TMD quark distributions at small x

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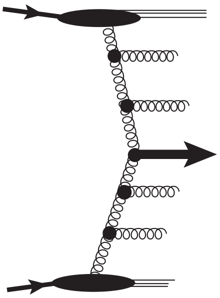
Why Transverse Momentum Dependent (TMD) pdfs?

p_T distribution of produced *e.g.* Z boson



conventional approach:

- collinear factorization, M^2 hard scale

$$d\sigma\left(\frac{s}{M^2}\right) = f(x_1,\mu_f^2)\otimes f(x_2,\mu_f^2)\otimes d\hat{\sigma}\left(\frac{x_1 x_2 s}{M^2}\right)$$

- strong ordering of transverse scales

LO partonic X-sec. $\hat{\sigma}$: $\frac{\mathbf{q}^2}{M^2} \to 0$

- correct kinematics ➡ higher order

goal of TMD pdfs: correct kinematics already at LO

- reduce size of higher order corrections

TMD parton distributions at small x

natural definition in high energy limit:

$$s \gg M_Z^2 \gg \Lambda_{\mathrm{QCD}}^2$$

perturbative QCD amplitudes: TM convolution (up to NLL)

$$\sigma_{a,b}(s, Q^2, Q_0^2) = \int d^2\mathbf{q} d^2\mathbf{p}\, \phi_a(\mathbf{q}, Q^2)\, G_{\mathrm{BFKL}}(s, \mathbf{p}, \mathbf{p})\, \phi_b(\mathbf{p}, Q_0^2)$$

Limit $Q^2 \gg Q_0^2$: natural starting point for definition of TMD distrubtions

caveats:

- t-channel purely gluonic
- plus momentum not conserved (strong ordering)
- resummation of $\ln s$, collinear $\ln \frac{Q^2}{Q_0^2}$ subleading

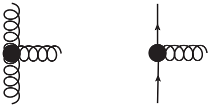
CCFM evolution and quark emission

- (partial) remedy: CCFM evolution ➞ resums both soft and small x logarithms + associated coherence effects
- TMD pdf required for coherence at $z \to 0$ [Ciafaloni (1988), (1998)])
- for inclusive observables: interpolation between DGLAP and BFKL
- realized in Monte Carlo event generator CASCADE [Jung, Salam (2001)]; [Jung et.al. (2010)]

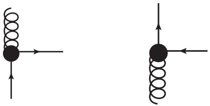
CCFM evolution based on coherence

➞ emissions **gluonic**

(LL)



gluon and valence quark naturally defined



no non-diagonal transitions

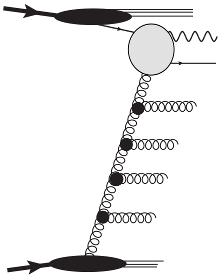
Consequences

parton shower (CASCADE)

- only gluonic emissions, no quarks
  ➡ jets purely gluonic

hard process:

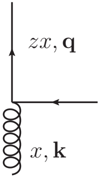
- quark $\equiv$ valence quark, seaquark $\mathcal{O}(\alpha_s, \alpha_s^2)$ etc.
- lose advantages of TMD approach



Goal of this study

Construct sea quark density
At first: quark from last splitting, on top of small x gluon

Current state of the art



- LO DGLAP splitting function $P_{qg}(z)$ on top of TMD gluon distribution [Gawron, Kwiecinski, Broniowski (2003)]; [Hoeche, Krauss, Teubner (2008)]
- quark and gluon TMD with last DGLAP evolution step unintegrated from collinear pdfs [Martin, Ryskin, Watt (2003, 2010)]

- correct collinear limit, works well; but miss corrections $\frac{\mathbf{k}^2}{\mathbf{q}^2}$
  → quark transverse momentum q_T: a 1-loop effect

The TMD splitting function

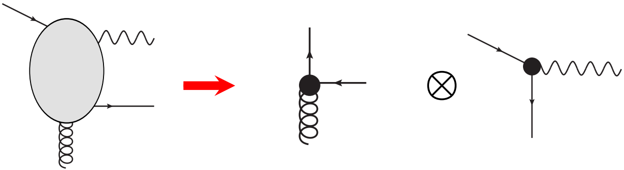
[Catani, Hautmann, (1994)]

$$P_{qg}^{\mathrm{CH}} = T_R \left(\frac{(\mathbf{q} - z\mathbf{k})^2}{(\mathbf{q} - z\mathbf{k})^2 + z(1-z)\mathbf{k}^2} \right)^2 \left[P_{qg}(z) + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{(\mathbf{q} - z\mathbf{k})^2} \right]$$

- through matching of high energy resummation on collinear factorization
  ➡ small x enhanced collinear logarithms included to all orders
- corrections $\frac{\mathbf{k}^2}{\mathbf{q}^2}$ to all orders included ➡ quark q_T follows from multiple small x enhanced branchings
- universal despite of off-shellness
  [Catani, Hautmann (1994)]; [Ciafaloni, Colferai (2005)]

Factorization of the partonic process

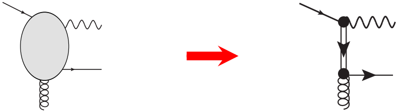
- Consider specific $2 \to 2$ process $qg^* \to Zq$ (forward Z)



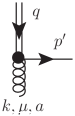
- forward Z: incoming quark valence like ➡ simplifies treatment
- Strategy: mimic gluonic case ➡ use factorization of $qg^* \to Zq$ in high energy limit

High energy limit for quark exchange: 'reggeized quark' formalism

- use formulations by [Bodgan, Fadin, (2005)],[Lipatov, Vyazosky (2001)], see also studies by [Saleev (2008)]; [Kniehl, Saleev, Shipilova (2008)]



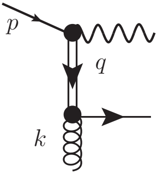
- achieve gauge invariant off-shell factorization through effective vertices $\equiv$ re-organization of QCD diagrams



$$= igt^a \left(\gamma^\mu - \not{q} \frac{(n^+)^\mu}{k^+} \right) \qquad \text{etc.}$$

- agree for on-shell quarks with QCD vertex, extra term provides gauge invariance $\rightarrow$ from expansion of Wilson line

Improved kinematics



Reggeized quark formalism: $\equiv$ high energy factorization

- strong ordering $\bar{z} = \frac{q^+}{p^+} \to 0, \;\; z = \frac{q^-}{k^-} \to 0$
- at finite energies: rough approximation, splitting function a constant

Observation: can relax ordering, while keeping gauge invariance (current conservation)

- relax ordering in $(-)$ momenta: recover TMD splitting function [Catani, Hautmann (1994)]
- relax ordering in $(+)$ momenta: correct t-channel virtuality
- note [Martin, Ryskin, Watt (2010)]: NLO DGLAP corrections with LO splitting kernel through correct virtuality alone

q_T versus q factorization

- $qg^* \to qZ$ factorized as convolution w.r.t $\mathbf{q}^2$ or $|q^2|$ ➡ different virtuality of the off-shell quark
- both imply approximation on transverse momentum
- Constraint $\mu_f^2 > q^2$ from collinear factorization [Catani, Hautmann (1994)]

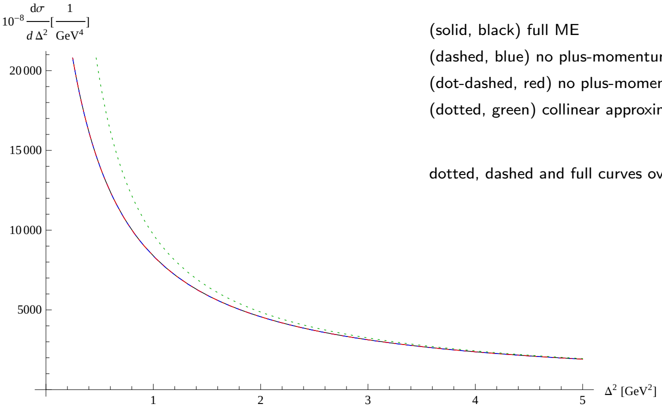
$$\sigma^{q_T-\text{fact.}}_{qg^*\to Zq} = \int_0^1 dz \int_0^{(1-z)(\mu^2 - z\mathbf{k}^2)} \frac{d\mathbf{\Delta}^2}{\mathbf{\Delta}^2}\,\hat{\sigma}_{qq^*\to Z}(z\nu, \mathbf{\Delta}^2)\cdot\frac{\alpha_s}{2\pi}P_{qg}\left(z, \frac{\mathbf{k}^2}{\mathbf{\Delta}^2}\right)$$

$$\sigma^{q-\text{fact.}}_{qg^*\to Zq} = \int_0^1 dz \int_{z\mathbf{k}^2}^{\mu^2} \frac{d|q^2|}{|q^2| - z\mathbf{k}^2}\,\hat{\sigma}_{qq^*\to Z}(z\nu, |q^2|)$$

$$\cdot\frac{\alpha_s}{2\pi}P_{qg}\left(z, \frac{\mathbf{k}^2}{(1-z)(|q^2| - z\mathbf{k}^2)}\right)$$

DGLAP versus q_T versus q versus exact - small $\mathbf{\Delta}^2$

study $\frac{d\sigma}{d\mathbf{\Delta}^2} \sim t$-channel virtuality; external: $x_1 x_2 s = 2.5 M_Z^2, \mathbf{k}^2 = 2\ \mathrm{GeV}^2$,



(solid, black) full ME

(dashed, blue) no plus-momentum ordering;

(dot-dashed, red) no plus-momentum and minus-momentum ordering;

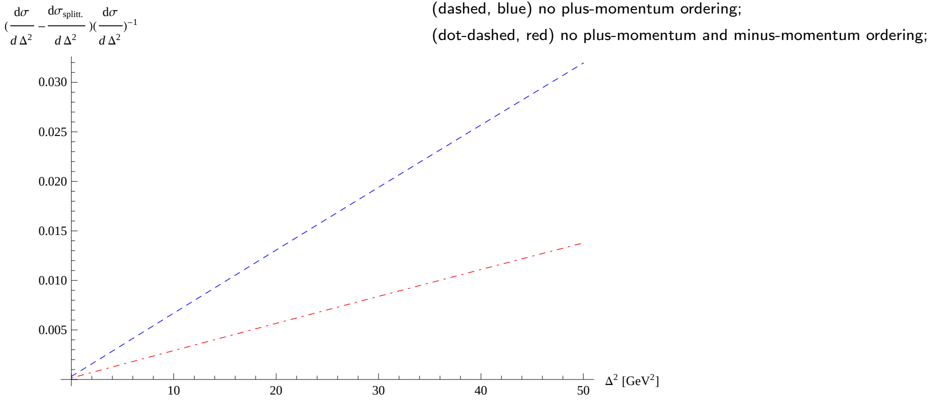
(dotted, green) collinear approximation.

dotted, dashed and full curves overlap

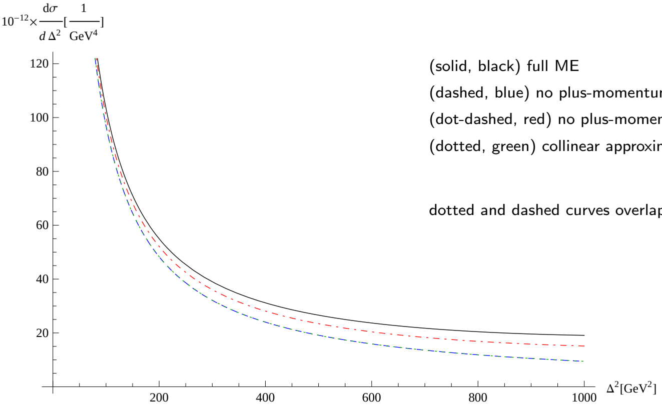
➡ only collinear approximation deviates for $\mathbf{\Delta}^2 \leq \mathbf{k}^2$

q_T versus q versus exact - intermediate Δ^2

absolute difference small, relative difference grows



DGLAP versus q_T versus q versus exact - large $\Delta^2 \sim M_Z^2$



(solid, black) full ME

(dashed, blue) no plus-momentum ordering;

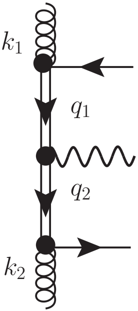
(dot-dashed, red) no plus-momentum and minus-momentum ordering;

(dotted, green) collinear approximation.

dotted and dashed curves overlap

→ exact kinematics becomes significant at finite $x_1 x_2 s$

Outlook: double off-shell coefficient $q^* q^* \to Z$



- Extend to central production and determine $q^* q^* \to Z$ coefficient
- to be used with sea- and valence quark (CCFM-parametrization) on equal footing
- again q_T (see also [Nefedov, Nikolaev, Saleev (2012)]) and q factorized expression
- new: combination with TMD splitting function

$$\hat{\sigma}^{q_T}_{q^* q^* \to Z} = \sigma_0 \frac{M_Z^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2}{M_Z^2 + (\mathbf{q}_1 + \mathbf{q}_2)^2} \delta\left(x_1 x_2 s - (\mathbf{q}_1 + \mathbf{q}_2)^2 - M_Z^2 \right),$$

$$\hat{\sigma}^{q}_{q^* q^* \to Z} = \sigma_0 \frac{s x_1 x_2 (\mathbf{q}_1^2 + \mathbf{q}_2^2 + M_Z^2)}{(\mathbf{q}_1^2 + x_1 x_2 s + q_1^2)(\mathbf{q}_2^2 + x_1 x_2 s + q_2^2)} \delta\left((q_1 + q_2)^2 - M_Z^2 \right)$$

Summary

- Current small-x parton showers include only gluon & valence quarks at TMD level
- Here: go beyond this approximation by including TMD sea-quark
- The presented method includes finite-k_T terms in the gluon - quark splitting P_{qg}, which control resummation of $\alpha_s(\alpha_s \ln 1/x)^n$ corrections to flavor-singlet observables
- We obtained an off-shell (but gauge-invariant) hard matrix element for coupling Z production to the TMD sea-quark using the "reggeized quark" formalism [Bodgan, Fadin (2005)],[Lipatov, Vyazosky (2001)]

Conclusions

Current results are based on: – 1 quark shower interaction only
– 1 off-shell quark only

➡ this needs extending; nevertheless, it is starting point to include systematically quark-initiated processes in small-x showers

➡ hopefully achieve more general defs of TMD pdfs: e.g., match on to SCET definitions [Stewart, Tackmann, Waalewijn (2009)]; [Garcia-Echevarria, Idilbi, Scimemi (2011)]; [Becher, Neubert (2011)]; [Mantry, Petriello (2011)] and TMD evolution and fits [Aybat, Rogers, 2011]

interesting study to compare with [Chiu, Jain, Neill, Rothstein (2012)]