Towards the Phenomenology of TMD's at NNLL

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The main object of this talk is

M. García Echevarría, A. Idilbi, **(EIS)** A. Schaefer, arXive:1208.1281

EIS:Our final definition of the TMD is given in arXive:1211.1947

Initial definition, calculation and properties of TMDs in JHEP 07(2012)002

Some questions …and our answers

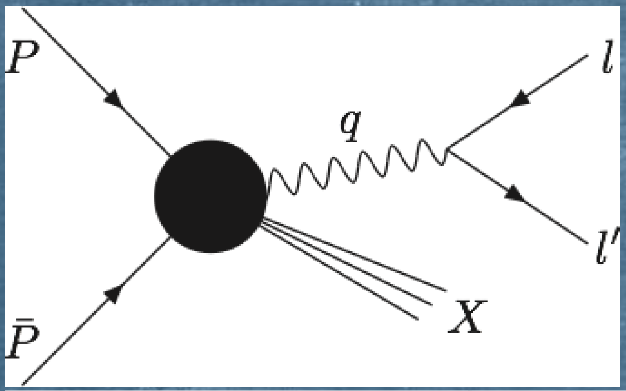
- Transverse Momentum distributions are fundamental in the factorization of DY at small qT and SIDIS and e+e- to 2j
- Can we formulate their definition independently of the IR/collinear regulators that we use? YES
- Are TMDs universal? See discussion
- How do we write the evolution of TMDs? Up to which order do we know their evolution?

We can up to NNLL..we could up NNNLL in some cases

- Is the evolution of all quark TMDs the same?YES
- Can we have a model independent evolution of the TMDs?YES, no effective strong coupling is necessary

Factorization in QCD

- Let's consider the inclusive Drell-Yan process:



$$q^2 = Q^2$$

Collins-Soper-Sterman '85, '88

$$\frac{d\sigma}{dQ^2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, \mathcal{H}_{ij}(x_1, x_2, Q^2, \mu^2)\, f_{i/P}(x_1, \mu^2)\, f_{j/\bar{P}}(x_2, \mu^2)$$

Short-distance physics.
Perturbative coefficient

Long-distance physics.
Non-perturbative PDFs

- The PDFs give us a good description of the inner structure of nucleons. But more information is gained if one considers the transverse momentum of partons as well.
- **Goal**: explore the internal structure of nucleons.
- Example: how is the nucleon spin originated by partons?

Naive TMDPDF...

- One could naively think of defining the TMDPDF by extending the PDF:

$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma|\ \left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{y}_\perp)\,\frac{\bar{\not{n}}}{2}\ \left[W_n^\dagger \xi_n\right](0)\,|P,\sigma\rangle$$

*We would also need **transverse** gauge links to maintain gauge invariance*

- If we calculate this matrix element we get:

$$\begin{aligned}\tilde{F}_n^{naive} = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\Bigg\{\delta(1-x)\Bigg[&\frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^+}{Q^2} + \frac{3}{2\varepsilon_{\rm UV}} \\ &-\frac{1}{4} + \frac{3}{2}L_T + 2L_T\ln\frac{\Delta^+}{Q^2}\Bigg] \\ &-(1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\Bigg\}\end{aligned}$$

- **It is ill-defined!! We cannot renormalize this quantity...**

Challenging Definition!!

- One can find many definitions of TMDPDF “in the market”:
  - Collins-Soper ’82: *just collinear (off-the-LC)*
  - Ji-Ma-Yuan ’05: *collinear with subtraction of complete soft function (off-the-LC)*
  - Cherednikov-Stefanis ’08: *collinear with subtraction of complete soft function (LC gauge)*
  - Mantry-Petriello ’10: *fully unintegrated collinear matrix element*
  - Collins ’11: *collinear with subtraction of square root of 3 soft functions (off-the-LC “strange”)*
  - Chiu-Jain-Neill-Rothstein ’12: *collinear matrix element (rapidity renormalization group)*

- The problem are the criteria to properly define the TMDPDF.

• A well-defined TMDPDF should:

1. Be compatible with a factorization theorem.

2. Have no mixed UV/nUV divergencies, i.e., be renormalizable

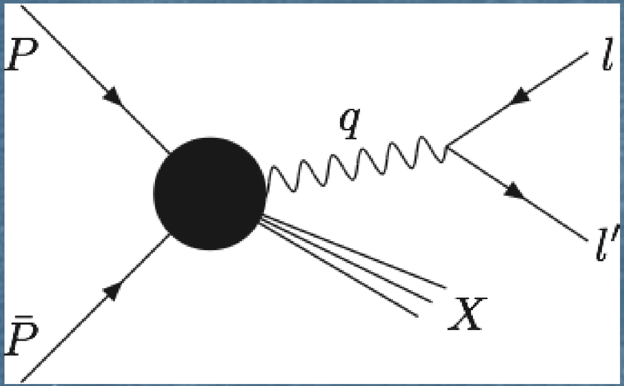
3. Have a matching coefficient onto PDFs independent of nUV regulators.

* By “nUV” I mean non-ultraviolet, i.e., infrared (IR) and rapidity.

- The definition we provide is the **only one** that fulfills all of them.

DY Factorization at Small q_T:

General Overview



$$q^2 = Q^2 \gg q_T^2$$

Problem with different scales... Perfect for Effective Field Theories approach!

QCD → SCET_{q_T} → $\mathrm{SCET}_{\Lambda_{QCD}}$

$$\tilde{M} = H(Q^2/\mu^2)\,\tilde{F}_n(x_n, b; Q^2, \mu^2)\,\tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$$q_T^2 \sim \Lambda_{QCD}^2$$

$$\tilde{M} = H(Q^2/\mu^2)\,\tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\,\tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\,f_n(x_n; \mu^2)\,f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

$$q_T^2 \gg \Lambda_{QCD}^2$$

- The IR has to be regulated consistently in the theories above and below every matching scale in order to properly extract the matching (Wilson) coefficients.

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Factorization of Modes (1/2)

The factorization of the relevant modes in tricky...

[Manohar-Stewart '06]

Soft and Collinear modes have the **same invariant mass**.
Only can be distinguished by their **relative rapidities**:

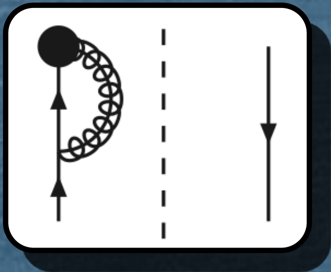
$$k_n \sim Q(1, \lambda^2, \lambda) \quad \rightarrow \quad y \gg 0$$
$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda) \quad \rightarrow \quad y \ll 0$$
$$k_s \sim Q(\lambda, \lambda, \lambda) \quad \rightarrow \quad y \approx 0$$

$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim q_T^2$$

$$\lambda \sim \frac{q_T}{Q}$$

$$y = \frac{1}{2} \ln \left| \frac{k^+}{k^-} \right|$$

Modes can be mixed under boosts, so we need rapidity cuts.



- Rapidity divergence when k^+ goes to 0
- We need a lower rapidity cutoff

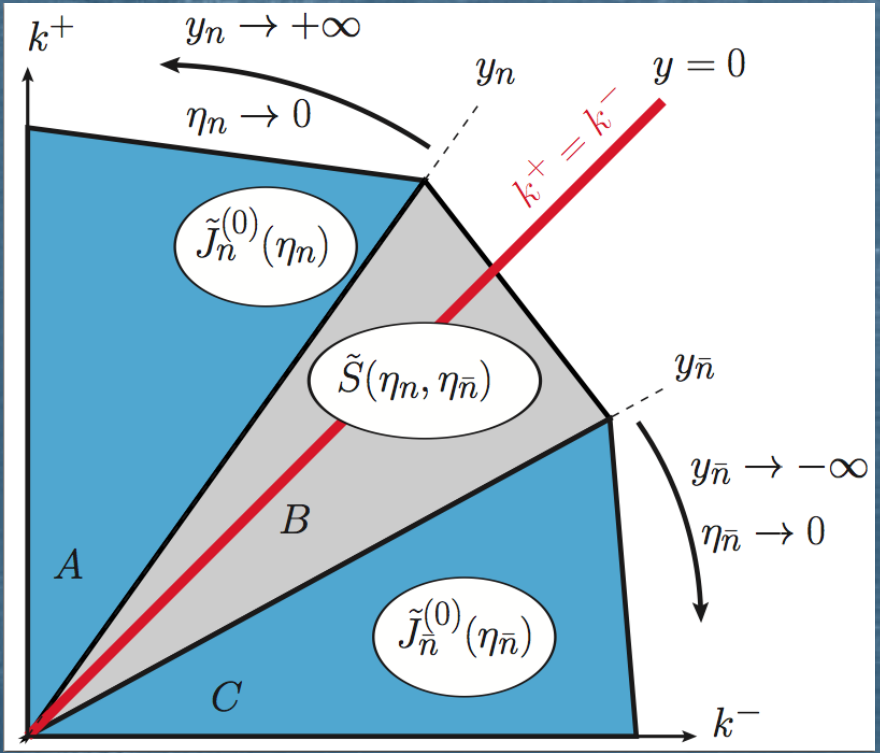
$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{[k^+ - i\varepsilon][(p+k)^2 + i\varepsilon][k^2 + i\varepsilon]}$$

Factorization of Modes (2/2)

- **We need to impose <u>rapidity cutoffs</u> to separate the modes:**

$$H(Q^2)\,\tilde{J}_n^{(0)}(\eta_n)\,\tilde{S}(\eta_n,\eta_{\bar{n}})\,\tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

Pure collinear!



$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

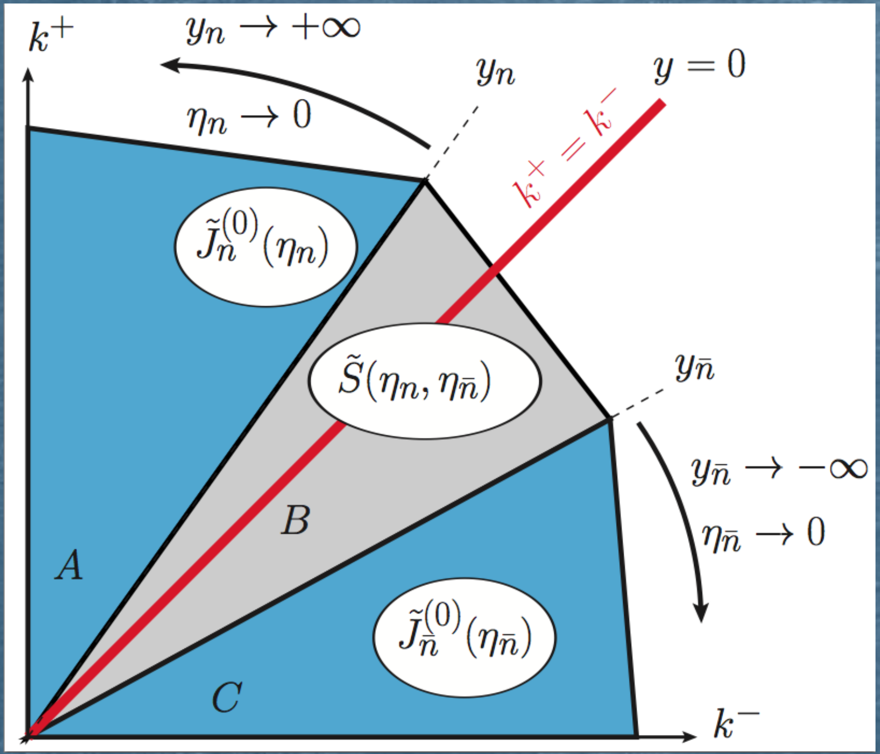
- A is collinear
- B is soft
- C is anti-collinear
- Soft function is NOT symmetric w.r.t. the "separating line" $k^+=k^-$ when $y^+ \neq y^-$.

- **We proved that the soft function can be split in two "hemispheres"**

- **And we will identify <u>positive</u> & <u>negative</u> rapidity quanta with each TMDPDF!!**

Definition of TMDPDF

Positive and negative rapidity quanta can be collected into two different TMDs because of the splitting of the soft function



$$\tilde{S}(\Delta^+, \Delta^-) = \sqrt{\tilde{S}(\Delta^-, \Delta^-)\, \tilde{S}(\Delta^+, \Delta^+)}$$

$$\tilde{F}_n(x_n, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^-) \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right)}$$

$$\tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^+) \sqrt{\tilde{S}\left(\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

No soft function in the factorization theorem!!

Evolution of the TMDPDF

- The hadronic tensor is RG scale independent

$$\tilde{M} = H(Q^2/\mu^2) F_n(x;\vec{b}_\perp, Q, \mu) F_{\bar{n}}(z;\vec{b}_\perp, Q, \mu)$$

$$\frac{d \ln \tilde{M}}{d \ln \mu} = 0 = \gamma_H + \gamma_n + \gamma_{\bar{n}} = \gamma_H + 2\gamma_{\bar{n}} = \gamma_H + 2\gamma_n$$

$$\gamma_H = A(\alpha_s) \ln \frac{Q^2}{\mu^2} + B(\alpha_s); \quad F_n(x;\vec{b}_\perp, Q, \mu) = \exp\left[\int_{\mu_I}^{\mu} \frac{d\mu'}{\mu'} \gamma_n\right] F_n(x;\vec{b}_\perp, Q, \mu_I)$$

$$H(Q^2/\mu^2) = |C(Q^2/\mu^2)|^2$$

Comes from the matching of currents: It is spin independent

The hard coefficient is the same as for inclusive DY! *Ergo,*

WE KNOW THE AD of the 8 TMDPDF up to 3-LOOPS

OPE of the TMDPDF on to the PDF

- When qT is in the perturbative region the TMDPDF can be factorized in a Wilson coefficient and a PDF like in OPE

$$F_f(x;\vec{b}_\perp, Q, \mu) = \sum_{j=q,g} \int_x^1 \frac{dx'}{x'} \tilde{C}_{f/j}\left(\frac{x}{x'}; b, Q, \mu\right) f_{j/P}(x';\mu)$$

The coefficient C works as any other Wilson coefficient
IT IS INDEPENDENT OF IR-SCALES

BUT THERE IS STILL A Q^2 DEPENDENCE

$$\tilde{C}_n(x;b,Q,\mu) = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left[-P_{q/q}L_T + (1-x) - \delta(1-x)\left(\frac{1}{2}L_T^2 - \frac{3}{2}L_T + \ln\frac{Q^2}{\mu^2}L_T + \frac{\pi^2}{12}\right)\right]$$

THESE TERMS HAVE TO BE RESUMMED!!

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

Q^2-Resummation

- Using Lorentz invariance and dimensional analysis

$$\ln F_n = \ln j_n - \frac{1}{2}\ln S$$

$$\ln j_n = R_n\left(x;\alpha_s, L_T, \ln\frac{\Delta}{Q^2}\right), \qquad \ln S = R_\phi\left(\alpha_s, L_T, \ln\frac{\Delta^2}{Q^2\mu^2}\right)$$

Since the TMDPDF (Wilson coefficients and PDFs) is free from rapidity divergences to all orders in perturbation theory:

$$\frac{d}{d\ln\Delta}\ln F_n = 0$$

Q^2-Resummation

- From the fact that the TMDPDF is free from rapidity divergencies we can extract and exponentiate the Q^2-dependence.

- But we can also extract it just applying the RGE to the hadronic tensor:

$$\frac{d\ln\tilde{F}_n}{d\ln\mu} = -\frac{1}{2}\gamma_H = -\frac{1}{2}A(\alpha_s)\ln\frac{Q^2}{\mu^2} - \frac{1}{2}B(\alpha_s)$$

$$\ln\tilde{F}_n = \ln\tilde{F}_n^{\not{Q}} - D(\alpha_s, L_T)\ln\frac{Q^2}{\mu^2}$$

Independent of Q^2!!

$$\tilde{C}_{f/j}(x,b;Q^2,\mu) = \left(\frac{Q^2}{\mu^2}\right)^{-D(b;\mu)} \tilde{C}^{\not{Q}}_{f/j}(x,b;\mu)$$

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$A(\alpha_s) = 2\Gamma_{cusp}$$

- The Q^2-factor is extracted for each TMDPDF individually.
- We do not need Collins-Soper evolution equation to resum the logs of Q^2.
- We know cusp AD at 3-loops, **so we know D at order α^2!!**

Q^2-Resummation

- The final form of the TMD in IPS is

$$\ln F_n = \ln F_n^{sub} - D(\alpha_s, L_T)\left(\ln\frac{Q^2}{\mu^2} + L_T\right)$$

$$F_n(x;\vec{b}_\perp, Q, \mu) = \left(\frac{Q^2 b^2 e^{2\gamma_E}}{4}\right)^{-D(\alpha_s, L_T)} C_n(x;\vec{b}_\perp, \mu) \otimes f_n(x;\mu)$$

$$\frac{dD(\alpha_s, L_T)}{d\ln\mu} = \Gamma_{\text{cusp}}(\alpha_s)$$

$$D(\alpha_s, L_T) = \sum_{n=1}^{\infty} d_n(L_T)\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$d_n'(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m} d_m(L_\perp)$$

The cusp AD is known at 3-loops!!

→ The function D is known up to order α^2

Resumming!

$$F_{f/P}(x;\vec{b}_\perp,Q^2,\mu=Q)=\sum_{j=q,g}\exp\left[\int_{\mu_I}^{\mu}\frac{d\mu'}{\mu'}\gamma_n\right]\left(\frac{Q^2}{\mu^2}\right)^{-D(b,\mu_I)} C_{f/j}\left(x;\vec{b}_\perp,\mu_I\right)\otimes f_{j/P}(x;\mu_I)$$

Order	γ	Γcusp	C	D
LL	-	α	tree	-
NLL	α	α^2	tree	α
NNLL	α^2	α^3	α	α^2
NNNLL	α^3	α^4	α^2	α^3

Green: Aybat, Collins , Qiu, Rogers; Aybat, Rogers; Anselmino, Boglione,Melis

Yellow: Our Group

Blue: Known pieces: C for unpolarized TMDs from Catani et al. ‘ 12 And Gehrmann et al. ‘12

The Evolution of all quark TMDs

- The hard matching coefficient H does not depend on spin! And its AD governs all evolution of the TMDs and also the evolution of the D-function! (EIS+S, ´12) even when the TMDs do not match on PDFs

$$F_{\alpha\beta}(x,\vec{k}_\perp) = \frac{1}{2}\int \frac{dr^- d^2\vec{r}_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}r^- xP^+ - \vec{r}_\perp\cdot\vec{k}_\perp)} \Phi^q_{\alpha\beta}(0^+,r^-,\vec{r}_\perp)\sqrt{S(0^+,0^-,\vec{r}_\perp)}$$

$$\Phi^q_{\alpha\beta}(0^+,r^-,\vec{r}_\perp) = \langle P\vec{S}\,\|[\bar{\xi}_{n\alpha} W_n^T](0^+,y^-,\vec{y}_\perp)[W_n^{T\dagger}\xi_{n\beta}](0)|\,P\vec{S}\rangle$$

$$S = \langle 0|\,\mathrm{Tr}\left[S_n^{T\dagger} S_{\bar{n}}^T\right](0^+,0^-,\vec{y}_\perp)[S_{\bar{n}}^{T\dagger} S_n^T](0)\,|\,0\rangle, \quad \alpha,\beta = \text{Dirac indeces}$$

THIS IS SPIN INDEPENDENT:
Same evolution for all 8 TMD's
Up to NNLL!

$$\gamma_F = \frac{-1}{2}\gamma_H$$

Evolution Kernel

- If we want to connect two TMDPDFs at two different scales:

$$\tilde{F}_n(x, b; Q_f^2) = \tilde{F}_n(x, b; Q_i^2)\, \tilde{R}(b; Q_i, Q_f)$$

$$\tilde{R}(b; Q_i, Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-D(\alpha_s(Q_i), L_T(Q_i))} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

- The evolution is given in terms of the function D and the AD

- When we Fourier transform back, we **need** to resum large logs in the D...

$$L_T = \ln\frac{Q^2 b^2}{4e^{-2\gamma_E}}$$

- I will show you **TWO** methods: the “traditional” CSS and the one we propose.

Resummation of R: CSS

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$D(b^*;Q_i) = D(b^*;\mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{\text{cusp}}$$

$$L_T = \ln \frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

$$\mu_{b^*} = \frac{2e^{-\gamma_E}}{b^*}$$

$$b_*(b) = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$

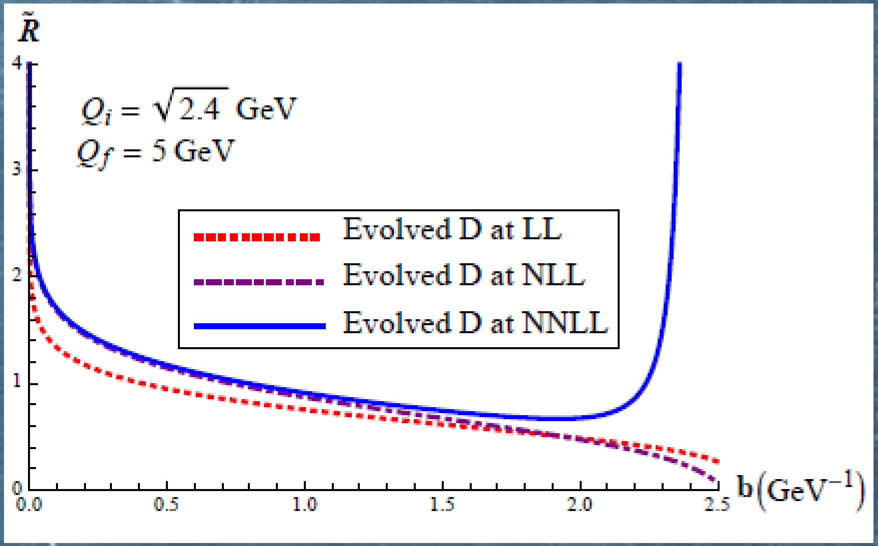
Non-perturbative model (BLNY)

$$\tilde{R}^{CSS}(b,Q_i,Q_f) = \exp\left\{-\frac{1}{2} g_2 b^2 \ln\frac{Q_f}{Q_i}\right\}$$
$$\times \left(\frac{Q_f^2}{Q_i^2}\right)^{-\left[D(b^*,\mu_b) - \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{cusp}\right]} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

Perturbative pieces

Resummation of R: CSS

$$\mu_b = \frac{2e^{-\gamma_E}}{b}$$



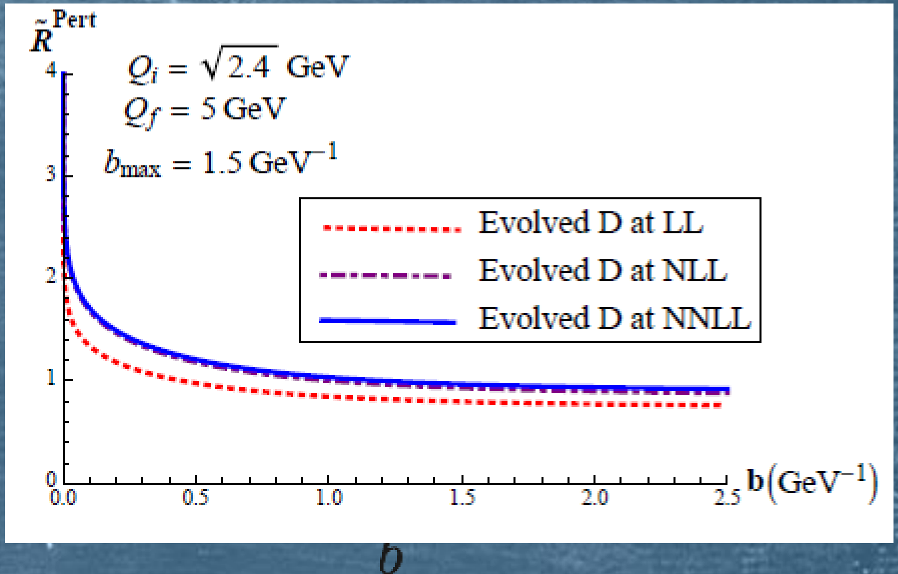
$$\tilde{R}^{Pert}(b, Q_i, Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-\left[D(b,\mu_b) - \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\mu}\Gamma_{cusp}\right]} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

The Evolution Kernel with the effective coupling hits the **Landau Pole!!** :-(

Resummation of R à la CSS

$$\mu_{b^*} = \frac{2e^{-\gamma_E}}{b^*}$$

$$b_*(b) = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$



$$\tilde{R}^{Pert}(b, Q_i, Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-\left[D(b^*,\mu_{b^*}) - \int_{\mu_{b^*}}^{Q_i} \frac{d\bar{\mu}}{\mu}\Gamma_{cusp}\right]} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'}\gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

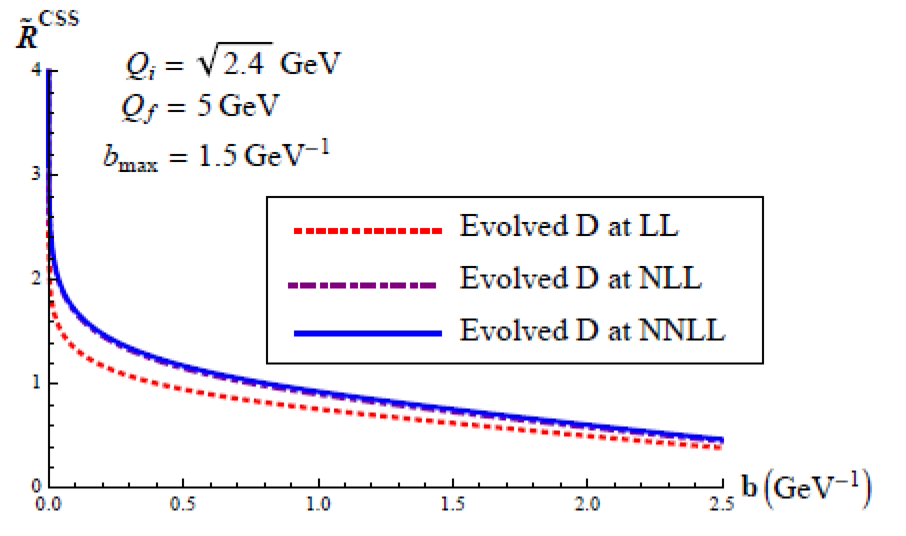
We impose a cutoff over b writing b*(b) instead of b.

But we loose information at large b!!

Resummation of R: CSS

$$\tilde{R}^{\rm NP} = \exp\left\{-\frac{1}{2} g_2 b^2 \ln \frac{Q_f}{Q_i}\right\}$$

The value of g_2 and b_{max} are extracted from fits to experimental data



$$\tilde{R}(b; Q_i, Q_f) = \tilde{R}\left(b^*(b); Q_i, Q_f\right)\ \tilde{R}^{\rm NP}(b; Q_i, Q_f)$$

We need to add a **non-perturbative model in the evolution** extracted from **data**...

- ***But there is a complete different way to resum the logs...***

D-Resummation

- We are going to write D as a **series** and resum it directly:

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$D(b;\mu) = \sum_{n=1}^{\infty} d_n(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\Gamma_{\rm cusp}(\alpha_s) = \sum_{n=1}^{\infty} \Gamma_{n-1}\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\beta(\alpha_s) = -2\alpha_s \sum_{n=1}^{\infty} \beta_{n-1}\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\frac{d}{dL_\perp} d_n(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m} d_m(L_\perp)$$

Recurrence relation

D-Resummation

$$D^R = \sum_{n=1}^{\infty} d_n(L_\perp)a^n =$$
$$\frac{1}{2}\sum_{n=1}^{\infty}\left\{X^n\left(\frac{\Gamma_0}{\beta_0}\frac{1}{n}\right) + aX^{n-1}\left(\frac{\Gamma_0\beta_1}{\beta_0^2}\left(-1+H_{n-1}^{(1)}\right)|_{n\geq 3} + \frac{\Gamma_1}{\beta_0}|_{n\geq 2}\right)\right.$$
$$+a^2X^{n-2}\left((n-1)2d_2(0)|_{n\geq 2} + (n-1)\frac{\Gamma_2}{2\beta_0}|_{n\geq 3} + \frac{\beta_1\Gamma_1}{\beta_0^2}s_n|_{n\geq 4} + \frac{\beta_1^2\Gamma_0}{\beta_0^3}t_n|_{n\geq 5}\right.$$
$$\left.\left.+\frac{\beta_2\Gamma_0}{2\beta_0^2}(n-3)|_{n\geq 4}\right) + \ldots\right\},$$

$$X = a\beta_0 L_\perp$$

$$D^R = -\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right]$$
$$+\frac{1}{2}\left(\frac{a}{1-X}\right)^2\left[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}(X(X-2) - 2\ln(1-X)) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2\right.$$
$$+\frac{\beta_1^2\Gamma_0}{\beta_0^3}\frac{+121X^6 - 188X^5 + 13X^4 + 30X^3 + 12X^2(1-\mathrm{Li}_2(X)) + 12X(X+1)\ln(1-X)}{24X^2}$$
$$\left.+\frac{\beta_1^2\Gamma_0}{2\beta_0^3}(1-X)^2\sum_{n=5}^{\infty}X^{n-2}(n-1)\left[H_{n-1}^{(1)}\right]^2\right] + \ldots,$$

D-Resummation

$$X = a\beta_0 L_\perp$$

$$X = 1 \rightarrow b_X = \frac{2e^{-\gamma_E}}{Q_i}\exp\frac{2\pi}{\beta_0\alpha_s(Q_i)}$$

In the IR region X~1

New expansion!

$$\begin{aligned}
D^R = & -\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right] \\
& + \frac{1}{2}\left(\frac{a}{1-X}\right)^2\Bigg[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}\left(X(X-2) - 2\ln(1-X)\right) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2 \\
& + \frac{\beta_1^2\Gamma_0}{\beta_0^3}\frac{+121X^6 - 188X^5 + 13X^4 + 30X^3 + 12X^2\left(1-\mathrm{Li}_2(X)\right) + 12X(X+1)\ln(1-X)}{24X^2} \\
& + \frac{\beta_1^2\Gamma_0}{2\beta_0^3}(1-X)^2\sum_{n=5}^{\infty}X^{n-2}(n-1)\left[H_{n-1}^{(1)}\right]^2\Bigg] + \dots,
\end{aligned}$$

D-Resummation

Properties of DR:

- The resummation works for all X<1
- The sign of DR is the same at all orders (that we checked)
- Asymptotically, when X→1

$$D^R|_{X\to 1^-} = -\frac{\Gamma_0}{2\beta_0}\ln(1-X)\left[1+\left(\frac{a}{1-X}\right)\frac{\beta_1}{\beta_0}+\left(\frac{a}{1-X}\right)^2\frac{\beta_1\Gamma_1}{\beta_0\Gamma_0}+\ldots\right]$$

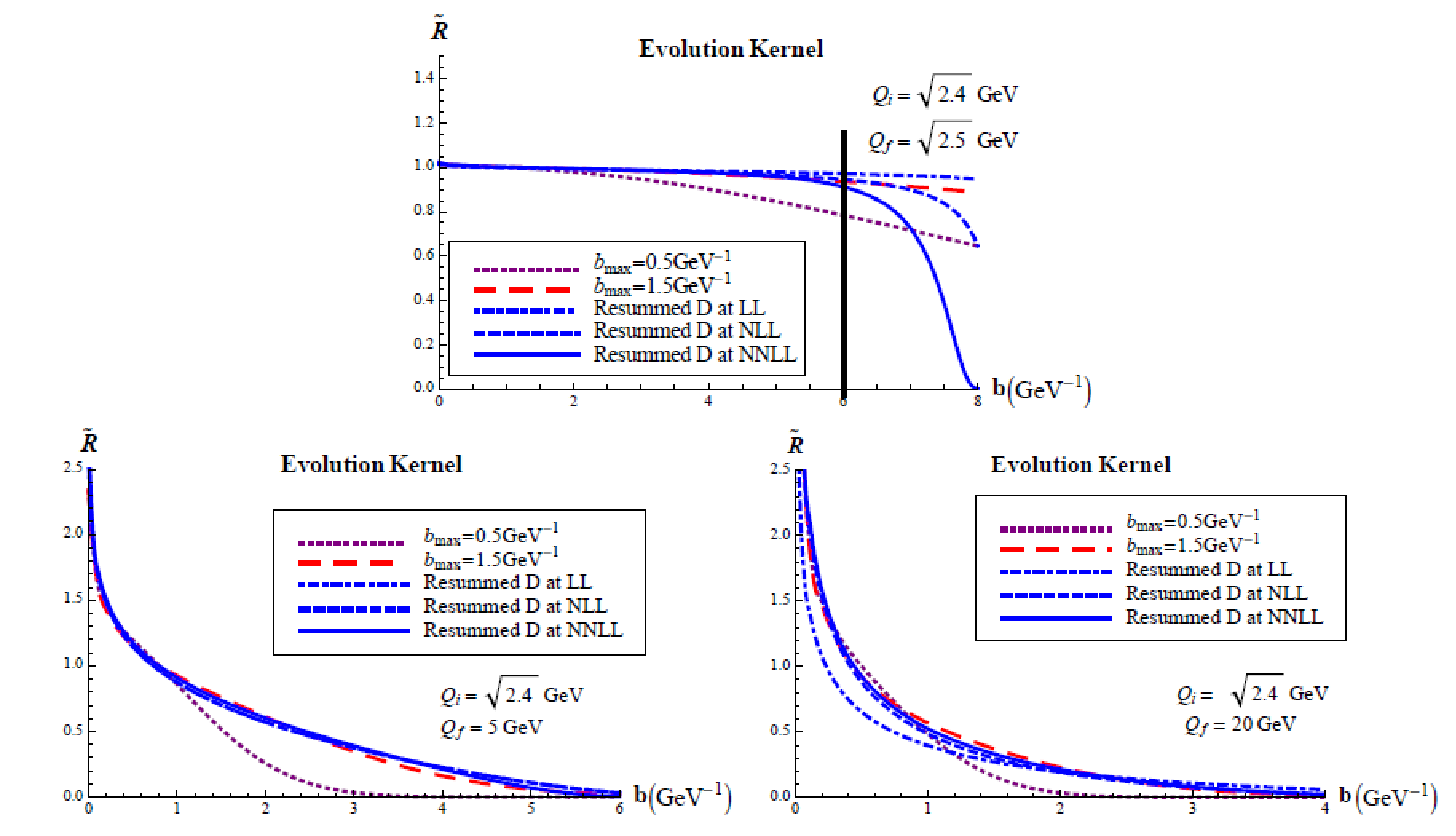
$$\overset{n_f=5}{=} -\frac{\Gamma_0}{2\beta_0}\ln(1-X)\left[1+\left(\frac{a}{1-X}\right)5.04+\left(\frac{a}{1-X}\right)^2 34.84+\ldots\right]$$

Truncation of DR:

- We can think to truncate DR when *a/(1-X)~1*
- We have tried the truncation at bc such that

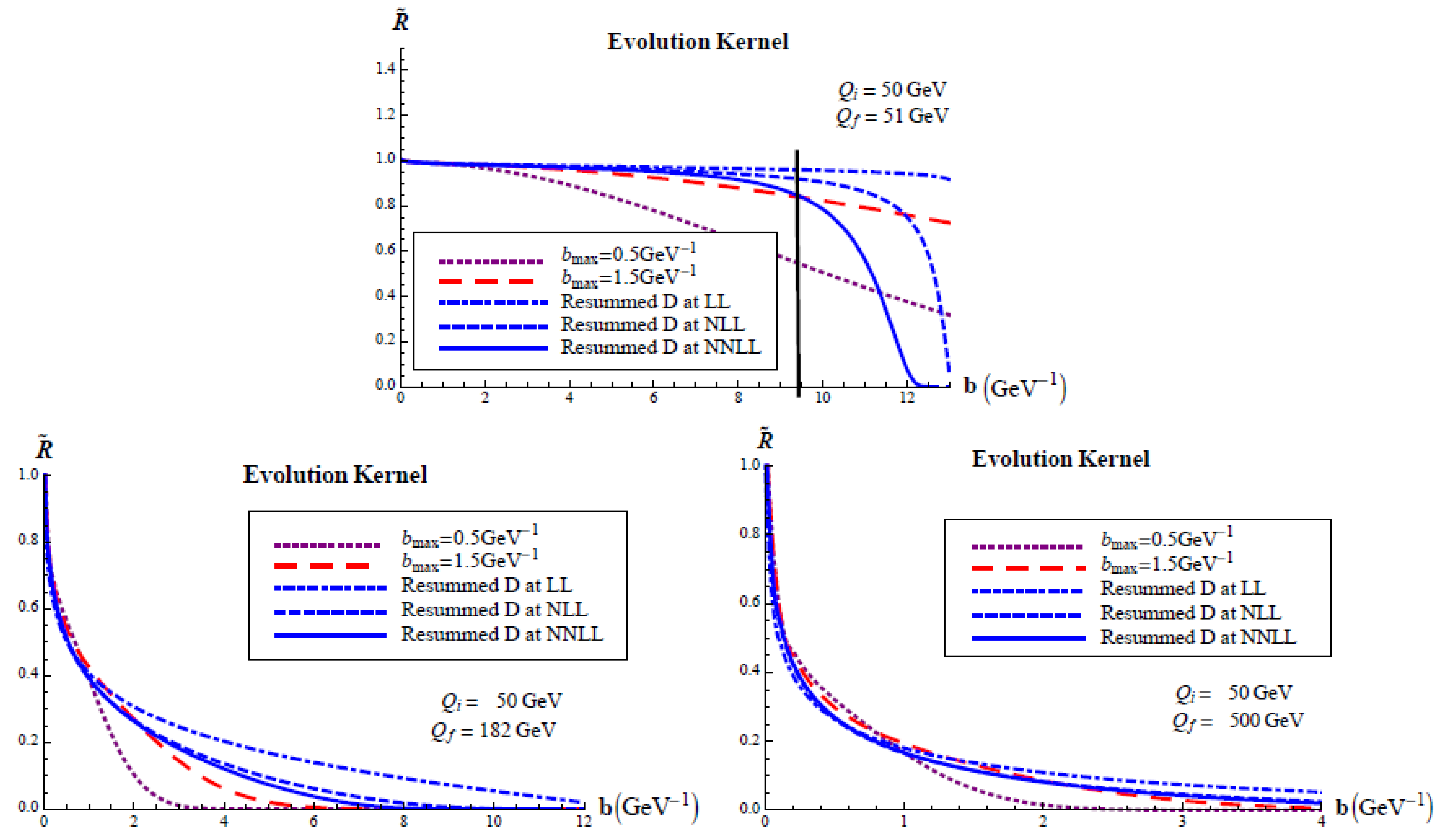
$$X(b_X)=1;\quad a(Q)/(1-X(b_{c1}))=1;\quad a(Q)/(1-X(b_{c2}))=0.2$$

Results



Evolution Kernel
$\tilde{R}$
$Q_i = \sqrt{2.4}$ GeV
$Q_f = \sqrt{2.5}$ GeV
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
b(GeV^{-1})
Evolution Kernel
$\tilde{R}$
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
$Q_i = \sqrt{2.4}$ GeV
Q_f = 5 GeV
b(GeV^{-1})
Evolution Kernel
$\tilde{R}$
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
$Q_i = \sqrt{2.4}$ GeV
Q_f = 20 GeV
b(GeV^{-1})

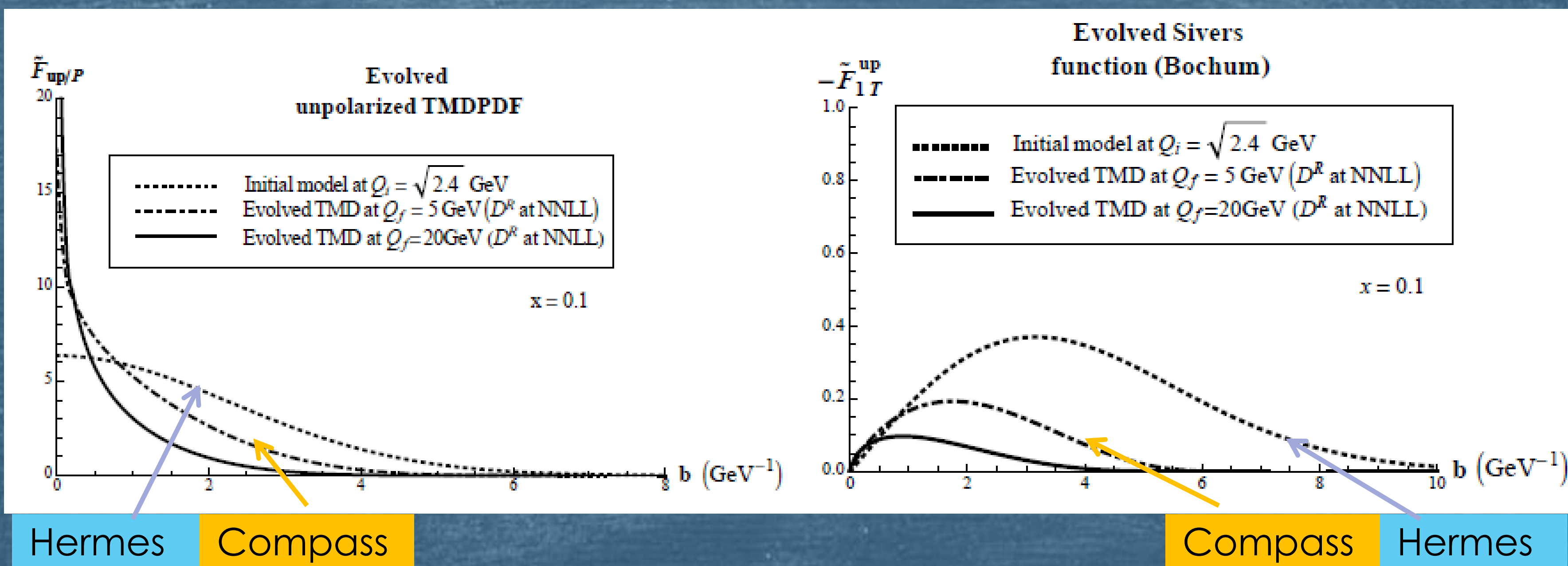
Results



Evolution Kernel
$\tilde{R}$
Q_i = 50 GeV
Q_f = 51 GeV
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
b (GeV^{-1})
Evolution Kernel
Q_i = 50 GeV
Q_f = 182 GeV
Evolution Kernel
Q_i = 50 GeV
Q_f = 500 GeV

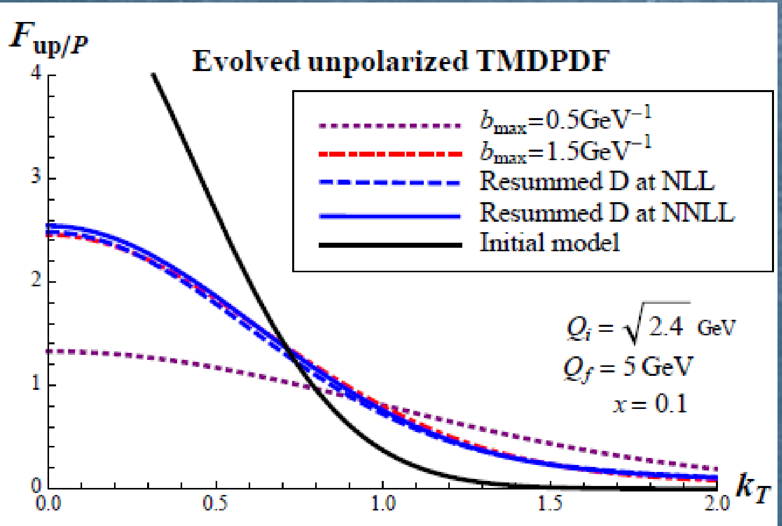
Results

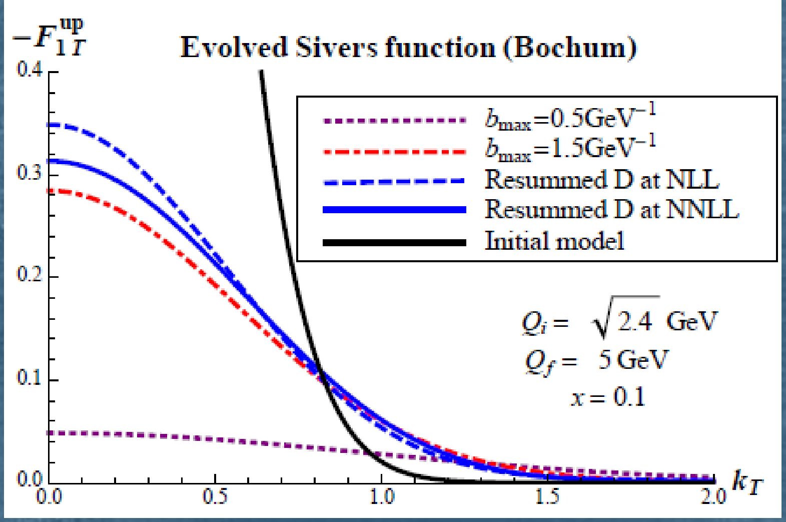
In practice the TMD are concentrated on a region of IPS shorter than the range of validity of the evolutor



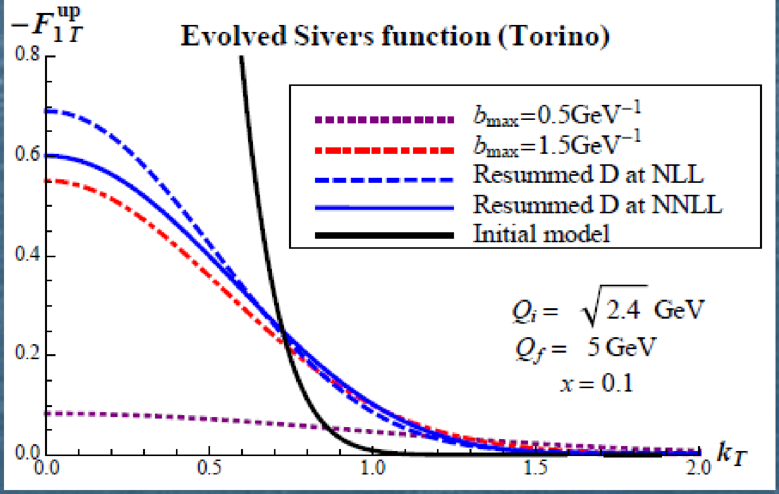
Results

We compare with CSS and bmax=0.5, Collins ideal bmax=1.5, fitted from Phenomenology (Konychev, Nadolsky'06)





All graphs show an agreement With the bmax=1.5 choice



CONCLUSIONS

- We have a formulation of factorization on-the-light-cone (no parameters on any matching coefficient!)
- We can relate the AD of the hard matching coefficient to the AD of the TMDPD's → WE KNOW THE EVOLUTION OF ALL TMDPDF UP TO NNLL
- We can build an evolutor for TMDPDF removing the problem of the Landau pole in a model independent way (agreement with fits that use bmax=1.5)
- We need experiments to get a mapping of TMDs as precise as for PDFs

BACKUP SLIDES

Splitting of the Soft Function

- The soft function can be split in two “pieces”.
- I will use the Δ-regulator, but the arguments are regulator-INdependent!!

- The hadronic tensor can be factorized in terms of two PDFs:

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(x_n; L_T, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(x_{\bar{n}}; L_T, Q^2/\mu^2) \times f_n(x_n; \Delta^-/\mu^2)\, f_{\bar{n}}(x_{\bar{n}}; \Delta^+/\mu^2)$$

- The hadronic tensor can be also written as:

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{J}_n^{(0)}(\Delta^-)\, \tilde{J}_{\bar{n}}^{(0)}(\Delta^+)\, \tilde{S}(\Delta^+, \Delta^-)$$

- The soft function can be split!!

$$\tilde{S}(\Delta^+, \Delta^-) = \sqrt{\tilde{S}(\Delta^-, \Delta^-)\, \tilde{S}(\Delta^+, \Delta^+)}$$

Formulas for Λ_{QCD}

$$\Lambda_{\mathrm{QCD}} = Q \exp[G(t_Q)]$$

$$t_Q \equiv -2\pi / (\beta_0 \alpha_s(Q))$$

$$G(t) = t + \frac{\beta_1}{2\beta_0^2} \ln(-t) - \frac{\beta_1^2 - \beta_0\beta_2}{4\beta_0^4}\frac{1}{t} - \frac{\beta_1^3 - 2\beta_0\beta_1\beta_2 + \beta_0^2\beta_3}{8\beta_0^6}\frac{1}{2t^2} + \ldots$$

$$\alpha_s(M_Z) = 0.117$$

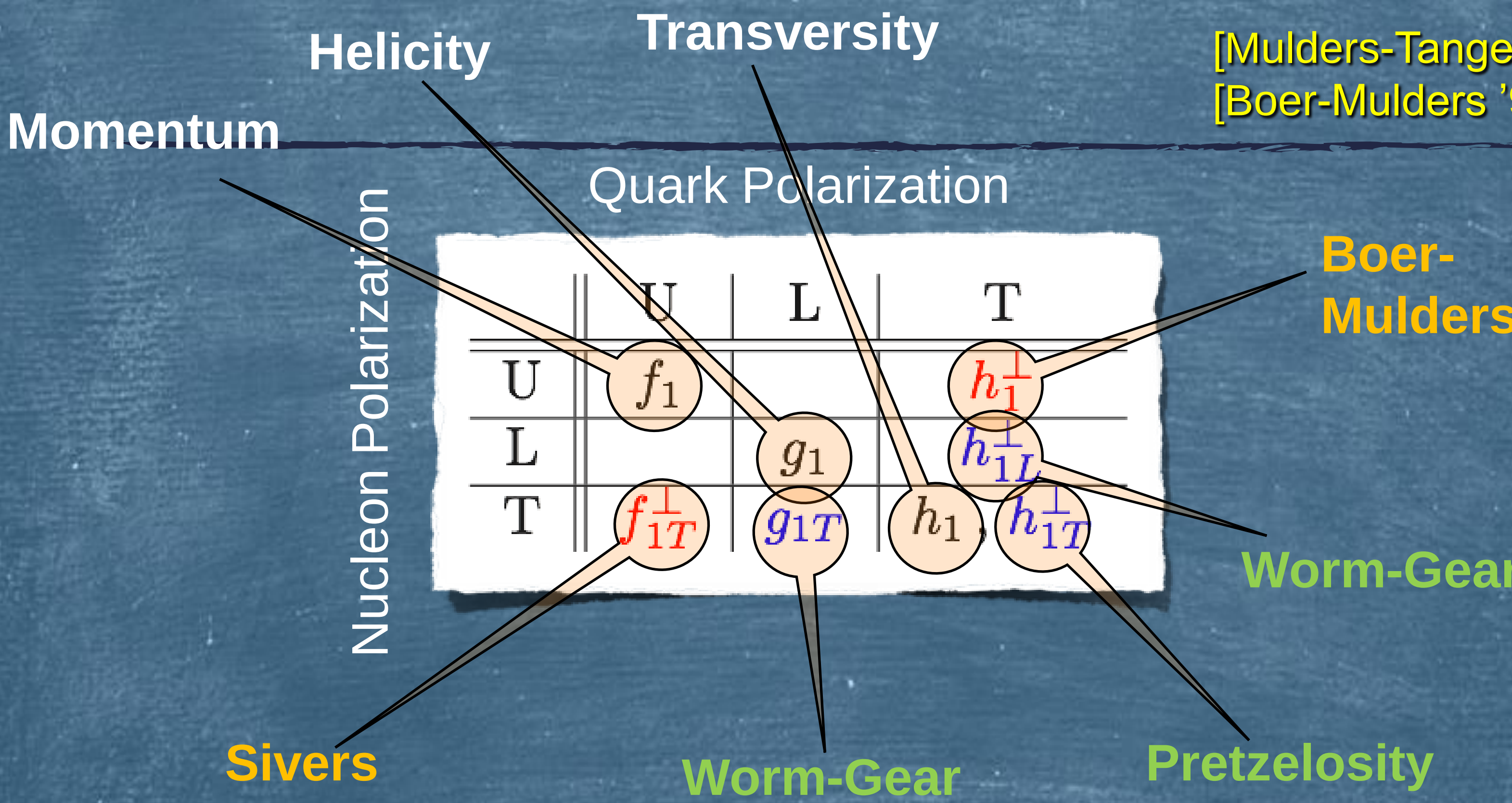
$$\mathrm{n}_f = 5$$

$$\Lambda_{\mathrm{QCD}} \approx 157\,\mathrm{MeV}$$

$$b_\Lambda = \frac{2e^{-\gamma_E}}{\Lambda_{\mathrm{QCD}}} \approx 7.15\,\mathrm{GeV}^{-1}$$

TMDPDFs at Leading Twist

[Mulders-Tangerman'96]
[Boer-Mulders '98]



- **The only ones that survive in the collinear limit (when we integrate over qT)** • **They are T-odd**

- *There are similar families for gluon-TMDPDFs and quark/gluon-TMDFFs*

- *They are **distributions** that give us information about the inner structure of the nucleons*