Scattering amplitudes for high-energy factorization

Andreas van Hameren

The Henryk Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences

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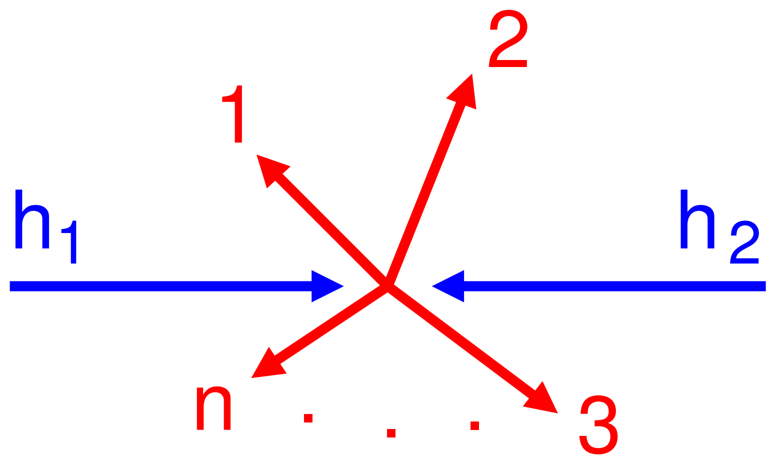
Hard scattering cross sections within collinear factorization



PDFs are related to the structure of the hadrons, universal to the scattering process

$$\sigma_{h_1,h_2\to n}(p_1,p_2) = \sum_{a,b}\int dx_1 dx_2 \, \boxed{f_a(x_1,\mu)\, f_b(x_2,\mu)} \, \hat{\sigma}_{a,b\to n}(x_1 p_1, x_2 p_2;\mu)$$

$$\hat{\sigma}_{a,b\to n}(p_a,p_b;\mu) = \boxed{\int d\Phi(p_a,p_b\to\{p\}_n)} \, \boxed{|\mathcal{M}_{a,b\to n}(p_a,p_b\to\{p\}_n;\mu)|^2} \, \boxed{\mathcal{O}(p_a,p_b,\{p\}_n)}$$

Phase space (includes spin/color summation) governs the kinematics

Matrix element (squared) contains model parameters, governs the dynamics

Observable, imposes phase space cuts

High–energy, or kT, factorization

Gribov, Levin, Ryskin 1983
Catani, Ciafaloni, Hautmann 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- to be applied in the 3-scale regime $s \gg m^2 \gg \Lambda^2_{QCD}$
- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$,
  but holds also for $s \gg m^2 \sim k_\perp^2$
- *unintegrated pdf* $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn...
- typically associated with small-x physics
- relevant for forward physics, saturation physics, heavy-ion physics...
- $k_\perp$ gives a handle on the size of the proton
- it is known how to construct the necessary gauge invariant matrix elements with off-shell gluons Lipatov 1995, Antonov, Lipatov, Kuraev,Cherednikov 2005

Lipatov's effective action

Effective action in terms of quarks $\psi, \bar{\psi}$ gluons v_μ and reggeized gluons $A_\pm$.

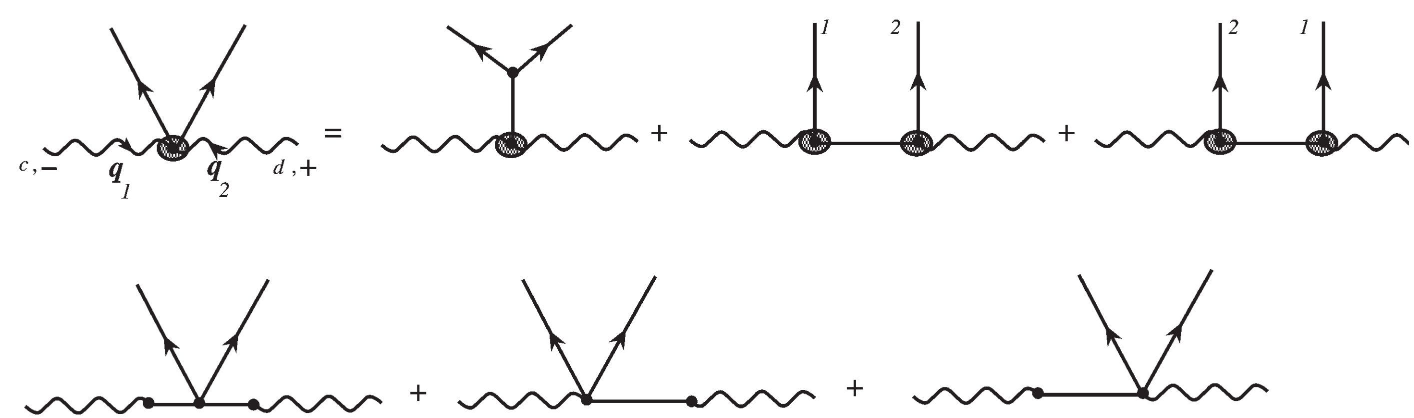
$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{ind}}$$

$$\mathcal{L}_{\mathrm{QCD}} = i\bar{\psi}\hat{D}\psi + \frac{1}{2}\mathrm{Tr}\,G_{\mu\nu}^2 \qquad D_\mu = \partial_\mu + g v_\mu \qquad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

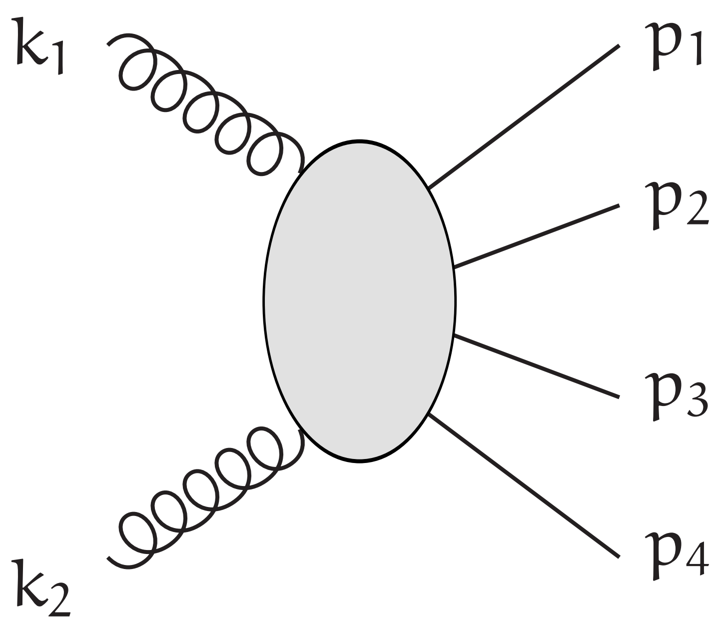
$$\mathcal{L}_{\mathrm{ind}} = -\mathrm{Tr}\left\{\frac{1}{g}\partial_+\left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} v_+(y)\,dy^+\right)\right]\cdot\partial_\sigma^2 A_-(x) + \frac{1}{g}\partial_-\left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^-} v_-(y)\,dy^-\right)\right]\cdot\partial_\sigma^2 A_+(x)\right\}$$

$$k_\pm = (n_\mu^\pm)k^\mu \qquad (n^-)^2 = (n^+)^2 = 0 \qquad n^+\cdot n^- = 2$$

Amplitudes are build up with the help of effective reggeon-gluon vertices.



Amplitudes for high–energy factorization



$$k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$

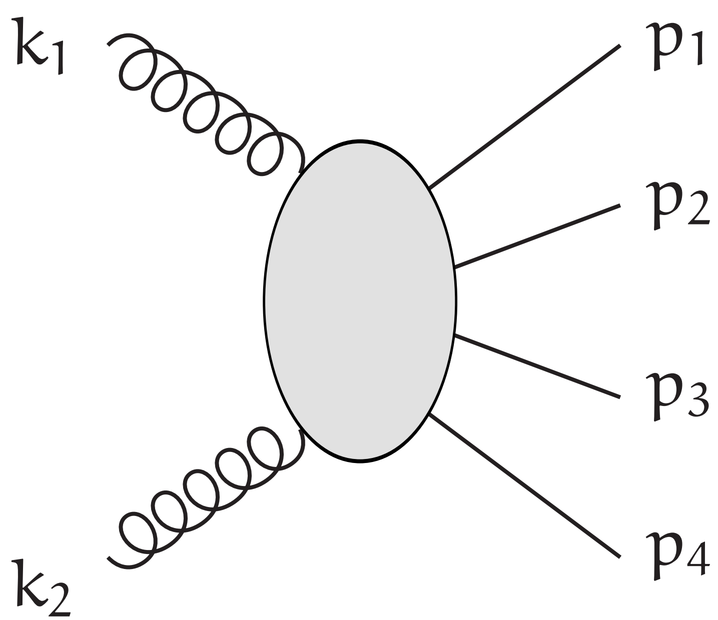
$$k_1 = x_1 p_A + k_{\perp 1} \qquad k_2 = x_2 p_B + k_{\perp 2}$$

$$p_A \cdot k_{\perp 1} = p_A \cdot k_{\perp 2} = p_B \cdot k_{\perp 1} = p_B \cdot k_{\perp 2} = 0$$
$$p_A^2 = p_B^2 = 0$$

$$k_1^2 = k_{\perp 1}^2 \qquad k_2^2 = k_{\perp 2}^2$$

Off-shell initial-state gluons $\Longrightarrow$ what about gauge invariance?

Amplitudes for high–energy factorization



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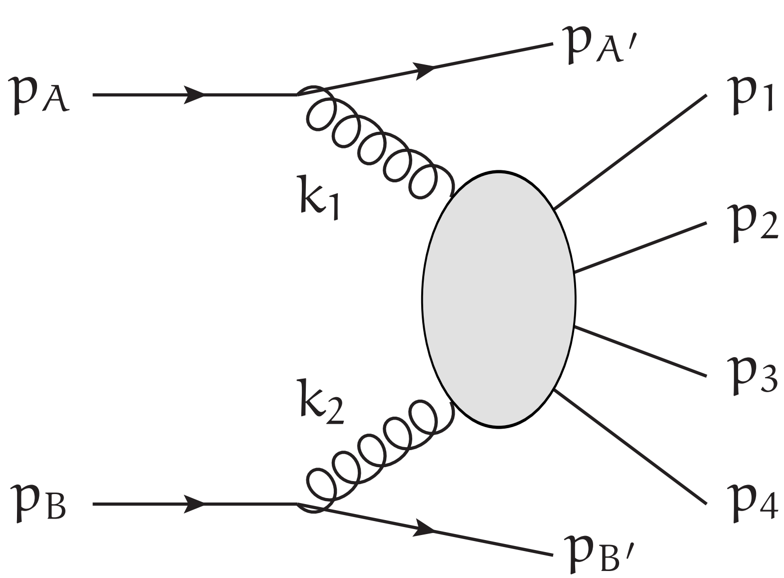
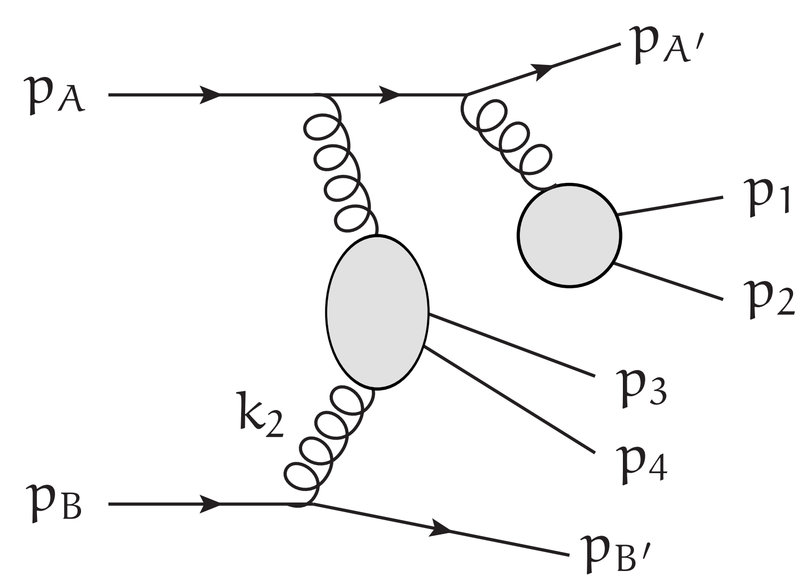
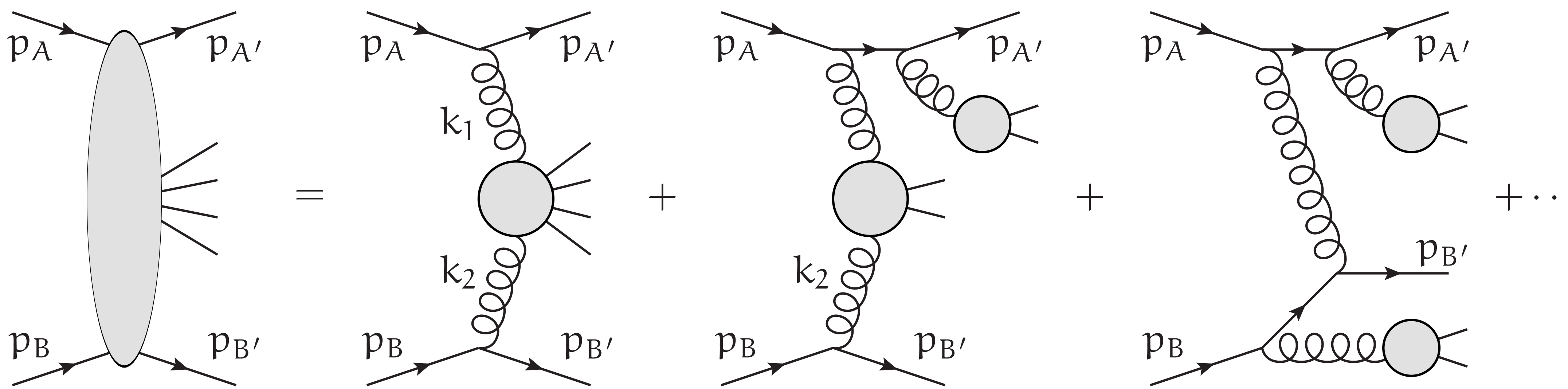




Two off–shell initial–state gluons

with P. Kotko
K. Kutak

Embed the process $g^*g^* \to X$ in the on-shell process $q_A q_B \to q_A q_B + X$.



$$\ell_1 = (E, 0, 0, E) \qquad \ell_2 = (E, 0, 0, -E)$$

$$p_A - p_{A'} = k_1 = x_1\ell_1 + k_{1\perp} + y_2\ell_2 \qquad p_B - p_{B'} = k_2 = x_2\ell_2 + k_{2\perp} + y_1\ell_1$$

The terms $y_2\ell_2$ and $y_1\ell_1$ are necessary to keep all quark momenta on-shell.
Usually, one takes $\ell_1 = p_A$ and $\ell_2 = p_B$, and extracts the amplitude for $g^*g^* \to X$ by neglecting terms proportional to $y_{1,2}$ $\Longrightarrow$ *high-energy limit*.

Continuation to complex momenta

- we are just interested in a gauge invariant amplitude $\mathcal{A}(g^*g^* \to X)$
- the amplitude $\mathcal{A}(q_A q_B \to q_A q_B + X)$ must be gauge invariant, must be completely on-shell, but does not have to be physical
- introduce complex on-shell momenta $p_A, p_{A'}, p_B, p_{B'}$

$$\ell_3^\mu = \tfrac{1}{2}\langle \ell_2|\gamma^\mu|\ell_1]$$
$$\ell_4^\mu = \tfrac{1}{2}\langle \ell_1|\gamma^\mu|\ell_2]$$

$$\ell_1^2 = \ell_2^2 = 0$$
$$\ell_3^2 = \ell_4^2 = 0$$
$$\ell_{1,2}\cdot\ell_{3,4} = 0$$

$$\ell_1\cdot\ell_2 = -\ell_3\cdot\ell_4$$

$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4\cdot k_{1\perp}}{\ell_1\cdot\ell_2}\,\ell_3^\mu \qquad p_{A'}^\mu = \Lambda\ell_1^\mu + \frac{\ell_3\cdot k_{1\perp}}{\ell_1\cdot\ell_2}\,\ell_4^\mu$$

$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3\cdot k_{2\perp}}{\ell_1\cdot\ell_2}\,\ell_4^\mu \qquad p_{B'}^\mu = \Lambda\ell_2^\mu + \frac{\ell_4\cdot k_{2\perp}}{\ell_1\cdot\ell_2}\,\ell_3^\mu$$

Now we have both the high-energy limit and on-shellness:

$$p_A^\mu - p_{A'}^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu$$

$$p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0$$

for any value of the dimensionless parameter Λ.

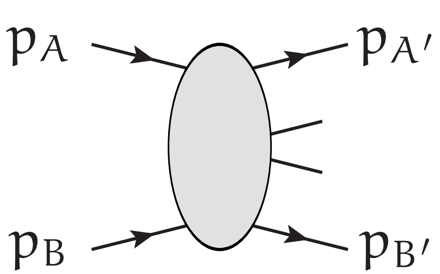
Extract physical amplitude

External spinors:

$$|p_A] = \frac{\sqrt{\Lambda + x_1} + \kappa_{13}}{\sqrt{|\sqrt{\Lambda + x_1} + \kappa_{13}|}}\,|\ell_1] \quad , \quad \langle p_{A'}| = \sqrt{|\sqrt{\Lambda} - \kappa_{14}|}\,\langle \ell_1|$$

$$|p_B] = \frac{\sqrt{\Lambda + x_2} + \kappa_{24}}{\sqrt{|\sqrt{\Lambda + x_2} + \kappa_{24}|}}\,|\ell_2] \quad , \quad \langle p_{B'}| = \sqrt{|\sqrt{\Lambda} - \kappa_{23}|}\,\langle \ell_2|$$

Take limit $\Lambda \to \infty$ to extract physical amplitude. This is not an approximation.



$$p_A = (\Lambda + x_1)\ell_1 + \kappa_{13}\ell_3$$
$$p_{A'} = \Lambda\ell_1 - \kappa_{14}\ell_4$$
$$p_A - p_{A'} = x_1\ell_1 + k_{1\perp}$$

$$p_B = (\Lambda + x_2)\ell_2 + \kappa_{24}\ell_4$$
$$p_{B'} = \Lambda\ell_2 - \kappa_{23}\ell_3$$
$$p_B - p_{B'} = x_2\ell_2 + k_{2\perp}$$

For for an A-quark line propagator, we get

$$\frac{\not{p}}{p^2} = \frac{(\Lambda + x)\not{\ell}_1 + y\not{\ell}_2 + \not{p}_\perp}{2(\Lambda + x)y\,\ell_1\cdot\ell_2 + p_\perp^2} \xrightarrow{\Lambda\to\infty} \frac{\not{\ell}_1}{2y\,\ell_1\cdot\ell_2} = \frac{\not{\ell}_1}{2\,\ell_1\cdot p}$$

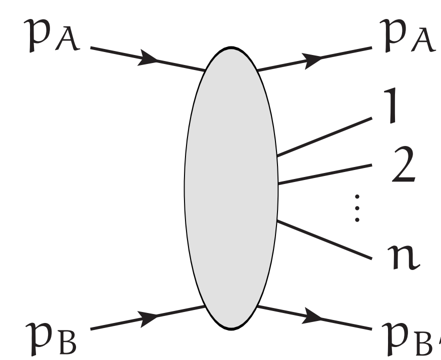
Gluons will attach to A-quark line via eikonal couplings

$$\langle \ell_1|\gamma^{\mu_1}\not{\ell}_1\gamma^{\mu_2}\not{\ell}_1 \cdots |\ell_1] = \langle \ell_1|\gamma^{\mu_1}|\ell_1]\langle \ell_1|\gamma^{\mu_2}|\ell_1]\langle \ell_1| \cdots |\ell_1]$$
$$= (2\ell_1^{\mu_1})(2\ell_1^{\mu_2}) \cdots .$$

Analogously for B-quark line.

The prescription to get $\mathcal{A}(g^* g^* \to X)$

1. Consider the process $q_A\, q_B \to q_A\, q_B\, X$, where q_A, q_B are distinguishable massless quarks not occurring in X, and with momentum flow as if the momenta p_A, p_B of the initial-state quarks and $p_{A'}, p_{B'}$ of the final-state quarks are given by



$$p_A^\mu = k_1^\mu \quad , \quad p_B^\mu = k_2^\mu \quad , \quad p_{A'}^\mu = p_{B'}^\mu = 0$$

2. Associate the number 1 instead of spinors with the end points of the A-quark line, interpret every vertex on the A-quark line as $g_s T^a_{ij}\, \ell_1^\mu$ instead of $-ig_s T^a_{ij}\, \gamma^\mu$, interpret every propagator on the A-quark line as $\delta_{ij}/\ell_1 \cdot p$ instead of $i\delta_{ij}/\not{p}$.

3. Associate the number 1 instead of spinors with the end points of the B-quark line, interpret every vertex on the B-quark line as $g_s T^a_{ij}\, \ell_2^\mu$ instead of $-ig_s T^a_{ij}\, \gamma^\mu$, interpret every propagator on the B-quark line as $\delta_{ij}/\ell_2 \cdot p$ instead of $i\delta_{ij}/\not{p}$.

4. Multiply the amplitude with $F = \dfrac{i\, x_1 \sqrt{-2k_{1\perp}^2}}{g_s} \times \dfrac{i\, x_2 \sqrt{-2k_{2\perp}^2}}{g_s}$.

5. For the rest normal Feynman rules apply.

In agreement with Lipatov's effective action.

Application

Ratio of observable distributions for p-p vs. p-Pb collisions.
Use unintegrated gluon densities from Phys.Rev.D86(2012)094043 for off-shell gluon.

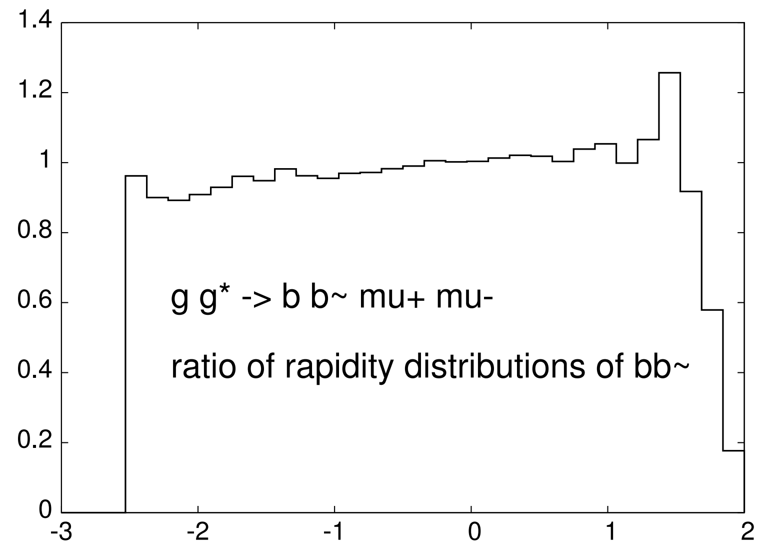
$$g g^* \to b\bar{b}\,\mu^+\mu^-$$

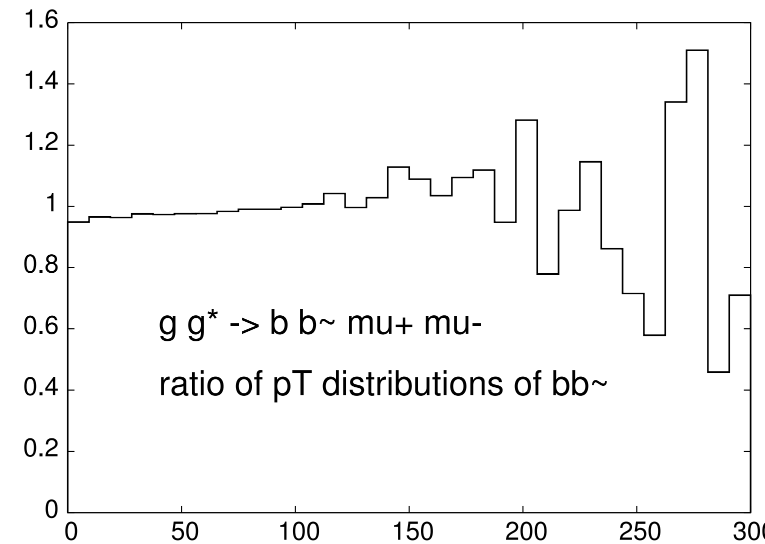
$$p_T(q) > 20\,\text{GeV} \quad y(q) < 2.5$$

$$p_T(\mu) > 20\,\text{GeV} \quad y(\mu) < 2.1$$

$$dR(q,q) > 0.4 \quad dR(q,\mu) > 0.4$$

$$\sqrt{s} = 8\,\text{TeV}$$





$$u g^* \to b\bar{b}\,\mu^+\nu_\mu\, d$$

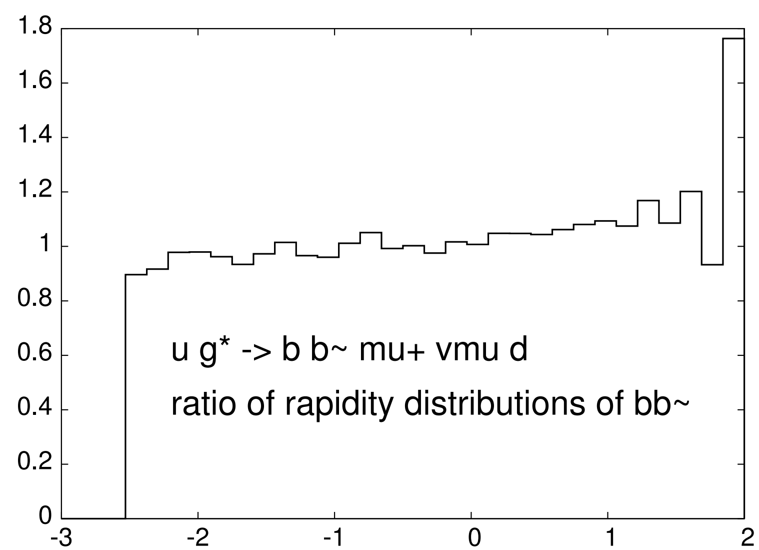
$$p_T(q) > 20\,\text{GeV} \quad y(q) < 2.5$$

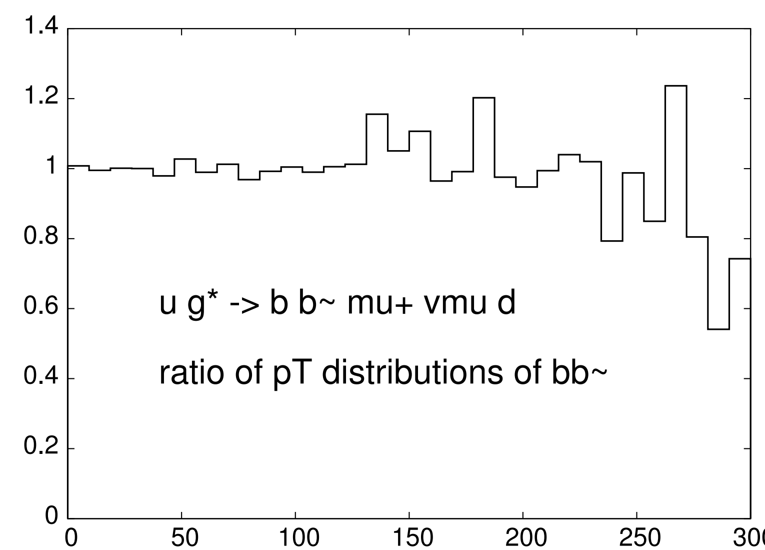
$$20\,\text{GeV} < p_T(\mu^+) < 50\,\text{GeV}$$

$$y(\mu) < 2.1$$

$$\not{E}_T > 20\,\text{GeV}$$

$$dR(q,q) > 0.4 \quad dR(q,\mu) > 0.4$$





Summary

- high-energy factorization
- gauge invariant off-shell helicity amplitudes for $g^* g \to n\, g$
  JHEP 1212 (2012) 029
- gauge invariant off-shell helicity amplitudes for $g^* g^* \to X$
  JHEP 1301 (2013) 078
- in agreement with Lipatov's effective action
- implemented in a Monte Carlo program, able to deal with processes like

$$
\begin{aligned}
&g^* g^* \to b\bar{b}Z \to b\bar{b}\,\mu^+\mu^- \\
&g^* g^* \to b\bar{b}Zg \to b\bar{b}\,\mu^+\mu^-\, g \\
&g^* g^* \to b\bar{b}g \\
&g^* g^* \to b\bar{b}gg
\end{aligned}
$$