Contribution of the Twist-3 Fragmentation Function to SSA in SIDIS

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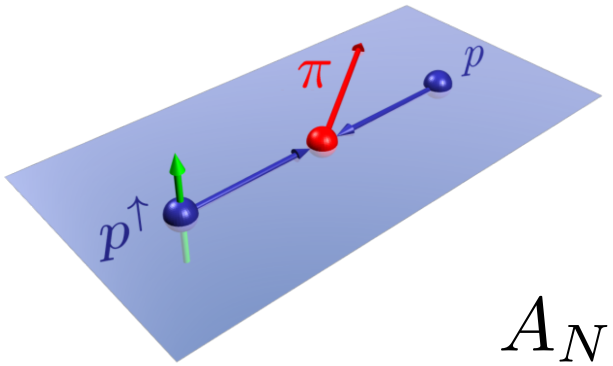
Single transverse-Spin Asymmetry (SSA)

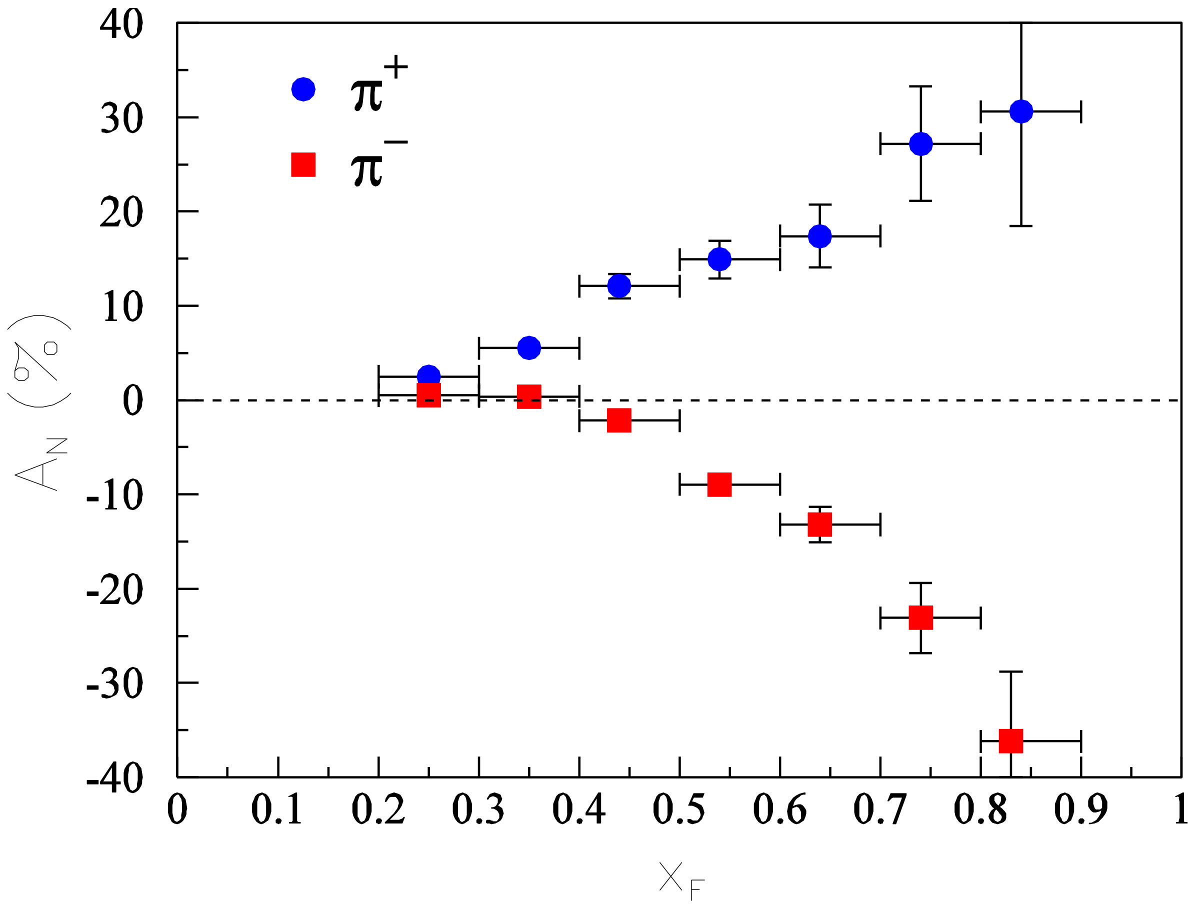
$p + p^{\uparrow} \to \pi + X$



$$A_N \equiv \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

$\sim 30\%$

$\sqrt{S} = 20\mathrm{GeV}$ E704 (1991)



E704, STAR, PHENIX...

$e + p^{\uparrow} \to e' + h + X$

HERMES, COMPASS, ...

cannot be understood by the collinear parton model.

⇒ Need extension of the framework for QCD hard process.

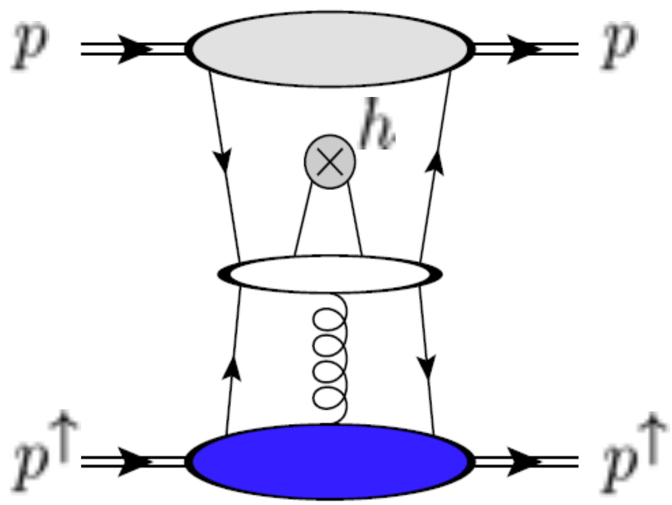
Collinear twist-3 approach for SSA

・Multi-parton correlation in collinear factorization . $(P_T \sim Q \gg \Lambda_{\rm QCD})$

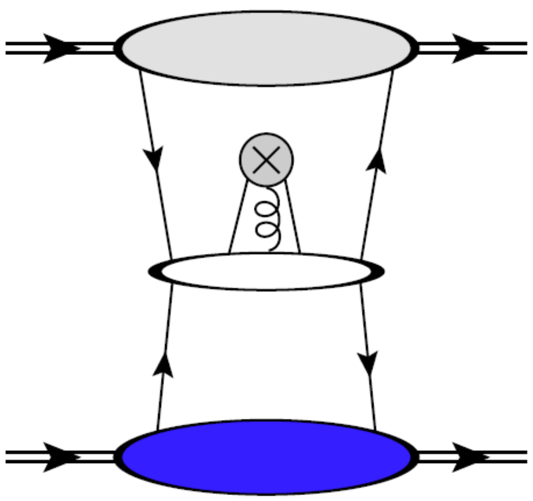
$p + p^{\uparrow} \to \pi + X$

Efremov-Teryaev, Qiu-Sterman, Eguchi-Koike-Tanaka, and many works.

Ex.



"twist-3 distribution"



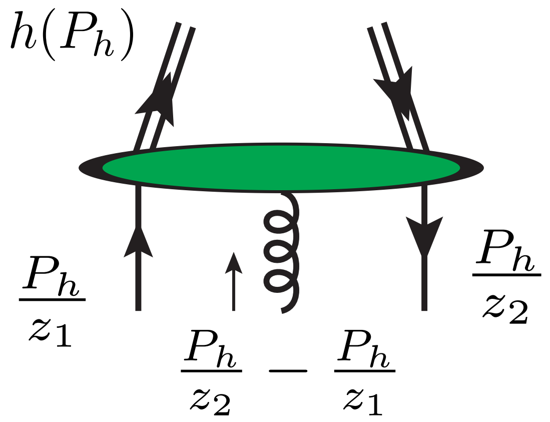
"twist-3 fragmentation"

This talk

▷ SSA can be described in term of these correlation effects.

Twist-3 Fragmentation Function

No constraint from T-invariance



$$\hat{E}_F(z_1, z_2) \sim \sum_X \langle 0|\psi|hX\rangle\langle hX|\bar{\psi}F|0\rangle$$

$$\hat{E}_F(z_1, z_2) = \mathrm{Re}[\hat{E}_F(z_1, z_2)] + i\mathrm{Im}[\hat{E}_F(z_1, z_2)]$$

⇒ "Pole" and "Non-pole" contributions.

"Pole" contribution to SSA

Meissner-Metz (2009)

Gamberg-Mukherjee-Mulders (2011)

At poles: $\frac{1}{z_1} = \frac{1}{z_2}$ or $\frac{1}{z_{1,2}} = 0$

$$\hat{E}_F(z, z) = \hat{E}_F(0, z) = 0$$

⇒ No contribution from poles.

(⇔ Twist-3 distributions)

"Non-pole" contribution to SSA

SIDIS : Collins azimuthal asymmetry — Yuan-Zhou (2009)

PP→hX : Derivative term — Kang-Yuan-Zhou (2010)

All contribution in LC gauge — Metz-Pitonyak (2013)

No formalism in covariant gauges.

Contents of this talk

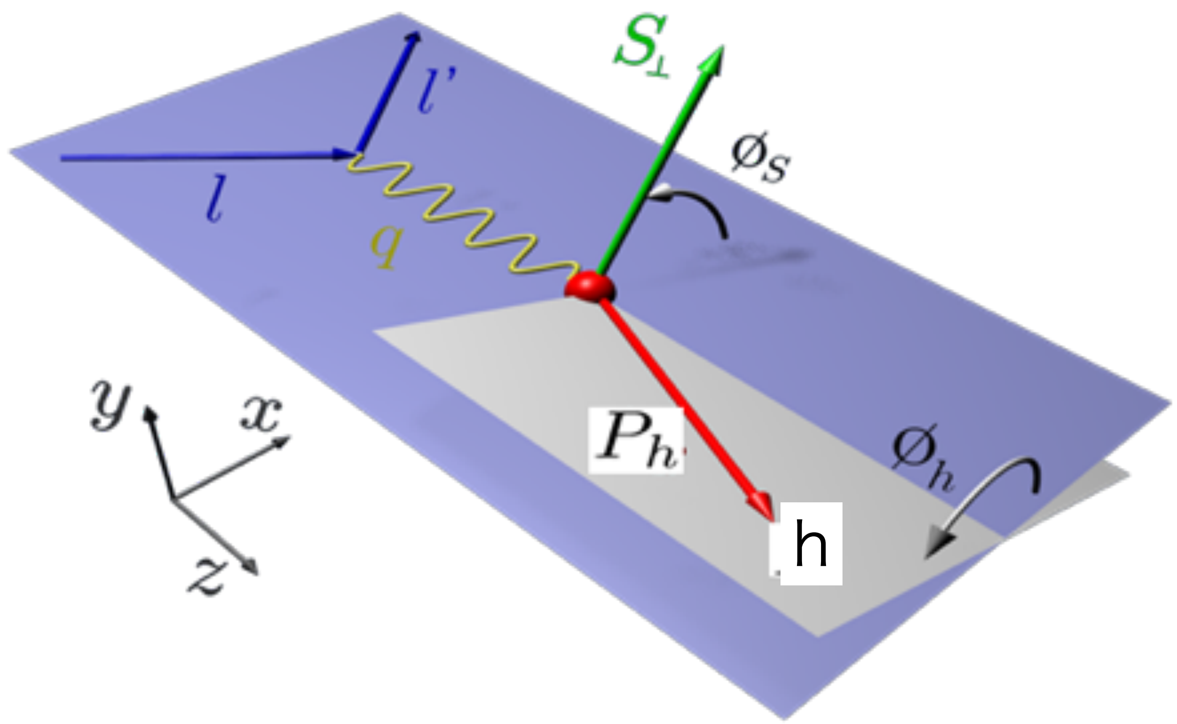
“Non-pole” Contribution of Twist-3 Fragmentation Function

- Collinear twist-3 formalism in Feynman gauge.

- Cross section formula in SIDIS.

Kinematics for SIDIS

$$e + p^{\uparrow} \to e' + h + X$$



kinematical variables

$$\begin{aligned} S_{ep} &= (p+l)^2, \\ x_{bj} &= \frac{Q^2}{2p\cdot q}, \\ Q^2 &= -q^2 = -(l-l')^2, \\ z_f &= \frac{p\cdot P_h}{p\cdot q}, \\ q_T &= \sqrt{-q_t^2}, \end{aligned}$$

ϕ, χ: azimuthal angles

$$d\sigma = \frac{1}{2S_{ep}} \frac{d^3\vec{P}_h}{(2\pi)^3 2P_h^0} \frac{d^3\vec{\ell}'}{(2\pi)^3 2\ell'^0} \frac{e^4}{q^4} L^{\mu\nu}(\ell,\ell') W_{\mu\nu}(p,q,P_h)$$

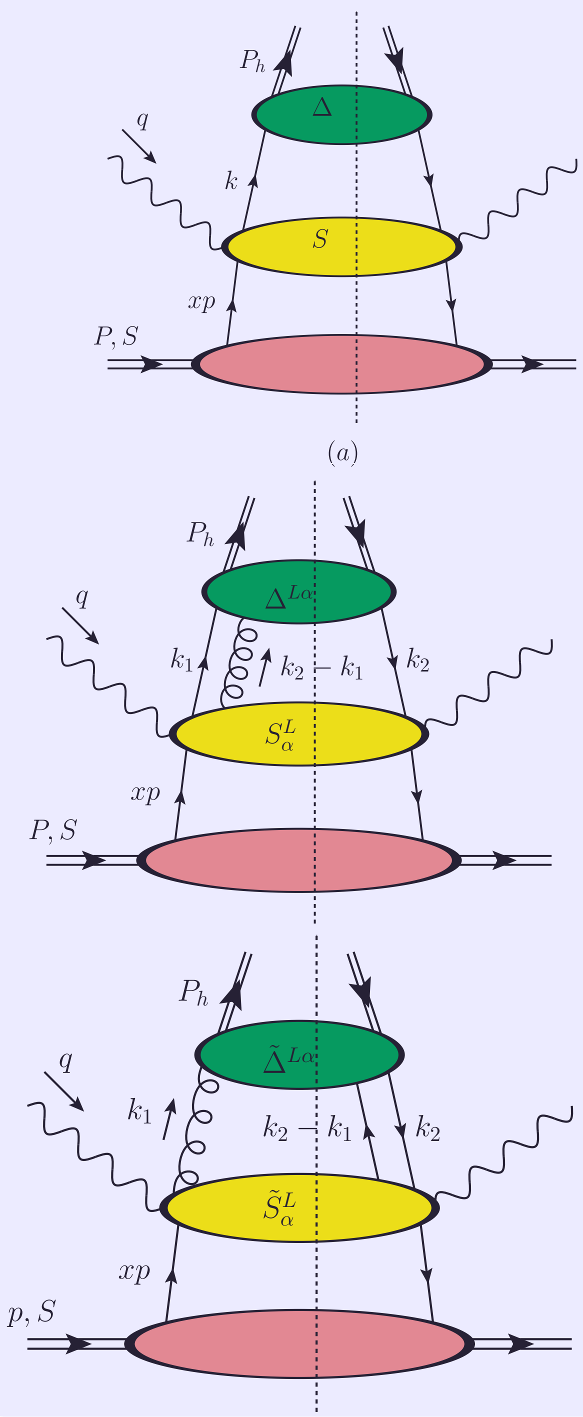
$$\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} = \frac{\alpha_{em}^2}{128\pi^4 x_{bj}^2 S_{ep}^2 Q^2} z_f L^{\mu\nu}(l,l') W_{\mu\nu}(p,q,P_h)$$

Let's focus on Twist-3 FF effect in hadronic tensor

Analysis of hadronic tensor

Factorize transversity h(x)

$$W_{\mu\nu}(p,q,P_h) = \int \frac{dx}{x} h(x) w_{\mu\nu}(xp,q,P_h)$$



+ Mirror diagram

+ Mirror diagram

$$= \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\Delta^{(0)}(k)S(k)\right]$$

$$+ \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\left\{\mathrm{Tr}\left[\Delta_A^{(1)\alpha}(k_1,k_2)S^L_\alpha(k_1,k_2)\right] + (\mu\leftrightarrow\nu)^*\right\}$$

$$+ \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\left\{\mathrm{Tr}\left[\tilde{\Delta}_A^{(1)\alpha}(k_1,k_2)\tilde{S}^L_\alpha(k_1,k_2)\right] + (\mu\leftrightarrow\nu)^*\right\}$$

Matrix elements for fragmentation:

$$\Delta^{(0)}(k) \sim \sum_X \langle 0|\psi|hX\rangle\langle hX|\bar{\psi}|0\rangle$$

$$\Delta_A^{(1)\alpha}(k_1,k_2) \sim \sum_X \langle 0|\psi|hX\rangle\langle hX|\bar{\psi}gA^\alpha|0\rangle$$

$$\tilde{\Delta}_A^{(1)\alpha}(k_1,k_2) \sim \sum_X \langle 0|\bar{\psi}\psi|hX\rangle\langle hX|gA^\alpha|0\rangle$$

Hard parts: $S(k), S^{L\alpha}(k_1,k_2), \tilde{S}^{L\alpha}(k_1,k_2)$

⇒ These diagrams contain twist-3 FF effects.

Identification of twist-3 contributions

Collinear expansion:

$$S(k) = S(z) + \frac{\partial S(k)}{\partial k^\alpha}\Big|_{\rm c.l.} \Omega^\alpha{}_\beta k^\beta + \cdots$$

$$S^L(k_1,k_2) = S^L(z_1,z_2) + \frac{\partial S^L(k_1,k_2)}{\partial k_1^\alpha}\Big|_{\rm c.l.} \Omega^\alpha{}_\beta k_1^\beta + \frac{\partial S^L(k_1,k_2)}{\partial k_2^\alpha}\Big|_{\rm c.l.} \Omega^\alpha{}_\beta k_2^\beta + \cdots$$

$\Omega^\alpha{}_\beta = g^\alpha{}_\beta - P_h^\alpha w_\beta$: Projection operator to extract twist-3 effects

$k^\alpha = \frac{P_h^\alpha}{z} + \Omega^\alpha{}_\beta k^\beta$

$P_h \cdot w = 1$

c.l. (collinear limit)

$k \to \frac{P_h}{z}$

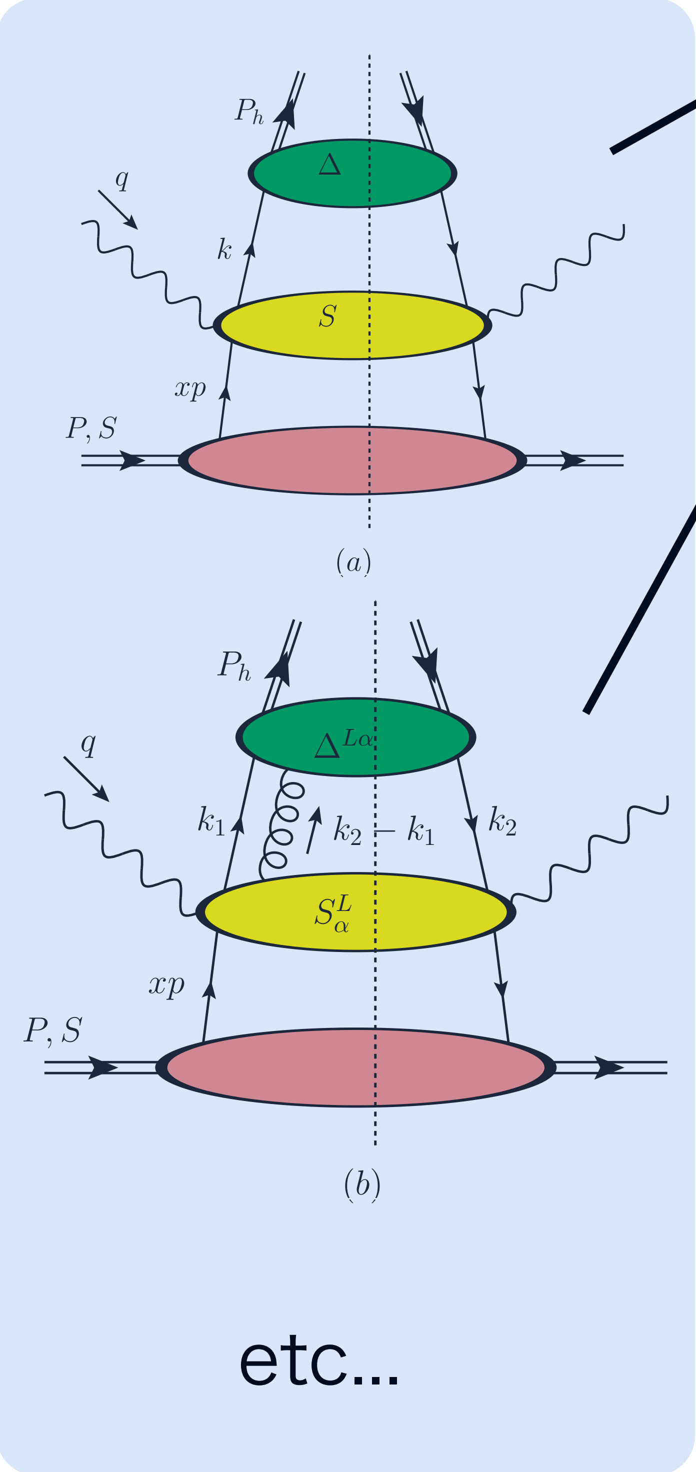
etc.

By partial integration: $k^\beta \to -i\partial^\beta$

⇒ produce derivative of field operators:

$\partial^\beta \bar{\psi}, \partial^\beta A^-,$ etc.

Twist-3 parts after collinear expansion



$$\Delta_F^{(1)\alpha} \sim \sum_X \langle p|\psi|hX\rangle\langle hX|\bar{\psi} g F^{\alpha -}|0\rangle$$

gauge-invariant

$$w^{(a)} = -i\Omega^\alpha{}_\beta \int \frac{dz}{z^2} \mathrm{Tr}\left[\Delta_\partial^{(0)\beta}(z) \frac{\partial S(k)}{\partial k^\alpha}\Big|_{\mathrm{c.l.}}\right]$$

$$w^{(b)} = -i\Omega^\alpha{}_\beta \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2} \mathrm{Tr}\left[\Delta_F^{(1)\beta}(z_1, z_2) \frac{\partial S^L(k_1,k_2)}{\partial k_2^\alpha}\Big|_{\mathrm{c.l.}}\right]$$

$$+\Omega^\alpha{}_\beta \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2} \mathrm{Tr}\left[\Delta_A^{(1)\beta}(z_1, z_2) \left(\left(\frac{1}{z_2} - \frac{1}{z_1}\right) \frac{\partial S^L(k_1,k_2)}{\partial k_2^\alpha}\Big|_{\mathrm{c.l.}} + S^L_\alpha(z_1, z_2)\right)\right]$$

$$-i\Omega^\alpha{}_\beta \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2} \mathrm{Tr}\left[\Delta_{\partial 1}^{(1)\beta}(z_1, z_2) \left(\frac{\partial S^L(k_1,k_2)}{\partial k_1^\alpha}\Big|_{\mathrm{c.l.}} + \frac{\partial S^L(k_1,k_2)}{\partial k_2^\alpha}\Big|_{\mathrm{c.l.}}\right)\right].$$

・Some matrix elements are gauge-dependent.

$$\Delta_\partial^{(0)\alpha} \sim \sum_X \langle p|\psi|hX\rangle\langle hX|(\partial^\alpha \bar{\psi})|0\rangle$$

$$\Delta_A^{(1)\alpha} \sim \sum_X \langle p|\psi|hX\rangle\langle hX|\bar{\psi} g A^\alpha|0\rangle$$

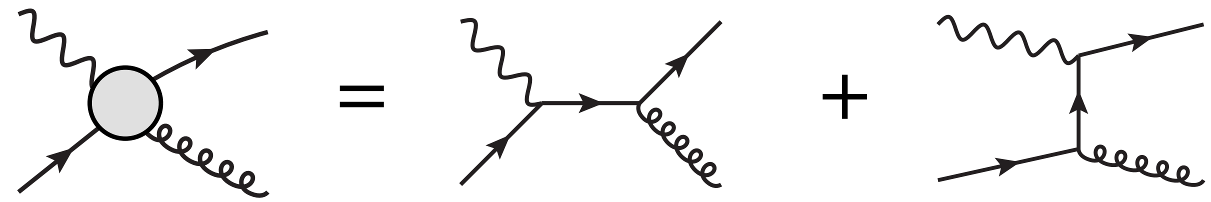
$$\Delta_{\partial 1}^{(1)\alpha} \sim \sum_X \langle p|\psi|hX\rangle\langle hX|(\partial^\alpha \bar{\psi}) g A^-|0\rangle$$

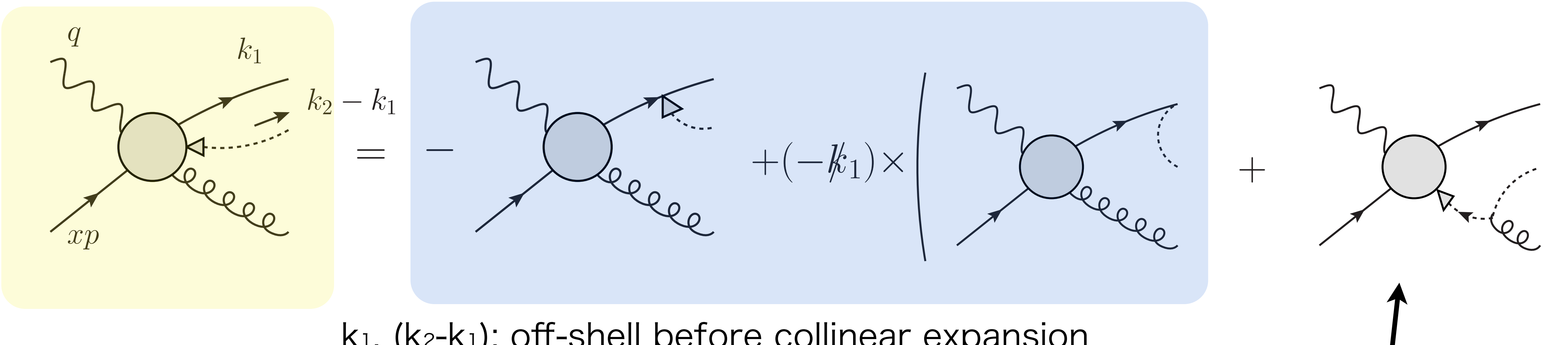
・Can we recast the MEs into gauge-invariant form?

⇒ Need relations among hard parts.

Ward-Takahashi (Slavnov-Taylor) identity

Relation between amplitudes:





k1, (k2-k1): off-shell before collinear expansion

No contribution from ghost-like term in tw3 accuracy.

Relation between hard parts:

$$(k_2 - k_1)^\sigma S^{La}_\sigma(k_1, k_2) = T^a S(k_2)$$

$$\left.\frac{\partial S^{La}(k_1,k_2)}{\partial k^\alpha_{1\perp}}\right|_{\rm c.l.} = \frac{1}{1/z_2 - 1/z_1 + i\epsilon} S^{La}_\alpha(z_1, z_2)$$

$$\left.\frac{\partial S^{La}(k_1,k_2)}{\partial k^\alpha_{2\perp}}\right|_{\rm c.l.} = \frac{1}{1/z_2 - 1/z_1 + i\epsilon}\left[T^a \left.\frac{\partial S(k_2)}{\partial k^\alpha_{2\perp}}\right|_{\rm c.l.} - S^{La}_\alpha(z_1, z_2)\right]$$

etc.

⇒ Let’s recast matrix elements.

Factorization property

Resultant formula

$$w^{(a)} + w^{(b)} + w^{(c)} = 2\Omega^{\alpha}{}_{\beta} \int \frac{dz}{z^2} \mathrm{Im}\mathrm{Tr}\left[\Delta^{\beta}_{\partial}(z) \frac{\partial S(k)}{\partial k^{\alpha}}\bigg|_{\text{c.l.}}\right]$$

$$-2\Omega^{\alpha}{}_{\beta} \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2} P\left(\frac{1}{1/z_2 - 1/z_1}\right) \mathrm{Im}\mathrm{Tr}\left[\Delta^{\beta}_{F}(z_1, z_2) S^{L}_{\alpha}(z_1, z_2)\right]$$

gauge-dependent terms construct (derivative of) gauge-links:

$$\left[D^{\alpha}(\lambda w) + ig \int_{-\infty}^{\lambda} d\mu F^{\alpha -}(\mu w)\right] = \partial^{\alpha}[\lambda w, -\infty w] + \mathcal{O}(g^2)$$

ex.

Matrix elements

$$\Delta^{\alpha}_{\partial ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|[-\infty w, 0]\psi_i(0)|hX\rangle\langle hX|\bar{\psi}_j(\lambda w)[\lambda w, -\infty w]|0\rangle \overleftarrow{\partial}^{\alpha}$$

$$= \frac{M_N}{2z}(\gamma_5 \not{P}_h \gamma_\lambda)\epsilon^{\lambda\alpha w P_h} \tilde{e}(z)$$

$$\Delta^{\alpha}_{Fij}(z_1, z_2) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|[-\infty w, 0]\psi_i(0)|hX\rangle$$

$$\times\langle hX|\bar{\psi}_j(\lambda w)[\lambda w, \mu w] g F^{\alpha\beta}(\mu w) w_\beta [\mu w, -\infty w]|0\rangle$$

$$= \frac{M_N}{2z_2}(\gamma_5 \not{P}_h \gamma_\lambda)\epsilon^{\lambda\alpha w P_h} \hat{E}_F(z_1, z_2)$$

$\tilde{e}(z), \hat{E}_F(z_1, z_2), \ldots$: color gauge-invariant tw3 FFs.

Cross section formula

$$\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_S M_N}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int_{x_{min}}^1\frac{dx}{x}\int_{z_{min}}^1\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\sum_a e_a^2\left[\frac{d}{d(1/z)}\left\{\frac{\mathrm{Im}\tilde{e}(z)}{z}\right\}\Delta\hat{\sigma}_k^1+\mathrm{Im}\tilde{e}(z)\Delta\hat{\sigma}_k^2\right.$$

$$\left.+2\int_z^\infty dz'\left\{\mathrm{Im}\hat{E}_F(z',z)\Delta\hat{\sigma}_k^3+\mathrm{Im}\tilde{E}_F(-z',(1/z-1/z')^{-1})\Delta\hat{\sigma}_k^4\right\}\right]$$

$$\cosh\psi=\frac{2x_{bj}S_{ep}}{Q^2}-1$$

$$\hat{x}\equiv\frac{x_{bj}}{x}\qquad \hat{z}\equiv\frac{z_f}{z}$$

$$\mathcal{A}_k\equiv\mathcal{A}_k(\phi-\chi)$$

$$\mathcal{S}_k\equiv\sin(\Phi_S-\chi)\ \text{for}\ k=1,\ldots,4$$

$$\mathcal{S}_k\equiv\cos(\Phi_S-\chi)\ \text{for}\ k=8,9$$

$$\begin{aligned}\mathcal{A}_1(\phi)&=1+\cosh^2\psi,\\ \mathcal{A}_2(\phi)&=-2,\\ \mathcal{A}_3(\phi)&=-\cos\phi\sinh 2\psi\\ \mathcal{A}_4(\phi)&=\cos 2\phi\sinh^2\psi,\\ \mathcal{A}_8(\phi)&=-\sin\phi\sinh 2\psi\\ \mathcal{A}_9(\phi)&=\sin 2\phi\sinh^2\psi.\end{aligned}$$

Five structure functions with different azimuthal dependence:

$$=\sin\Phi_S\left(\mathcal{F}_1+\mathcal{F}_2\cos\phi+\mathcal{F}_3\cos 2\phi\right)+\cos\Phi_S\left(\mathcal{F}_4\sin\phi+\mathcal{F}_5\sin 2\phi\right).$$

$$=\sin(\phi_h-\phi_S)\,F^{\sin(\phi_h-\phi_S)}+\sin(2\phi_h-\phi_S)\,F^{\sin(2\phi_h-\phi_S)}+\sin\phi_S\,F^{\sin\phi_S}$$
$$+\sin(3\phi_h-\phi_S)\,F^{\sin(3\phi_h-\phi_S)}+\sin(\phi_h+\phi_S)\,F^{\sin(\phi_h+\phi_S)},$$

: The same as TMD approach.

Summary

“Non-pole” Contribution of Twist-3 Fragmentation Functions to SSA

- We have established collinear twist-3 formalism in Feynman gauge.

  Ward-Takahashi identity ⇒ MEs are combined into CGI one .

- Derivation of the cross section for SSA in SIDIS.

  Contribution to five azimuthal asymmetries.

  EIC would give an opportunity to determine these functions.