Theoretical foundations of the quantum statistical approach to parton distributions and recent results

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Outline

- Basic procedure to construct the statistical polarized parton distributions
- Essential features from unpolarized and polarized Deep Inelastic Scattering data
- Predictions tested against new data : DIS, Semi-inclusive DIS and several hadronic processes
- Extension to transverse momentum dependence (TMD):
  Transverse energy sum rule
  Gaussian shape with no x, k_T factorization
  Melosh-Wigner effects mainly in low x, Q^2 region
- Conclusions

Collaboration with Claude Bourrely and Franco Buccella

- A Statistical Approach for Polarized Parton Distributions
  Euro. Phys. J. C23, 487 (2002)
- Recent Tests for the Statistical Parton Distributions
  Mod. Phys. Letters A18, 771 (2003)
- The Statistical Parton Distributions: status and prospects
  Euro. Phys. J. C41,327 (2005)
- The extension to the transverse momemtum of the statistical parton distributions
  Mod. Phys. Letters A21, 143 (2006)
- Strangeness asymmetry of the nucleon in the statistical parton model
  Phys. Lett. B648, 39 (2007)
- How is transversity related to helicity for quarks and antiquarks in a proton?
  Mod. Phys. Letters A24, 1889 (2009)
- Semiinclusive DIS cross sections and spin asymmetries in the quantum statistical parton distributions approach
  Phys. Rev. D83, 074008 (2011)
- The transverse momentum dependent statistical parton distributions revisited
  Int. Journal of Mod. Phys. A28, 1350026 (2013)

Our motivation and goals

- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features

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- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features
- Will parametrize our PDF in terms of a very few number of physical parameters, at variance with standard polynomial type parametrizations
- Will be able to construct simultaneously unpolarized and polarized PDF:
  A UNIQUE CASE ON THE MARKET!
- Will be able to describe physical observables both in DIS and hadronic collisions
- Will make some very specific challenging predictions

Basic procedure

Use a simple description of the PDF, at input scale Q_0^2, proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, *plus* sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and *minus* sign for gluons, corresponds to a Bose-Einstein distribution. X_{0p} is a constant which plays the role of the *thermodynamical potential* of the parton *p* and $\bar{x}$ is the *universal temperature*, which is the same for all partons.

NOTE: x is indeed the natural variable, since all the sum rules we will use are expressed in terms of x

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From the chiral structure of QCD, we have two important properties, allowing to relate quark and antiquark distributions and to restrict the gluon distribution:
- Potential of a quark q^h of helicity *h* is opposite to the potential of the corresponding antiquark $\bar{q}^{-h}$ of helicity *-h*, $X_{0q}^h = -X_{0\bar{q}}^{-h}$.
- Potential of the gluon G is zero, $X_{0G} = 0$.

The polarized PDF $q^{\pm}(x, Q_0^2)$ at initial scale Q_0^2

For light quarks *q = u, d* of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{A X_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ ,$$

consequently for antiquarks of helicity $h = \mp$

$$x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A} (X_{0q}^h)^{-1} x^{\bar{b}}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ .$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for $\bar{q}$).
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For strange quarks and antiquarks, s and $\bar{s}$, use the same procedure which leads to $xs(x, Q_0^2) \neq x\bar{s}(x, Q_0^2)$ and $x\Delta s(x, Q_0^2) \neq x\Delta\bar{s}(x, Q_0^2)$ (Phys. Lett. B648, 39 (2007)).

For gluons we use a Bose-Einstein expression given by $xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}$, with a vanishing potential and the same temperature $\bar{x}$. We also need to specify the polarized gluon distribution and we take for consistency $x\Delta G(x, Q_0^2) = 0$ only at initial scale

Essential features from the DIS data

From well established features of u and d extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have $X^+_{0u} + X^-_{0u} > X^+_{0d} + X^-_{0d}$
- $\Delta u(x) > 0$, therefore $X^+_{0u} > X^-_{0u}$
- $\Delta d(x) < 0$, therefore $X^-_{0d} > X^+_{0d}$.

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- $\Delta u(x) > 0$, therefore $X_{0u}^{+} > X_{0u}^{-}$
- $\Delta d(x) < 0$, therefore $X_{0d}^{-} > X_{0d}^{+}$.

So we expect X_{0u}^{+} to be the largest potential and X_{0d}^{+} the smallest one. In fact, from our fit we have obtained the following ordering

$$X_{0u}^{+} > X_{0d}^{-} \sim X_{0u}^{-} > X_{0d}^{+} \; .$$

This ordering has important consequences for $\bar{u}$ and $\bar{d}$, namely

Essential features from DIS data

- $\bar{d}(x) > \bar{u}(x)$, flavor symmetry breaking expected from Pauli exclusion principle. This was already confirmed by the violation of the Gottfried sum rule (NMC).
- $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, a PREDICTION from 2002, in agreement with polarized DIS (see below) and will be more precisely checked at RHIC-BNL from $W^{\pm}$ production, already in active running phase.

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- Note that since $u^{-}(x) \sim d^{-}(x)$, it follows that $\bar{u}^{+}(x) \sim \bar{d}^{+}(x)$, so we have

$$\Delta\bar{u}(x) - \Delta\bar{d}(x) \sim \bar{d}(x) - \bar{u}(x) \ ,$$

i.e. the flavor symmetry breaking is almost the same for unpolarized and polarized distributions ($\Delta\bar{u}$ and $\Delta\bar{d}$ contribute to about 10% to the Bjorken sum rule).

Very few free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on $F_2^p(x,Q^2), F_2^n(x,Q^2), xF_3^{\nu N}(x,Q^2)$ and $g_1^{p,d,n}(x,Q^2)$, in correspondance with ten free parameters for the light quark sector with some physical significance:

* the four potentials $X_{0u}^+,\ X_{0u}^-,\ X_{0d}^-,\ X_{0d}^+,$
* the universal temperature $\bar{x}$,
* and $b,\ \bar{b},\ \tilde{b},\ b_G,\ \tilde{A}.$

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* the universal temperature $\bar{x}$,
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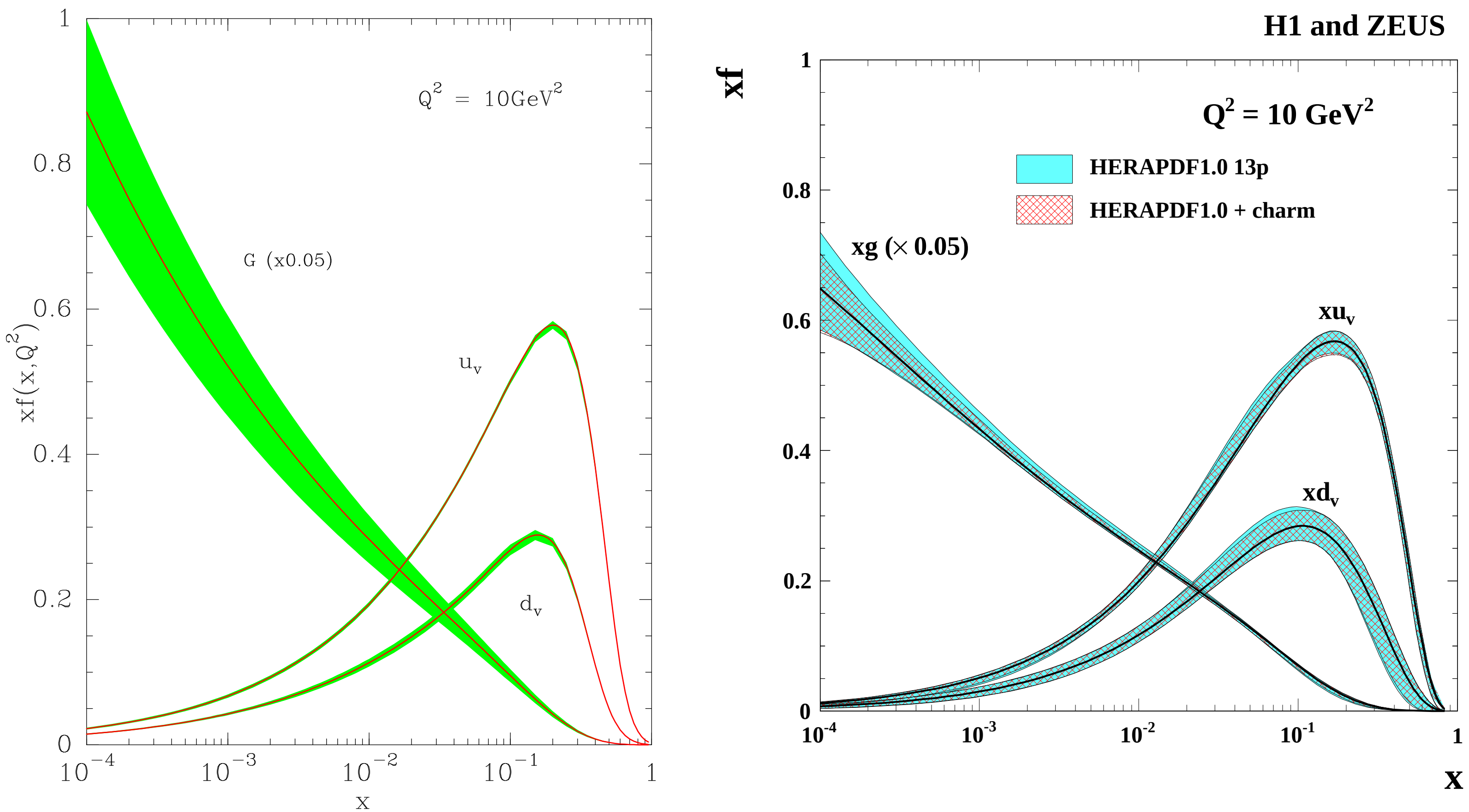
We also have three additional parameters, $A,\ \bar{A},\ A_G$, which are fixed by 3 normalization conditions .

$$u-\bar{u}=2,\ \ d-\bar{d}=1$$

and the momentum sum rule.

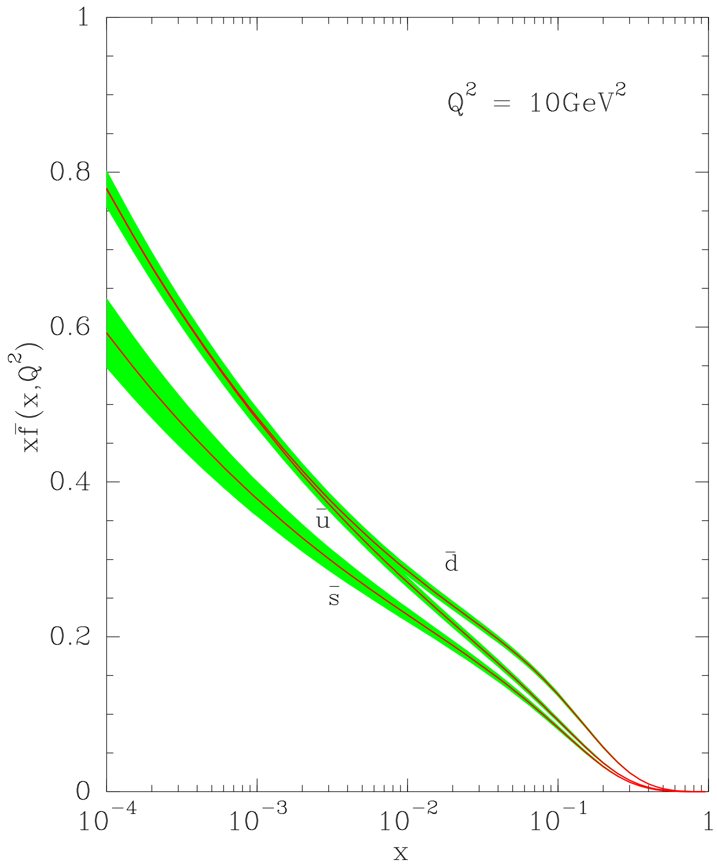
There are 4 parameters to describe the strange quark-antiquark sector

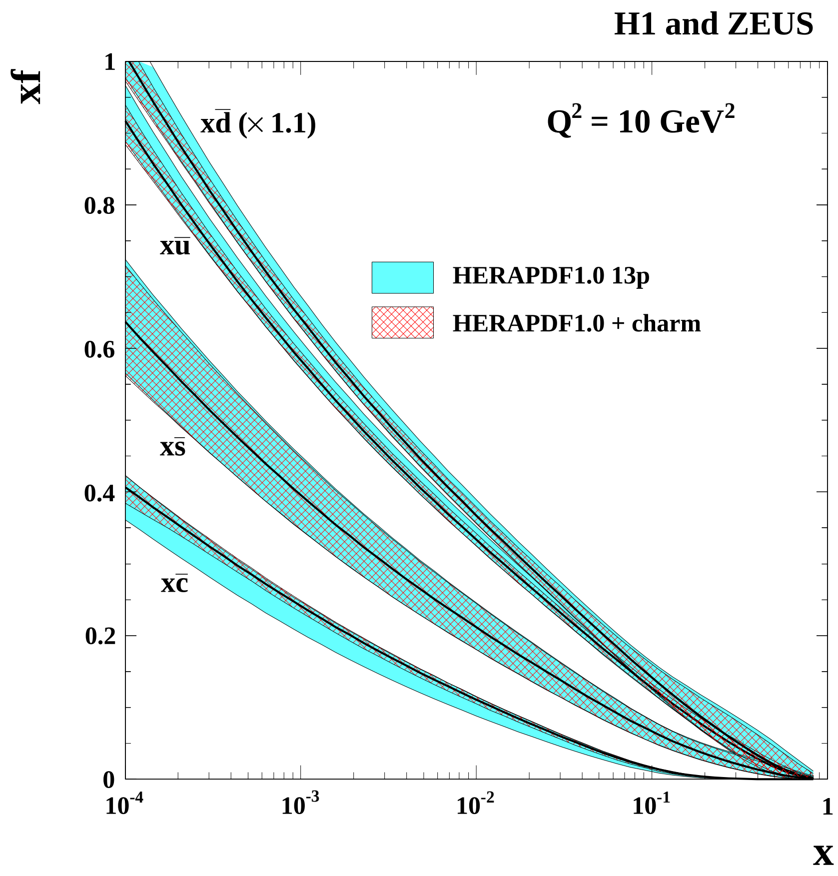
A global view of the unpolarized parton distributions



u_v, d_v agree well, but G grows faster

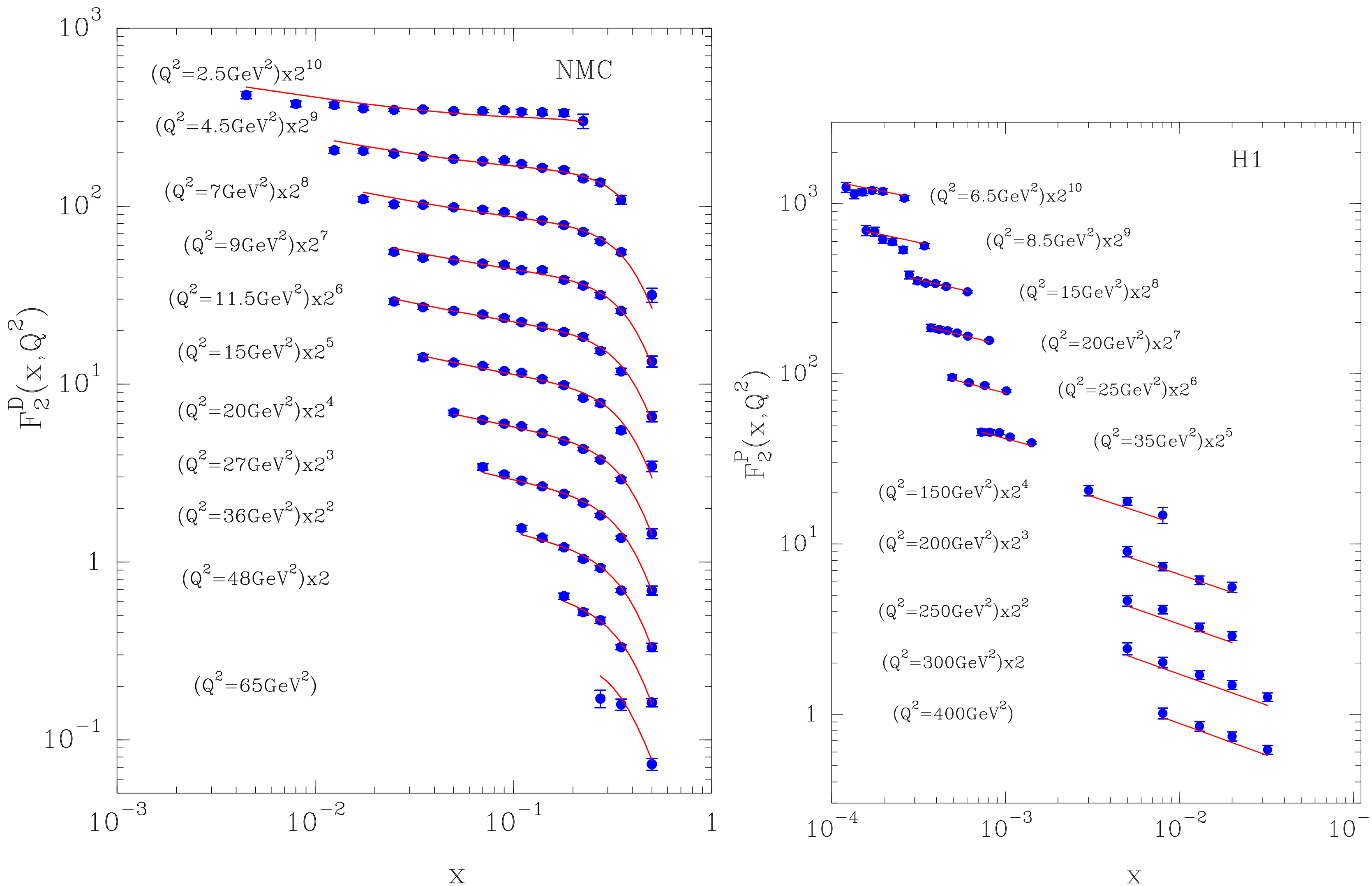
A global view of the unpolarized sea parton distributions



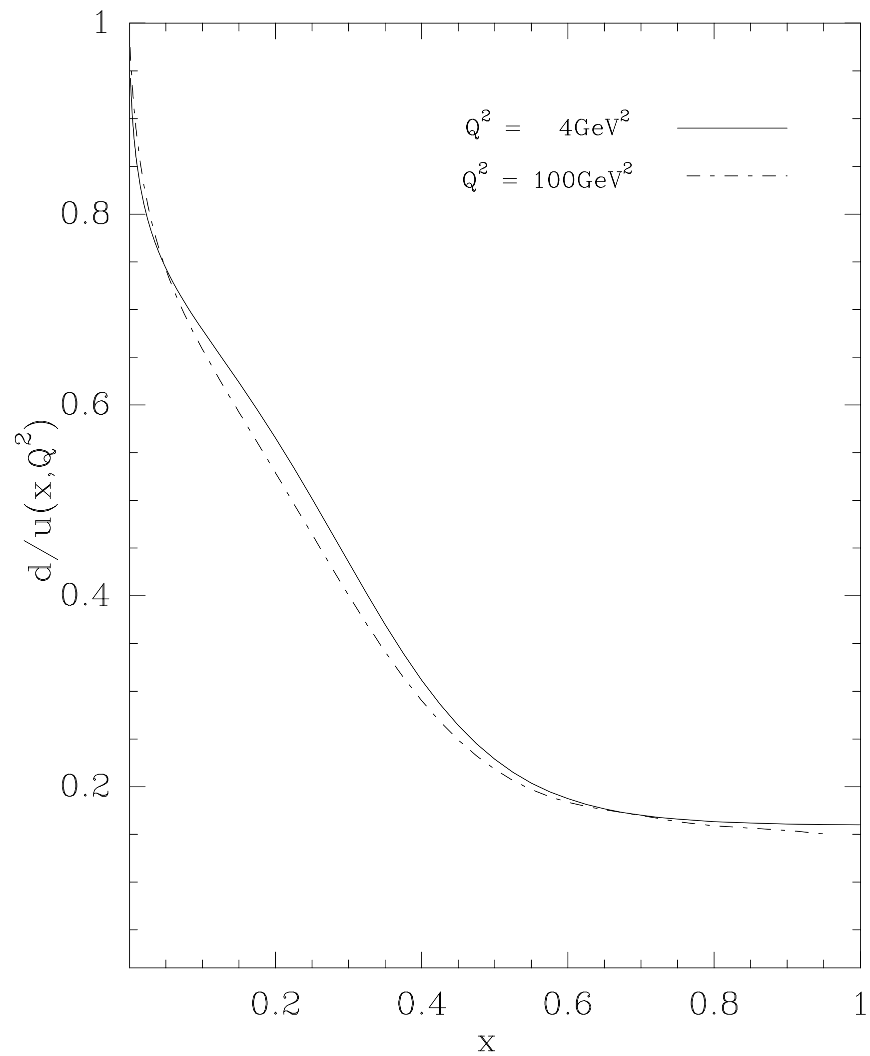


$\bar{s}$ agrees well but $\bar{u}, \bar{d}$ grow faster and we expect $\bar{d} > \bar{u}$ for all x

Some data on $F_2^D(x,Q^2)$, $F_2^p(x,Q^2)$

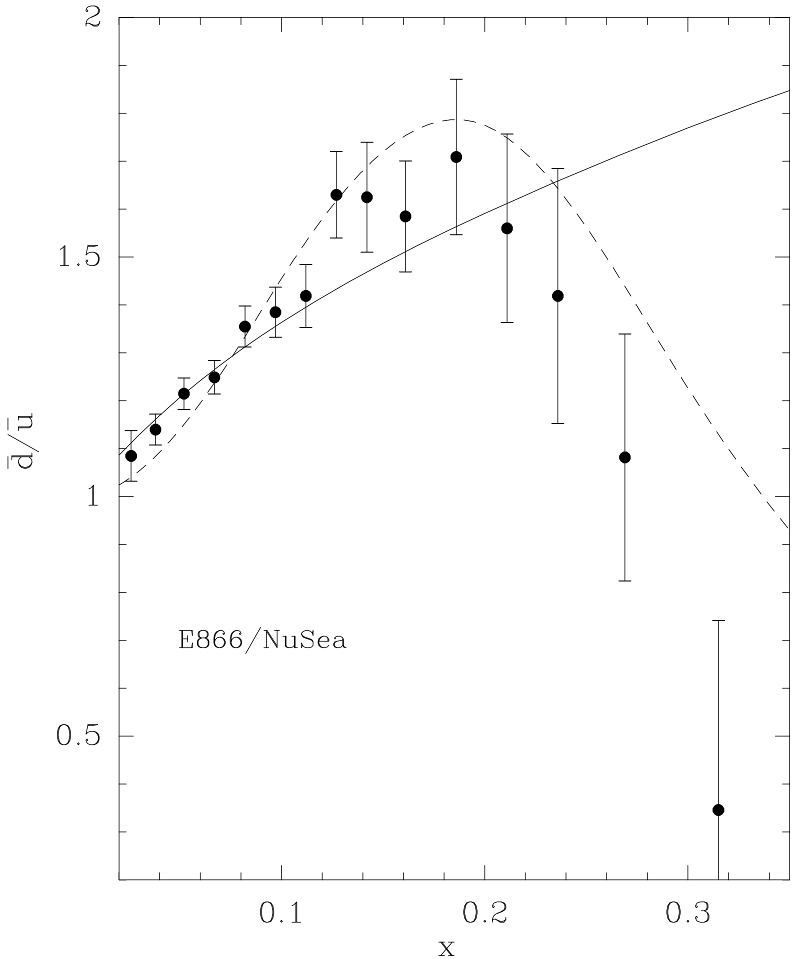


The predicted d/u ratio versus x

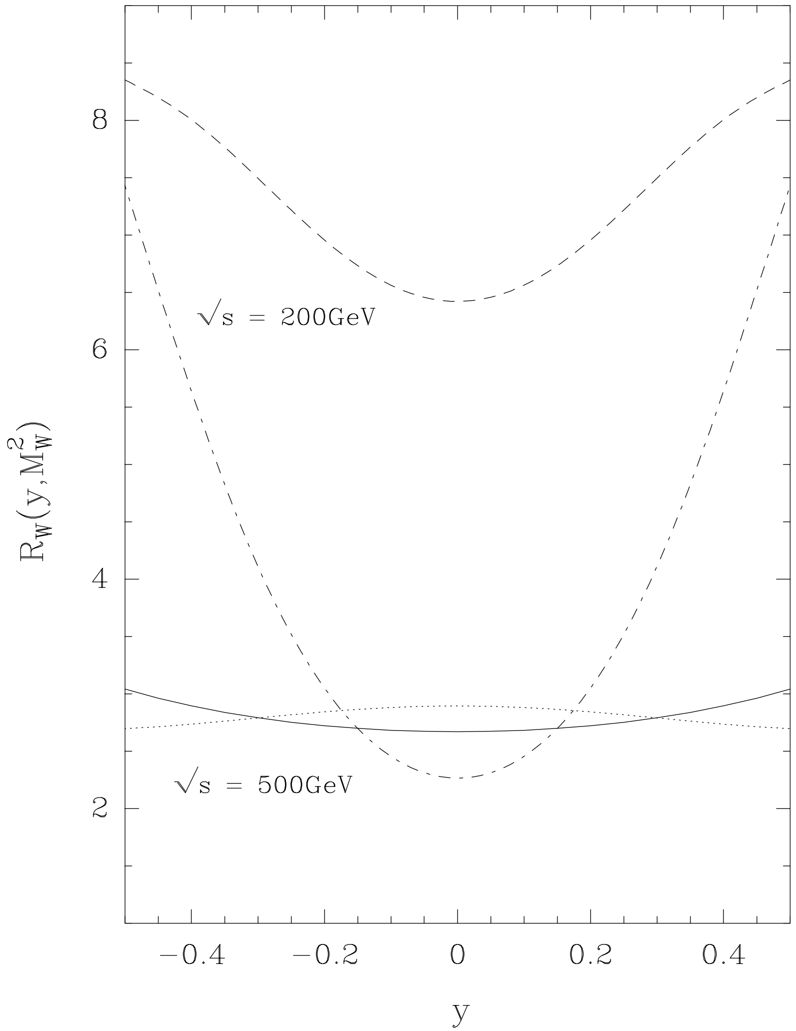


An important issue: $\bar{d}/\bar{u}$ at large x

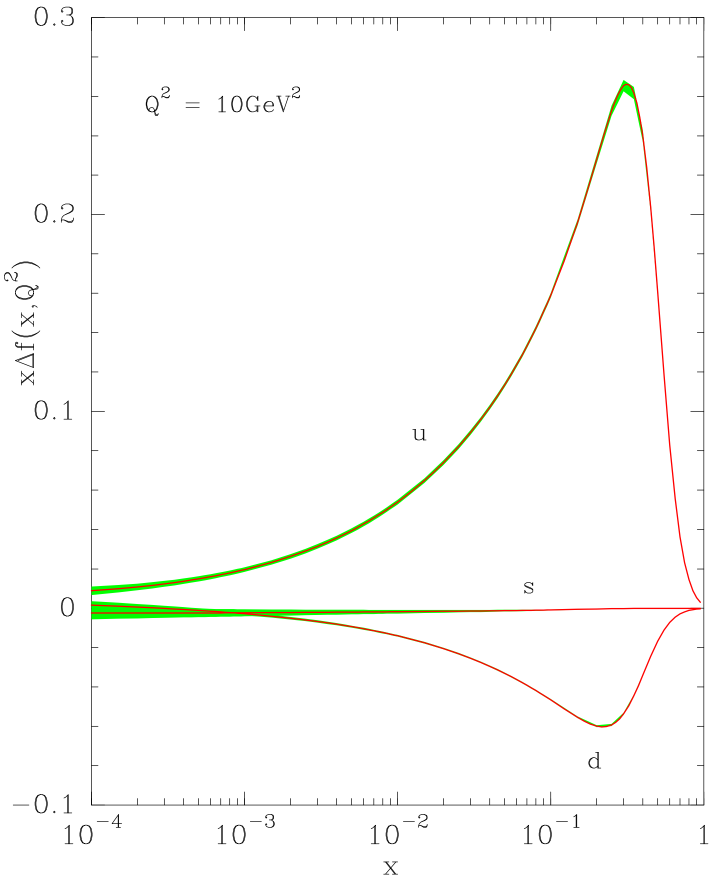
$Q^2 = 54\text{GeV}^2$



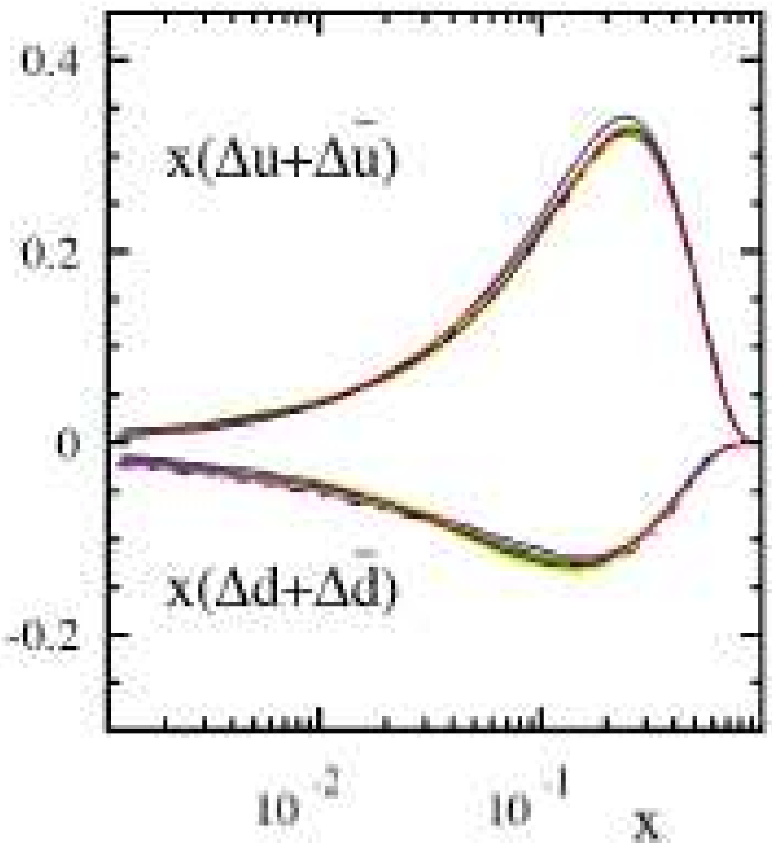
$R_W = (d\sigma_W^+/dy)/(d\sigma_W^-/dy)$ $\quad$ $Q^2 = M_W^2$



A global view of the polarized parton distributions

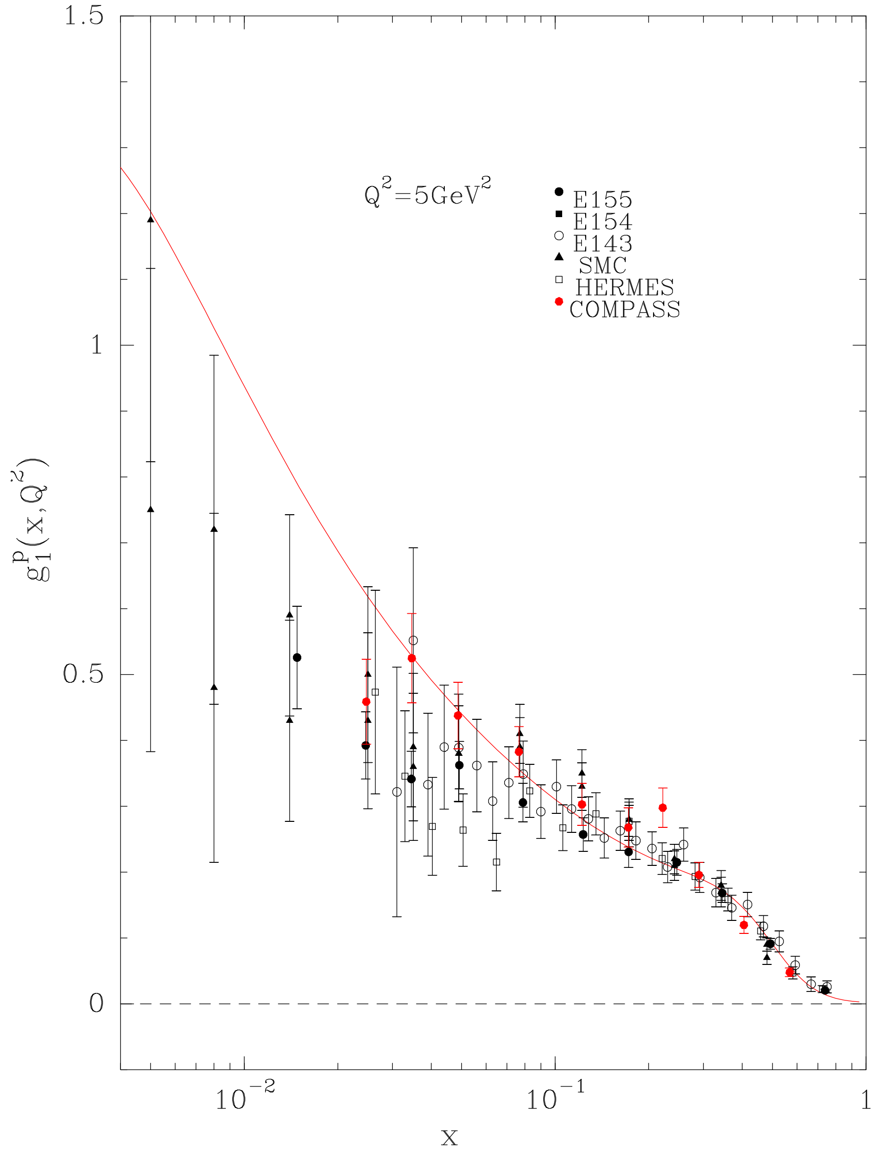


BBS

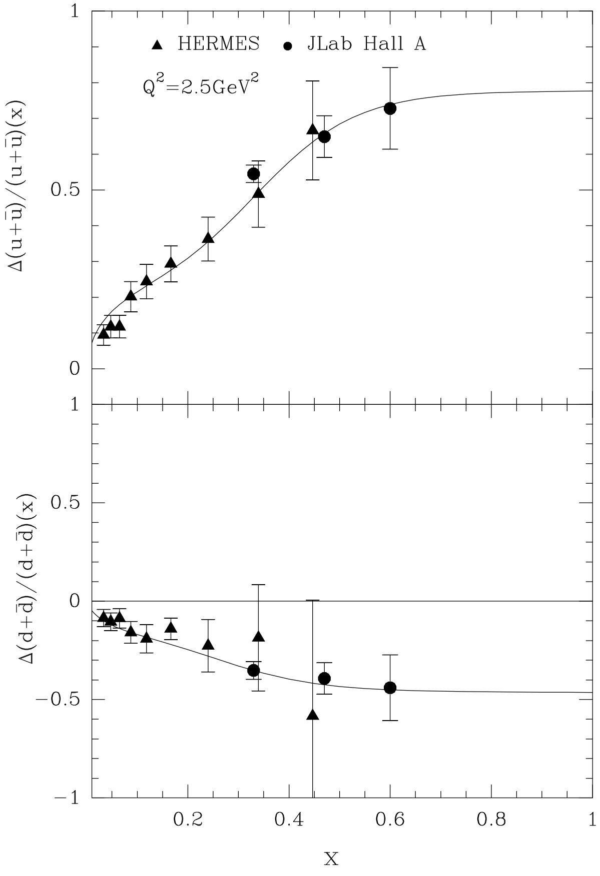


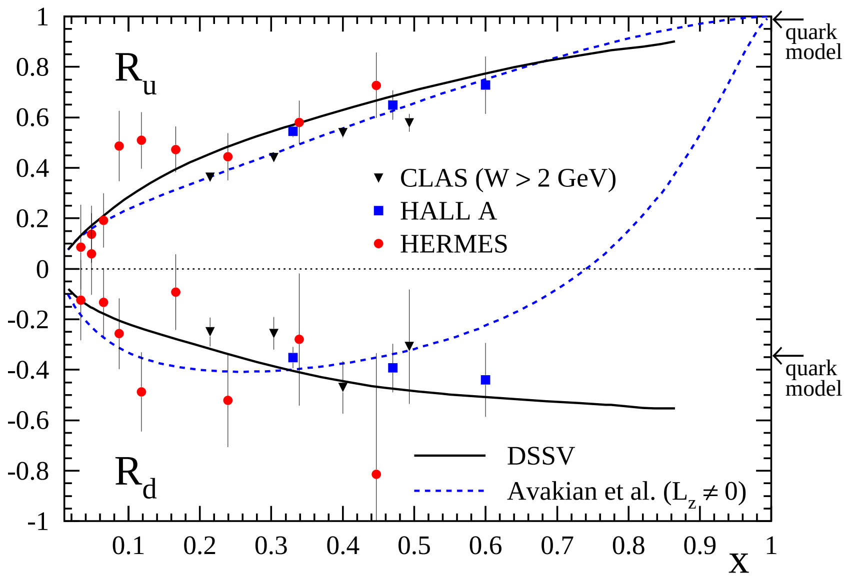
DSSV

A compilation of data on $g_1^p(x, Q^2)$

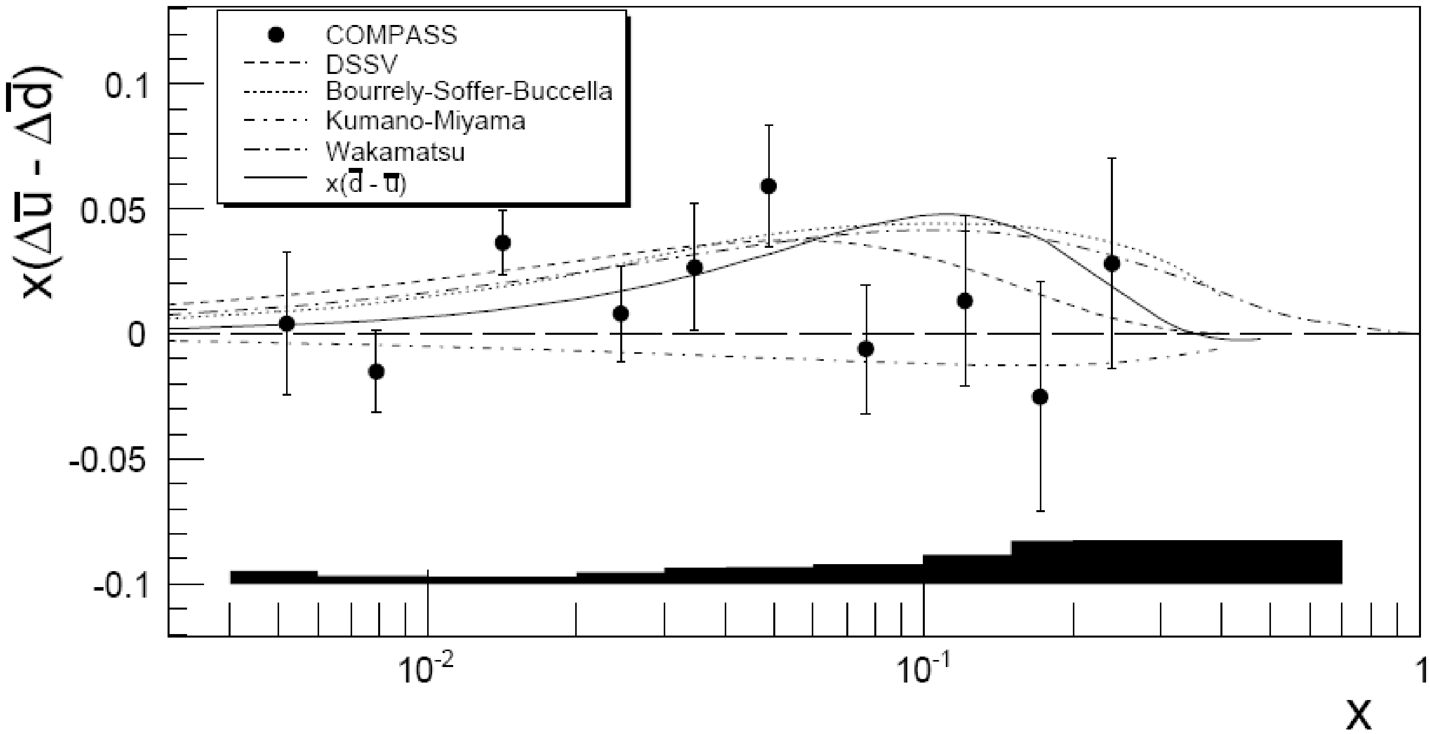


Helicity distributions versus x at DESY and JLab (2004)





The difference $x\Delta\bar{u}(x) - x\Delta\bar{d}(x)$ versus x



Comparison with recent COMPASS data

Transverse momentum dependence (TMD) of the PDF

How to introduce the TMD of the PDF ?

There are several possibilities

- Assume factorization and simple Gaussian behavior for the PDF

$$q(x,k_T) = q(x)\frac{1}{\pi\mu_0^2}\exp[-k_T^2/\mu_0^2]\ ,$$

  and also for the fragmentation function

$$D(z,q_T) = D(z)\frac{1}{\pi\mu_D^2}\exp[-q_T^2/\mu_D^2]\ .$$

  A naive assumption which has no theoretical justification

- No factorization: Covariant approach, derivative method
- No factorization: The statistical distributions for quarks and antiquarks

(TMD) in the statistical approach

The parton distributions $p_i(x, k_T^2)$ of momentum k_T, must obey the momentum sum rule

$$\sum_i \int_0^1 dx \int x p_i(x, k_T^2) dk_T^2 = 1 \ ,$$

and also the transverse energy sum rule

$$\sum_i \int_0^1 dx \int p_i(x, k_T^2) \frac{k_T^2}{x} dk_T^2 = M^2 \ .$$

From the general method of statistical thermodynamics we are led to put $p_i(x, k_T^2)$ in correspondance with the following expression

$$\exp(\frac{-x}{\bar{x}} + \frac{-k_T^2}{x\mu^2}) \ ,$$

where μ^2 is a parameter interpreted as the transverse temperature.
So we have now the main ingredients for the extension to the TMD of the statistical PDF.
We obtain in a natural way the Gaussian shape with NO x, k_T factorization

(TMD) in the statistical approach

The quantum statistics distributions for quarks and antiquarks read in this case

$$xq^h(x,k_T^2) = \frac{F(x)}{\exp(x - X_{0q}^h)/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 - Y_{0q}^h) + 1} \ ,$$

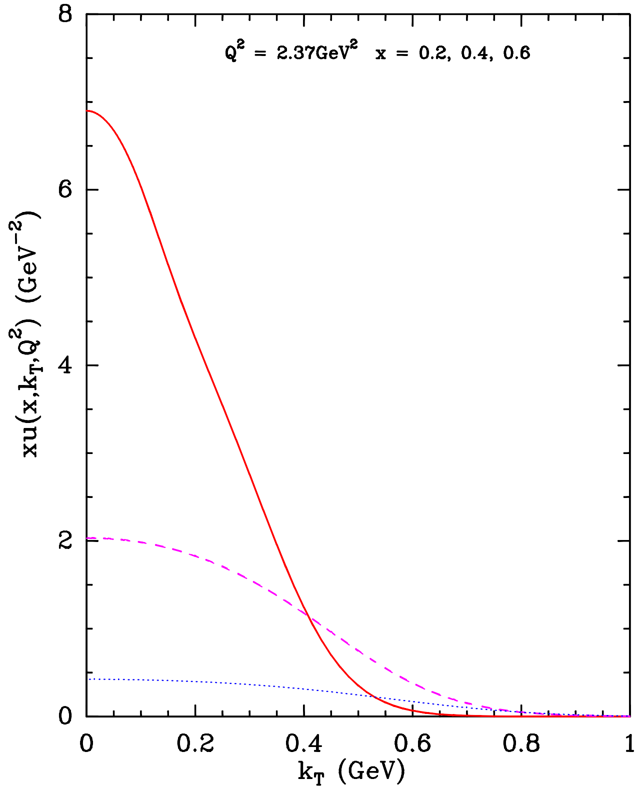
$$x\bar{q}^h(x,k_T^2) = \frac{\bar{F}(x)}{\exp(x + X_{0q}^{-h})/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 + Y_{0q}^{-h}) + 1},$$

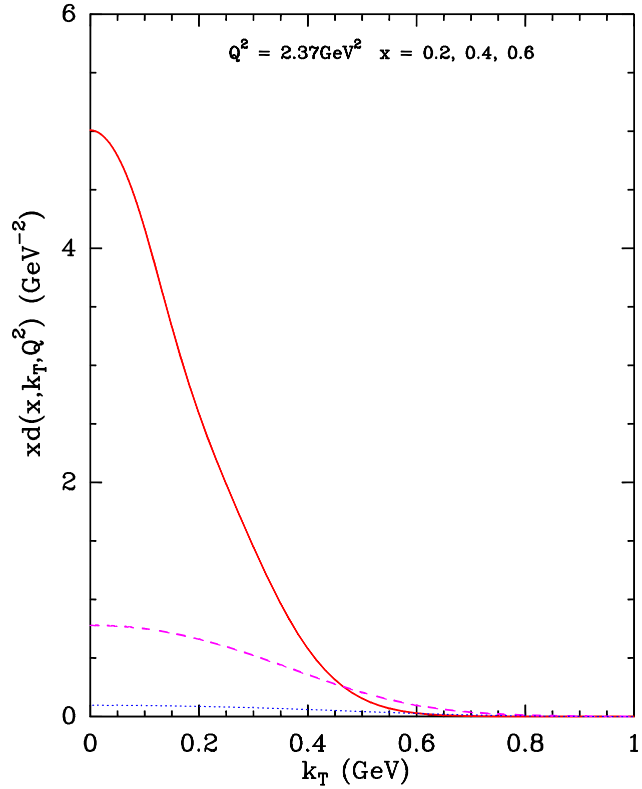
where

$$F(x) = \frac{Ax^{b-1}X_{0q}^h}{\ln(1 + \exp Y_{0q}^h)\mu^2} = \frac{Ax^{b-1}}{k\mu^2} \ ,$$

because Y_{0q}^h are the thermodynamical potentials chosen such that $\ln(1 + \exp Y_{0q}^h) = kX_{0q}^h$, in order to recover the factors X_{0q}^h, introduced earlier. Similarly for $\bar{q}$ we have $\bar{F}(x) = \bar{A}x^{2b-1}/k\mu^2$. This determination of the 4 potentials Y_{0q}^h can be achieved with the choice $k = 3.05$. Finally μ^2 will be determined by the transverse energy sum rule and one finds $\mu^2 = 0.198\text{GeV}^2$.

The statistical distributions u and d vs k_T





Melosh-Wigner effects

So far in all our quark or antiquark TMD distributions, the label "'h"' stands for the helicity along the longitudinal momentum and not along the direction of the momentum, as normally defined for a genuine helicity. The basic effect of a transverse momentum $k_T \neq 0$ is the Melosh-Wigner rotation, which mixes the components $q^{\pm}$ in the following way

$$q^{+MW} = \cos^2\theta\, q^+ + \sin^2\theta\, q^- \quad \text{and} \quad q^{-MW} = \cos^2\theta\, q^- + \sin^2\theta\, q^+ \ ,$$

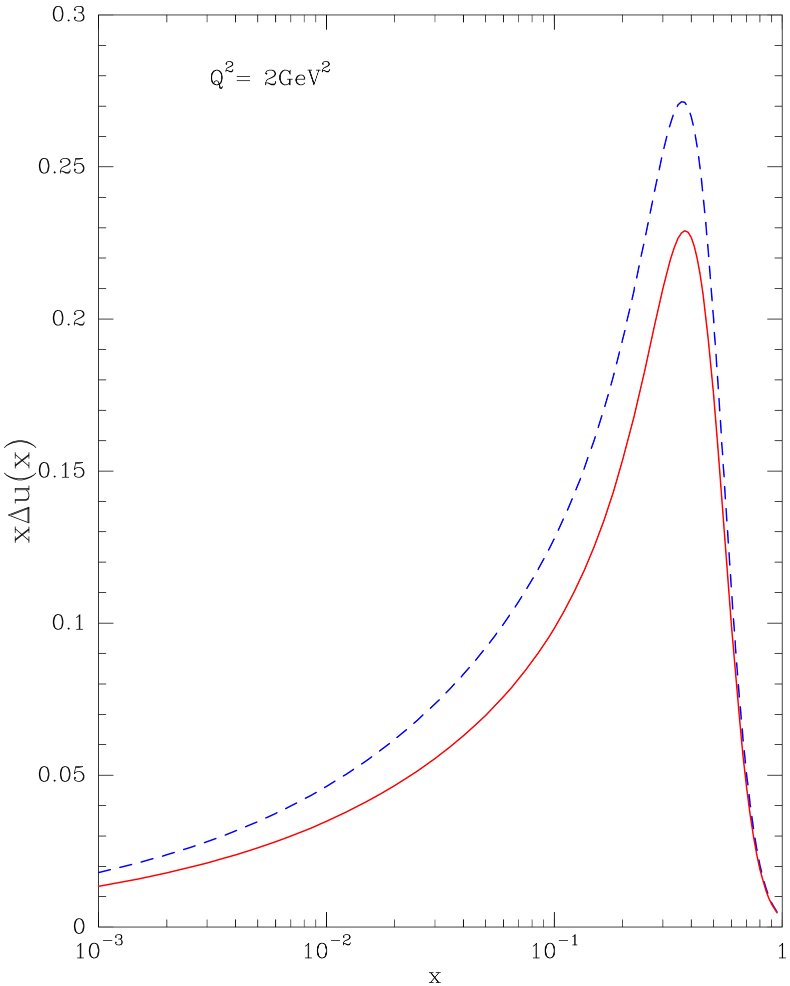
where, for massless partons, $\theta = \arctan\left(\frac{k_T}{p_0+p_z}\right)$, with $p_0 = \sqrt{k_T^2 + p_z^2}$.

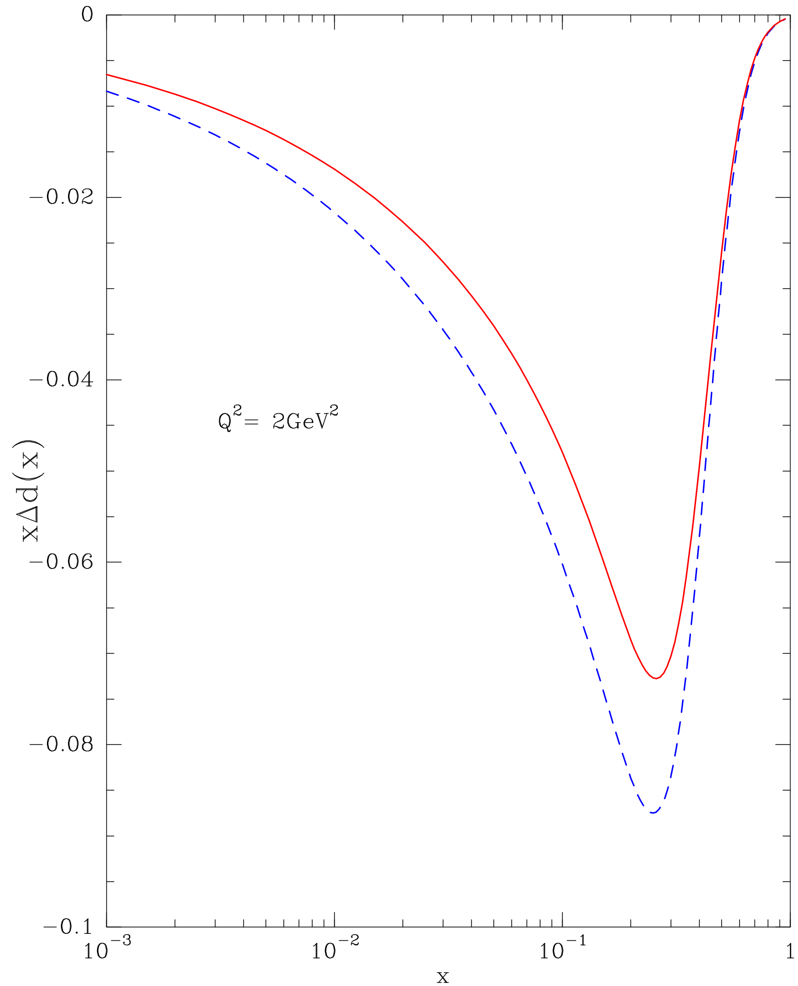
It vanishes when either $k_T = 0$ or p_z goes to infinity.

Consequently $q = q^+ + q^-$ remains unchanged since $q^{MW} = q$,

whereas we have $\Delta q^{MW} = (\cos^2\theta - \sin^2\theta)\Delta q$.

Predicted quark helicity distributions





The effect is relevant for small Q^2 and mainly in the low x region

Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon.
- All unpolarized and polarized distributions depend upon a small number of free parameters, with some physical meaning.
- New tests against experimental (unpolarized and polarized) data on DIS, Semi-inclusive DIS and hadronic processes are very satisfactory.
- Good predictive power but some special features remain to be verified, specially in the high x region.
- Extension to TMD has been achieved and must be checked more accurately together with Melosh-Wigner effects in the low x region