Saturation, coherence and exclusive final states

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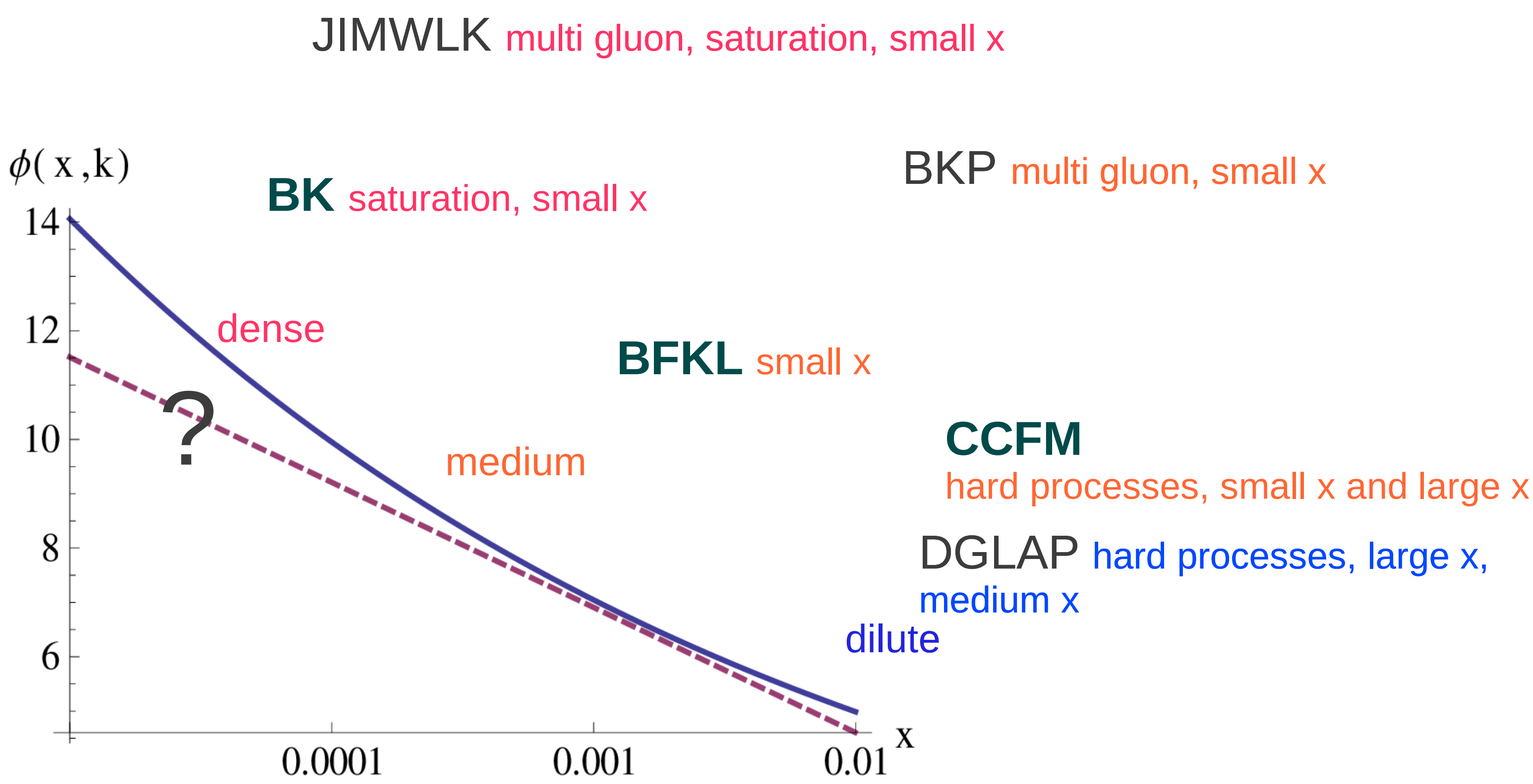
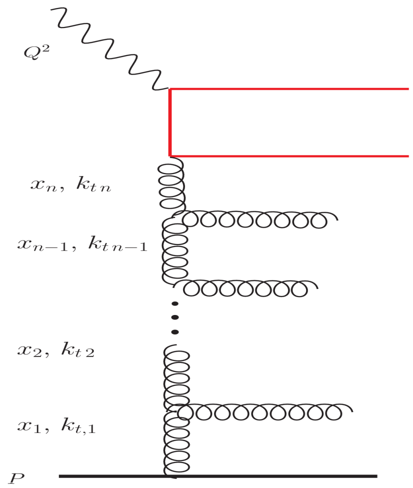
Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011

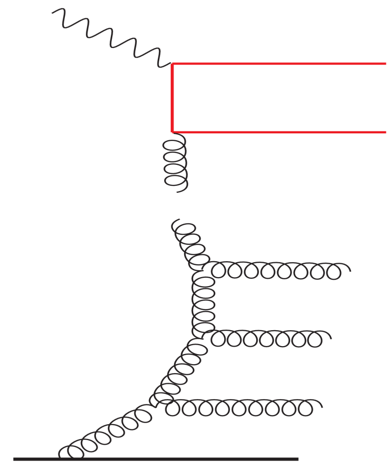
The motivation – to understand gluon at low x



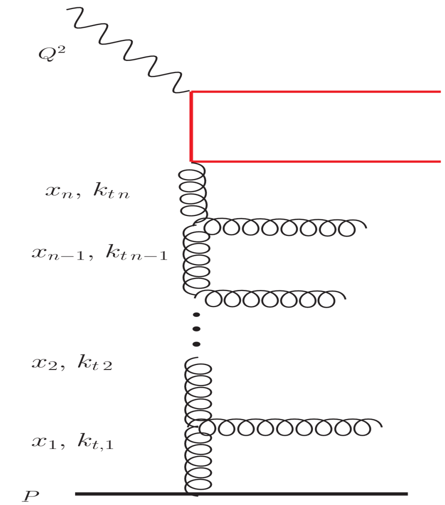
Schematic illustration of evolution scheems in pQCD



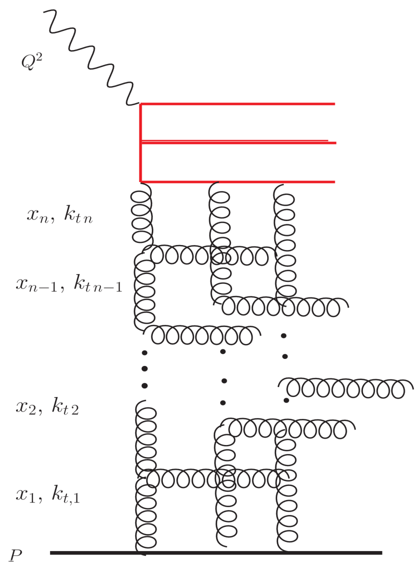
DGLAP (ordering in hard scale)



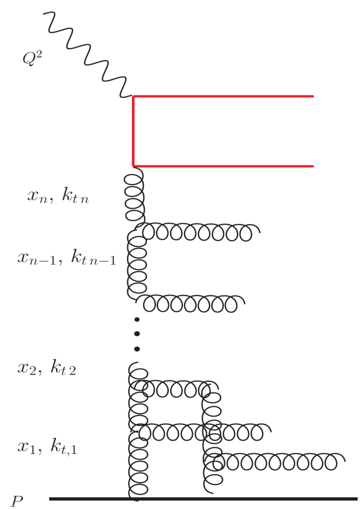
CCFM (ordering in angle)



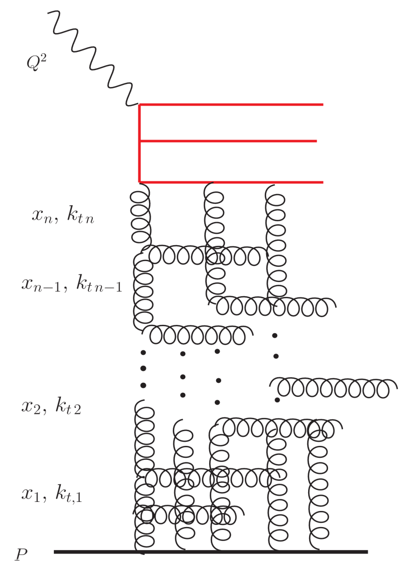
BFKL (ordering in x)



BKP (multi gluons ord. In x)

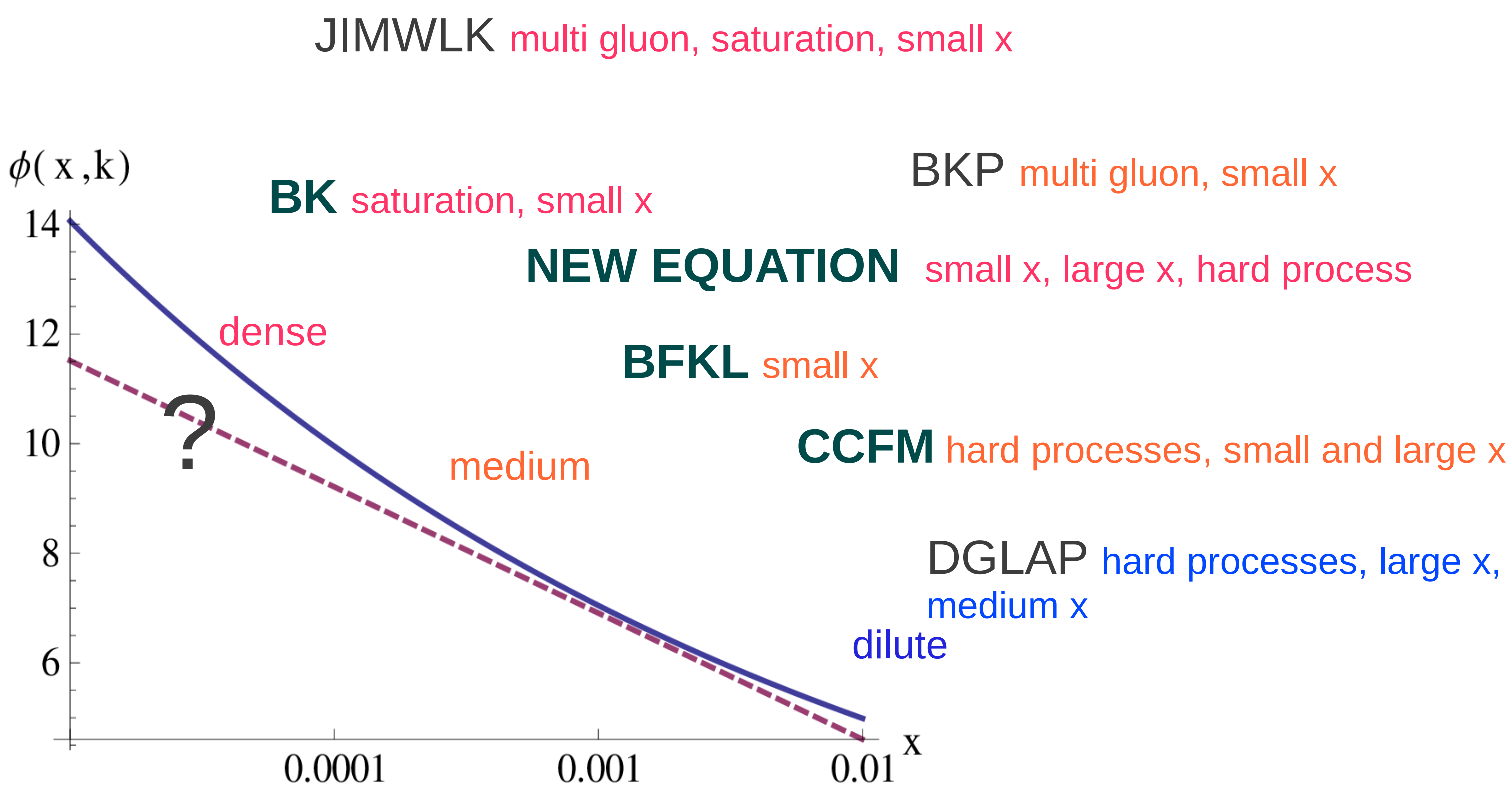


BK (ordering in x, fusion)

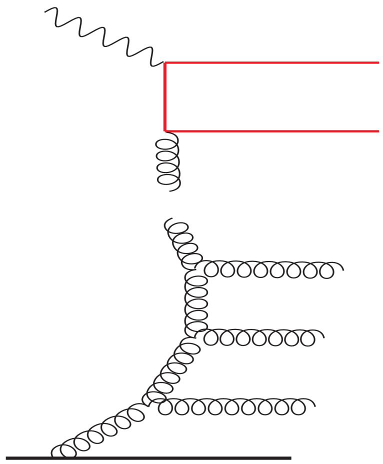


JIMWLK (multigluon,fusion)

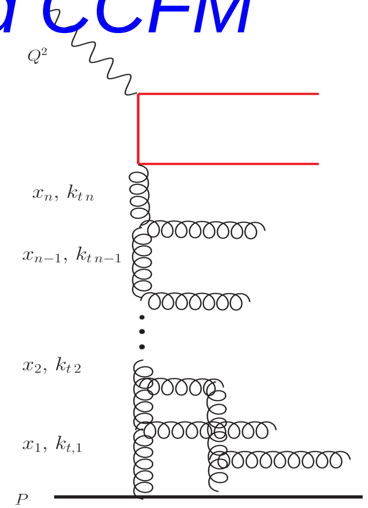
The motivation – to understand gluon at low x with the help of exclusive processes



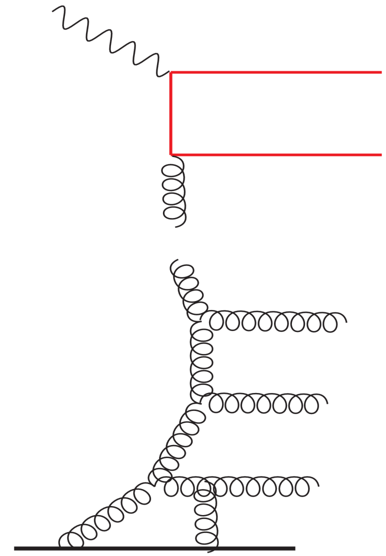
Schematic illustration of proposed recently new equation combining BK and CCFM



CCFM



BK



New equation

QCD at high energies – high energy factorization

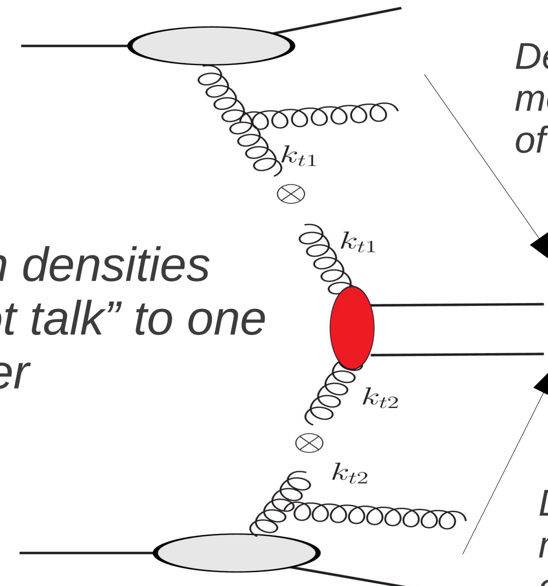
$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2S)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t})$$
$$\times\, \mathcal{F}_{a/A}(x_1,k_{1t}^2,\mu^2)\,\mathcal{F}_{b/B}(x_2,k_{2t}^2,\mu^2)\,\frac{1}{1+\delta_{cd}}$$

$$k_1^\mu = x_1P_1^\mu + \bar{x}_1P_2^\mu + k_{1t}^\mu \qquad k_2^\mu = x_2P_2^\mu + \bar{x}_2P_1^\mu + k_{2t}^\mu$$

$$\bar{x}_1 = \frac{k_1^2+\mathbf{k}^2}{Sx_1} \qquad \bar{x}_2 = \frac{k_2^2+\mathbf{k}^2}{Sx_2}$$

$$|\mathcal{M}_{ab\to cd}|^2 = \frac{2x_1k_1^{\mu_1}k_1^{\nu_1}}{k_1^2}\frac{2x_2k_2^{\mu_2}k_2^{\nu_2}}{k_2^2}\mathcal{M}_{ab\to cd\,\mu_1\nu_1}\mathcal{M}^*_{ab\to cd\,\mu_2\nu_2}$$

Parton densities "do not talk" to one another



Decreasing longitudianl momentum fractions of off-shell partons

Decreasing longitudianl momentum fractions of off-shell partons

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

Originally derived for quarks in final state.
Lipatov provided general framework.

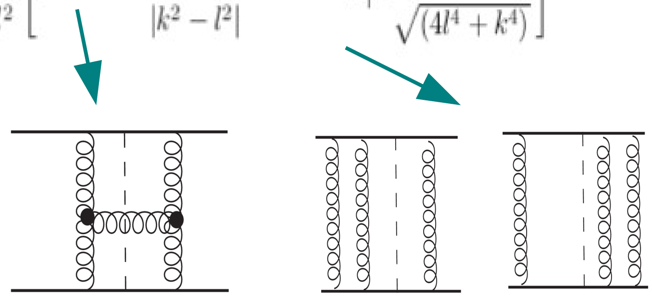
Recently new approach consistent with Lipatov's action allowed for formulation and numerical calculation of any tree level amplitude with off-shell gluons in initial state
Van Hameren, Kotko, KK '12

Generalized to p-A
Dominguez, Huan, Marquet,Xiao '10

o be be presented by A. van Hameren

The BFKL evolution - solution

$$\mathcal{F}(x,k^2) = \mathcal{F}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}(x/z,l^2) - k^2\mathcal{F}(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\mathcal{F}(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]$$



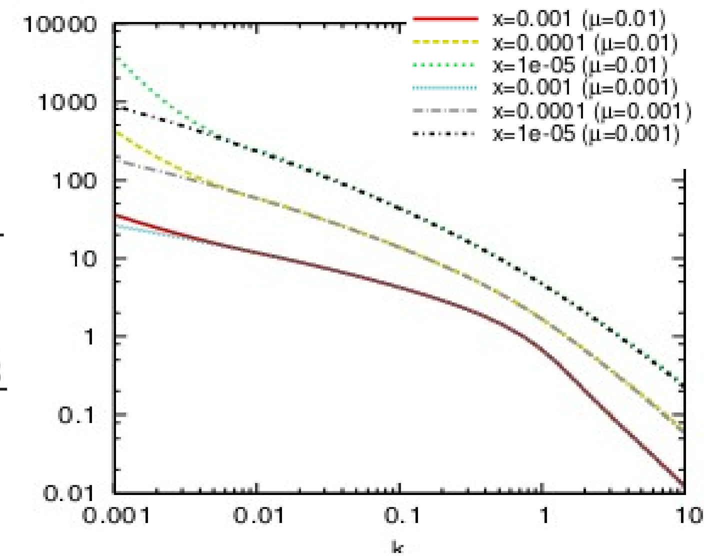
$$\mathcal{F}(x,k^2) \equiv \frac{f(x,k^2)}{k^2}$$

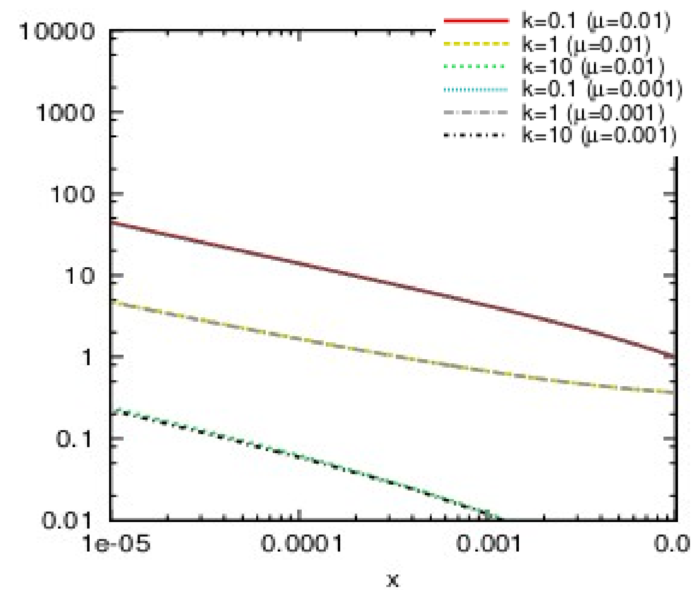
$$f(x,k^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\gamma (k^2)^\gamma \left(\frac{x}{x_0}\right)^{-K(\gamma)}$$

$$f(x,\gamma) = \int_1^\infty dk^2 (k^2)^{-\gamma-1} f(x,k^2) \qquad K(\gamma) = \frac{N_c\alpha_s}{\pi}[2\psi(1) - \psi(\gamma) - \psi(1-\gamma)]$$

$$\mathcal{F}(x,k^2) = \frac{x^{-\lambda}}{k^2} \qquad \alpha_P(0) = 1 + \lambda$$

$$\lambda = 0.5$$

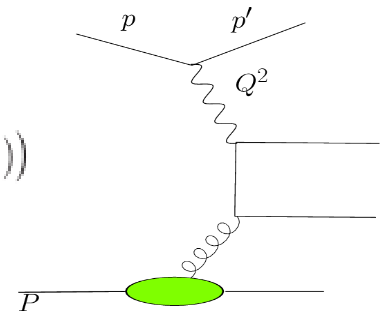


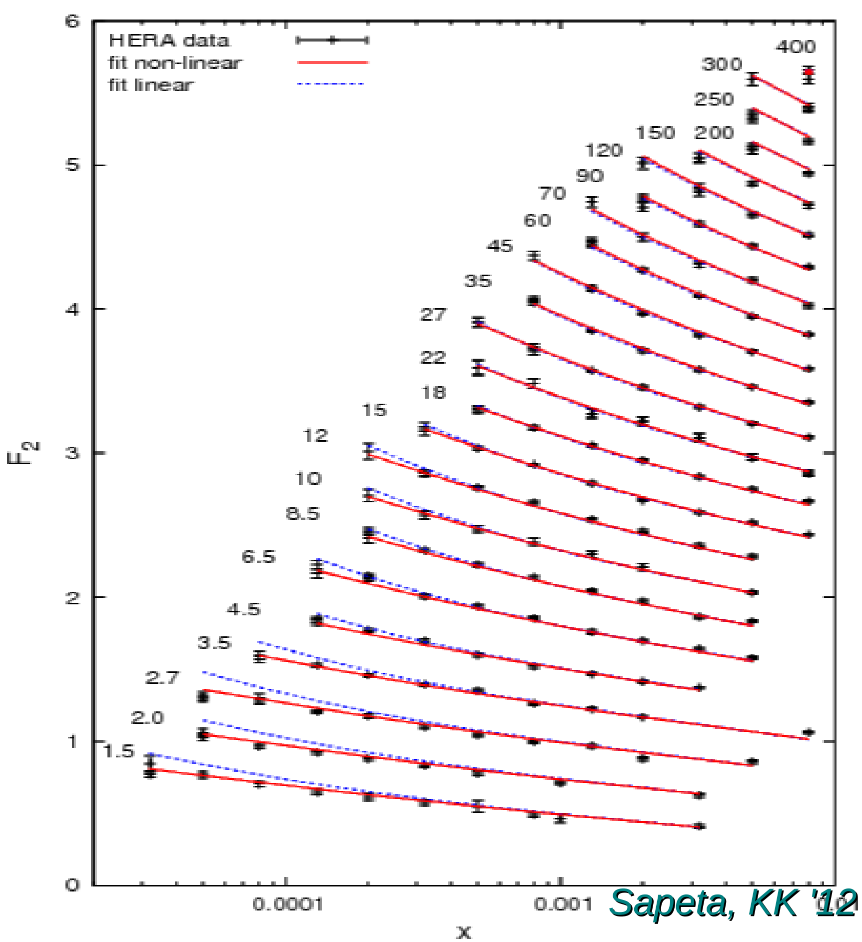


Common work with postdoc Dawid Toton

BFKL applied to DIS - some recent results

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2}\alpha_s \sum_q e_q^2 \int d^2k\, \mathcal{F}(x,k^2)\left(S_L(k^2,Q^2,m_q^2) + S_T(k^2,Q^2,m_q^2)\right)$$





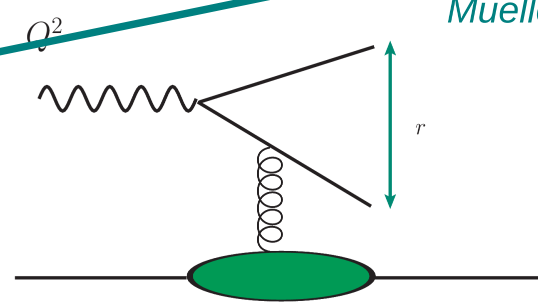
Sapeta, KK '12

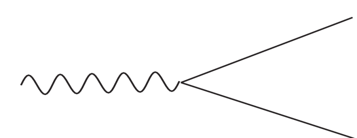
In the dipole formalism

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2b \int_0^1 dz \int d^2r \left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right) N(x,r,b)$$

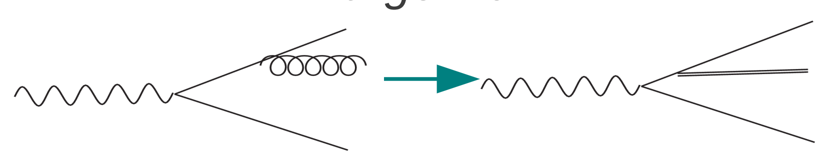
Mueller, Patel '95

Forward scattering amplitude





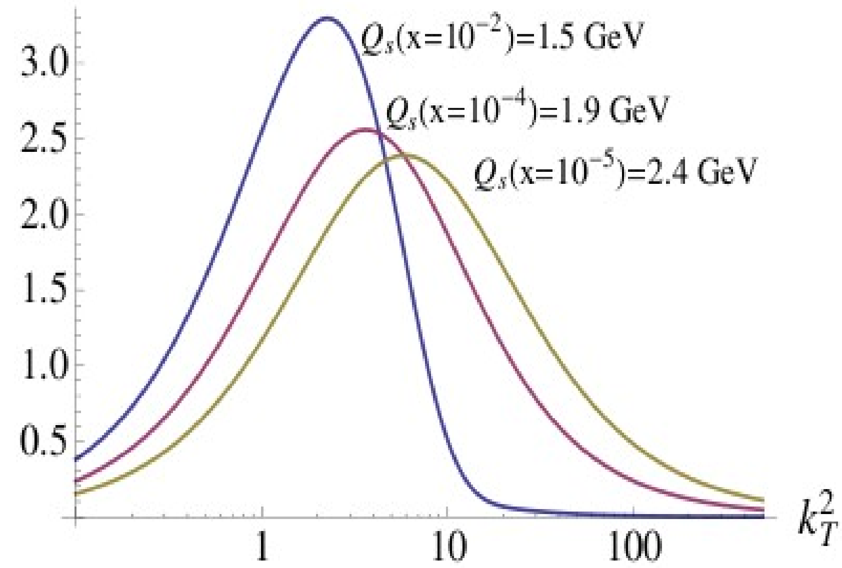
Large Nc



$$\mathcal{F}(x,k^2) = \frac{N_c}{\alpha_s(2\pi)^3} \int d^2b \int d^2r\, e^{ik\cdot r} \nabla_r^2 N(r,b,x)$$

High energy factorizable gluon density with saturation

$$\mathcal{F}(x,k^2) = \frac{N_c}{\alpha_s(2\pi)^3}\int d^2b \int d^2r\, e^{ik\cdot r}\nabla_r^2 N(r,b,x)$$



$$\mathcal{F}(x,k^2) = \mathcal{F}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}(x/z,l^2) - k^2\mathcal{F}(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\mathcal{F}(x/z,k}{\sqrt{(4l^4+k}}\right.$$

$$-\frac{2\alpha_s^2\pi}{N_c R^2}\int_{x/x_0}^1 \frac{dz}{z}\left\{\left[\int_{k^2}^\infty \frac{dl^2}{l^2}\mathcal{F}(x/z,l^2)\right]^2 + \mathcal{F}(x/z,k^2)\int_{k^2}^\infty \frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x/z,l\right.$$

target's radius

$$\Phi_b(x,k^2) = \Phi_{0\,b}(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\Phi_b(x/z,l^2) - k^2\Phi_b(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi_b(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right] - \overline{\alpha}_s\int_{x/x_0}^1 \frac{dz}{z}\Phi_b^2(x/z,k^2)$$

Kwiecinski, KK '03
Nikolaev, Schaffer '04

Balitsky 96, Kovchegov 99

$$\Phi_b(x,k^2) = \Phi(x,k^2)S(b) \qquad \int d^2\mathbf{b}\, S(b) = 1, \quad \int d^2b\, S^2(b) = \frac{1}{\pi R^2}$$

Interesting properties
travelling wave solution
Munier, Peschanski '03

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right] - \frac{\overline{\alpha}_s}{\pi R^2}\int_{x/x_0}^1 \frac{dz}{z}\Phi^2(x/z,k^2)$$

$$\mathcal{F}(x,k^2) = \frac{N_c}{4\alpha_s\pi^2}k^2\nabla_k^2\Phi(x,k^2)$$

inversion

$$\Phi(x,k^2) = \frac{\alpha_s\pi^2}{N_c}\int_{k^2}^\infty \frac{dl^2}{l^2}\ln\frac{l^2}{k^2}\mathcal{F}(x,l^2)$$

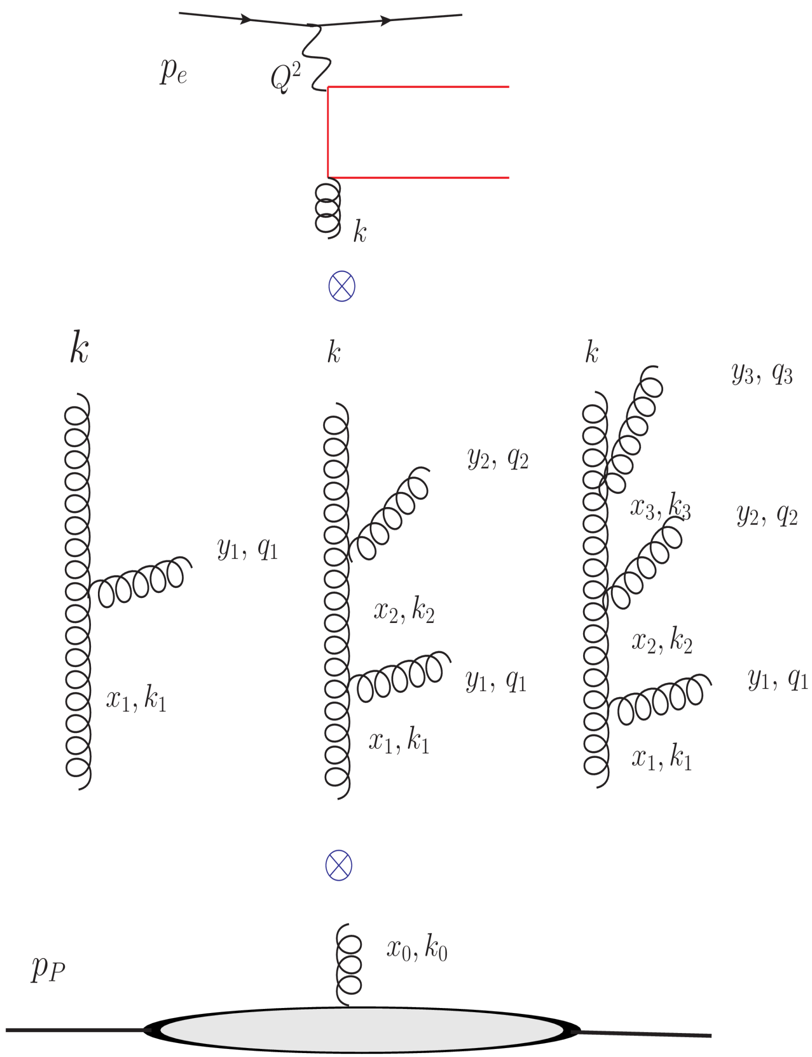
density potential

Unique transformationpossible due to color transparency of the dipole amplitude

CCFM evolution equation - evolution with observer

Catani,Ciafaloni,Fiorani Marchesin '88

Recent review: *Avsar, Iancu '09*



In x → 1region where emitted gluons are soft the dominant contribution to the amplitude comes from the angular ordered region.

$$\bar{\xi} > \xi_i > \xi_{i-1} > \ldots > \xi_1 > \xi_0$$

The same structure for x → 0 although the softest emitted gluons are harder than internal.

$$q_i = \alpha_i\, p_P + \beta_i\, p_e + q_{t\,i} \qquad s = (p_P + p_e)^2$$

$$\eta_i = \frac{1}{2}\ln(\xi_i) \equiv \frac{1}{2}\ln\left(\frac{\beta_i}{\alpha_i}\right) = \ln\left(\frac{|\mathbf{q}_i|}{\sqrt{s}\,\alpha_i}\right) \qquad \tan\frac{\theta_i}{2} = \frac{|\mathbf{q}_i|}{\sqrt{s}\,\alpha_i}$$

$$\bar{\xi} = p^2/(x^2 s)$$

$$z_i = x_i/x_{i-1}$$

$$dP_i^{\theta} = \frac{\alpha_S}{2\pi} dz_i \frac{d^2 q_i}{q_i^2} P_{gg}(z_i)\theta(q_i - z_{i-1}q_{i-1})(1 - z_i)$$

Implemented in CASCADE Monte Carlo **H. Jung 02**
New program CohRad developed by
Magdalena Slawinska at al

CCFM evolution equation - evolution with observer

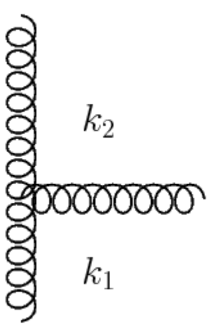
$$\mathcal{A}(x,k^2,p) = \mathcal{A}(x,k^2,p) + \bar{\alpha}_s \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \int_x^{1-Q_0/|\bar{q}|} dz\, \theta(p - z\bar{q}) P_{gg}(z,k^2,\bar{q}) \mathcal{A}(x/z, k', \bar{q})$$

In DIS $p^2 = \frac{Q^2}{z(1-z)}$ $\bar{q} = q/(1-z)$

$$P_{gg}(z,k^2,p) = \frac{\alpha_S}{2\pi} 2C_A\, \Delta_S(zq,p) \left(\frac{\Delta_{NS}(z,q,k^2)}{z} + \frac{1}{1-z} \right),$$

non-eikonal emission eikonal emission

regulates 1/(1-z) regulates 1/z



~ 1/z+1/(1-z)

$$\Delta_{ns}(z_i, q_i, k_i) = \exp\left(- \int_{z_i}^{1} dz' \frac{\bar{\alpha}_s}{z'} \int \frac{dq'^2}{q'^2} \theta(k_i - q')\theta(q' - z' q_i) \right)$$

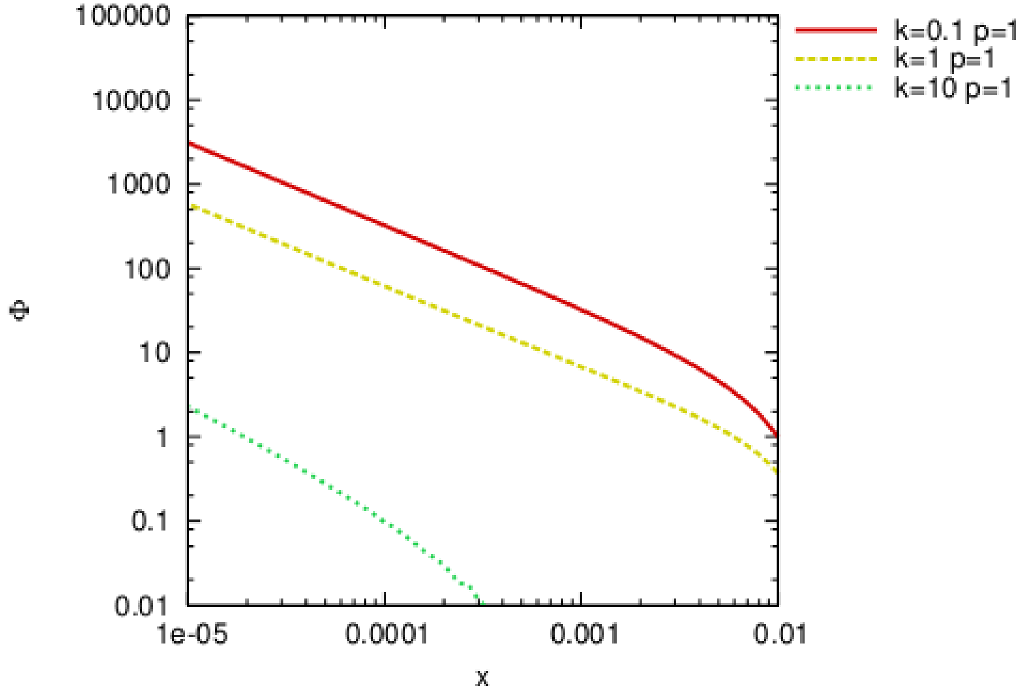
No emission of gluons with $x' = z' x_{i-1}$

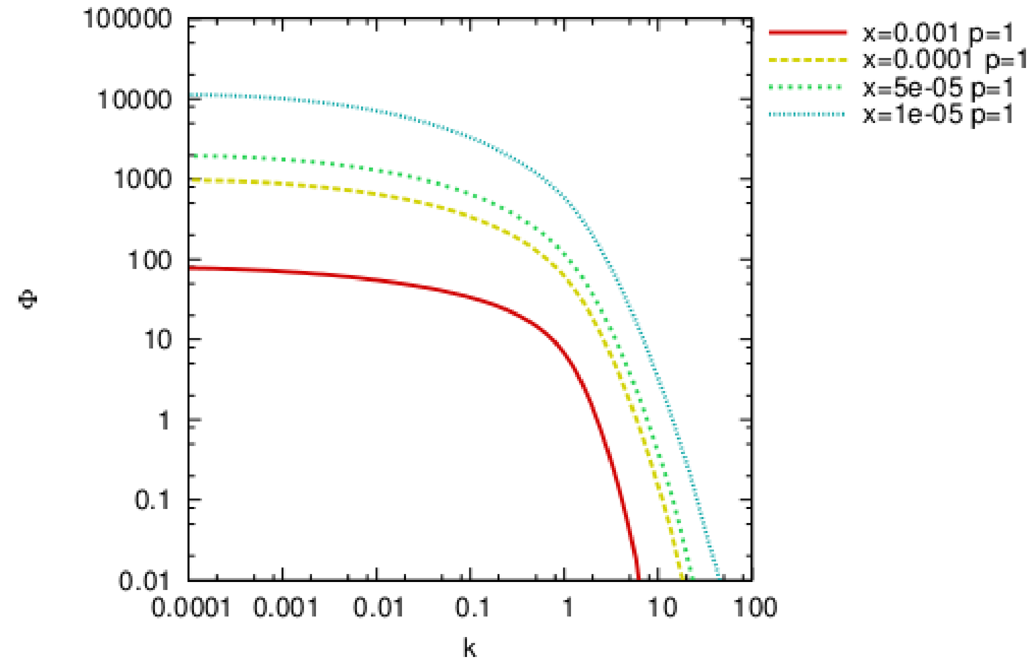
in region $x_i < x' < x_{i-1}$

and with momentum q' smaller than k_i

and with angle $\theta' > \theta_i$

Solution of the CCFM equation

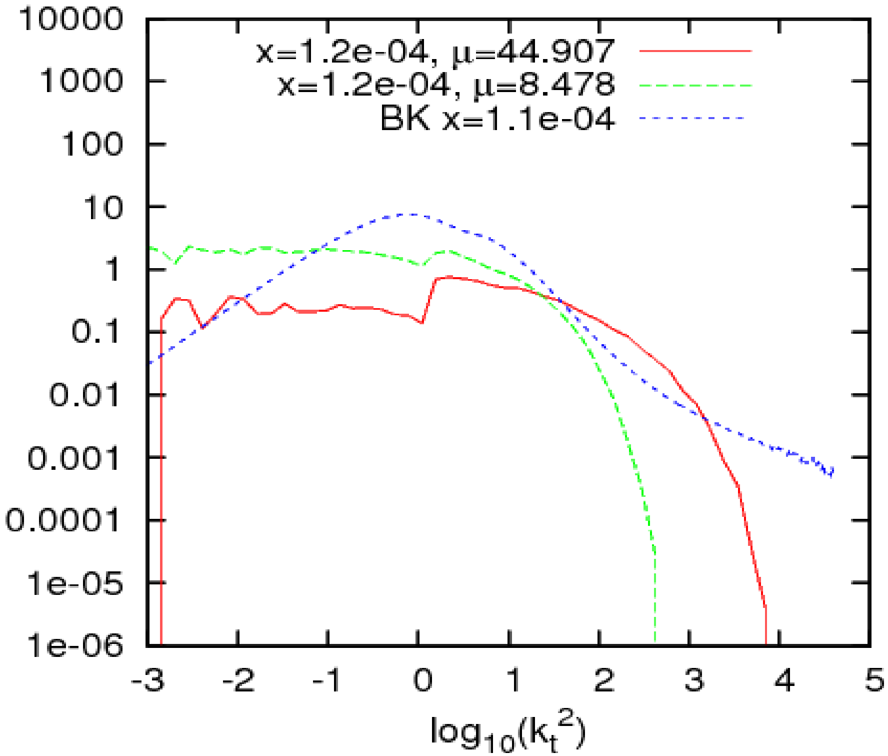


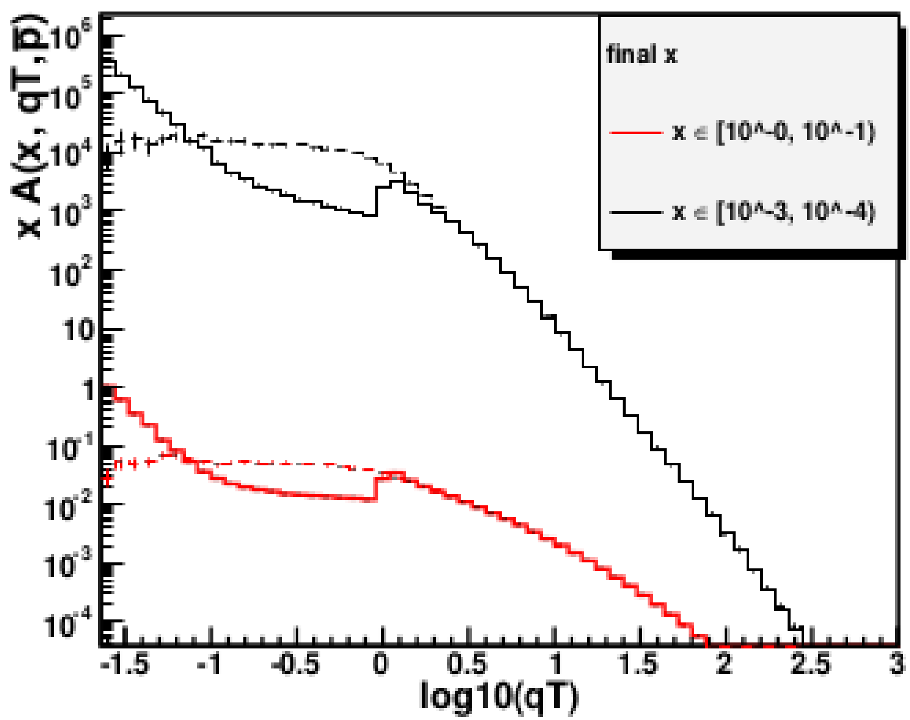


Common work with postdoc
Dawid Toton

To simplify numerics only non-Sudakov
and 1/z pole.

Forward physics as the way to constrain gluon both at large and small pt





Slawinska, Jadach, KK,Phys. Lett. B

- *Too flat behavior of at large k_t in BK*

- *No universality of CCFM distribution at small k_t*

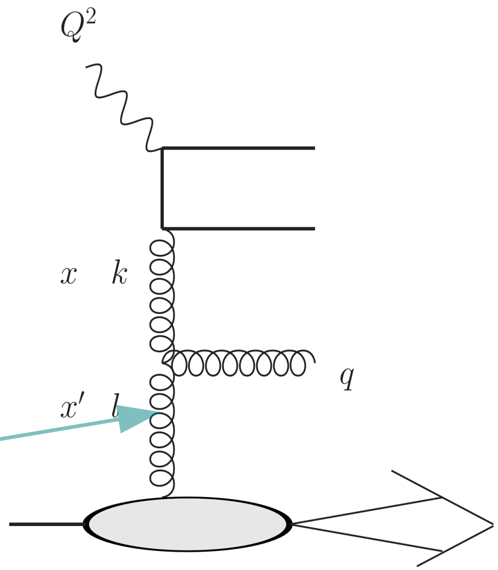
- *Lack of saturation in CCFM at at small k_t*

Needed framework which unifies both correct behaviors

Resummed form of the BK

The strategy:

- *Use the BK equation for gluon number density. Simple nonlinear term*
- *Split linear kernel into resolved and unresolved parts*
- *Resumm the virtual contribution and unresolved ones in th*
- *Use analogy to postulate a nonlinear CCFM*



The starting point:

Integration over l

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^{\infty} \frac{dl^2}{l^2} \left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}} \right] - \frac{\overline{\alpha}_s}{\pi R^2} \int_{x/x_0}^{1} \frac{dz}{z} \Phi^2(x/z,k^2)$$

let $\pi R^2 = 1$

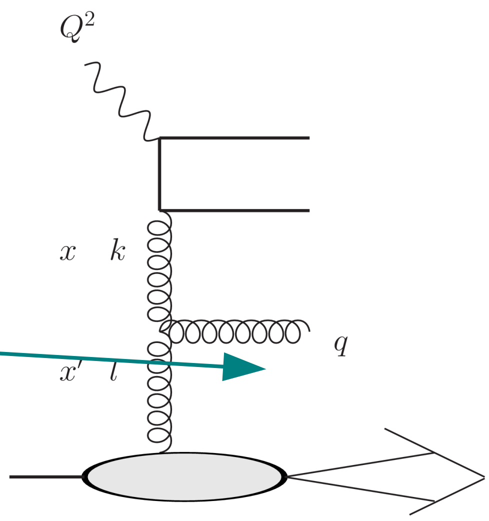
Integro differentaial form solved in: *Phys. Rev. D 65:074037,2002*
Golec-Biernat, Motyka, Stasto

Resummed form of the BK

K.K. JHEP '12
K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek '12

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z, |\mathbf{k}+\mathbf{q}|^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)$$

Write in exclusive form. Combine virtual cont



Integration over q

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \Phi(x/z, |\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z, |\mathbf{k}+\mathbf{q}|^2)\theta(\mu^2-q^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)\,.$$

Resolution scale introduced

Perform Mellin transform w.r.t x to get rid of "z" integral

$$\overline{\Phi}(\omega,k^2) = \int_0^1 dx x^{\omega-1}\Phi(x,k^2)$$

$$\Phi(x,k^2) = \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega}\overline{\Phi}(\omega,k^2)$$

Resummed form of the BK

Using in unresolved real part $|\mathbf{k}+\mathbf{q}|^2 \approx \mathbf{k}^2$ ⟵ $q^2 < \mu^2$

$$\overline{\Phi}(\omega,k^2) = \overline{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\omega}\int\frac{d^2\mathbf{q}}{q^2}[\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2)] + \frac{\overline{\alpha}_s}{\omega}\int\frac{d^2\mathbf{q}}{q^2}\overline{\Phi}(\omega,k^2)[\theta(\mu^2-q^2)-\theta(k^2 \cdots - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2)$$

$$\overline{\Phi}(\omega,k^2) = \overline{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\omega}\int\frac{d^2\mathbf{q}}{\pi q^2}\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) - \frac{\overline{\alpha}_s}{\omega}\overline{\Phi}(\omega,k^2)\ln\frac{k^2}{\mu^2} - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2)\,.$$

Inverting the transform we get:...

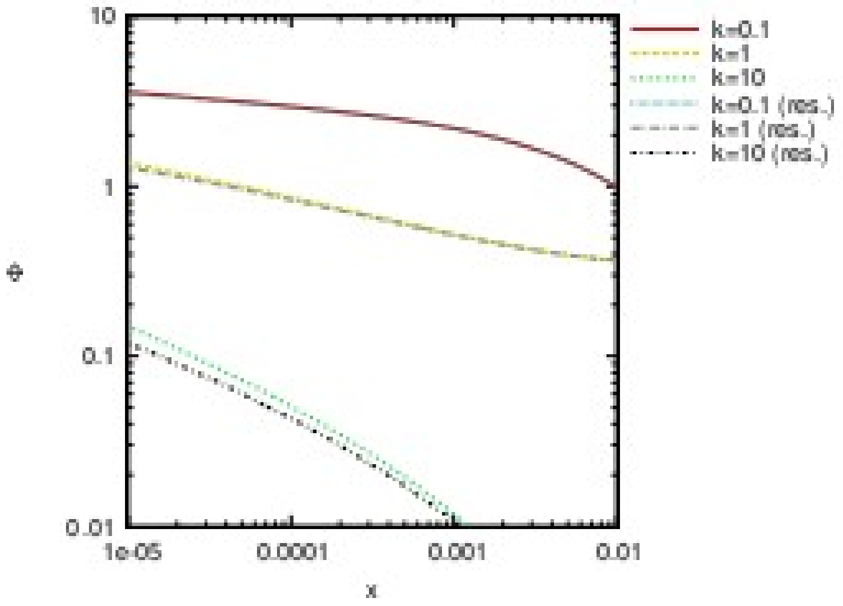
BK equation in the resummed exclusive form

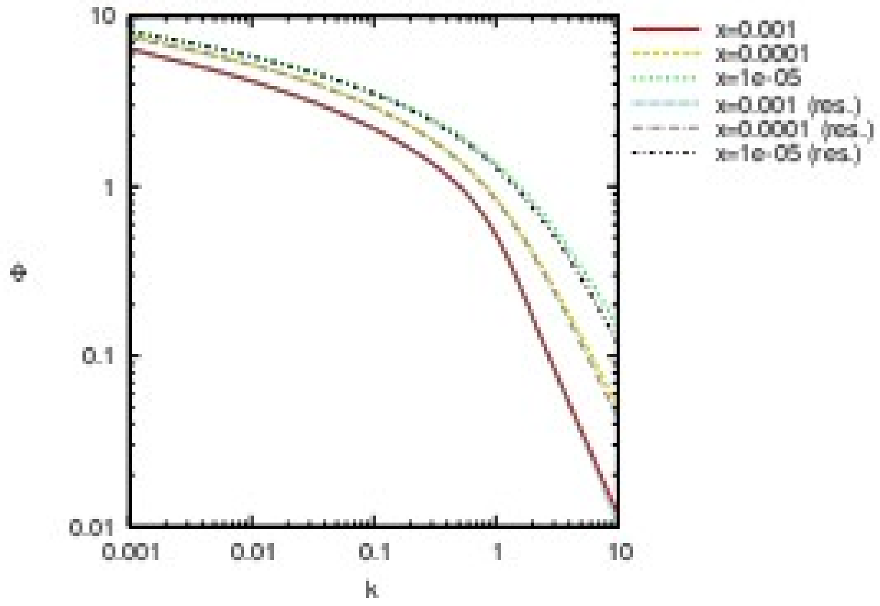
$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\,\frac{\Delta_R(z,k,\mu)}{z}\Big[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\Big]$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln\tfrac{1}{z}\ln\tfrac{k^2}{\mu^2}\right)$$

- *The same resumed piece for linear* and *nonlinear*
- *Initial distribution also gets multiplied by the Regge form factor*
- *New scale introduced to equation. One has to check dependence of the solution on it*
- *Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states*

BK before and after resummation



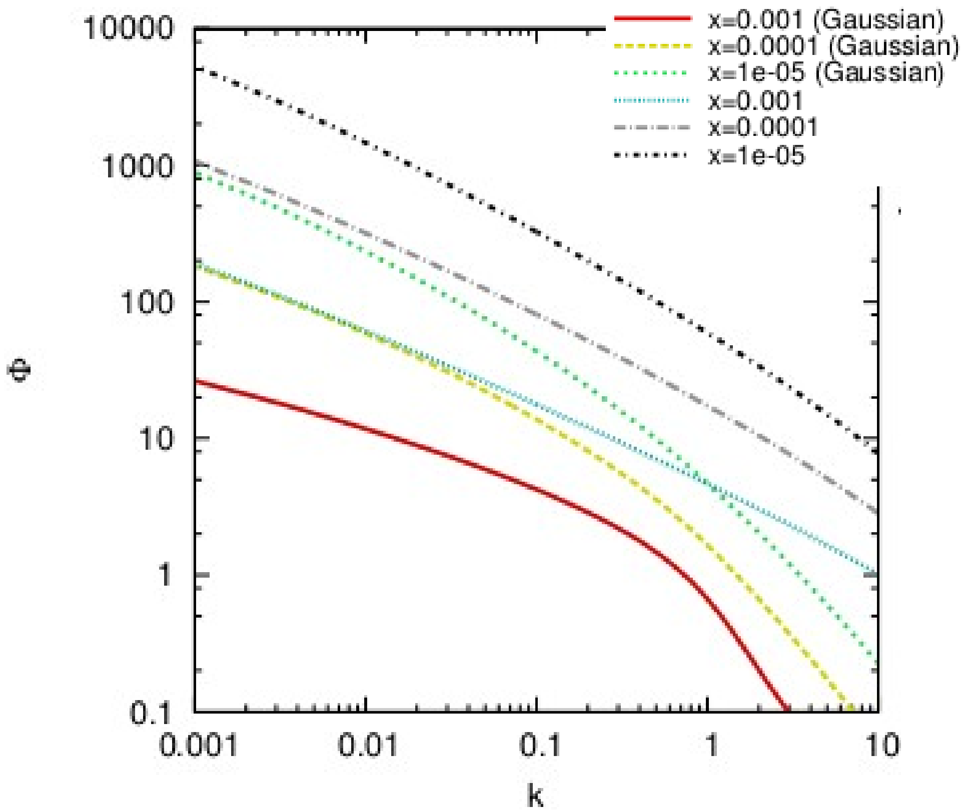


K.K, W. Placzek, D. Toton,
Arxiv 1303.0431

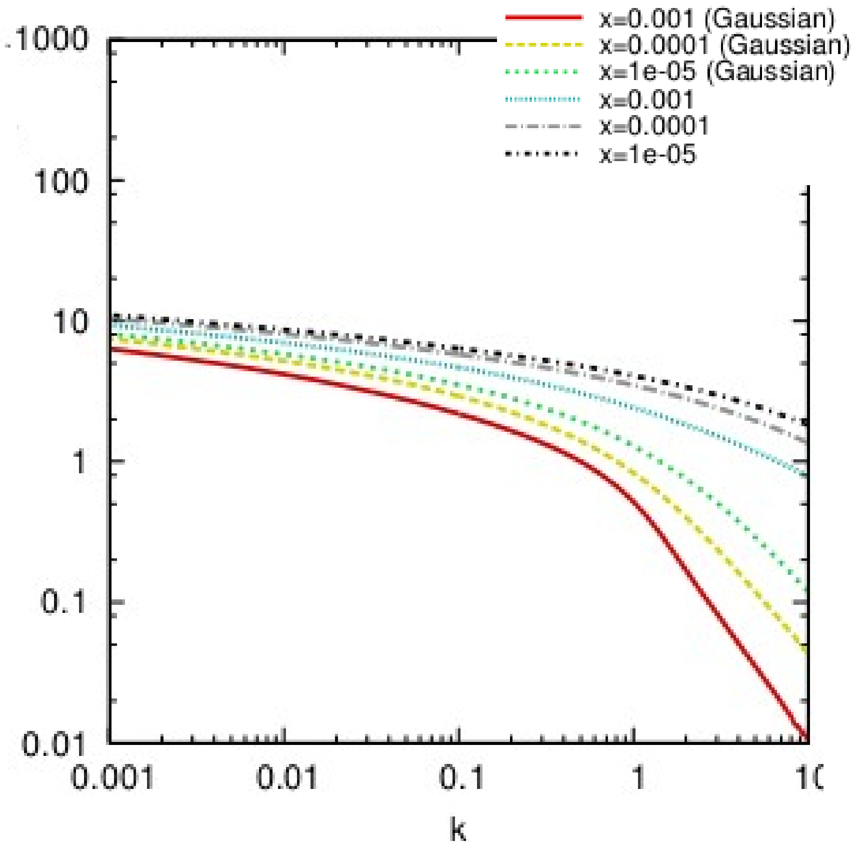
Agreement within a 1%

Dependence on the initial conditions

BFKL



BK



Two types of initial distributions

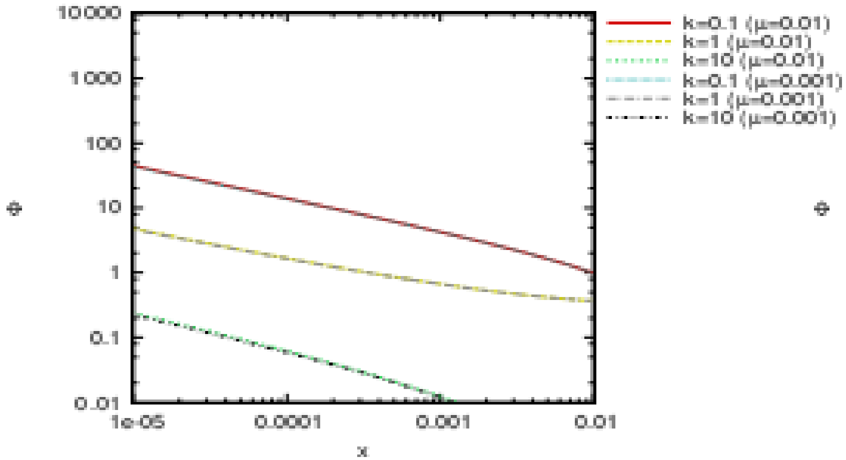
$$\Phi_0 = x^{\bar{\omega}} \exp\left(-\frac{k^2}{1\text{GeV}}\right)$$

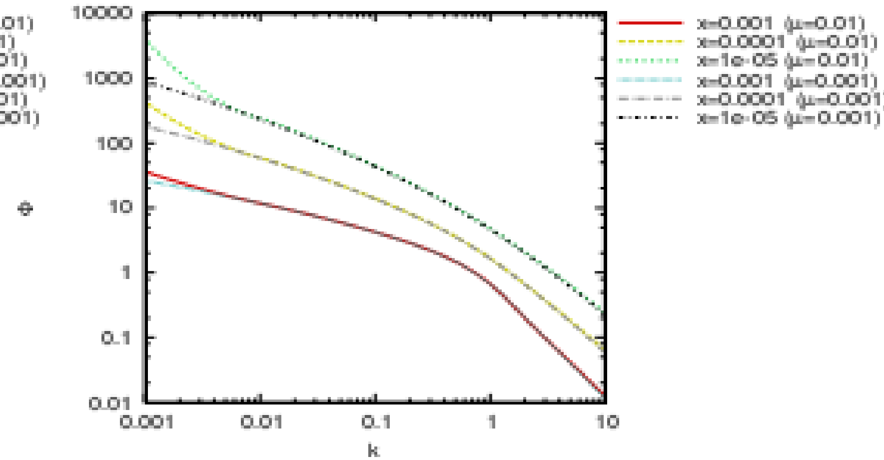
$$\Phi_0 = x^{\bar{\omega}} \sqrt{\frac{1\text{GeV}}{k}}$$

Common work with postdoc Dawid Toton

Stability againist variation of the resolution scale

BFKL





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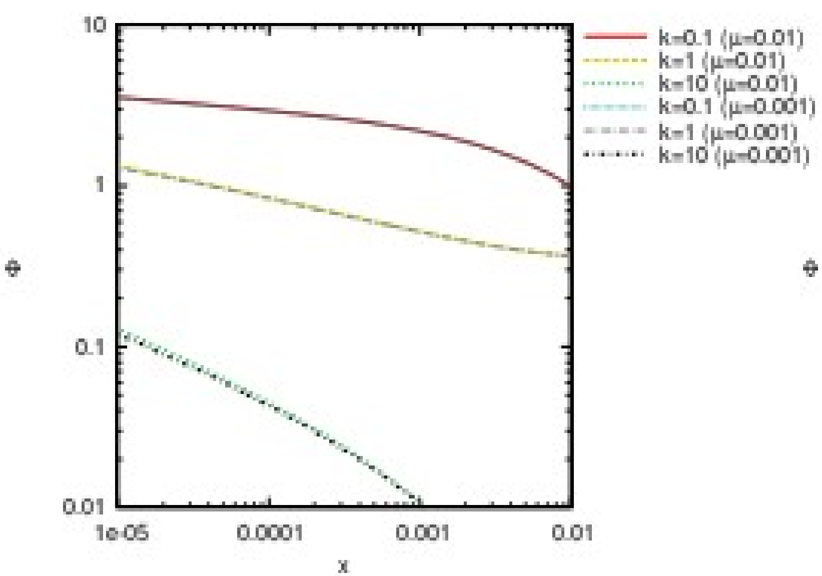

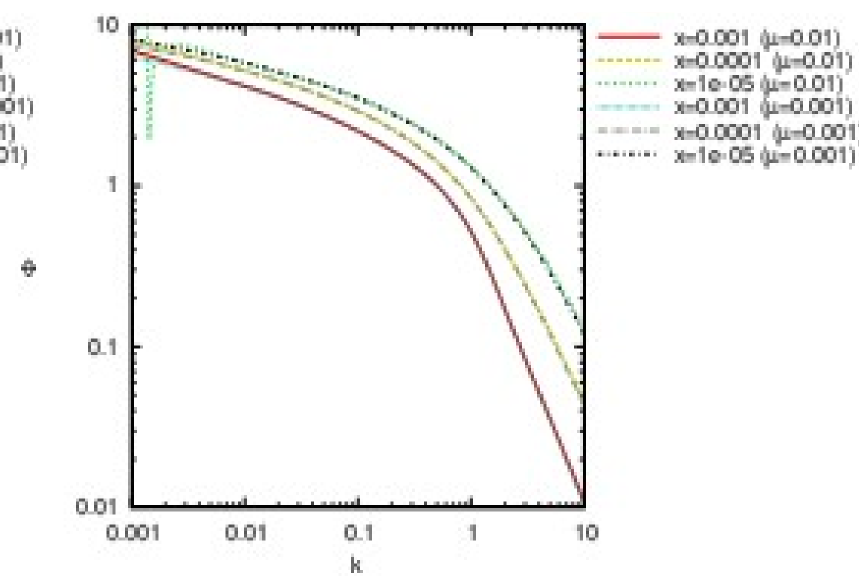

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Arxiv 1303.0431

Extension of CCFM to non linear equation for gluon number density

K.K. '12
K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek '12

Unifies saturation and exclusiveness
Structure motivated by the form of resummed BK and CCFM

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\frac{\Delta_R(z,k,\mu)}{z}\left[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right]$$

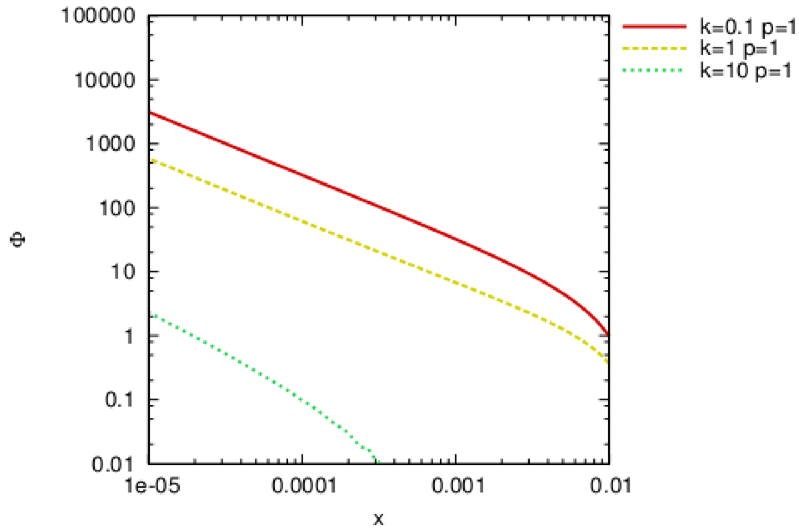
$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \int \frac{d^2\bar{\mathbf{q}}}{\pi\bar{q}^2}\int_x^{1-Q_0/|\bar{\mathbf{q}}|} dz\,\theta(p-z\bar{q})\Delta_s(p,z\bar{q})\left(\frac{\Delta_{ns}(z,k,\bar{q})}{z}+\frac{1}{1-z}\right)\left[\mathcal{E}\left(\frac{x}{z},k'^2,\bar{q}\right) - \bar{q}^2\delta(\bar{q}^2-k^2/(1-z)^2)\,\mathcal{E}^2(\frac{x}{z},\bar{q}^2,\bar{q})\right]$$

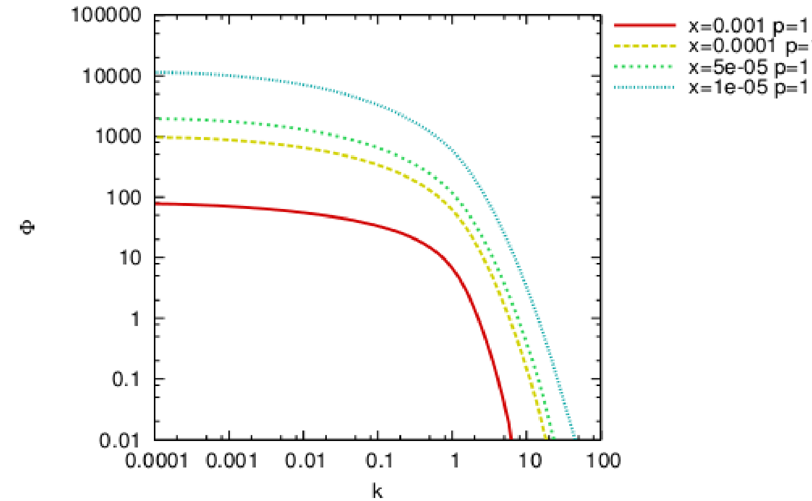
Modiffication of the nonlinear term to remove pole 1/(1-z) proposed by Deak '12

Solution of the KGBJS equation

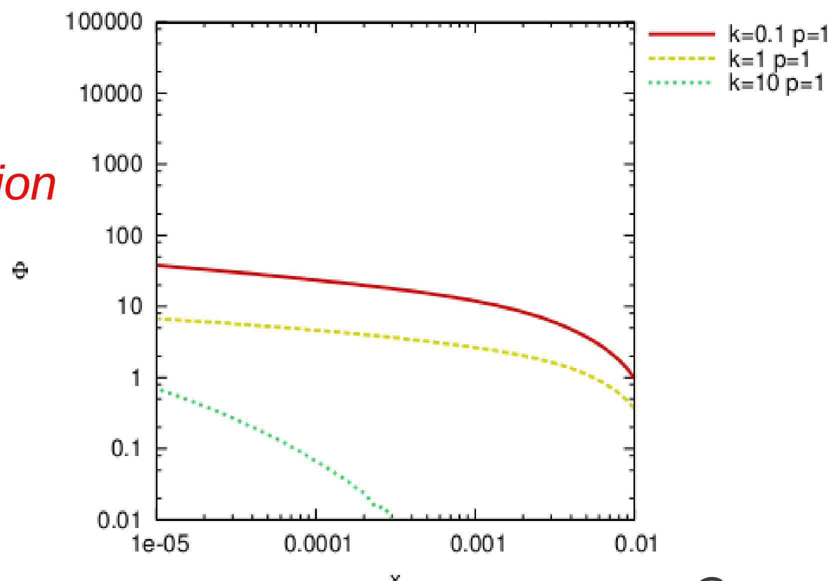
To be released soon

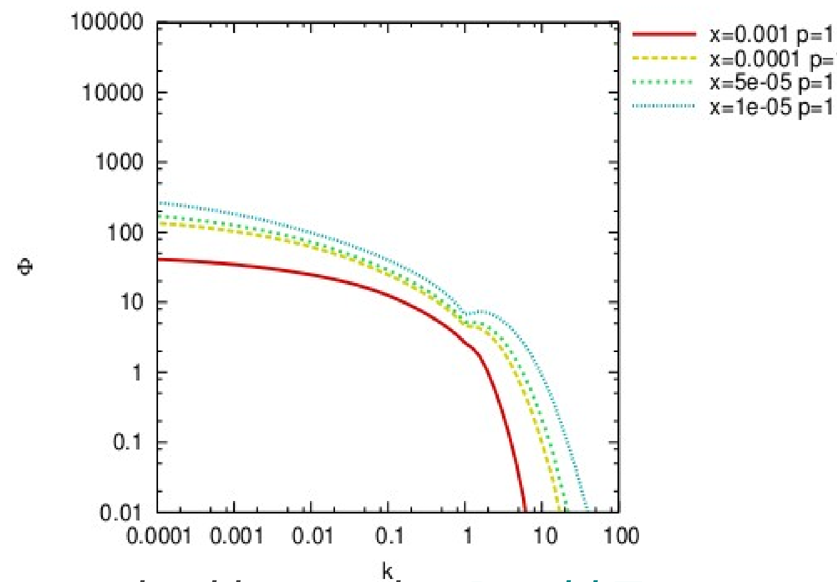
CCFM





New equation





Common work with postdoc Dawid Toton.
*Solution also obtained by M. **Deak '12***

To simplify numerics only non-Sudakov and 1/z pole.

Extension of CCFM to nonlinear equation for gluon momentum density

The same procedure of resummation can be applied to the high energy factorizable gluon density. The stucture of nonlinearity does not introduce complications:

KK JHEP 12

$$\mathcal{F}(x,k^2) = \tilde{\mathcal{F}}_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\Delta_R(z,k,\mu)\left\{\int\frac{d^2\mathbf{q}}{\pi q^2}\theta(q^2-\mu^2)\mathcal{F}(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2)\right.$$
$$\left.-\frac{\pi\alpha_s^2}{4N_cR^2}k^2\nabla_k^2\left[\int_{k^2}^\infty\frac{dl^2}{l^2}\ln\frac{l^2}{k^2}\mathcal{F}(x,l^2)\right]^2\right\}$$

And resummed TPV

$$\mathcal{V}_{\text{resummed}} = \frac{\pi\alpha_s^2}{4\,N_c\,R^2}k^2\Delta_R(z,k,\mu)\nabla_k^2\ln\frac{l_1^2}{k^2}\theta(l_1^2-k^2)\ln\frac{l_2^2}{k^2}\theta(l_2^2-k^2)$$

Include coherence

$$\mathcal{F}(x,k^2,p) = \tilde{\mathcal{F}}_0(x,k^2,p)+\overline{\alpha}_s\int\frac{d^2\mathbf{q}}{\pi q^2}\int_{x/x_0}^1\frac{dz}{z}\theta(p-q\,z)\Delta_{ns}(z,k,q)\left\{\mathcal{F}(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2,q)\right.$$
$$\left.-\frac{\pi\alpha_s^2}{4N_cR^2}q^2\delta(q^2-k^2)\nabla_q^2\left[\int_{q^2}^\infty\frac{dl^2}{l^2}\ln\frac{l^2}{k^2}\mathcal{F}(x/z,l^2,l)\right]^2\right\}$$

Can be further extended to include Sudakov and 1/(1-z) terms

Conclusions and outlook

- *LHC gives oportunity to test parton densities both when the parton density is probed at low x and at low, medium and large kt at some external scale.*

- *The new representation of the BK equation has been found*

- *Well motivated anzatz for the equations which incorporate both saturation effects and coherence. Solutions are still to be understood*

- *Prospects for Monte Carlo simulation of physics of saturation*