Inclusive Cross Sections in ME+PS Merging

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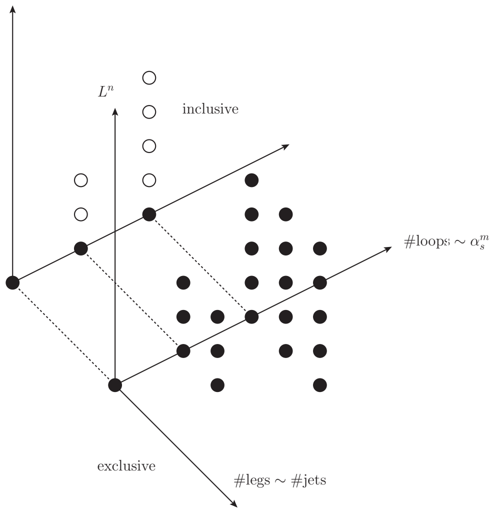
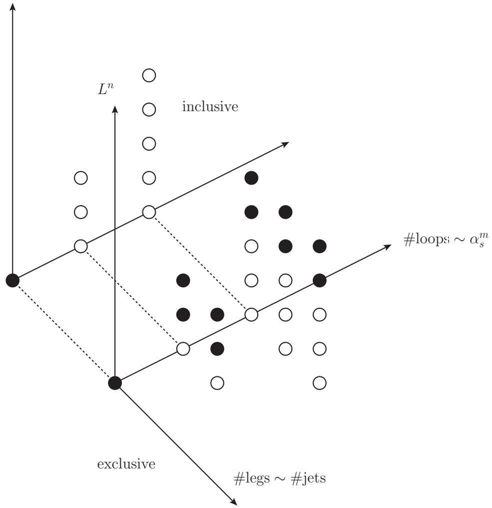
SP, 1211.5467

Warmup – the pQCD landscape.



Warmup – the (ideal) shower landscape.



Warmup – some notation.

Parton shower action on events with n additional jets ϕ_n:

generic splitting kernels P and evolution variable q

$$\mathrm{PS}_\mu\left[\mathrm{d}\sigma(\phi_n, q_n)\right] = \mathrm{d}\sigma(\phi_n, q_n)\Delta_n(\mu|q_n) + \mathrm{PS}_\mu\left[\mathrm{d}\sigma(\phi_n, q_n)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}P_\mu(\phi_n, q_{n+1})\Delta_n(q_{n+1}|q_n)\right]$$

[no emission] + [zero or more emission off one emission]

- Hard scale $q_n \sim$ min jet $p_\perp$, may be lowest scale in a clustering procedure,
- infrared cutoff μ.

Sudakov form factor a.k.a. no emission probability:

$$\Delta_n(q|Q) = \exp\left(-\int_q^Q \mathrm{d}k\ \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n \mathrm{d}k}\ P(\phi_n, k)\right)$$

Warmup – shower cross sections.

Shower evolution is unitary ($\sim$ Markov process):

$$\int_q^{q_{k-1}} \mathrm{d}q_k \frac{\mathrm{d}\phi_k}{\mathrm{d}\phi_{k-1}\mathrm{d}q_k} P(\phi_{k-1}, q_k)\Delta_{k-1}(q_k|q_{k-1}) = 1 - \Delta_{k-1}(q|q_{k-1})$$

$\mathrm{Pr}\left[\text{emit somewhere between } q_{k-1} \text{ and } q \textit{ and anything more}\right] = 1 - \mathrm{Pr}\left[\text{no emission between } q_{k-1} \text{ and } q\right]$

n (parton shower) jet cross sections starting from LO configuration ϕ_0:

$\Delta_{n-1}(q_n|\cdots|q_0) = \Delta_{n-1}(q_n|q_{n-1})\cdots\Delta_0(q_1|q_0)$

$$= n \qquad \mathrm{d}\sigma^{(0)}(\phi_0, q_0)\frac{\mathrm{d}\phi_n}{\mathrm{d}\phi_0} P_\mu(\phi_{n-1}, q_n)\cdots P_\mu(\phi_0, q_1)\Delta_n(\mu|q_n|\cdots|q_0)$$

$\mathrm{Pr}\left[\text{emit @ } q_1, \ldots, q_n \text{ with nothing in between}\right] \times \mathrm{Pr}\left[\text{no emission down to } \mu\right]$

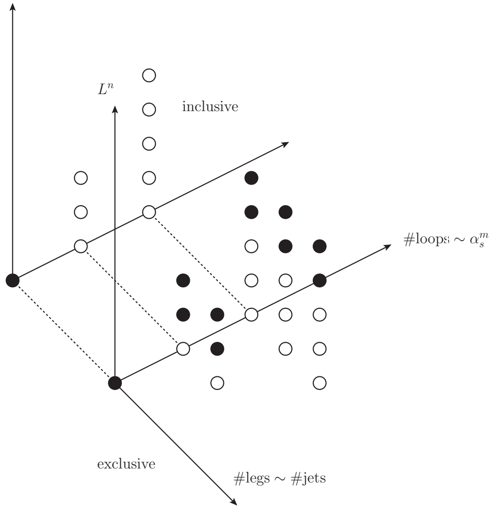
$$\geq n \qquad \mathrm{d}\sigma^{(0)}(\phi_0, q_0)\frac{\mathrm{d}\phi_n}{\mathrm{d}\phi_0} P_\mu(\phi_{n-1}, q_n)\cdots P_\mu(\phi_0, q_1)\Delta_{n-1}(q_n|\cdots|q_0)$$

$\mathrm{Pr}\left[\text{emit @ } q_1, \ldots, q_n \text{ with nothing in between}\right]$

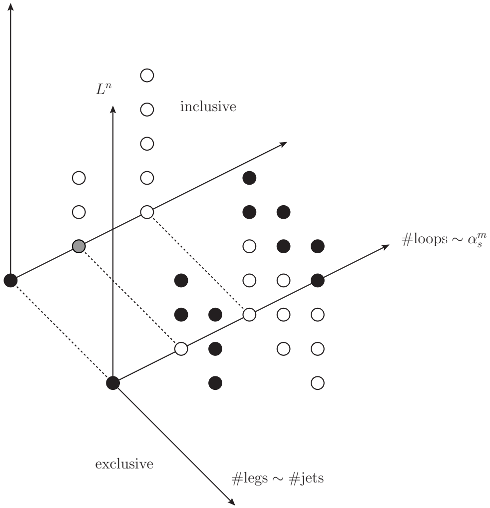
Outline.

- Tree level merging, revisited.
- Inclusive cross sections as a guide to NLO merging.
- Same procedure, one order higher.
- Conclusions & Outlook.

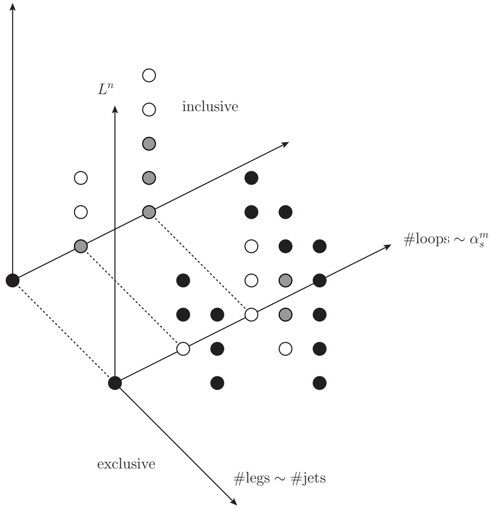
Tree level merging.



Tree level merging.



Tree level merging.



Tree level merging.

Merging condition: LO $\times$ products of splitting kernels $\to$ exact tree level ME.

$$\mathrm{PS}_\mu \left[\mathrm{d}\sigma_{N,\mu}^{\mathsf{merged}} \right] = \sum_{k=0}^{N-1} \mathrm{d}\sigma_\mu^{(0)}(\phi_k, q_k)\Delta_k(\mu|q_k|\cdots|q_0) \; + \mathrm{PS}_\mu \left[\mathrm{d}\sigma_\mu^{(0)}(\phi_N, q_N)\Delta_{N-1}(q_N|\cdots|q_0) \right]$$

- Parton shower infrared cutoff applied to reclustered tree level matrix elements,
- proper Sudakov form factors to account for exclusiveness,
- no merging scale required in the first place.

Tree level merging.

Cut off matrix elements at $\rho > \mu$:

$$\mathrm{PS}_\mu \left[\mathrm{d}\sigma_{N,\rho}^{\text{merged}} \right] =$$

$$\sum_{k=0}^{N-1} \mathrm{PS}_{\mu|\rho} \left[\mathrm{d}\sigma_\rho^{(0)}(\phi_k, q_k) \Delta_k(\rho|q_k|\cdots|q_0) \right] + \mathrm{PS}_\mu \left[\mathrm{d}\sigma_\rho^{(0)}(\phi_N, q_N) \Delta_{N-1}(q_N|\cdots|q_0) \right]$$

- ‘Traditional’ ME+PS merging, [CKKW, /Lönnblad, ...]
- no restriction on showering off the highest multiplicity.

Tools & side remarks.

$$\mathrm{d}\sigma^{\text{merged}}_{N,\rho} = \sum_{k=0}^{N-1} \mathrm{PS}^{-1}_{\rho}\left[\mathrm{d}\sigma^{(0)}_{\rho}(\phi_k, q_k)\Delta_k(\rho|q_k|\cdots|q_0)\right] + \mathrm{d}\sigma^{(0)}_{\rho}(\phi_N, q_N)\Delta_{N-1}(q_N|\cdots|q_0)$$

$$\mathrm{PS}_{\mu}\left[\mathrm{PS}^{-1}_{\rho}[\mathrm{d}\sigma_{\rho}(\phi_n, q_n)]\right] = \mathrm{PS}_{\mu|\rho}\left[\mathrm{d}\sigma_{\rho}(\phi_n, q_n)\right]$$

Inverse of the shower action is

$$\mathrm{PS}^{-1}_{\mu}\left[\mathrm{d}\sigma_{\mu}(\phi_n, q_n)\right] = \frac{\mathrm{d}\sigma_{\mu}(\phi_n, q_n)}{\Delta_n(\mu|q_n)} - \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n} P_{\mu}(\phi_n, q_{n+1}) \frac{\mathrm{d}\sigma_{\mu}(\phi_n, q_n)}{\Delta_n(\mu|q_{n+1})}$$

- Can be sampled, if needed, and
- expansions can be used for fixed-order matching subtractions.

Inclusive cross sections?

Exclusive cross sections are fine by the very definition of the merging condition.

Inclusive cross sections are generally spoiled, say $\geq N-1$ (parton shower) jets:

$$\mathrm{d}\sigma_\rho^{(0)}(\phi_{N-1}, q_{N-1})\Delta_{N-2}(q_{N-1}|\cdots|q_0)+$$

$$\int_\rho^{q_{N-1}} \mathrm{d}q_N \left(\frac{\mathrm{d}\sigma_\rho^{(0)}(\phi_N, q_N)}{\mathrm{d}q_N} - \frac{\mathrm{d}\phi_N}{\mathrm{d}\phi_{N-1}\mathrm{d}q_N} P_\rho(\phi_{N-1}, q_N)\mathrm{d}\sigma_\rho^{(0)}(\phi_{N-1}, q_{N-1}) \right) \times$$

$$\Delta_{N-1}(q_N|\cdots|q_0)$$

Natural consequence of replacing splitting kernels by matrix elements *except for the Sudakov exponents.* [cf. matrix element correction approaches like Vincia, Skands et al.]

Not a problem as long as the shower kernels approximate the singly-unresolved limits of the *tree level matrix elements* sufficiently good.

From nLO merging to NLO merging.

Constrain the matching condition to preserve inclusive cross section:
Except for the highest multiplicity, replace

$$\mathrm{d}\sigma_\rho^{(0)}(\phi_k, q_k) \to \mathrm{d}\sigma_\rho^{(0)}(\phi_k, q_k) - \int_\rho^{q_k} \mathrm{d}q_{k+1} \frac{\mathrm{d}\sigma_\rho^{(0)}(\phi_{k+1}, q_{k+1})}{\mathrm{d}q_{k+1}} \Delta_k(q_{k+1}|q_k)$$

Fixed order expansion is a variant of the LoopSim nLO exclusive k jet cross section.

[Rubin, Salam, Sapeta – 1006.2144]

After showering we get precisely this contribution with the proper Sudakov supression.

LO merging with inclusive cross sections preserved $\to$ nLO merging.

From nLO merging to NLO merging.

Replace the nLO approximate α_s correction by the NLO exact α_s correction.

In a nutshell: Where available add

$$\mathrm{PS}_\rho^{-1}\left[\left(\mathrm{d}\sigma_\rho^{(1)}(\phi_n, q_n) + \int_0^{q_n} \mathrm{d}q_{n+1}\frac{\mathrm{d}\sigma^{(0)}(\phi_{n+1}, q_{n+1})}{\mathrm{d}q_{n+1}}\theta(q_n-\rho)\right)\Delta_{n-1}(q_n|\cdots|q_0)\right]$$

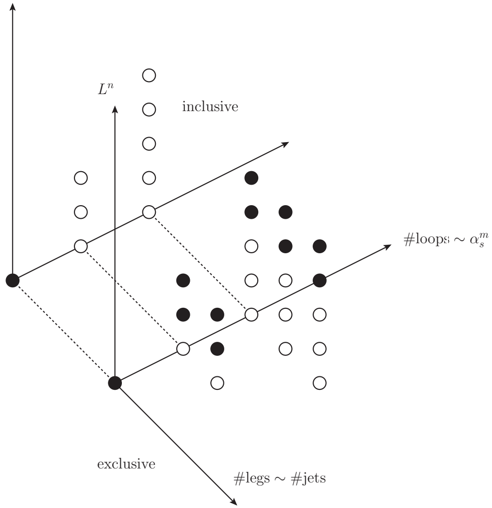
to the merged cross section.

This is NLO merging, as recently discussed in several variants.

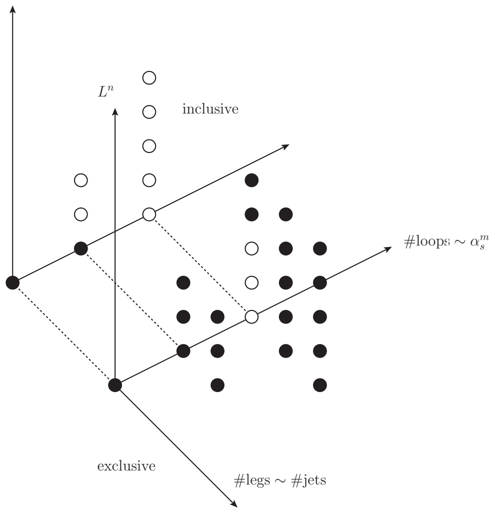
[Höche et al. – 1207.5030, Frederix, Frixione – 1209.6215, Lönnblad, Prestel – 1211.7278, Hamilton et al. – 1212.4504]

- Recover exclusive NLO *n*-jet cross sections above the merging scale.
- NLO accuracy below the merging scale by constrained NLO matching.

The landscape for NLO merging.



The landscape for NLO merging.



The landscape for NLO merging.

Exclusive cross sections are fine by the very definition of the merging condition.

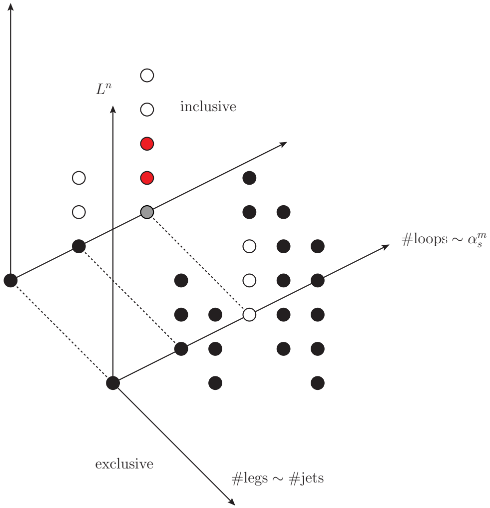
Inclusive cross sections are generally spoiled, say $\geq N-1$ (parton shower) jets:

$$\mathrm{d}\sigma^{\mathrm{NLO}}_{\rho,\mathrm{incl}}(\phi_{N-1}, q_{N-1})\Delta_{N-2}(q_{N-1}|\cdots|q_0)+$$
$$\int_{\rho}^{q_{N-1}} \mathrm{d}q_N \left(\frac{\mathrm{d}\sigma^{\delta\mathrm{NLO}}_{\rho,\mathrm{excl}}(\phi_N, q_N)}{\mathrm{d}q_N} - \frac{\mathrm{d}\phi_N}{\mathrm{d}\phi_{N-1}\mathrm{d}q_N} P_\rho(\phi_{N-1}, q_N)\mathrm{d}\sigma^{\delta\mathrm{NLO}}_{\rho,\mathrm{excl}}(\phi_{N-1}, q_{N-1}) \right) \times$$
$$\Delta_{N-1}(q_N|\cdots|q_0)$$

Similar to the tree level problems.

But now a serious problem unless we have a shower which knows about the singly unresolved limits of *virtual contributions*.

The landscape for NLO merging.



From NLO merging to nNLO merging.

"Same procedure as one order lower, Miss Sophy?"

"Same procedure as at any order, James."

Summary.

Formulate (N)LO merging algorithms in a very generic way.

Constrain inclusive cross sections to the respective input calculations.

[very similar approach by Lönnblad and Prestel]

- Generates approximate higher order contributions a la LoopSim.
- Algorithmic approach towards heven higher orders:
  LO $\rightarrow$ nLO $\rightarrow$ NLO $\rightarrow$ nNLO $\rightarrow$ ...

Should significantly reduce the merging scale uncertainty.

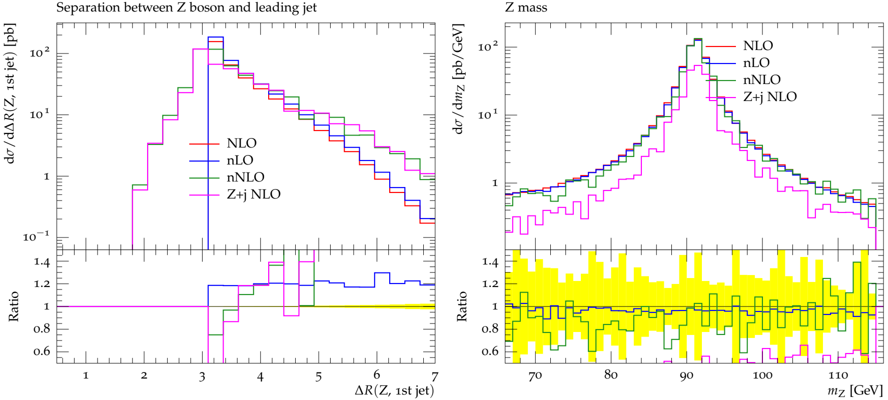
Outlook.

Implementation in progress based on Herwig++'s Matchbox and DipoleShower modules.

[SP & S. Gieseke, 1109.6256]

Step one: Re-interpret the LoopSim algorithm with (dipole) shower clusterings.

[work in progress with J. Bellm & S. Gieseke]



[preliminary]

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