# Recent results within Lipatov's high energy effective action

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in collaboration with

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based on **PRD85 (2012) 056006, NPB859 (2012) 129, NPB (2012) 133, PRDXX (2013)** and work in progress

# Outline

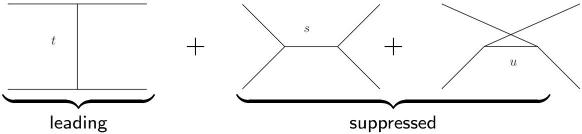
- High energy effective action - an introduction
- a scheme for NLO calculations: example $qq \to qq$
- the quark induced Mueller-Tang jet impact factor at NLO
- Summary

# Regge limit of high energy collisions

simplest process: elastic scattering, Regge limit $s \sim -u \gg -t, m^2$

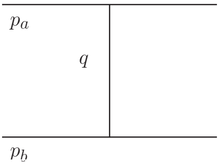
$\phi^3$ theory:

even more simpler



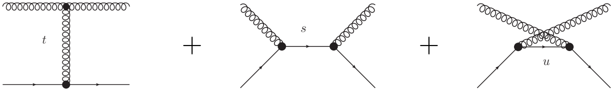

kinematics

- scattering particles separated by large relative boost factor
- amplitude factorizes into '$(+)$' and '$(-)$' cluster
- inside each 'cluster': expansion parameter $\lambda^{\pm} = \frac{q_j^{\pm}}{p_i^{\pm}}$



# QCD $\equiv$gauge theory

**less simple:**




- covariant gauge: all diagrams contribute for $s \gg -t$
- (quasi-)elastic scattering: light-cone gauge ➡ $t$-channel diagram only

# QCD ≡gauge theory

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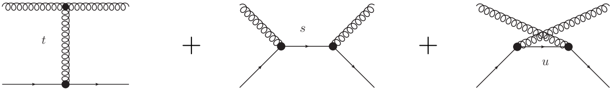
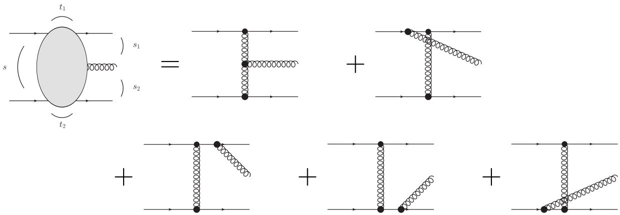



- covariant gauge: all diagrams contribute for $s \gg -t$
- (quasi-)elastic scattering: light-cone gauge ➡ $t$-channel diagram only

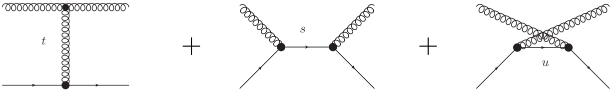
**even harder:**




double Regge kinematics $s \gg s_1, s_2 \gg -t_1, -t_2$

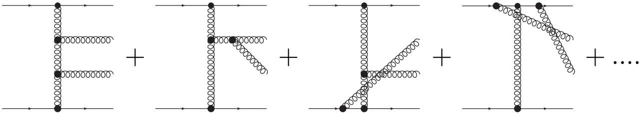
# QCD $\equiv$gauge theory

**less simple:**




- covariant gauge: all diagrams contribute for $s \gg -t$
- (quasi-)elastic scattering: light-cone gauge ➡ $t$-channel diagram only

**and so on ...**



triple Regge kinematics:

$$s \gg s_1, s_2, s_3 \gg -t_1, -t_2, -t_3$$

$+$ some particles in same rapidity cluster ...

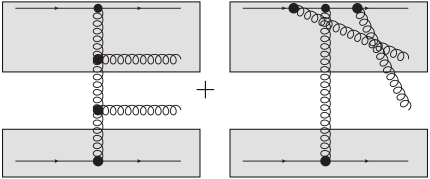
# Effective field theory: integrate out fast fields

divide final state particles into clusters of particles “local in rapidity”

for each cluster

- integrate out specific details of fast $+/-$ fields
- dynamics in local cluster: QCD Lagrangian + universal eikonal factor

(up to power suppressed corrections)

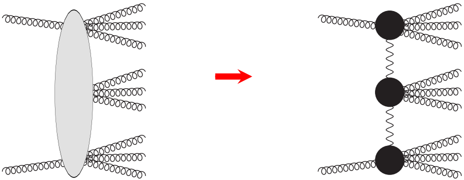


→ effective field theory for **each cluster** of particles local in rapidity

# A new field: the reggeized gluon

to re-construct QCD scattering amplitude:

new (scalar) auxiliary field $A_\pm$



properties:

- $\delta A_\pm = 0$ — invariant w.r.t local gauge transformations, but charged under $SU(N_c)$

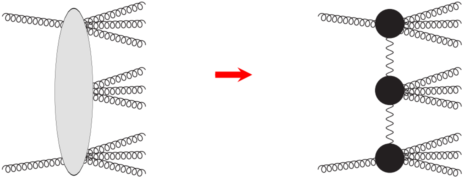
  manifest gauge invariance factorization

- $\partial_+ A_- = 0 = \partial_- A_+$ — strong ordering in rapidity between different clusters

# A new field: the reggeized gluon

to re-construct QCD scattering amplitude:

new (scalar) auxiliary field $A_{\pm}$



effective action: interaction of QCD fields with reggeized gluon [Lipatov (1995)]:

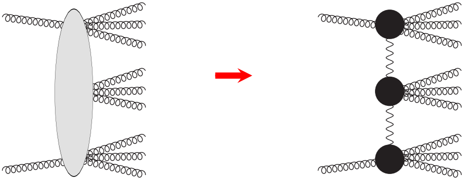
$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}$$

$S_{\text{ind.}}$: coupling of reggeized gluon ($A_{\pm}$) to QCD gluon ($v_\mu$)

# A new field: the reggeized gluon

to re-construct QCD scattering amplitude:

new (scalar) auxiliary field $A_\pm$



$$S_{\text{ind.}}[v_\mu, A_\pm] = \int d^4x \, \text{tr} \left[ \left( W_+[v(x)] - A_+(x) \right) \partial_\perp^2 A_-(x) \right] + (+) \leftrightarrow (-)$$

$$W_\pm[v] = \mathcal{P}_A \left( v_\pm \frac{1}{D_\pm} \partial_\pm \right) \qquad \text{with} \qquad D_\pm = \partial_\pm + g v_\pm .$$

Wilson line, but projected on asymmetric color ($\mathcal{P}_A$) → crossing symmetry

trivial for tree-level amplitudes, matters for pole prescription [MH; (2012)]

# Locality in rapidity versus a possible over-counting

$$S_{\text{eff}} = S_{\text{QCD}}[v, \psi, \phi] + S_{\text{ind.}}[v, A]$$

[Lipatov, (1995, 1997)]: each field carries implicit rapidity label, locality in rapidity

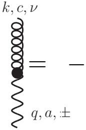
→ effective action: interaction restricted to narrow width in rapidity

**practice:** need scheme to take care of this in systematic manner

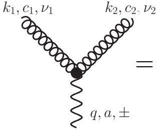
- particularly relevant to go beyond tree-level

# Feynman rules

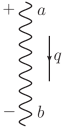
new vertices and propagator:




$$= -i\mathbf{q}^2 \delta^{ac} (n^\pm)^\nu, \qquad k^\pm = 0.$$

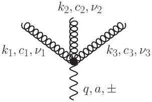



$$= g\mathbf{q}^2 \frac{f^{c_1 c_2 a}}{k_1^\pm} (n^\pm)^{\nu_1} (n^\pm)^{\nu_2}, \qquad k_1^\pm + k_2^\pm = 0$$




$$= \delta^{ab} \frac{i/2}{\mathbf{q}^2}$$

in general: whole tower of induced vertices

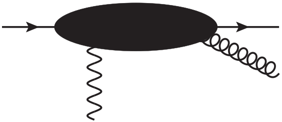



$$\sim g^2 \mathbf{q}^2 \left( \frac{f^{c_3 c_1 e} f^{e c_2 a}}{k_3^\pm k_2^\pm} + \frac{f^{c_3 c_2 e} f^{e c_1 a}}{k_3^\pm k_1^\pm} \right)$$

independently re-derivation for quasi-elastic processes **[Hameren, Kotko, Kutak (2012)]**

# The propsed scheme

## Rules for calculation of vertices of reggeized gluons and QCD fields



Disclaimer: obtained and verified from study of explicit examples; likely to be true in general, no general proof so far, report current stage of our understanding

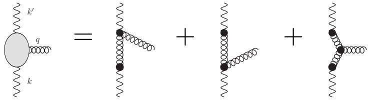
in perturbation theory:

- desired correlator (with regulator), from effective action Feynman rules, reggeized gluon $\equiv$ background field
- subtract all contributions with internal reggeized gluon lines ➡ coefficient
- use subtracted coefficients to construct QCD scattering amplitudes in a diagramatic way ($\sim$ Regge field theory on the level of diagrams)

# An example: quark-quark scattering at NLO (real part)

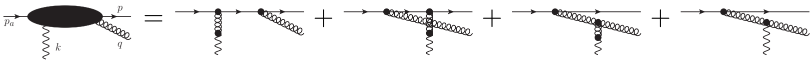
real NLO corrections: $qq \to qqg$ amplitude
Regge limit from following elements

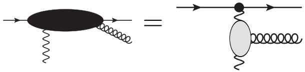
**central production vertex:**



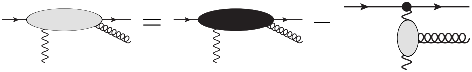

- Lipatov vertex
- $\mathcal{O}(g)$ - LO vertex

**quasi-elastic production vertex** [MH, Sabio Vera (2012)]:



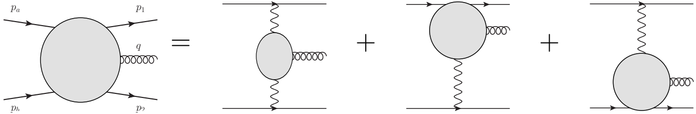

agrees with [Ciafaloni (1998)], [Ciafaloni, Colferai (1999)], [Bartels, Colferai, Vacca (2002)] property: $\lim\limits_{s_{qg}\to\infty}$  violates locality in rapidity!

➡ subract diagrams with internal reggeized gluon
(≡ central production contribution)



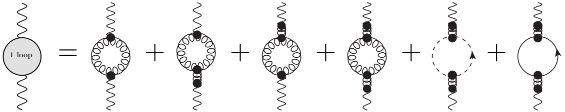
by construction: “local in rapidity”

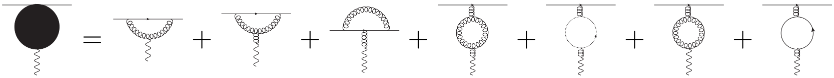
use to build the $qq \to qqg$ amplitude (reggeized gluon diagrams)



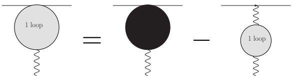
# Virtual corrections

for details: next talk by Grigorios Chachamis

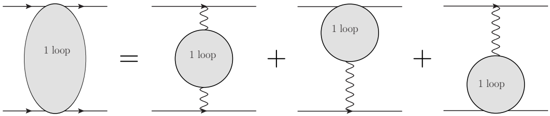


subtract diagrams with internal reggeized gluon

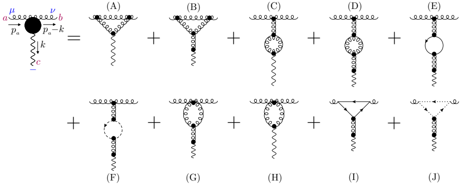


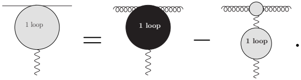

1-loop scattering amplitude
agrees with [Fadin, Fiore, Quartarolo (1994)]

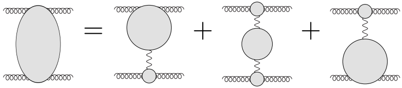


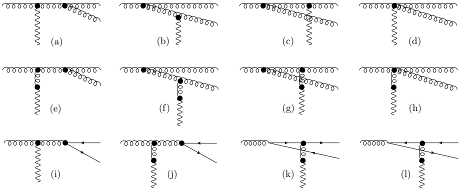
# Now also verified for the gluon

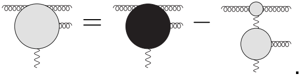
**[Chachamis, MH, Madrigal Martinez, Sabio Vera (2012)]**

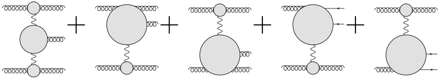




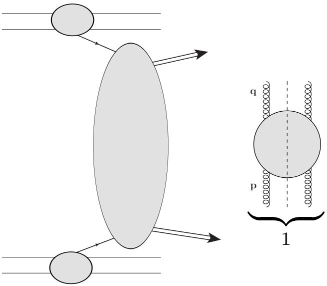





# Forward-backward jets with rapidity gap

[Mueller, Tang (1992)], [Engberg, Ingelmann, Motyka (2001)], [Chevallier, Kepka, Marquet, Royon (2009)], [Kepka, Marquet, Royon (2010)]

- no radiation ➡ exchange of color singlet
- jets widely separated in rapidity ➡ high energy factorization + BFKL resummation




**why interesting?**

- diffraction in perturbation theory
- slope and intercept of perturbative Pomeron (non-forward BFKL)

**description**

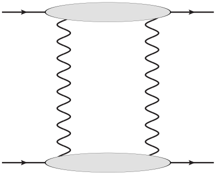
- non-forward BFKL at NLO, impact factors LO only; expect large correction

**phenomenology can be difficult** but also allows to learn ...

- soft re-scattering $\Leftrightarrow$ need rapidity gap survival probability
- no real gap ➡ take care! becomes a NNLO correction! [Y. Hatta, C. Marquet, C. Royon, G. Soyez, T. Ueda, D. Werder, (2013)]

# Impact factor for the perturbative Pomeron

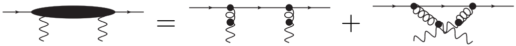
perturbative Pomeron $\equiv$ exchange of 2 reggeized gluons in color singlet (BFKL: resummation of $\alpha_s \Delta y_{\text{jet}}$ )



- effective action: longitudinal loop integrals $dl^- dl^+$ factorize
- move them to quark-2 reggeized gluon vertex
- remain with convolution in transverse momentum

# LO straight forward

in agreement with [Mueller, Tang (1992)]



$\left[\int \frac{dl^-}{4\pi}$  $\right]^2$ → $$h_{MT}^{(0)} = C_f^2 \cdot \frac{\alpha_s^2(\mu^2)}{\mu^{4\epsilon}\Gamma^2(1-\epsilon)(N_c^2-1)}$$

no transverse momentum dependence - a constant

cross-section

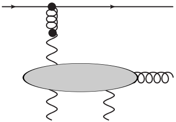
$$d\sigma_{ab} = h_{aMT}^{(0)} h_{bMT}^{(0)} \left[\int \frac{d^{2+2\epsilon}\mathbf{l}_1}{\mathbf{l}_1^2(\mathbf{k}-\mathbf{l}_1)^2}\right]\left[\int \frac{d^{2+2\epsilon}\mathbf{l}_2}{\mathbf{l}_2^2(\mathbf{k}-\mathbf{l}_2)^2}\right] d[\mathbf{k}] \qquad d[\mathbf{k}] \equiv d^{2+2\epsilon}\mathbf{k}$$

same for gluon with $C_f^2 \leftrightarrow C_a^2$ in $h_{MT}^{(0)}$

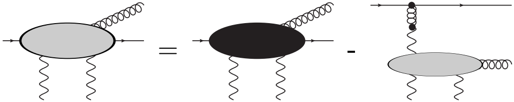
# real NLO corrections from the effective action

Determine the $qr^* r^* \to qg$ amplitude – 2 contributions

(A)

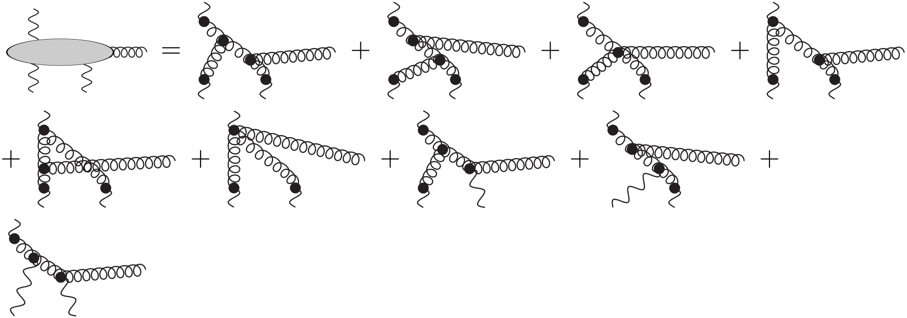


(B)



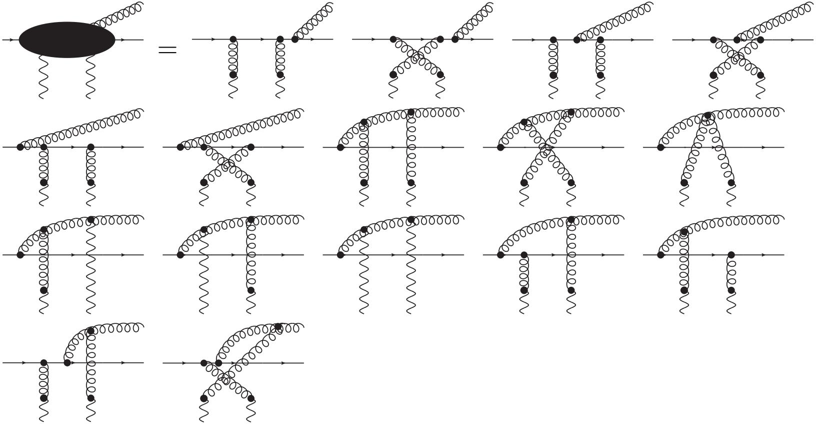
- $s_{qg}$ requires regulator ➡ finite diffractive mass $M_X^2 > s_{qg}$ of quark-gluon system
- finite $M_X^2$ fits with experimental observation: $\Delta y_{\text{jets}} > \Delta y_{\text{gap}}$

# effective diagrams - factorized NLO corrections



- the Reggeon-Particle-2 Reggeon Vertex (in the color singlet) [Bartels (1980)]
- the 'real' part of the Triple-Pomeron-Vertex [Bartels, Wusthoff (1995)]
- from the effective action [Braun, Vyazovsky (2006)]

# effective diagrams - real NLO corrections



# partonic impact factor with quasi-elastic corrections

After integration over long. loop momenta: quasi-elastic cross-section

[MH, Murdaca, Sabio Vera (prep.)]

$$d\sigma_{ab}^{qea} = h_{aMT}^{(0)} h_{bMT}^{(0)} \left[ \int \frac{d^{2+2\epsilon}\mathbf{l}}{\mathbf{l}^2(\mathbf{k}-\mathbf{l})^2} \right] \left[ \int \frac{d^{2+2\epsilon}\mathbf{l}'}{\mathbf{l}'^2(\mathbf{k}-\mathbf{l}')^2} \right] \mathcal{F}_{qqg}^{MT}(\mathbf{l},\mathbf{l}') d[\mathbf{k}] d[\mathbf{p}] d\eta$$

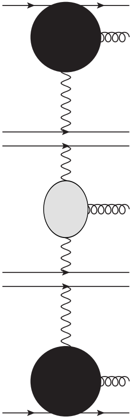
with

$$\mathcal{F}_{qqg}^{MT} = \frac{\alpha_s(\mu^2)\mathcal{P}_{qg}(z,\epsilon)}{C_f^2 \pi^{1+\epsilon}\mu^{2\epsilon}\Gamma(1-\epsilon)} \left[ C_f \left( \frac{\boldsymbol{\Delta}}{\boldsymbol{\Delta}^2} - \frac{\mathbf{q}}{\mathbf{q}^2} \right) + C_a \left( \frac{\mathbf{p}}{\mathbf{p}^2} - \frac{1}{2}\frac{\mathbf{l}_1 - \mathbf{q}}{(\mathbf{q}-\mathbf{l}_1)^2} - \frac{1}{2}\frac{\mathbf{p}-\mathbf{l}_1}{(\mathbf{p}-\mathbf{l}_1)^2} \right) \right]$$
$$\cdot \left[ C_f \left( \frac{\boldsymbol{\Delta}}{\boldsymbol{\Delta}^2} - \frac{\mathbf{q}}{\mathbf{q}^2} \right) + C_a \left( \frac{\mathbf{p}}{\mathbf{p}^2} - \frac{1}{2}\frac{\mathbf{l}_2 - \mathbf{q}}{(\mathbf{q}-\mathbf{l}_2)^2} - \frac{1}{2}\frac{\mathbf{p}-\mathbf{l}_2}{(\mathbf{p}-\mathbf{l}_2)^2} \right) \right], \qquad \boldsymbol{\Delta} = \mathbf{q} - z\mathbf{k}$$

$$P_{qg}(z,\epsilon) = C_f \frac{1 + (1-z)^2 + \epsilon z^2}{z}$$

# Subtraction and regulators - Mueller Navelet method (1)

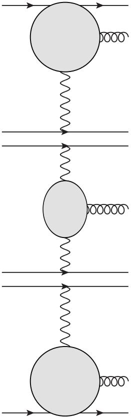
method proposed in [MH, Sabio Vera; (2011)]:
cutoffs $\eta_{a,b} \to \pm\infty$ to regularize integral over gluon rapidity $\eta_{\text{gluon}}$



$\eta_{\text{gluon}} > \eta_b$
$\Rightarrow$ central $\times(\ln \frac{p_a^+}{|\mathbf{q}|} - \eta_b) + \dots$

$\eta_a > \eta_{\text{gluon}} > \eta_b$
$\Rightarrow$ central $\times(\eta_a - \eta_b)$

$\eta_{\text{gluon}} < \eta_a$
$\Rightarrow$ central $\times(\eta_a + \ln \frac{p_b^-}{|\mathbf{q}|})\dots$



$\Rightarrow$ central $\times(\ln \frac{p_a^+}{|\mathbf{q}|} - \eta_a) + \dots$

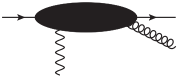
$\Rightarrow$ central $\times(\eta_a - \eta_b)$

$\Rightarrow$ central
$\times(\ln \frac{p_b^-}{|\mathbf{q}|} + \eta_b) + \dots$

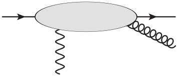
method: introduce regulator ➡ subtraction with regulated phase space
➡ cancellation of regulator overall $\ln s/\mathbf{q}^2$,

# Alternative - Mueller Navelet method (2)

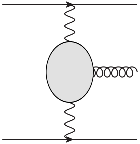
- subtraction without regulator
- regulation in close analogy to "+" prescription



divergence: central $\times \int_0^1 \frac{1}{z} f(z), \quad f(z)$ reg. at $z = 0$



regularized: central $\times \int_0^1 dz \frac{f(z) - f(0)}{z}$

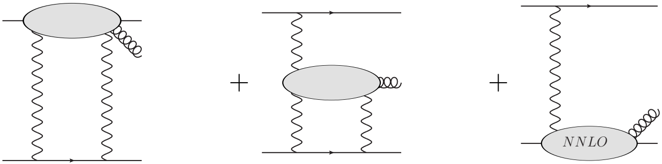


integrate over entire phase space: term $\times \int_{\ln \frac{|\mathbf{q}|}{p_b}}^{\ln \frac{p_a^+}{|\mathbf{q}|}} d\eta$

final result agrees!

# Mueller Tang - method (1) & (2)

in reality 3 contribution, ...




- without last diagram no cancellation of cut-offs
- method (1) with naïve rapidity cut: collinear finiteness in danger ...

method (2)

- upper cut-off $M_X^2 > s_{qg}$ on central diagram
- divergences in coefficients regulated through '$(+)$' type prescription.

# Finiteness of the jet impact factor

- virtual corrections [Fadin, Fiore, Kotsky, Papa (1999)], using conventional methods, based on $s$-channel unitarity; divergent parts

$$h^{(1)}_{\text{virt.}} = h^{(0)}_{\text{MT}} \frac{\alpha_s}{2\pi} \left[ -2\frac{C_f}{\epsilon^2} - \frac{\beta_0}{\epsilon} + \frac{3C_f}{\epsilon} - 2\frac{C_f}{\epsilon} \ln \frac{\mathbf{k}^2}{\mu^2} \right]$$

- cancellation with real correction and charge renormalization

$$h^{(1)}_{\text{real}} = h^{(0)}{}_{\text{MT}} \frac{\alpha_s}{2\pi} \left[ 2\frac{C_f}{\epsilon^2} + \frac{3C_f}{\epsilon} + 2\frac{C_f}{\epsilon} \ln \frac{\mathbf{k}^2}{\mu^2} + \frac{C_a^2}{C_f} \left( \ln \frac{M_X^2 + \mathbf{k}^2}{\mathbf{k}^2} - \frac{3}{2} \right) \right]$$

- remainder canceled from collinear counterterm:

$$h^{(1)}_{\text{coll. ct}} = -h^{(0)}{}_{\text{MT}} \frac{\alpha_s}{2\pi} \frac{C_a^2}{C_f} \left( \ln \frac{M_X^2 + \mathbf{k}^2}{\mathbf{k}^2} - \frac{3}{2} \right)$$

finite terms ready, but lengthy .... refer to the paper in prep. for details ...

# Summary

- High energy limit of QCD non-trivial ➡ convenient tool: gauge invariant high energy effective action (HEA) [Lipatov (1995)]
- Locality in rapidity of the HEA ➡ need proper scheme for NLO corrections
- developed such a scheme from explicit example ($qq \to qq$); subsequently confirmed for ($gg \to gg$)
- new result: real NLO for quark induced Mueller-Tang jet impact factor
- finite after combination with virtual corrections [Fadin, Fiore, Kotsky, Papa (1999)] & collinear counterterm + QCD charge renormalization

# Virtual corrections

determined in [Fadin, Fiore, Kotsky, Papa (1999)], using conventional methods, based on $s$-channel unitarity

$$H_v^{(1)} = H_{v,q}^{(1)}(\mathbf{k}, \mathbf{l}_1) + H_{v,qg}^{(1)}(\mathbf{k}, \mathbf{l}_1) + H_{v,q}^{(1)}(\mathbf{k}, \mathbf{l}_2) + H_{v,qg}^{(1)}(\mathbf{k}, \mathbf{l}_2),$$

with 'quark-gluon' intermediate contribution

$$H_{v,qg}^{(1)}(\mathbf{k}, \mathbf{l}_1) = C_f^2 H^{(0)} (4\pi)^{1+\epsilon} \left\{ - C_f I_B^{(+)}(\mathbf{l}_1, \mathbf{k}) - \frac{C_a}{2} \left[ \tilde{I}_A^{(+)}(\mathbf{l}_1, \mathbf{k}) - I_B^{(+)}(\mathbf{l}_1, \mathbf{k}) + \tilde{I}_C^{(+)}(\mathbf{l}_1, \mathbf{k}) \right] \right\} \quad (1)$$

quark intermediate state

$$H_{v,q}^{(1)}(\mathbf{k}, \mathbf{l}_1) = C_f^2 H^{(0)} \frac{\alpha_s c_\Gamma(\epsilon)}{4\pi \Gamma(1-\epsilon)(-\epsilon)} \left\{ \left[ \left( \frac{\mathbf{l}_1^2}{\mu^2} \right)^\epsilon + \left( \frac{(\mathbf{k} - \mathbf{l}_1)^2}{\mu^2} \right)^\epsilon \right] \left[ \frac{-n_f(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)} \right. \right.$$

$$+ (2C_f - C_a) \left( \frac{1}{\epsilon(1+2\epsilon)} + \frac{1}{2} \right) + C_a \left( \psi(1-\epsilon) - 2\psi(\epsilon) + \psi(1) \right.$$

$$\left. \left. + \frac{1}{4(1+2\epsilon)(3+2\epsilon)} - \frac{1}{\epsilon(1+2\epsilon)} - \frac{7}{4(1+2\epsilon)} \right) \right]$$

$$\left. + C_a \left[ \ln \frac{s_0}{\mathbf{l}_1^2} \left( \frac{\mathbf{l}_1^2}{\mu^2} \right)^\epsilon + \ln \frac{s_0}{(\mathbf{k} - \mathbf{l}_1)^2} \left( \frac{(\mathbf{k} - \mathbf{l}_1)^2}{\mu^2} \right)^\epsilon \right] \right\}.$$

# A list of (master) integrals

calculated in [Fadin, Fiore, Kotsky, Papa (2000)]

$$\tilde{I}_A(\mathbf{l}_1, \mathbf{k}, s_0) = \frac{\Gamma(-\epsilon)\Gamma(1+\epsilon)^2}{4\pi\Gamma(1+2\epsilon)}\Bigg\{ -2\left(\mathbf{k}^2\right)^2\left[\frac{1}{2}\ln\left(\frac{s_0}{\mathbf{k}^2}\right) + \psi(1) - \psi(1+2\epsilon) - \frac{3}{4(1+2\epsilon)}\right] + \mathbf{l}_1^2$$

$$\left[-\frac{1}{\epsilon} - \frac{3}{1+2\epsilon} + 2\psi(1-\epsilon) - 2\psi(1+2\epsilon) + 2\psi(1) - 2\psi(\epsilon) + 2\ln\left(\frac{s_0}{\mathbf{l}_1^2}\right) - \epsilon K_1\right] + (\mathbf{l}_1 \leftrightarrow \mathbf{k} - \mathbf{l}_1)\Bigg\}$$

The integral $K_1$ is only available as an expansion in $\epsilon$. It reads

$$K_1 = \frac{1}{2}\left(\mathbf{k}^2\right)^\epsilon \left\{ \frac{1}{\epsilon^2}\left[2 - \left(\frac{\mathbf{l}_1^2}{\mathbf{k}^2}\right)^\epsilon - \left(\frac{(\mathbf{k}-\mathbf{l}_1)^2}{\mathbf{k}^2}\right)^\epsilon\right] + 4\psi''(1)\epsilon + \ln\left(\frac{\mathbf{l}_1^2}{\mathbf{k}^2}\right)\ln\left(\frac{(\mathbf{k}-\mathbf{l}_1)^2}{\mathbf{k}^2}\right)\right\}$$

Furthermore

$$I_B^{(+)}(\mathbf{l}_1, \mathbf{k}) = -\frac{2\Gamma(-\epsilon)\Gamma(1+\epsilon)^2}{4\pi\Gamma(1+2\epsilon)}\left[\frac{1}{\epsilon(1+2\epsilon)} + \frac{1}{2}\right]\cdot\left[\left(\mathbf{k}^2\right)^\epsilon - \left(\mathbf{l}_1^2\right)^\epsilon - \left((\mathbf{k}-\mathbf{l}_1)^2\right)^\epsilon\right],$$

$$\tilde{I}_C^{(+)}(\mathbf{l}_1, \mathbf{k}) = \tilde{I}_A^{(+)}(\mathbf{l}_1, \mathbf{k}) + \frac{2\Gamma(-\epsilon)\Gamma(1+\epsilon)^2}{4\pi\Gamma(1+2\epsilon)}\left[-\left(\frac{1}{\epsilon(1+2\epsilon)+\frac{1}{2}}\right)\left(\mathbf{k}^2\right) + K_2'\right].$$

The integral $K_2'$ is only know as an expansion in $\epsilon$, $K_2'(\mathbf{k}, \mathbf{l}_1, \mathbf{k}-\mathbf{l}_1) =$

$$= \epsilon\left[1 + \frac{\ln^2\left(\frac{\mathbf{l}_1^2}{(\mathbf{k}-\mathbf{l}_1)^2}\right)}{2} - \frac{3}{2}\frac{\mathbf{l}_1^2 - (\mathbf{k}-\mathbf{l}_1)^2}{\mathbf{k}^2}\ln\left(\frac{\mathbf{l}_1^2}{(\mathbf{k}-\mathbf{l}_1)^2}\right) - 6\frac{|\mathbf{l}_1||\mathbf{k}-\mathbf{l}_1|}{\mathbf{k}^2}\theta\sin\theta + 8\psi'(1) - 2\theta^2\right]$$

with the angle $\theta$ betwen $\mathbf{l}_1$ and $\mathbf{k}-\mathbf{l}_1$ such that $|\theta| \leq \pi$.