaunt, Stirling, Korotkikh, Snigirev)

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1) D_{f_2}(x_2) \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1} (1 - x_2)^{2+n_2}}$$

- ▶ Symmetric

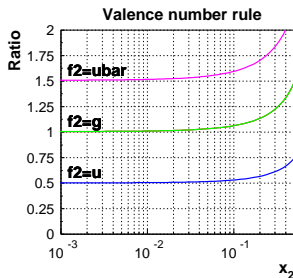
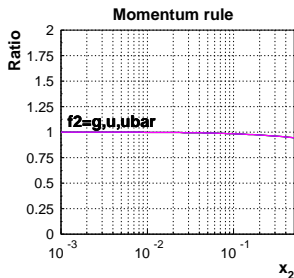
$$D_{f_1 f_2}(x_1, x_2) = D_{f_2 f_1}(x_2, x_1)$$

and positive definite ansatz which **does not fulfill** sum rules.

Sum rule violation by the Gaunt-Stirling input

$$\text{Ratio}_{\text{Mom}}(x_2, f_2) = \frac{\sum_{f_1} \int_0^{1-x_2} dx_1 x_1 D_{f_1 f_2}(x_1, x_2)}{(1-x_2) D_{f_2}(x_2)} = 1$$

$$\text{Ratio}_{\text{Val}}(x_2, f_2) = \frac{\int_0^{1-x_2} dx_1 \{D_{uf_2}(x_1, x_2) - D_{\bar{u}f_2}(x_1, x_2)\}}{N_{uf_2} D_{f_2}(x_2)} = 1$$



Valence number sum rule badly violated for the GS input.

How to fulfill the sum rules?

- ▶ Nonsymmetric ansatz to fulfill the momentum sum rule:

$$D_{f_1 f_2}(x_1, x_2) = \frac{1}{1-x_2} D_{f_1}\left(\frac{x_1}{1-x_2}\right) \cdot D_{f_2}(x_2)$$

- ▶ Corrections for identical quark flavours/antiflavours to obey the valence number sum rule:

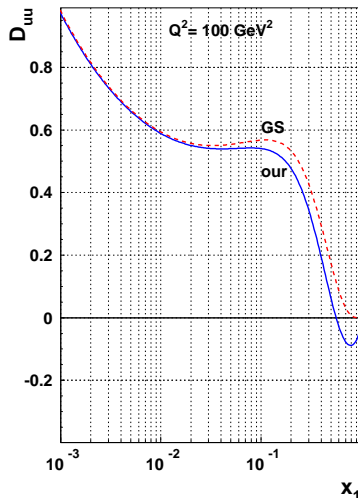
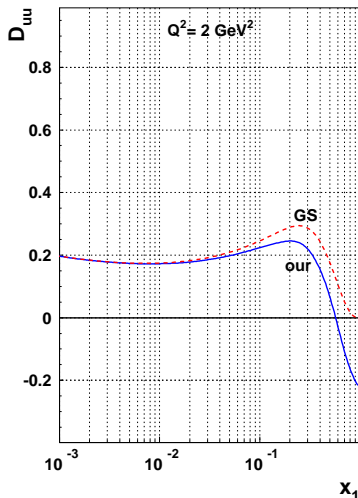
$$D_{f_i f_i}(x_1, x_2) = \frac{1}{1-x_2} \left\{ D_{f_i}\left(\frac{x_1}{1-x_2}\right) - \frac{1}{2} \right\} D_{f_i}(x_2)$$

$$D_{f_i \bar{f}_i}(x_1, x_2) = \frac{1}{1-x_2} \left\{ D_{f_i}\left(\frac{x_1}{1-x_2}\right) + \frac{1}{2} \right\} D_{\bar{f}_i}(x_2)$$

- ▶ DPDF for identical flavours, $D_{f_i f_i}(x_1, x_2)$, **are not** positive definite. This is the **price to pay** for the construction with single PDFs !

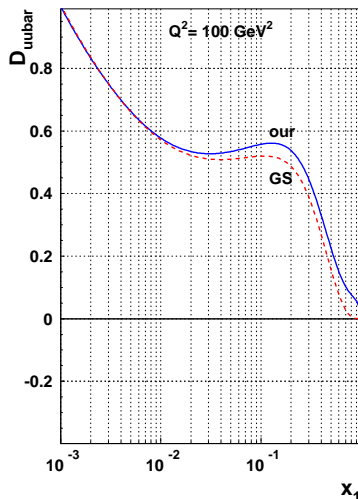
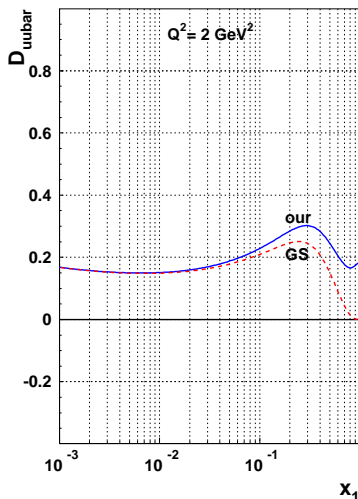
Our input vs. Gaunt-Stirling input for D_{uu}

$$D_{uu}(x_1, x_2=10^{-3})$$



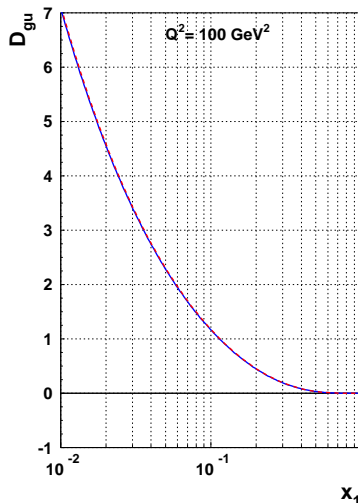
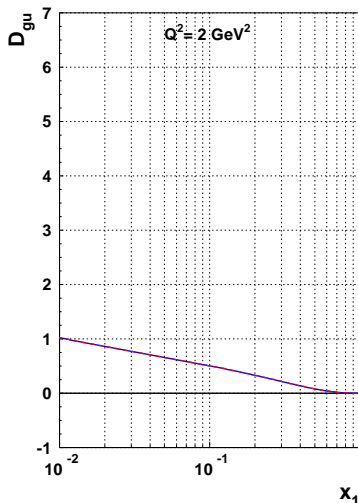
Our vs. Gaunt-Stirling input for $D_{u\bar{u}}$

$$D_{u\bar{u}\text{bar}}(x_1, x_2=10^{-3})$$



Our vs. Gaunt-Stirling input for D_{gu}

$$D_{gu}(x_1, x_2=10^{-3})$$



Summary

- ▶ If input DPDF are constructed from known single PDF:

	GS input	Our input
Parton symmetry	yes	no
Positivity	yes	no
Sum rules	no	yes

- ▶ **Alternative:** specify positive initial DPDF and generate initial PDF using sum rules. Unfortunately, no experimental knowledge on DPDF.
- ▶ We stay with a dilemma how to specify initial conditions.
- ▶ However, for $x_1, x_2 \rightarrow 0$ factorized form is a good approximation:

$$D_{f_1 f_2}(x_1, x_2, Q) \approx D_{f_1}(x_1, Q) D_{f_2}(x_2, Q)$$

BACKUP

Evolution equations for DPDF

- ▶ For unequal hard scales, $Q_1 < Q_2$, two step evolution:

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \rightarrow D_{f_1 f_2}(x_1, x_2, Q_1, Q_1) \rightarrow D_{f_1 f_2}(x_1^{\text{fixed}}, x_2, Q_1, Q_2)$$

- ▶ Single PDF evolution, $Q_1 \rightarrow Q_2$, with respect to the second variable x_2 in the second step:

$$\partial_{t_2} D_{f_1 f_2}(x_1, x_2, t_1, t_2) = \sum_{f'} \int_0^{1-x_1} du \mathcal{K}_{f_2 f'}(x_2, u, t) D_{f_1 f'}(x_1, u, t, t_2)$$