arXiv:1303.6952

Using 1-jettiness to Measure 2 Jets in DIS in 3 Ways

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In collaboration with

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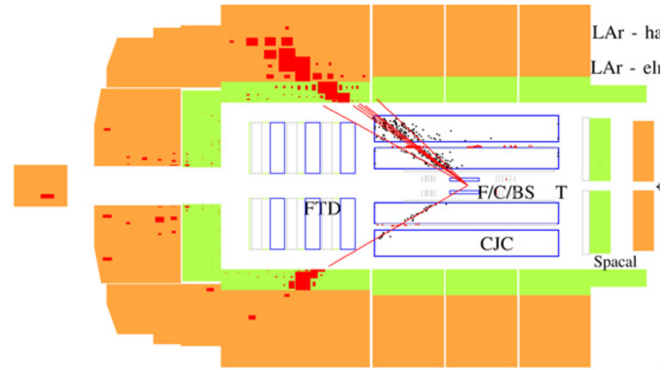
DIS 2013

Outline

- Deep inelastic scattering (DIS)
  - **2**-jets event: 1 ISR+1 jet



H1 Event from *www-h1.desy.de*

- Event shape*:*

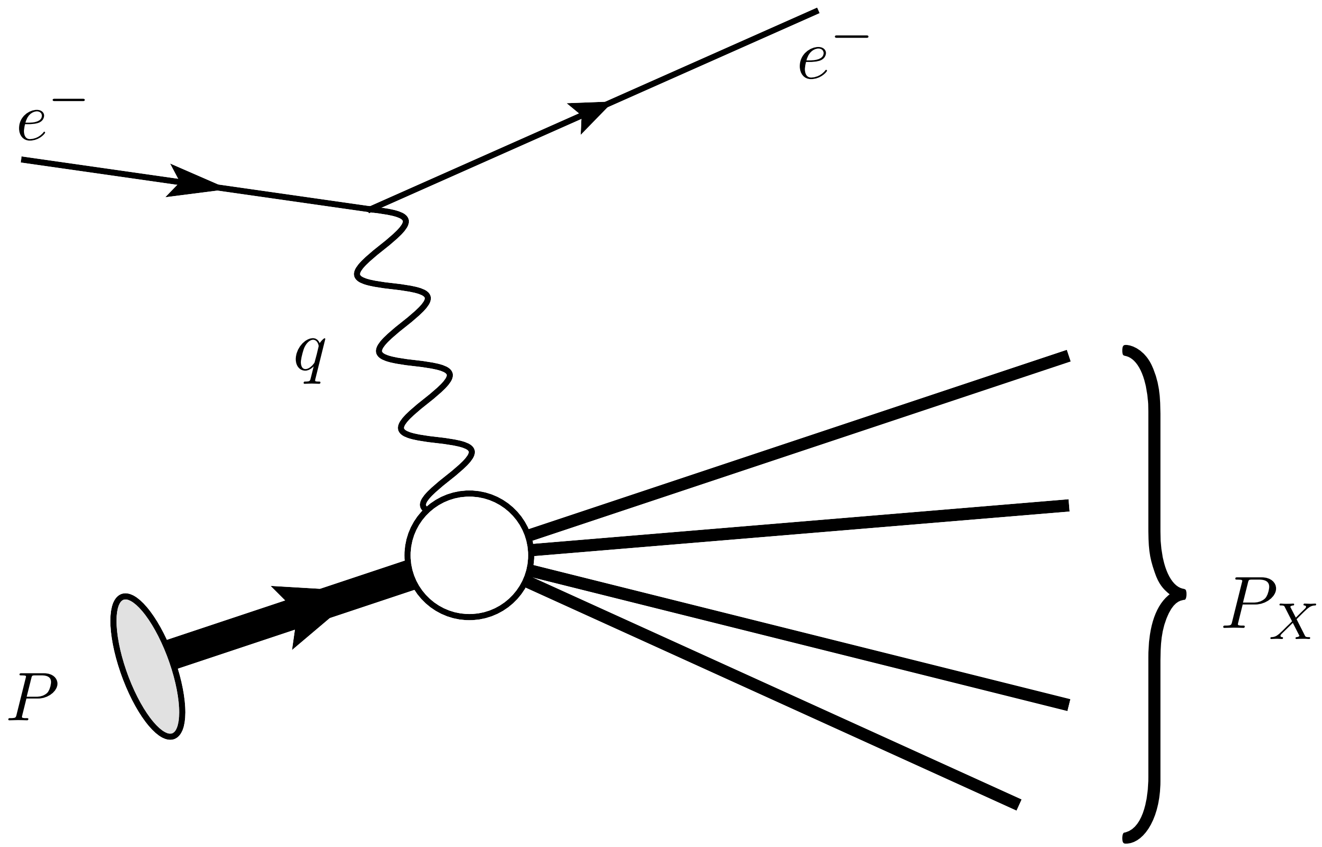
  1*-jettiness* in **3** ways

- Derivation of *factorization thm.* to all orders in α_s

$$\sigma \sim H \times B \otimes J \otimes S$$

- Resummed predictions at **NNLL**
  - Higher logarithmic accuracy than previous work at NLL
- Summary

Deep Inelastic Scattering



DIS variables

$$q^2 = -Q^2$$

Björken scaling:

$$x = \frac{Q^2}{2P.q}$$

Inelasticity:

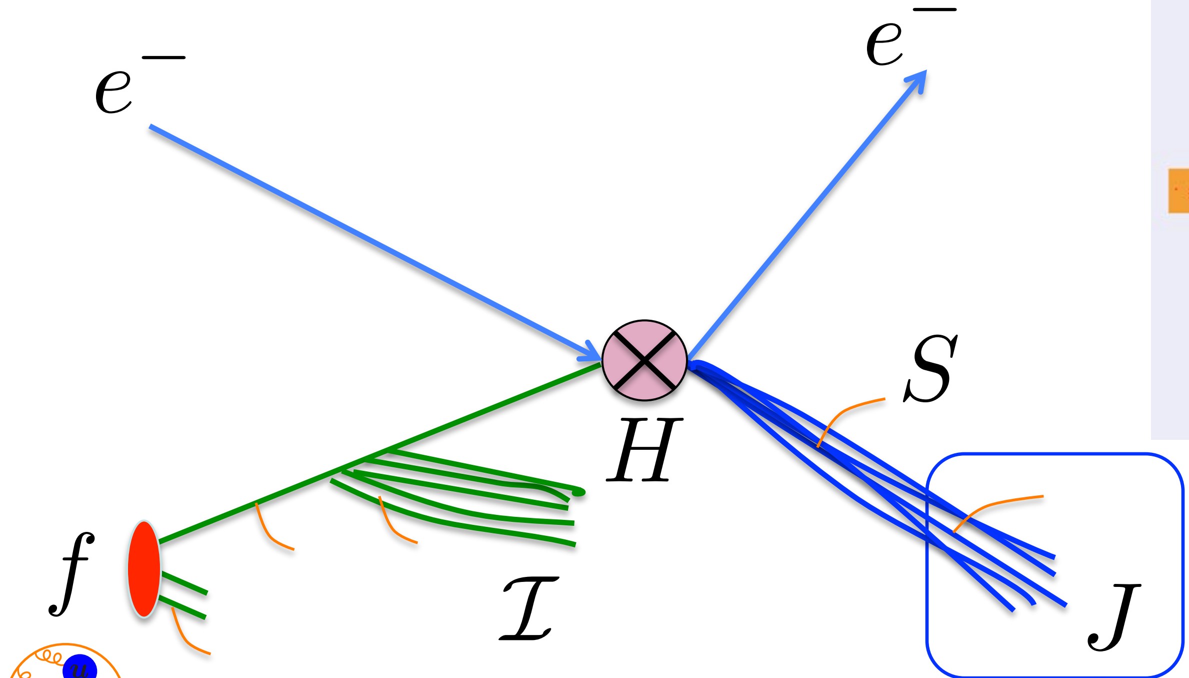
$$y = \frac{Q^2}{xs}$$

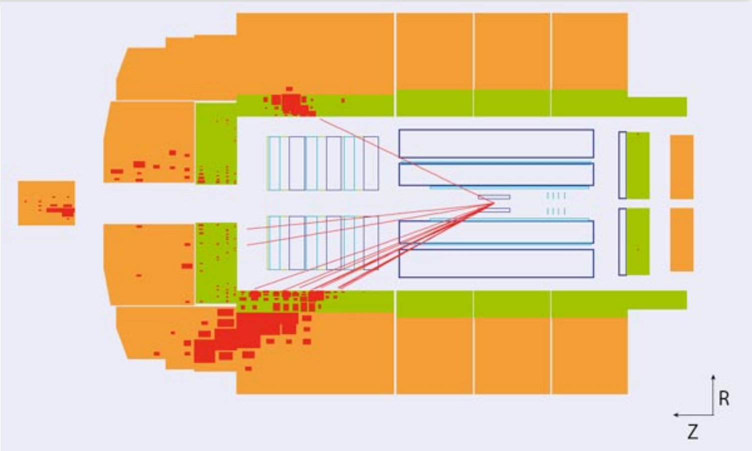
$$P_X^2 = \frac{1-x}{x} Q^2$$

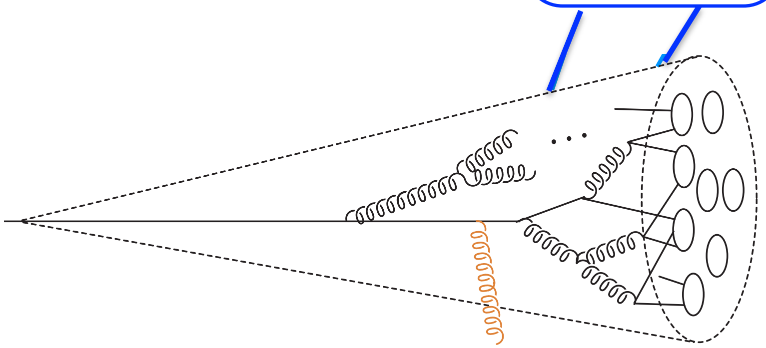
- **Large Q transfer from e^- to proton**
- **Traditional factorization for large Q >> Λ_{QCD}**

$$\sigma = f \otimes \sigma_{\text{parton}}$$

Typical jet event







- ***quarks and gluons form jets***

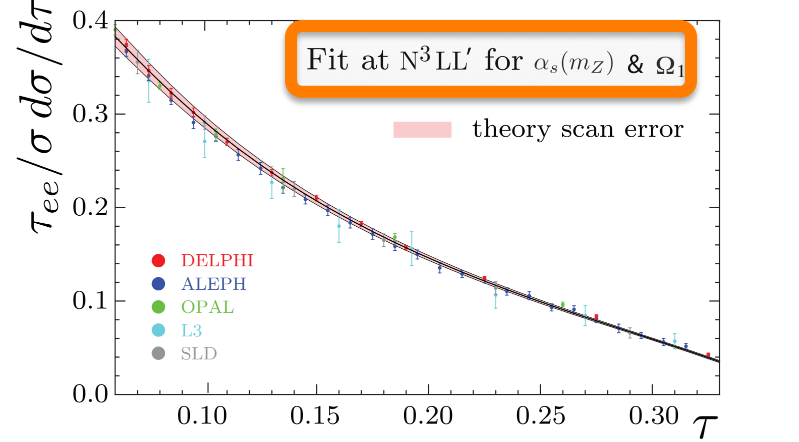
$$\sigma \sim H \times B \otimes J \otimes S$$

$$B = f \otimes \mathcal{I}$$

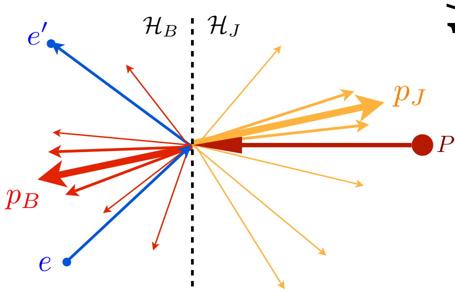
Event shape: Thrust

- e^+e^-: $\tau_{ee} = 1 - \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{p}_i \cdot \vec{n}|$
  - *dijet limit:* $\tau_{ee} \to 0$
  - $\log \tau_{ee}$ should be resummed
  - Up to $N^3LO(\alpha_s^3)$+N^{3}LL

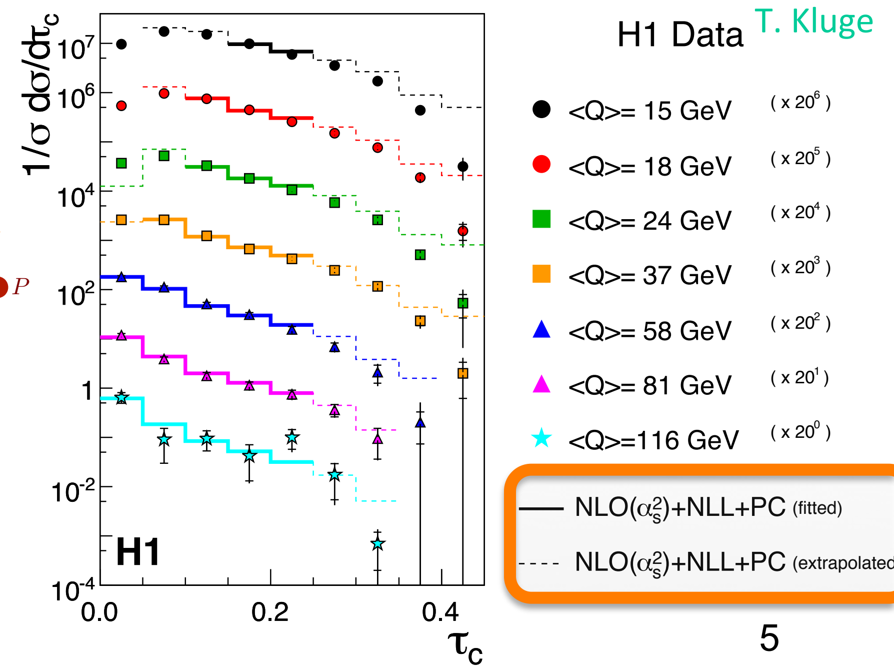
Abbate, Fickinger, Hoang, Mateu, Stwart



- *DIS :* $\tau_{\mathrm{DIS}} = 1 - \frac{1}{E_J} \sum_{i \in \mathcal{H}_J} |\vec{p}_i \cdot \vec{n}|$
  - 2 choices for $\vec{n}$
    - Axis minimizing thrust
    - Virtual photon axis
  - one hemisphere
  - Up to NLO(α_s^2)+NLL



Antonelli, Dasgupta, Salam



T. Kluge

- Uncertainty dominated by theory

$$\alpha_s(m_Z) = 0.1198 \pm 0.0013(\mathrm{exp.})\ {}^{+0.0056}_{-0.0043}(\mathrm{th.})$$

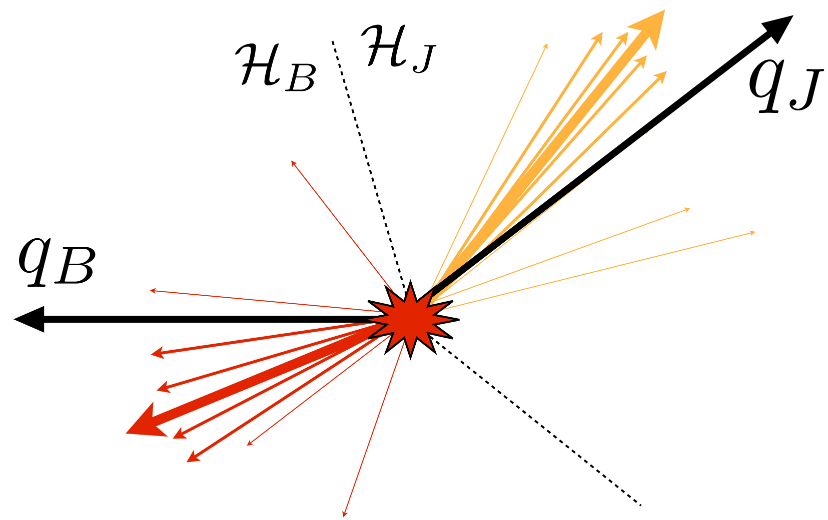
- *Higher precision?* N^{2}LL or higher ?

ZEUS also has done similar analysis.

Event shape: 1-jettiness

- **N-jettiness**
  $$\tau_N = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_1 \cdot p_i, \dots, q_N \cdot p_i\}$$
  Stewart, Tackmann, Waalewijn
  - Generalization of thrust
  - N-jet limit: $\tau_N \to 0$

- **1-jettiness:** 1 jet + 1 ISR
  $$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$
  - q_B, q_J are axes to project particle mom.
  - Considering 3 ways to define q_J
  - min. groups particles into 2 regions



Why 1-jettiness?

DIS thrust: Non-Global Log beyond NLL Dasgupta, Salam

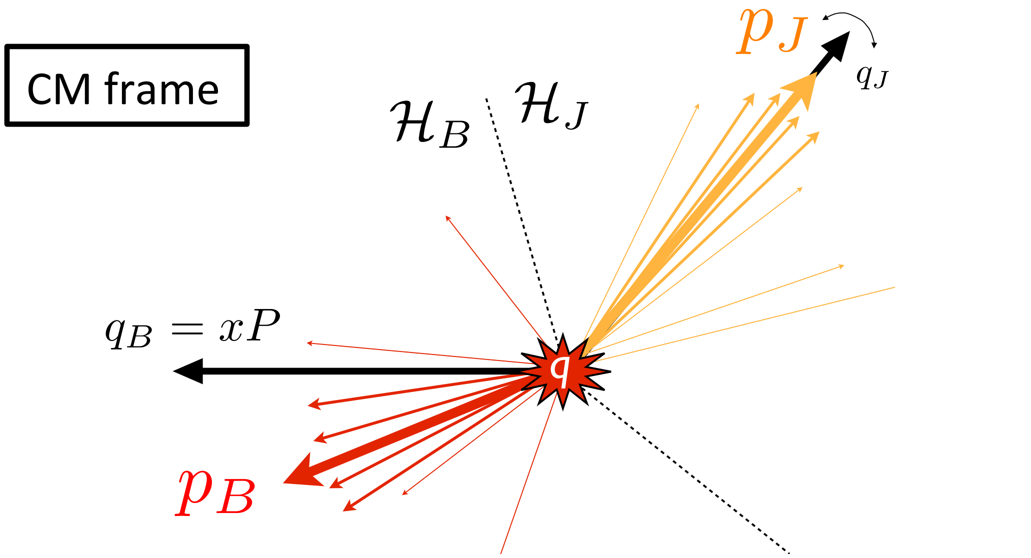
Unknown how to resum NGL

1-jettiness: No NGL, N^nLL (n>1) accessible

derive factorization thm. by using SCET

accuracy systematically improved with higher order ME's

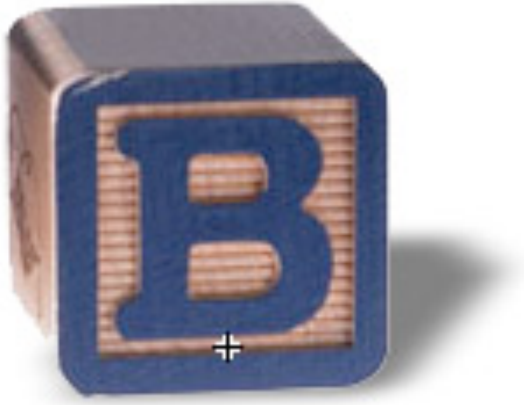
1-jettiness in 3 ways



$$\tau_1^a = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B^a \cdot p_i, q_J^a \cdot p_i\}$$

Kang, Mantry, Qiu PRD2012, 2013

- q_J^a :
  - **"Aligned"** with jet mom.
  - Determined from jet algorithm or min. procedure
- $q_B^a = xP$: proton direction (similar for q_B^b, q_B^c)



1-jettiness in 3 ways

$$\tau_1^b = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B^b \cdot p_i, q_J^b \cdot p_i\}$$

- q_J^b : *z*-axis in **Breit** frame
  - not aligned to jet
  - Zero transverse mom.
    $$(p_J + p_B)_\perp = 0$$

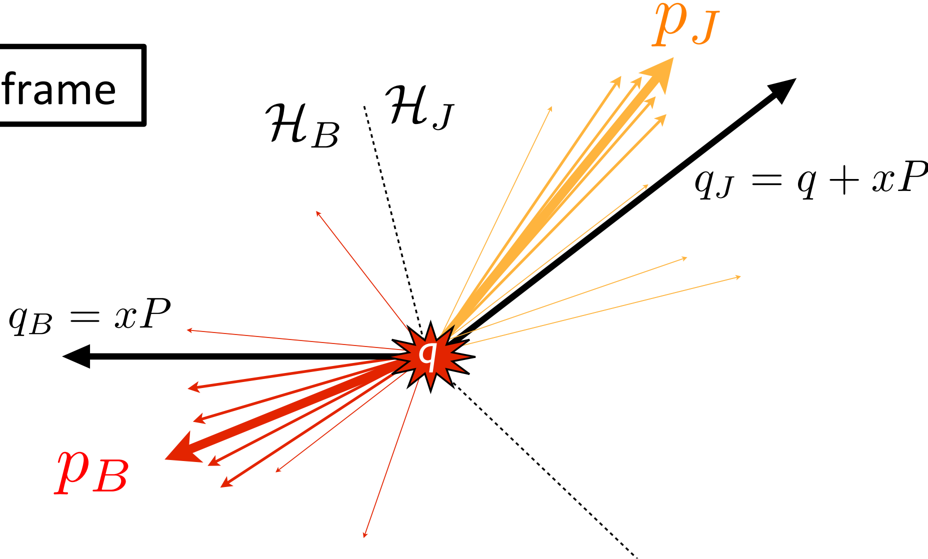
$$\tau_1^b \stackrel{\text{Breit}}{=} \frac{1}{Q}\left[\sum_{i \in \mathcal{H}_B} \bar{n}_z \cdot p_i + \sum_{i \in \mathcal{H}_J} n_z \cdot p_i\right]$$

Antonelli, Dasgupta, Salam JHEP 2000

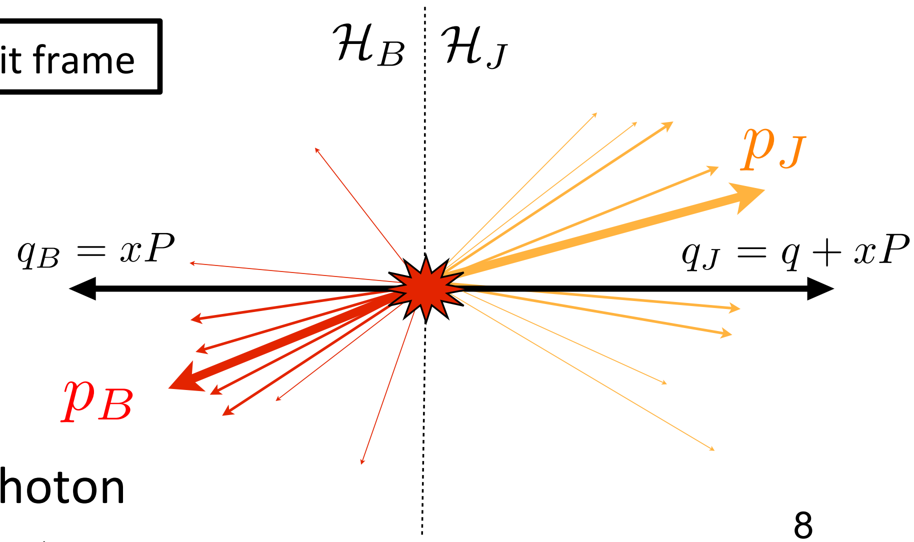
Breit frame: collision frame of proton and photon

$$P = 1/2x\,(Q, 0, 0, -Q) \qquad q = (0, 0, 0, Q)$$

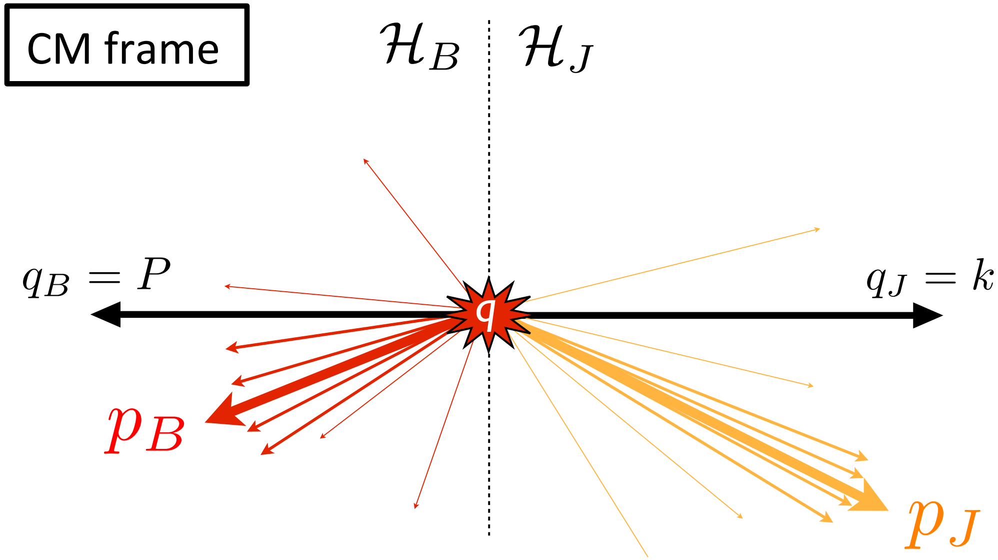
CM frame



Breit frame



1-jettiness in 3 ways



$$\tau_1^c = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B^c \cdot p_i, q_J^c \cdot p_i\}$$

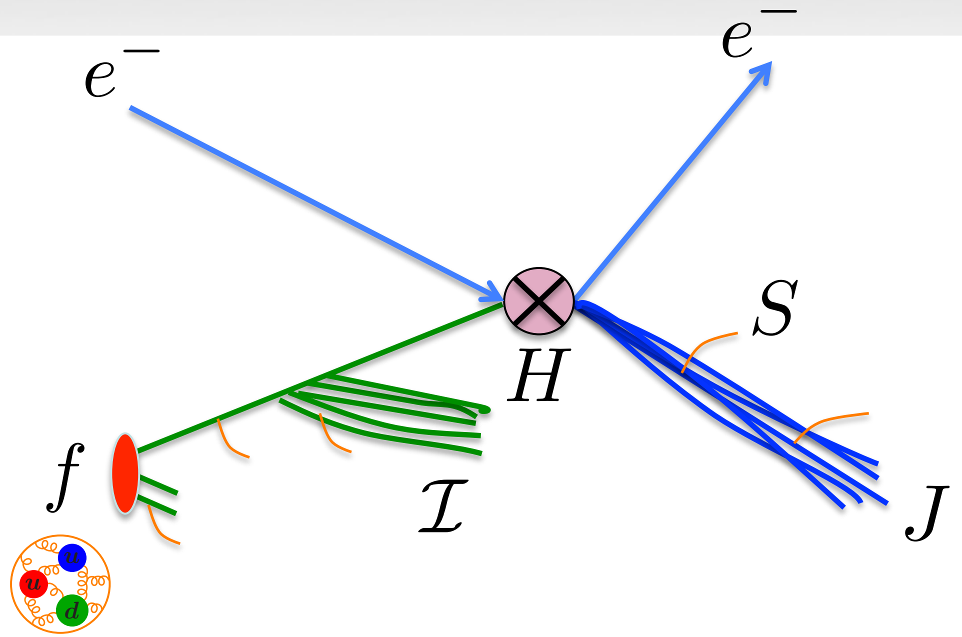
- q_J^c : *z*-axis in **CM** frame
  - electron direction
  - transverse mom. conservation

$$q_\perp = (p_J + p_B)_\perp$$

$$\tau_1^c \overset{\text{CM}}{=} \frac{1}{xy\sqrt{s}} \left[\sum_{i \in \mathcal{H}_B} \bar{n}_z \cdot p_i + \sum_{i \in \mathcal{H}_J} n_z \cdot p_i \right]$$

- Small τ_1^c region
  - Small $q_\perp = \sqrt{1-y}Q$, $\quad y \to 1$
  - dijet events in longitudinal direction

Factorization theorems

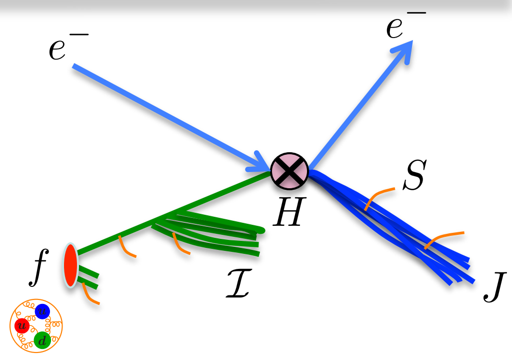


$$\sigma \sim H \times B \otimes J \otimes S$$

$$B = f \otimes \mathcal{I}$$

Factorization proof using SCET

$$\frac{d\sigma}{dx\, dQ^2\, d\tau_1} = L_{\mu\nu}\, W^{\mu\nu}(x, Q^2, \tau_1)$$



$$W^{\mu\nu} = \int d^4x e^{iq\cdot x} \langle P | J^{\dagger\,\mu}(x) \delta(\tau_1 - \hat{\tau}_1) J^\nu(0) | P \rangle$$

$$J^\mu(x) = \sum_{n_1, n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2\, e^{i(\tilde{p}_1 - \tilde{p}_2)\cdot x}\, C^\mu_{q\bar{q}}\, \bar{\chi}_{n_1,\tilde{p}_1} T\, [Y^\dagger_{n_1} Y_{n_2}]\, \chi_{n_2,\tilde{p}_2}$$

Wilson coefficient | Quark jet field | Soft gluon Wilson line

$$W^{\mu\nu} = 2(2\pi)^4\, Q_J^2 Q_B^2 \int d^2\tilde{p}_\perp \frac{2}{n_J \cdot n_B} \int d\tau_B d\tau_J\, d\tau_s^B d\tau_s^J \delta(\tau_1 - \tau_B - \tau_J - \tau_s^J - \tau_s^B)$$

$$\times\, C^{\dagger\,\mu}_{q\bar{q}} C^\nu_{q\bar{q}}$$ **Hard function**

$$\times \langle P_{n_B} | \bar{\chi}_{n_B}\, \delta(Q_B \tau_B - n_B \cdot \hat{p}_{n_B})\, [\delta(\bar{n}_B \cdot q - \bar{n}_B \cdot \mathcal{P})\, \delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\, \chi_{n_B}] | P_{n_B} \rangle$$ **Beam func.** **(PDF + ISR)**

$$\times \langle 0 | \chi_{n_J}\, \delta(Q_J \tau_J - n_J \cdot \hat{p}_{n_J})\, [\delta(\bar{n}_J \cdot q + \bar{n}_J \cdot \mathcal{P})\, \delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\, \bar{\chi}_{n_J}] | 0 \rangle$$ **Jet function**

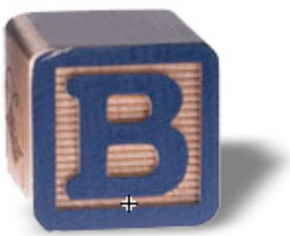
$$\times\, \langle 0 |\, [Y^\dagger_{n_B} Y_{n_J}]\, \delta(Q_J \tau_s^J - n_J \cdot \hat{p}^s_J) \delta(Q_B \tau_s^B - n_B \cdot \hat{p}^s_B)\, [Y^\dagger_{n_B} Y_{n_J}]\, | 0 \rangle$$ **Soft function**

Factorization theorems

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx\,dQ^2\,d\tau_1^a} = H_q(\mu)\int dt_B\,dt_J\,dk_s\,\delta\left(\tau_1^a - \frac{t_B}{Q^2} - \frac{t_J}{Q^2} - \frac{k_s}{Q}\right)$$

$$\times B_q\left(t_B, x, \mu\right)\; J_q\left(t_J, \mu\right)\; S\left(k_s, \mu\right) + \left(q \leftrightarrow \bar{q}\right)$$

Kang, Mantry, Qiu PRD2012, 2013



$$\frac{1}{\sigma_0}\frac{d\sigma}{dx\,dQ^2\,d\tau_1^b} = H_q(\mu)\int dt_B\,dt_J\,dk_s\,\delta\left(\tau_1^a - \frac{t_B}{Q^2} - \frac{t_J}{Q^2} - \frac{k_s}{Q}\right)$$

$$\times \int d^2\vec{p}_\perp\, B_q\left(t_B, x, \vec{p}_\perp^{\,2}, \mu\right)\; J_q\left(t_J - \vec{p}_\perp^{\,2}, \mu\right)\; S\left(k_s, \mu\right) + \left(q \leftrightarrow \bar{q}\right)$$

Transverse momentum dependent Beam function

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx\,dQ^2\,d\tau_1^c} = H_q(\mu)\int dt_B\,dt_J\,dk_s\,\delta\left(\tau_1^a - \frac{t_B}{Q^2} - \frac{t_J}{xQ^2} - \frac{k_s}{\sqrt{x}Q}\right)$$

$$\times \int d^2\vec{p}_\perp\, B_q\left(t_B, x, \vec{p}_\perp^{\,2}, \mu\right)\; J_q\left(t_J - (\vec{q}_\perp + \vec{p}_\perp)^2, \mu\right)\; S\left(k_s, \mu\right) + \left(q \leftrightarrow \bar{q}\right)$$

Resummation and RGE

- Fourier transformation

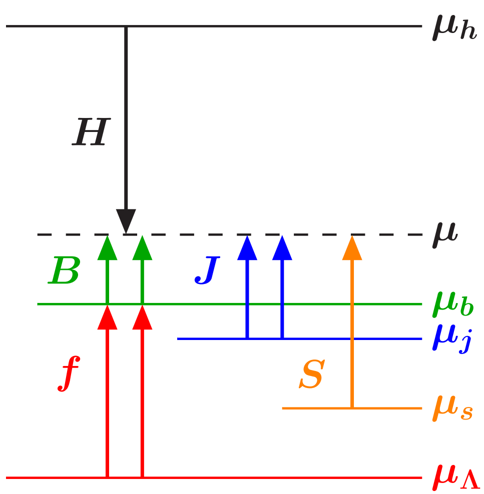
y : conjugate variable of τ_1

$$\frac{d\tilde{\sigma}}{dy} = \int d\tau_1 \, e^{-iy\tau_1} \frac{d\sigma}{d\tau_1} = H(\mu) \, \widetilde{B}_q(y,x,\mu) \, \widetilde{J}_q(y,\mu) \, \widetilde{S}(y,\mu)$$

$$\ln \frac{d\tilde{\sigma}}{dy} = \underbrace{L \sum_{k=1}^{\infty} (\alpha_s L)^k}_{LL} + \underbrace{\sum_{k=1}^{\infty} (\alpha_s L)^k}_{NLL} + \underbrace{\alpha_s \sum_{k=0}^{\infty} (\alpha_s L)^k}_{NNLL} + \cdots$$

$L = \log(iy)$

- Resumming large logs
  - No large logs in each function at its natural scale μ_i
  - *RG evolution* from μ_i to common scale μ



$O(\alpha_s)$+NNLL Predictions

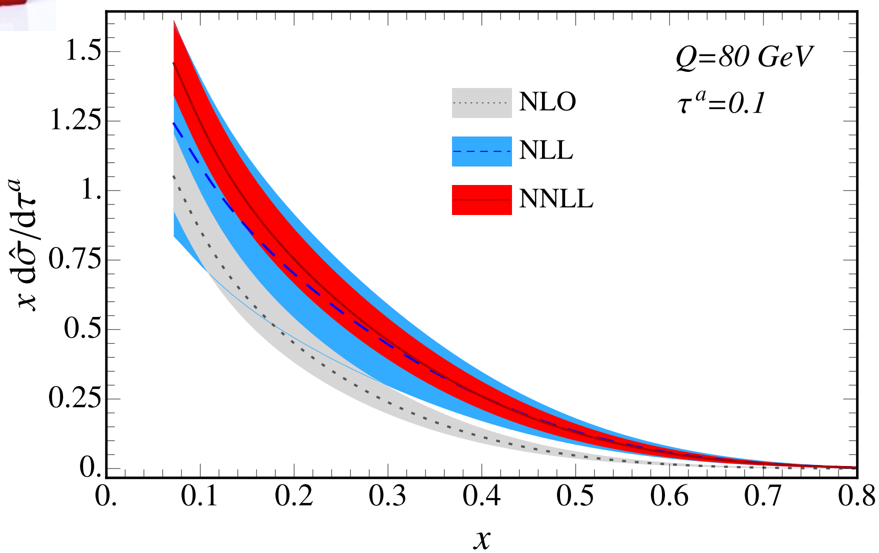
All functions *H, B, J, S* are known up to O(αs)

Cusp and non-cusp anomalous dim. are known to up to $O(a_s^3)$ and $O(a_s^2)$

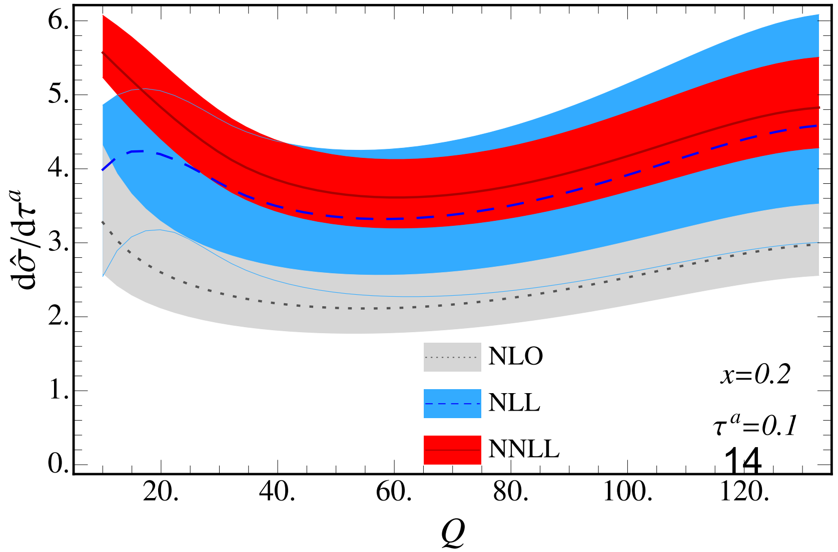
HERA energy:

$\sqrt{s} \approx 300$ GeV

x distribution



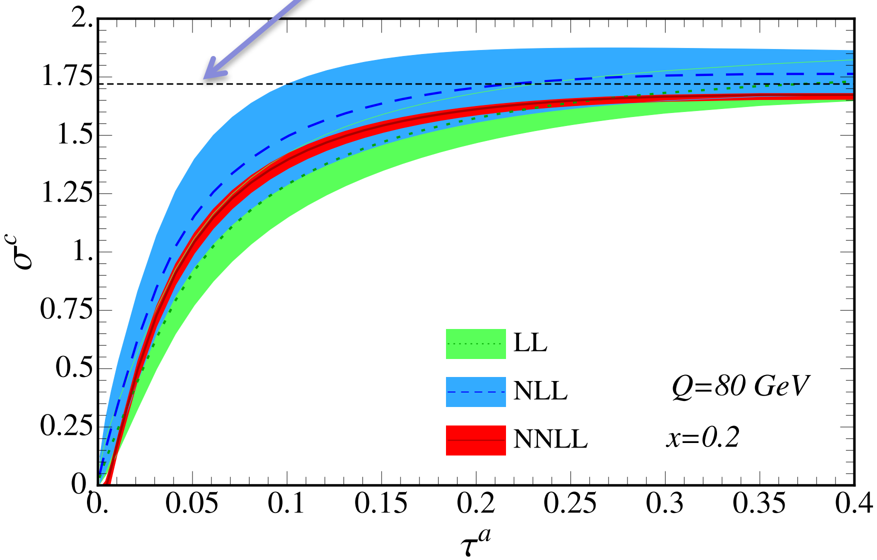
Q distribution



$\tau_1{}^a$ distribution

total at NLO

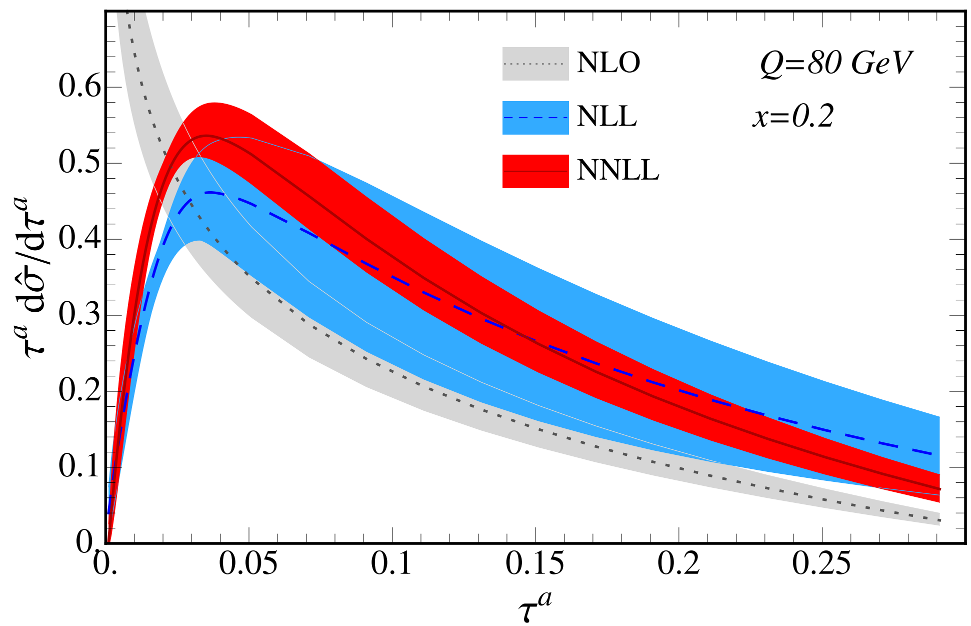
Cumulant cross section



$$\sigma_c(x, Q^2, \tau_1) = \frac{1}{\sigma_0} \int_0^{\tau_1} d\tau_1' \frac{d\sigma}{dx\, dQ^2\, d\tau_1'}$$

- Good convergence LL, NLL, NNLL
- Small nonsingular corrections

Differential cross section



- Resummation cures singular behavior in NLO

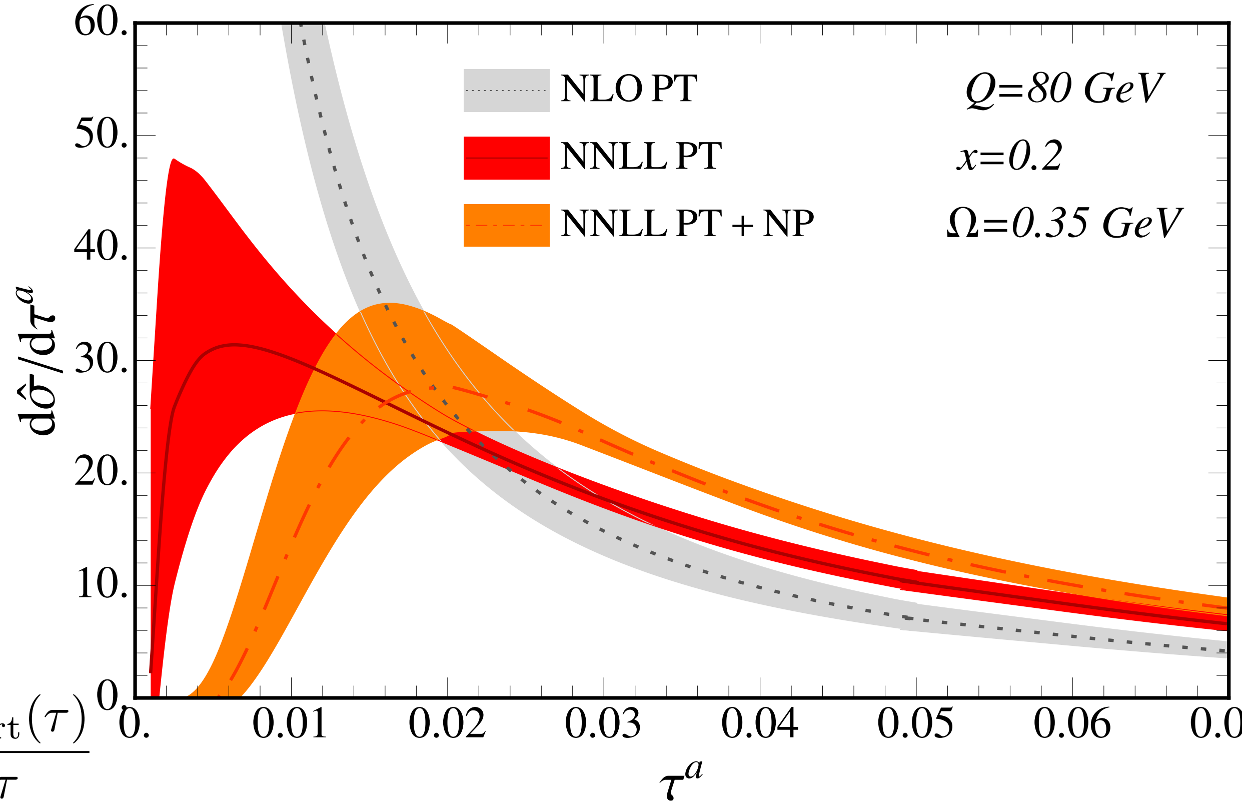
Nonperturbative effect

$$\int dk\, \sigma_{\rm pert}\left(\tau_1 - \frac{k}{Q}\right) F(k)$$

$$F(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{N} c_n f_n\left(\frac{k}{\lambda}\right)\right]^2$$

Ligeti, Tackmann, Stewart

Convolution with NP shape function

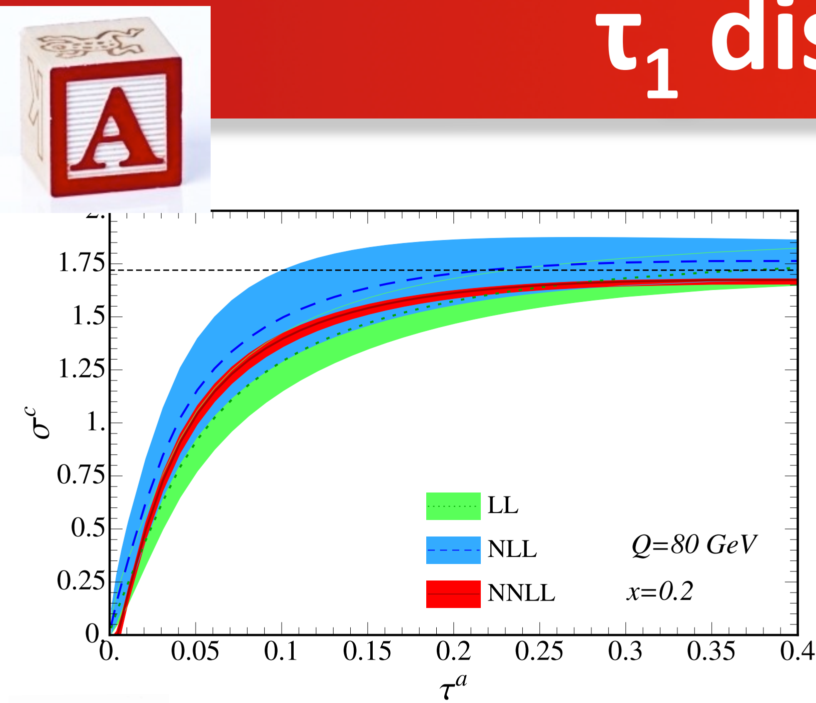


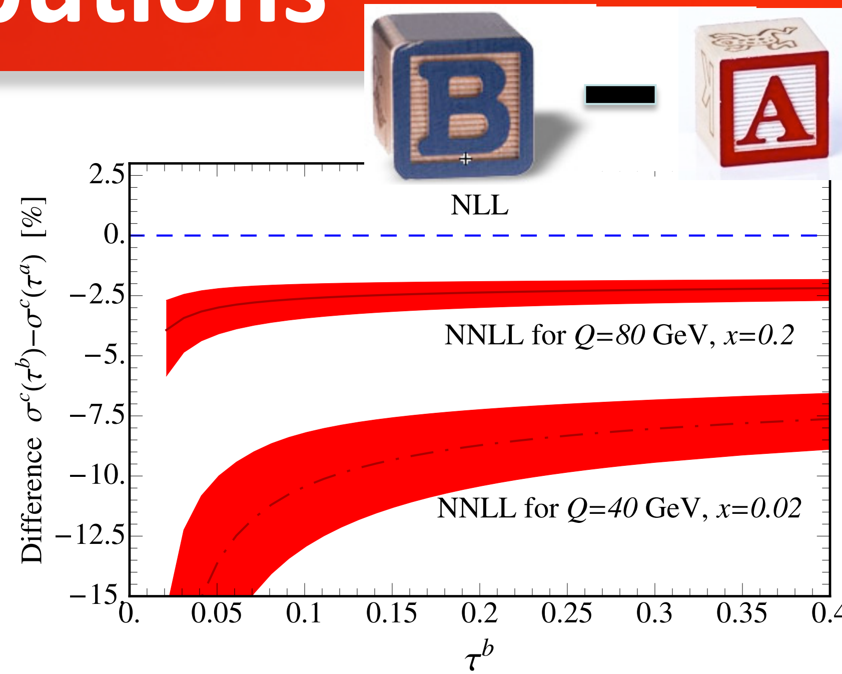
- N=0 for illustration
- Tail region:
  power correction

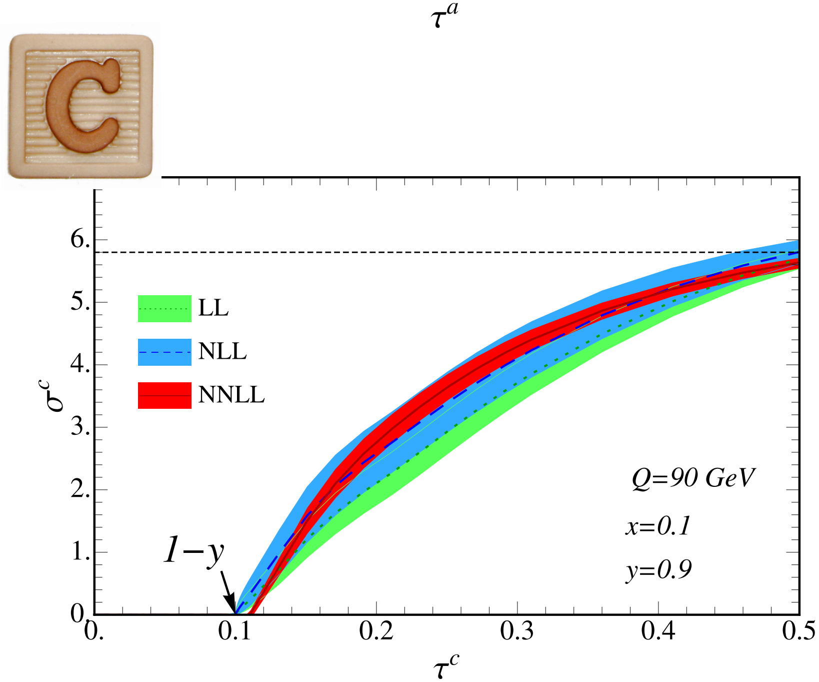
$$\sigma(\tau) = \sigma_{\rm pert}(\tau) - \frac{2\Omega}{Q}\frac{d\sigma_{\rm pert}(\tau)}{d\tau}$$

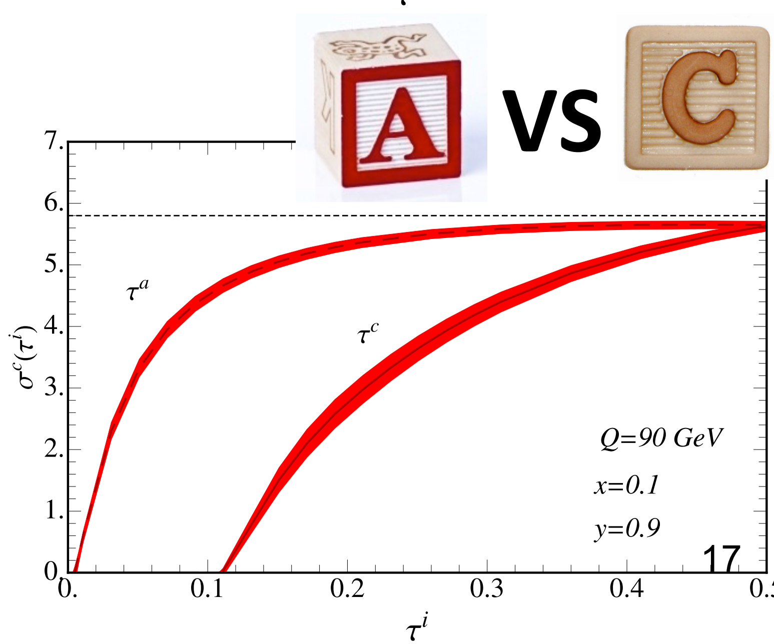
- Universality of Ω including hadron mass: $\Omega = \Omega_1^a = \Omega_1^b = \Omega_1^c$
  - Independence of axes q_B, q_J
  - Interesting to measure the universality

τ_1 distributions









Summary

arXiv:1303.6952

- **Factorization thms for three 1-jettiness** $\tau_1^a \quad \tau_1^b \quad \tau_1^c$

$$\sigma \sim H \times B \otimes J \otimes S \qquad B = f \otimes \mathcal{I}$$

  - Systematically improving accuracy with higher order functions

- **NNLL+O(α_s) predictions:** *x, Q, 1-jettiness* spectrum

  - Universal nonperturbative correction

- Useful for $\boldsymbol{\alpha_s}$ determination, measurement of **universal hadronization effects**, improved (nuclear) **PDF** extraction

Kang, Mantry, Qiu

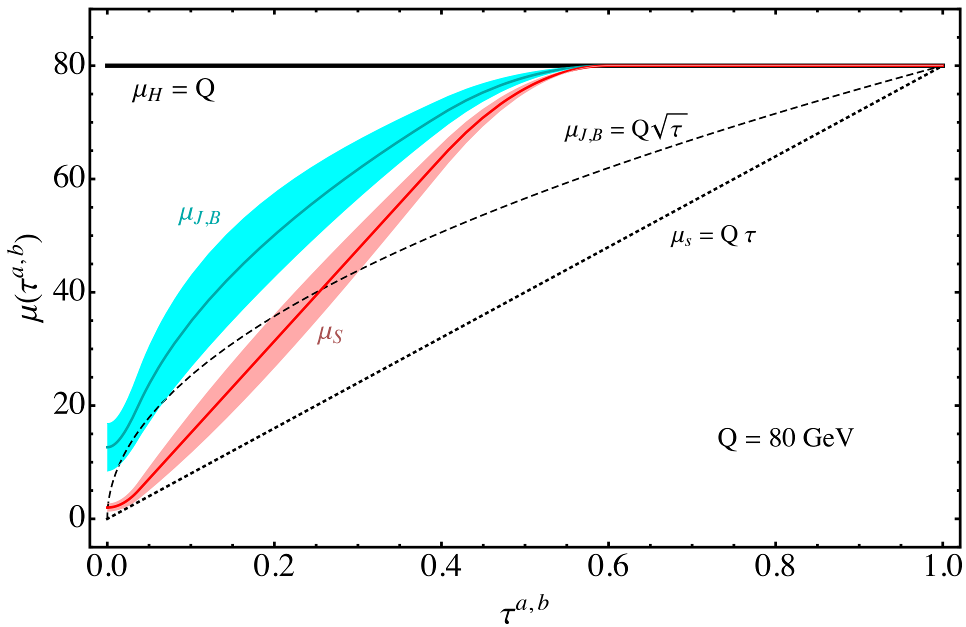
- Higher precision? O(α_s^2) terms, N^{3}LL

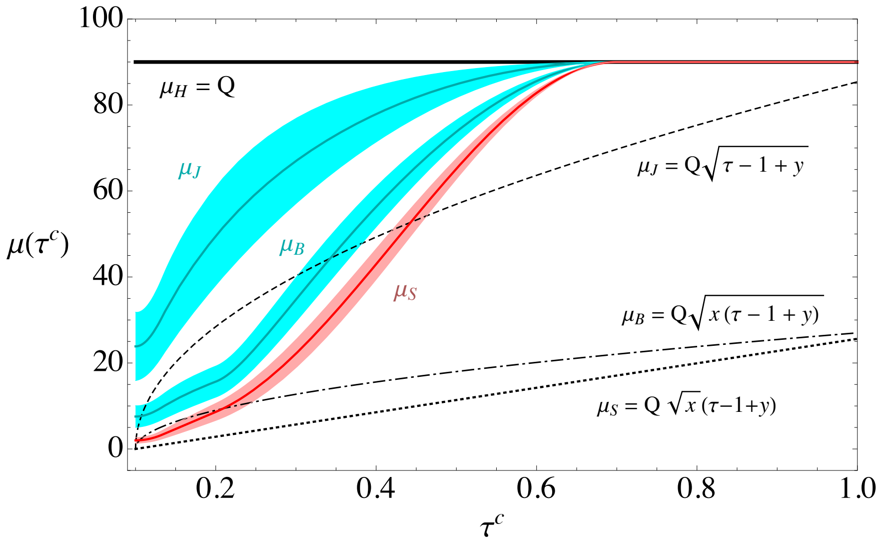
DISASTER, DISENT, DISPATCH

- Beyond 1-jettiness: ***N*-jettiness factorization** (N>1)

Backup

Choice of scales





- For $\Lambda_{QCD} \ll \tau \ll 1$

  $\mu_H = Q \quad \mu_{B,J} = \sqrt{\tau} Q$

  $\mu_S = \tau Q$

- For $\tau \sim \Lambda_{QCD}/Q$

  significant nonperturbative effect

  soft scale freezing at $\mu_S \sim \Lambda_{QCD}$

  $\mu_{B,J} \sim \sqrt{\Lambda_{QCD} Q}$

- For $\tau \sim 1$

  no hierarchy in scales

  no large logs

  $\mu_H \sim \mu_{B,J} \sim \mu_S \sim Q$

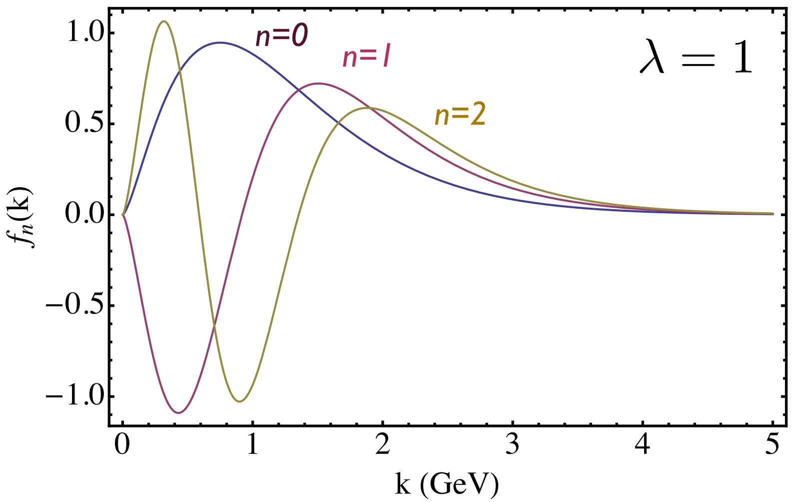
Nonpertubative Effect

- Estimating nonperturbative part of soft function
- For $\tau \gg \Lambda_{QCD}/Q$
  OPE gives power correction with $\mathcal{O}(\Lambda_{QCD}/\tau Q)$ suppression

$$\sigma(\tau) = \sigma_{\text{pert}}(\tau) - \boxed{\frac{2\Omega}{Q}} \frac{d\sigma_{\text{pert}}(\tau)}{d\tau} \approx \sigma_{\text{pert}}(\tau - 2\Omega/Q)$$

  - $\Omega \sim \Lambda_{QCD}$: nonpertubative matrix element

- For $\tau \geq \Lambda_{QCD}/Q$
  significant nonpertubative effect
  convolving shape function
  consistent with power correction



$$\sigma(\tau) = \int dk \sigma_{\text{pert}}(\tau - k/Q)\, F(k)$$

$$\to \sigma_{\text{pert}}(\tau) - \boxed{\left(\int dk \, \frac{k}{Q} F(k) \right)} \frac{d\sigma_{\text{pert}}(\tau)}{d\tau}$$

$$F(k) = \frac{1}{\lambda} \left[\sum_{n=0}^{N} c_n f_n \left(\frac{k}{\lambda} \right) \right]^2$$

Ligeti, Tackmann, Stewart

missing particles in forward region

$$\eta = -\ln(\tan\theta/2)$$

- Proton remnants and particles moving very forward region out of detector coverage: $0 < \theta < \theta_{\rm cut}\,,\ \eta > \eta_{\rm cut}$
  - H1 : $\theta_{\rm cut} = 4^\circ(0.7^\circ)$ and $\eta_{\rm cut} = 3.4(5.1)$ for main cal. (PLUG cal.)
  - ZEUS: $\theta_{\rm cut} = 2.2^\circ$ and $\eta_{\rm cut} = 4.0$ for FCAL

$$\Delta\eta = \ln\frac{E_p^{\rm lab}}{E_p^{\rm CM}} = \ln\frac{920}{157} = 1.8$$

- Boost to CM frame: $\eta^{\rm CM} = \eta - \Delta\eta$
  - H1: $\eta_{\rm cut}^{\rm CM} = 1.6(3.3)$, $e^{-\eta_{\rm cut}^{\rm CM}} = 0.2(0.04)$
  - ZEUS: $\eta_{\rm cut}^{\rm CM} = 2.2$, $e^{-\eta_{\rm cut}^{\rm CM}} = 0.1$

Suppression factor!

- Maximum missing measurement: $\tau_{\rm miss} = \dfrac{2q_B\cdot p_{\rm miss}}{Q^2} = \dfrac{m_T}{Q_B}e^{-\eta}$
  - $m_T^{\rm max} = E_p^{\rm lab}\sin\theta_{\rm cut}$

    about 64(11) GeV for H1 and 32 GeV for ZEUS

$$Q_B = \sqrt{y/x}Q,\ xQ$$

1-jettiness from jet region

- If only jet region ($\mathcal{H}_J$) can be measured

  Use mom. conservation of two hemispheres

  - τ_1^b and τ_1^c can be exactly reproduced

$$\tau_1^b \overset{\text{Breit}}{=} \frac{1}{Q} \sum_{i \in X} \min\{n_z \cdot p_i, \bar{n}_z \cdot p_i\}$$
$$= \frac{1}{Q} \Big[\sum_{i \in \mathcal{H}_J^b} (E_i - p_{z\,i}) + \sum_{i \in \mathcal{H}_B^b} (E_i + p_{z\,i}) \Big]$$
$$= \frac{1}{Q} \Big[\sum_{i \in X} (E_i + p_{z\,i}) - 2 \sum_{i \in \mathcal{H}_J^b} p_{z\,i} \Big],$$

$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q} \sum_{i \in \mathcal{H}_J^b} p_{z\,i}$$

Antonelli, Dasgupta, Salam JHEP 2000

$$\tau_1^c \overset{\text{CM}}{=} \frac{1}{xy\sqrt{s}} \sum_{i \in X} \min\{n_z \cdot p_i, \bar{n}_z \cdot p_i\}$$
$$= \frac{1}{xy\sqrt{s}} \Big[\sum_{i \in X} (E_i + p_{z\,i}) - 2 \sum_{i \in \mathcal{H}_J^c} p_{z\,i} \Big]$$

$$\tau_1^c \overset{\text{CM}}{=} \frac{1}{x} \Big(1 - \frac{2}{y\sqrt{s}} \sum_{i \in \mathcal{H}_J^c} p_{z\,i} \Big)$$

- τ_1^a can be reproduced for dijet limit

$$\tau_1^a = \tau_1^b + \frac{2}{Q^2} \sum_{i \in \mathcal{H}_J^b} (q_J^a - q_J^b) \cdot p_i + \mathcal{O}(\lambda^3)$$

Beam, Jet, Soft functions

from Chris Lee's talk

in SCET 2013