Proton structure functions and physical evolution kernels

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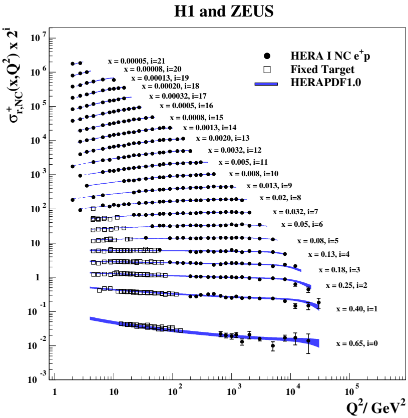
Deep Inelastic Scattering



[H1 and ZEUS collaboration, JHEP 1001:109 (2010)]

description with collinear factorization: a success!

$$F_2(x, Q^2) = \text{coeff. funct.} \otimes \text{pdfs}$$

- Q^2 dependence: DGLAP evolution
- essential building block: parton distribution functions

Factorization and Parton distribution functions

$$F_2(x, Q^2) = \text{coeff. funct.} \otimes \text{pdfs}$$

the upside ...

- collinear factorization ➡ universal pdfs $= q, g$ (we like that!)
- dependence on factorization scale described by DGLAP

the downside ....

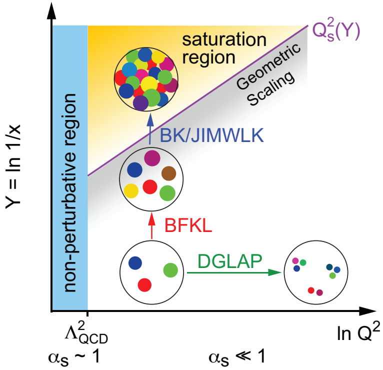
- q, g given by theory definition (➡ factorization scheme dependence)
- differences between schemes can be large

$$e.g. \sim (\alpha_s \ln 1/x)^n \text{ at small } x$$

(we don't like that so much ....)

don't care so much if I only want to parametrize my data ....

.... but if I want to understand physics at small x, this matters



[EIC White Paper, arXiv:1212.1701]

DGLAP: rise at small x

- (perturbative) DGLAP splitting functions
- (non-pert.) x-dependence of pdfs at initial scale

Question:

- What is physics (DGLAP, BFKL, saturation)
- what is clever choice of scheme?

solution: direct evolution of structure functions

$$\text{“} \quad \partial_{\ln Q^2} F(x, Q^2) = \quad K \quad \otimes \quad F(x, Q^2) \quad \text{”}$$

$$\begin{array}{ccccc} \uparrow & & \uparrow & & \uparrow \\ \text{obs.} \rightarrow & & \text{obs.} & \leftarrow & \text{obs.} \end{array}$$

evolution kernels K

- physical
- no factorization scheme ambiguity (only renormalization scale)

in principle equivalent to [Catani (1996)]

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

idea around for a while: [Bardeen, Buras (1979)], [Floratos, Kounnas, Lacaze (1981)], [Grunberg (1984)], [Catani (1996)], [Blümlein, Ravindran, van Neerven (2000)], [Vogt, Moch, Soar, Vermaseren (2010)]

a first study: α_s determination from polarized DIS [Blümlein, Böttcher; (2002)]

goal of this study: detailed numerical analysis, particular for the small x limit

issue 1

DGLAP equation = matrix equation:

$$\partial_{\ln \mu_f^2} f = P \otimes f \qquad\qquad f = q, g$$

$n_f = 3$
3 quark, 1 gluon pdf → singlet $= (\Sigma, g)$
NS_1, NS_2

need 4 observables to disentangle this (F_2^p, F_2^n, F_L, F_3) → possible, but involved
need all observables at high accuracy

small x: Σ = sea quark, NS$= 0$

$$(F_2, F_L) \Leftrightarrow (\Sigma, g)$$

future facilities (EIC, LHeC): accurate determination of both F_2, F_L

Determination of physical evolution kernels

define:

$$\mathbf{F} = \begin{pmatrix} F_2 \\ F_L \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} C_{2q} & C_{2g} \\ C_{Lq} & C_{Lg} \end{pmatrix} \quad \mathbf{f} = \begin{pmatrix} \Sigma \\ g \end{pmatrix} \quad \mathbf{P} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}$$

requires Mellin/Moment space with $a(N) = \int_0^1 dx x^{N-1} a(x)$ ➡ $\otimes \to \cdot$

$$\begin{aligned} \partial_{\ln Q^2} \mathbf{F} &= \partial_{\ln Q^2} \left(\mathbf{C} \cdot \mathbf{f} \right) \\ &= \beta \frac{d\mathbf{C}}{da_s} \cdot \mathbf{f} + \mathbf{C} \cdot \mathbf{P} \cdot \mathbf{f} \\ &= \underbrace{\left[\beta \frac{d\mathbf{C}}{da_s} \cdot \mathbf{C}^{-1} + \mathbf{C} \cdot \mathbf{P} \cdot \mathbf{C}^{-1} \right]}_{\tilde{\mathbf{K}}} \mathbf{F} \end{aligned}$$

with $a_s \equiv \frac{\alpha_s}{4\pi}$

issue 2

- coefficent matrix: $C_{2q} = \mathcal{O}(1)$ while $C_{Lq}, C_{Lg} = \mathcal{O}(a_s)$
- $\tilde{\mathbf{K}} = \left[\beta \frac{d\mathbf{C}}{da_s} \cdot \mathbf{C}^{-1} + \mathbf{C} \cdot \mathbf{P} \cdot \mathbf{C}^{-1}\right]$ mixes different orders of a_s ➡ scale dependent kernel!

solution: scheme independence of $C_{Lq}^{(1)}, C_{Lg}^{(1)}$ ➡ evolve $\tilde{\mathbf{F}} = (F_2, \tilde{F}_L)$

$$\tilde{F}_L^{(g)} = \frac{F_L}{a_s C_{Lg}^{(1)}}$$

[Blümlein, Ravindran, van Neerven (2000)]

or

$$\tilde{F}_L^{(q)} = \frac{F_L}{a_s C_{Lq}^{(1)}}$$

[Vogt, Moch, Soar, Vermaseren (2010)]

➡ modified coefficient matrix $\tilde{\mathbf{C}}$ & $\mathbf{K} = \left[\beta \frac{d\tilde{\mathbf{C}}}{da_s} \cdot \tilde{\mathbf{C}}^{-1} + \tilde{\mathbf{C}} \cdot \mathbf{P} \cdot \tilde{\mathbf{C}}^{-1}\right]$

➡ up to NLO both “schemes” with identical results

perturbative expansion: LO

$$\mathbf{K} = \begin{pmatrix} K_{22} & K_{2L} \\ K_{L2} & K_{LL} \end{pmatrix}$$

perturbative expansion

$$\mathbf{K} = a_s \mathbf{K}^{(0)} + a_s^2 \mathbf{K}^{(1)} \quad \mathbf{C} = \mathbf{C}^{(0)} + a_s \mathbf{C}^{(1)} + a_s^2 \mathbf{C}^{(2)} \quad \mathbf{P} = a_s \mathbf{P}^{(0)} + a_s^2 \mathbf{P}^{(1)}$$

LO kernels (using the quark convention)

$$K_{22}^{(0)} = P_{\rm qq}^{(0)} - \frac{C_{\rm Lq}^{(1)} P_{\rm qg}^{(0)}}{C_{\rm Lg}^{(1)}} \qquad K_{2L}^{(0)} = \frac{C_{\rm Lq}^{(1)} P_{\rm qg}^{(0)}}{C_{\rm Lg}^{(1)}}$$

$$K_{L2}^{(0)} = \frac{C_{\rm Lg}^{(1)} P_{\rm gq}^{(0)}}{C_{\rm Lq}^{(1)}} - \frac{C_{\rm Lq}^{(1)} P_{\rm qg}^{(0)}}{C_{\rm Lg}^{(1)}} - P_{\rm gg}^{(0)} + P_{\rm qq}^{(0)} \qquad K_{LL}^{(0)} = \frac{C_{\rm Lq}^{(1)} P_{\rm qg}^{(0)}}{C_{\rm Lg}^{(1)}} + P_{\rm gg}^{(0)}$$

LO solution

standard solution for $\partial_{\ln Q^2} \mathbf{F} = \mathbf{K} \cdot \mathbf{F}$:

$$\mathbf{F}(N, Q^2) = \mathbf{D}^{-1}(Q^2) \left[\left(\frac{a_s}{a_0} \right)^{\lambda_+(N)} e_+(N) + \left(\frac{a_s}{a_0} \right)^{\lambda_-(N)} e_-(N) \right] \mathbf{D}(Q_0^2) \cdot \mathbf{F}(N, Q_0^2)$$

$$a_s = a_s(Q^2) \qquad a_0 \equiv a_s(Q_0^2)$$

$\lambda_\pm(N)$ eigenvalues of the matrix $\mathbf{K}^{(0)}$, $e_\pm(N)$ projectors on eigenspaces
matrix $\mathbf{D}$ achieves rotation $\tilde{\mathbf{F}} = \mathbf{D} \cdot \mathbf{F}$

Observation:

- $\lambda_\pm(N)$ agree for $\mathbf{K}^{(0)}$ and $\mathbf{P}^{(0)}$
- eigenspaces differ, but final result for (F_2, F_L) agrees exactly!

→ at LO, physical and DGLAP evolution are the same, if $\mu_f^2 \equiv Q^2$

NLO Kernels obtained easily ....

for the 'quark' convention

$$K_{22}^{(1)} = -\frac{C_{2g}^{(1)} C_{Lq}^{(1)} P_{gg}^{(0)}}{C_{Lg}^{(1)}} + C_{2g}^{(1)} P_{gq}^{(0)} + \frac{\beta_0 C_{2g}^{(1)} {C_{Lq}}^{(1)}}{{C_{Lg}}^{(1)}}$$

$$- \frac{{C_{2g}}^{(1)} C_{Lq}^{(1)2} P_{qg}^{(0)}}{C_{Lg}^{(1)2}} + \frac{C_{2g}^{(1)} C_{Lq}^{(1)} P_{qq}^{(0)}}{C_{Lg}^{(1)}} - \beta_0 C_{2q}^{(1)}$$

$$+ \frac{C_{Lg}^{(2)} C_{Lq}^{(1)} P_{qg}^{(0)}}{C_{Lg}^{(1)2}} - \frac{C_{Lq}^{(1)} P_{qg}^{(1)}}{C_{Lg}^{(1)}} - \frac{C_{Lq}^{(2)} P_{qg}^{(0)}}{C_{Lg}^{(1)}} + P_{qq}^{(1)}$$

$$K_{2L}^{(1)} = \frac{C_{2g}^{(1)} C_{Lq}^{(1)} P_{gg}^{(0)}}{C_{Lg}^{(1)}} - \frac{\beta_0 C_{2g}^{(1)} C_{Lq}^{(1)}}{C_{Lg}^{(1)}} + \frac{C_{2g}^{(1)} C_{Lq}^{(1)2} P_{qg}^{(0)}}{C_{Lg}^{(1)2}}$$

$$- \frac{C_{2g}^{(1)} C_{Lq}^{(1)} P_{qq}^{(0)}}{C_{Lg}^{(1)}} + \frac{C_{2q}^{(1)} C_{Lq}^{(1)} P_{qg}^{(0)}}{C_{Lg}^{(1)}} - \frac{C_{Lg}^{(2)} C_{Lq}^{(1)} P_{qg}^{(0)}}{C_{Lg}^{(1)2}} + \frac{C_{Lq}^{(1)} P_{qg}^{(1)}}{C_{Lg}^{(1)}}$$

$$
\begin{aligned}
K^{(1)}_{L2} = & -\frac{C^{(1)}_{2g} C^{(1)}_{Lq} P^{(0)}_{gg}}{C^{(1)}_{Lg}} + C^{(1)}_{2g} P^{(0)}_{gq} - \frac{C^{(1)}_{2g} C^{(1)\,2}_{Lq} P^{(0)}_{qg}}{C^{(1)\,2}_{Lg}} + \frac{C^{(1)}_{2g} C^{(1)}_{Lq} P^{(0)}_{qq}}{C^{(1)}_{Lg}} \\
& + C^{(1)}_{2q} P^{(0)}_{gg} - \frac{C^{(1)}_{2q} C^{(1)}_{Lg} P^{(0)}_{gq}}{C^{(1)}_{Lq}} + \frac{C^{(1)}_{2q} C^{(1)}_{Lq} P^{(0)}_{qg}}{C^{(1)}_{Lg}} - C^{(1)}_{2q} P^{(0)}_{qq} - \frac{C^{(2)}_{Lq} P^{(0)}_{gg}}{C^{(1)}_{Lq}} \\
& + \frac{C^{(2)}_{Lg} P^{(0)}_{gq}}{C^{(1)}_{Lq}} + \frac{C^{(1)}_{Lg} P^{(1)}_{gq}}{C^{(1)}_{Lq}} + \frac{\beta_0 C^{(2)}_{Lg}}{C^{(1)}_{Lg}} + \frac{C^{(2)}_{Lg} C^{(1)}_{Lq} P^{(0)}_{qg}}{C^{(1)\,2}_{Lg}} - \frac{C^{(1)}_{Lq} P^{(1)}_{qg}}{C^{(1)}_{Lg}} \\
& - \frac{2 C^{(2)}_{Lq} P^{(0)}_{qg}}{C^{(1)}_{Lg}} - \frac{\beta_0 C^{(2)}_{Lq}}{C^{(1)}_{Lq}} + \frac{C^{(2)}_{Lq} P^{(0)}_{qq}}{C^{(1)}_{Lq}} - P^{(1)}_{gg} + P^{(1)}_{qq} \\
K^{(1)}_{LL} = & \frac{C^{(1)}_{2g} C^{(1)}_{Lq} P^{(0)}_{gg}}{C^{(1)}_{Lg}} - C^{(1)}_{2g} P^{(0)}_{gq} + \frac{C^{(1)}_{2g} C^{(1)\,2}_{Lq} P^{(0)}_{qg}}{C^{(1)\,2}_{Lg}} - \frac{C^{(1)}_{2g} C^{(1)}_{Lq} P^{(0)}_{qq}}{C^{(1)}_{Lg}} \\
& - \frac{\beta_0 C^{(2)}_{Lg}}{C^{(1)}_{Lg}} - \frac{C^{(2)}_{Lg} C^{(1)}_{Lq} P^{(0)}_{qg}}{C^{(1)\,2}_{Lg}} + \frac{C^{(1)}_{Lq} P^{(1)}_{qg}}{C^{(1)}_{Lg}} + \frac{C^{(2)}_{Lq} P^{(0)}_{qg}}{C^{(1)}_{Lg}} + P^{(1)}_{gg}
\end{aligned}
$$

NLO solution

no exact analytic solution at NLO, use solution by [Glück, Reya Vogt (1990)] , also used in PEGASUS [Vogt (2004)] :

$$\mathbf{F}(N, Q^2) = \mathbf{D}^{-1}\Big\{ \left(\frac{a_s}{a_0}\right)^{\lambda_+} \Big[e_+(N) + (a_0 - a_s)e_+ \cdot \mathbf{R}^{(1)} \cdot e_+ + \left[a_0 - a_s\left(\frac{a_s}{a_0}\right)^{\lambda_- - \lambda_+}\right] \frac{e_- \cdot \mathbf{R}^{(1)} \cdot e_-}{\lambda_+ - \lambda_- + 1}\Big] + (+) \leftrightarrow (-)\Big\} \mathbf{D} \cdot \mathbf{F}(N, Q_0^2)$$

NLO corrections contained in

$$\mathbf{R}^{(1)} = \frac{1}{\beta_0}\mathbf{K}^{(1)} - \frac{\beta_1}{\beta_0^2}\mathbf{K}^{(0)}$$

running coupling at NLO

$$\frac{da_s}{d\ln\mu^2} = -a_s^2\beta_0 - a_s^3\beta_1$$ – solved numerically

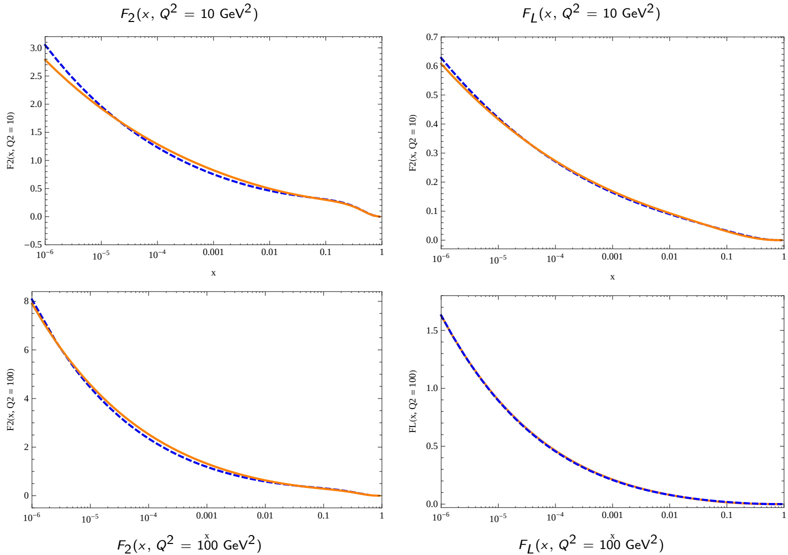
Numerical implementation

- parallel implementation for conventional 'coeff $\otimes$ pdf' and physical evolution kernels
- two independent codes (FORTRAN, MATHEMATICA) + pdf evolution cross-checked with PEGASUS [Vogt (2004)]
- use same input at $Q_0^2 = 2\text{GeV}^2$, given in terms of toy-pdfs (PEGASUS default intial parton distributions) with $n_f = 3$ and $\alpha_s(Q_0^2) = 0.35$

$$\begin{aligned}
xu_v(x, Q_0^2) &= 5.10722x^{0.8}(1-x)^3 \\
xd_V(x, Q_0^2) &= 3.064320x^{0.8}(1-x)^4 \\
xg(x, Q_0^2) &= 1.70000x^{-0.1}(1-x)^5 \\
x\bar{d}(x, Q_0^2) &= 0.1939875x^{-0.1}(1-x)^6 \\
x\bar{u}(x, Q_0^2) &= (1-x)x\bar{d}(x, Q_0^2) \\
xs(x, Q_0^2) &= x\bar{s}(x, Q_0^2) = 0.2(\bar{u}+\bar{d})(x, Q_0^2)
\end{aligned}$$

- $\mathbf{C}^{(0)}$, $\mathbf{C}^{(1)}$ and $\mathbf{P}^{(0)}$, $\mathbf{P}^{(1)}$ taken from [Floratos, Kounas, Lacaze (1981)], $C^{(2)}_{Lq,g}$ in the parametrized version of [van Neerven, Vogt (1999), (2000)]

The doublet (F_2, F_L) at NLO (with toy input)



physical kernels

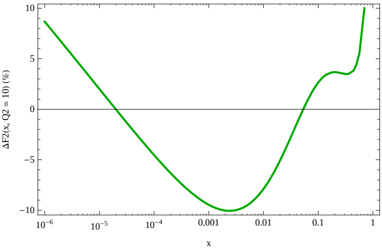
(**straight**)

coeff ⊗ pdf

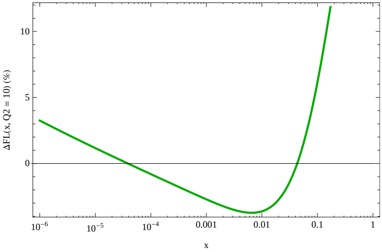
(**dashed**)

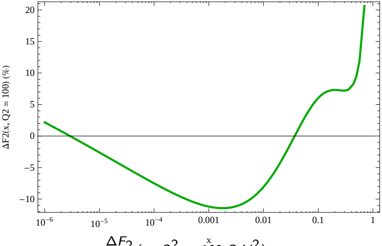
The doublet (F_2, F_L) at NLO – deviation in %

$\frac{\Delta F_2}{F_2}(x,\, Q^2 = 10 \text{ GeV}^2)$

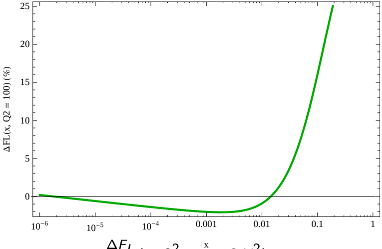


$\frac{\Delta F_L}{F_L}(x,\, Q^2 = 10 \text{ GeV}^2)$





$\frac{\Delta F_2}{F_2}(x,\, Q^2 = 100 \text{ GeV}^2)$



$\frac{\Delta F_L}{F_L}(x,\, Q^2 = 100 \text{ GeV}^2)$

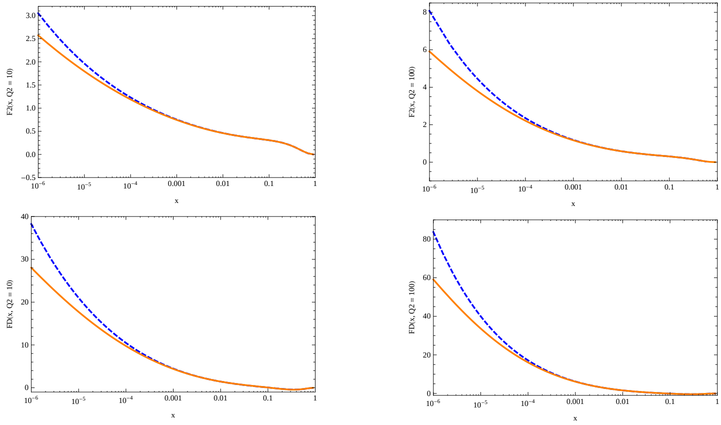
An alternative: the doublet (F_2, F_D)

scaling violations $F_D(x, Q^2) = -\frac{a_s \beta_0}{\beta} \frac{dF_2(x, Q^2)}{d \ln Q^2}$

- definition of F_D ➡ coefficent matrix $\begin{pmatrix} C_{2q} & C_{2g} \\ C_{Dq} & C_{Dg} \end{pmatrix}$
- physical evolution kernels analogous to doublet (F_2, F_L)
  [Furmanski, Petronzio; (1982)], [Blümlein, Ravindran, van Neerven (2000)].

The doublet (F_2, F_D) at NLO (with toy input)

$Q^2 = 10$ GeV^2 $\qquad$ $Q^2 = 100$ GeV^2



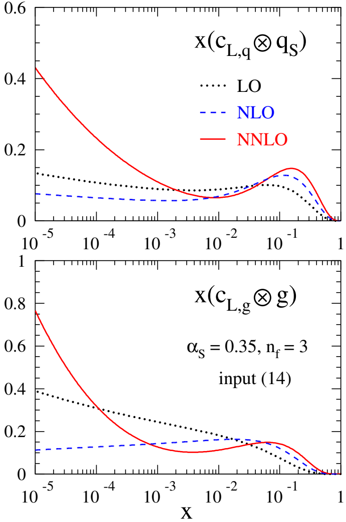
physical kernels

(**straight**)

coeff $\otimes$ pdf

(**dashed**)

Why are differences so large?



[Moch, Vogt, Vermaseren (2004)]

- coefficents $C^{(2)}_{L,qg}$ give large corrections ➡ rotated into evolution kernels
- similar: large scaling violations at small x

A closer look at the NLO solution

solution (schematic)

$$F = (1 + a_s C)\left(\frac{a_s}{a_0}\right)^{R^{(0)}}\left[\text{"}1\text{"} + (a_0 - a_s)R_{\mathbf{P}}^{(1)}\right]f(Q_0^2) \quad \text{convent. DGLAP "P"}$$

$$F = \left(\frac{a_s}{a_0}\right)^{R^{(0)}}\left[\text{"}1\text{"} + (a_0 - a_s)R_{\mathbf{K}}^{(1)}\right](1 + a_0 C)f(Q_0^2) \quad \text{phys. evolution "K"}$$

contains spurious NNLO terms

- convent. DGLAP: $\mathcal{O}(a_s a_0)$, $\mathcal{O}(a_s^2)$ – final scale Q^2
- phys. evolution: $\mathcal{O}(a_s a_0)$, $\mathcal{O}(a_0^2)$ – intital scale Q_0^2

The “higer order zero” (HOzero) solution

an old idea: possible to avoid terms beyond *NLO* entirely

at least for the theoretical study, evolution of experimental data can be more involved

schematic:

$$F^{\text{convent.}}_{\text{HOzero}} = \left(\frac{a_s}{a_0}\right)^{R^{(0)}} \left["1" + (a_0 - a_s) R^{(1)}_{\mathbf{P}} + a_s C \right] f(Q_0^2)$$

$$F^{\text{phys. evolv.}}_{\text{HOzero}} = \left(\frac{a_s}{a_0}\right)^{R^{(0)}} \left["1" + (a_0 - a_s) R^{(1)}_{\mathbf{K}} + a_0 C \right] f(Q_0^2)$$

both solutions agree exactly over the entire phase space (for $\mu_f^2 = Q^2$)

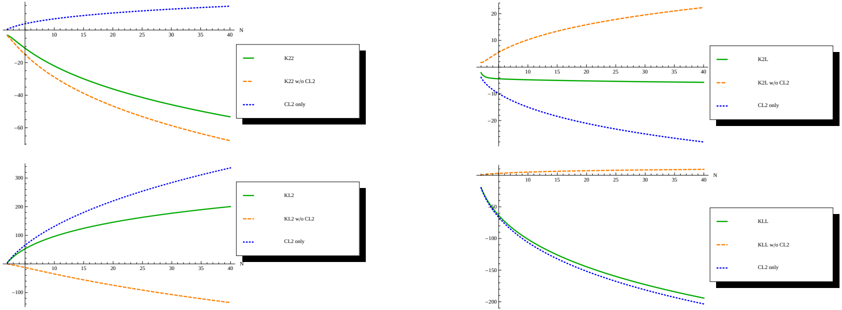
→ no intrinsic problem, but in certain region of phase space higher order corrections are sizeable

Summary and Conclusions

numerical study of physical anomalous dimensions

- use small x approximation ➞ (F_2, F_L)
- at LO identical to conventional DGLAP ‘coeff. $\otimes$ pdf’ (for $\mu_f^2 = Q^2$)
- at NLO, ‘quark’ and ‘gluon’ convention for $\tilde{F}_L$ give same result
- at NLO, difference between ‘coeff. $\otimes$ pdf’ and physical kernels sizeable
  (F_2, F_L) over the full phase space, (F_2, F_D) particularly the small x region
- exact equivalence (for $\mu_f^2 = Q^2$) for “higher order zero” solution

Why is this difference so large?



- coefficents $C^{(2)}_{L,qg}$ are large corrections ➡ dominate the NLO evolution kernels
- can one control this ...?