On a relation between production processes and total cross sections

Stéphane Munier

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QCD at high density

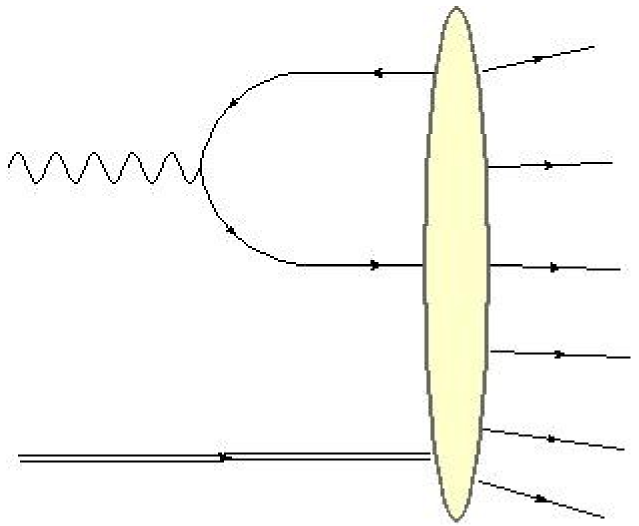
Parton densities "seen" in hadronic collisions increase with the energy. This growth is predicted by linear evolution equations such as the BFKL equation, established in QCD.

At very high energies, parton densities may become so large that they **saturate**, which means that the evolution equations become **nonlinear** (BK, JIMWLK equations), and predict the emergence of a new hard, energy-dependent, momentum scale called the **saturation scale** Q_s .

This regime is very interesting theoretically. Parton saturation may also have important phenomenological consequences at the LHC.

QCD at high density: How to test it?

At an ***electron-hadron collider:***



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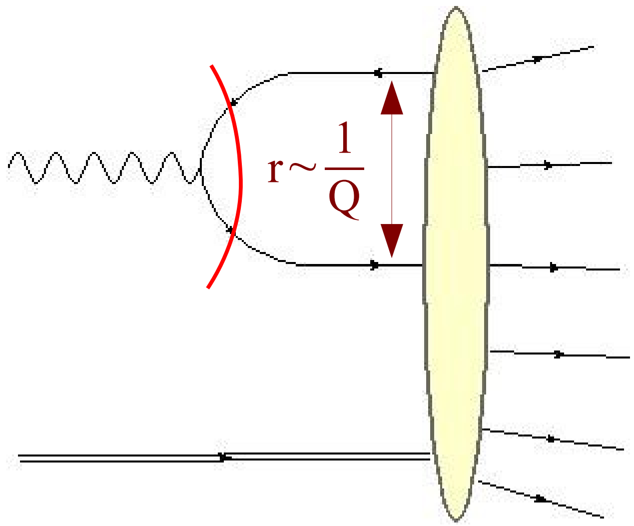
DIS can be understood as a dipole of "tunable" size r interacting with the target (p or A), that gets denser at higher energies.



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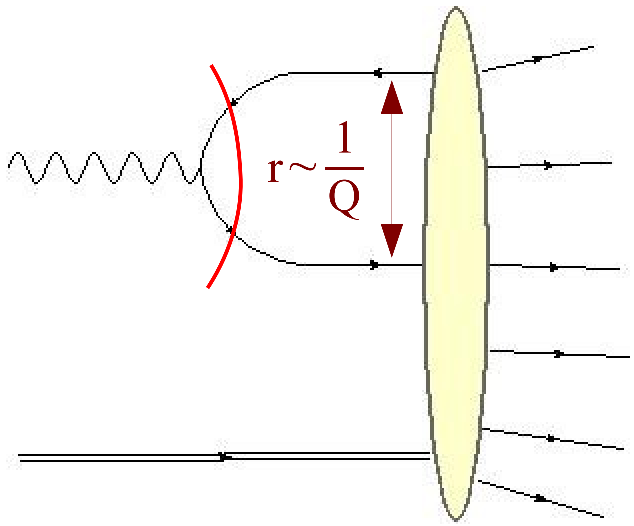

A lot of understanding of the dipole scattering amplitude was gained at HERA, at the border of the dense/saturation regime of QCD!

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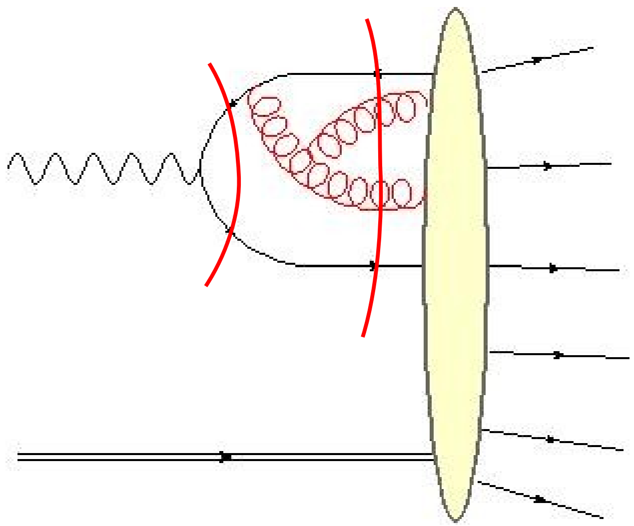


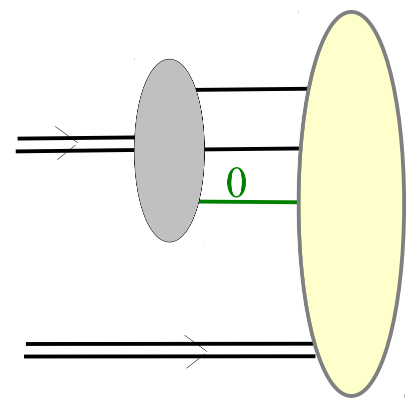
A lot of understanding of the dipole scattering amplitude was gained at HERA, at the border of the dense/saturation regime of QCD!

On the theoretical size, it is "easy" to formulate the QCD evolution of the dipole amplitude with the energy as **radiative corrections to the dipole wave function**.

BFKL (at low density), BK, JIMWLK equations (accounting for high-density effects)

QCD at high density: How to test it?

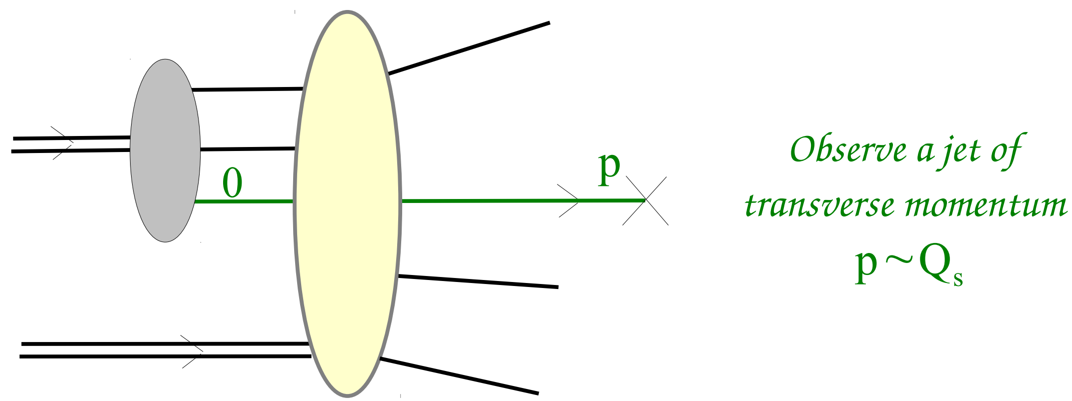
At a **hadron collider**, we need to find appropriate production processes:



QCD at high density: How to test it?

At a ***hadron collider****, we need to find appropriate production processes:*

- p_T *-broadening:*



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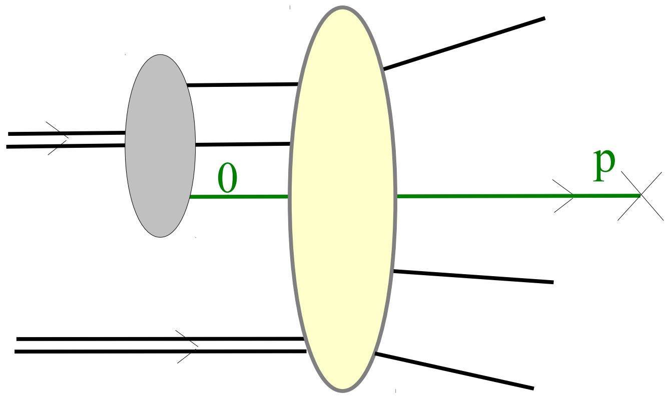
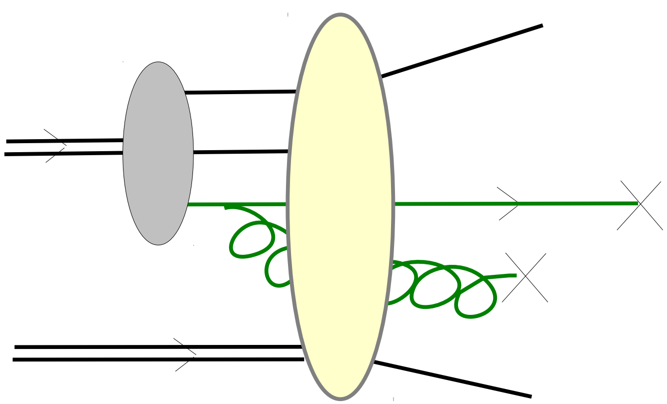


Observe a jet of transverse momentum $p \sim Q_s$

- *Forward dijet azimuthal correlations:*



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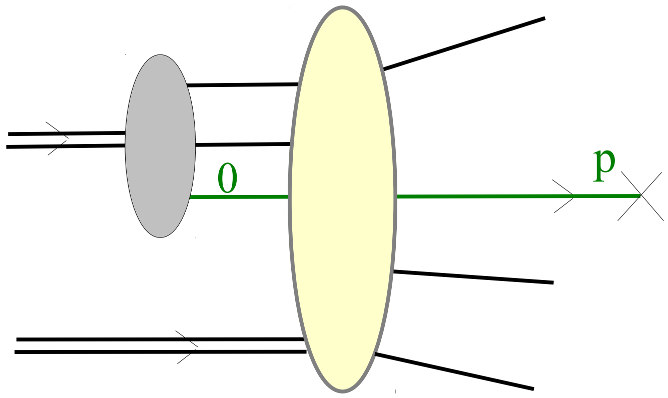

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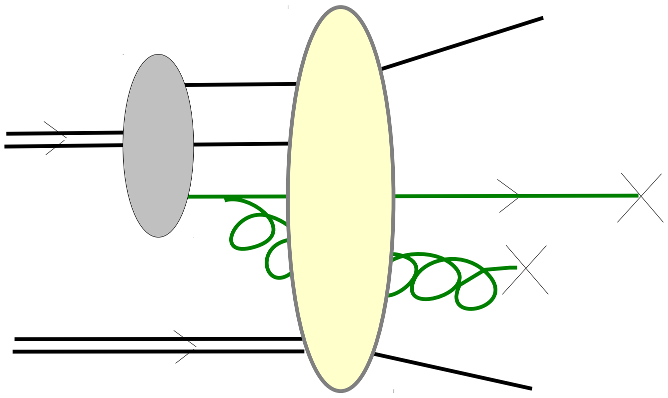

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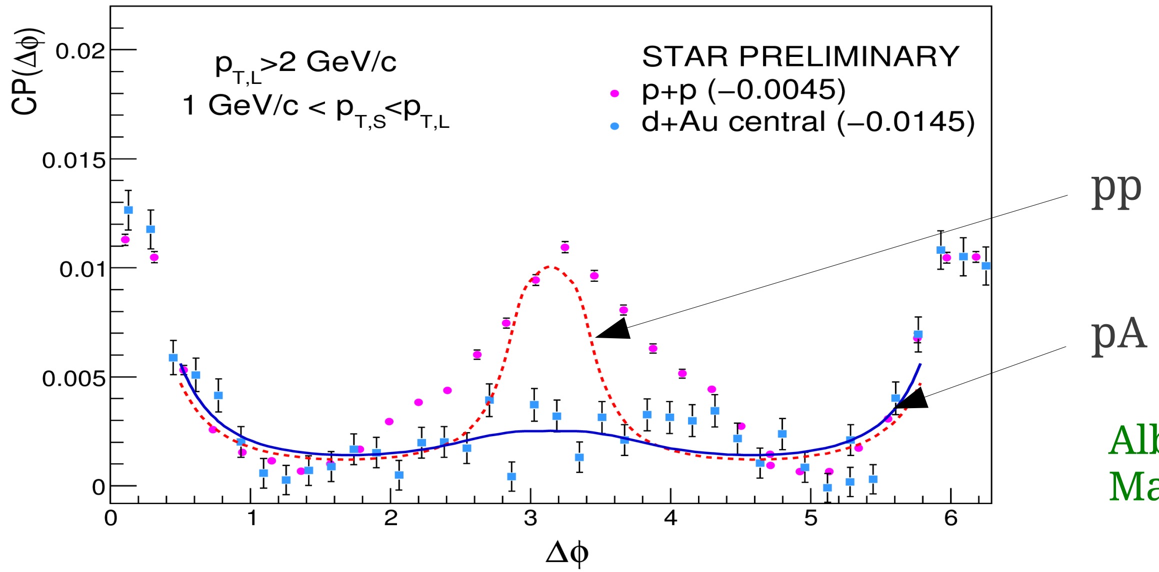
These observables are more tricky to formulate in QCD!

Forward dijet azimuthal correlations

Approximate formulation

Marquet (2007)

Prediction that some ***"jet" correlations*** *are different between pA and pp:*



Albacete,
Marquet PRL (2010)

- *Qualitative agreement with experimental results*
- *A better description needs more theoretical work*

See also Dominguez, Marquet, Xiao, Yuan (2011)

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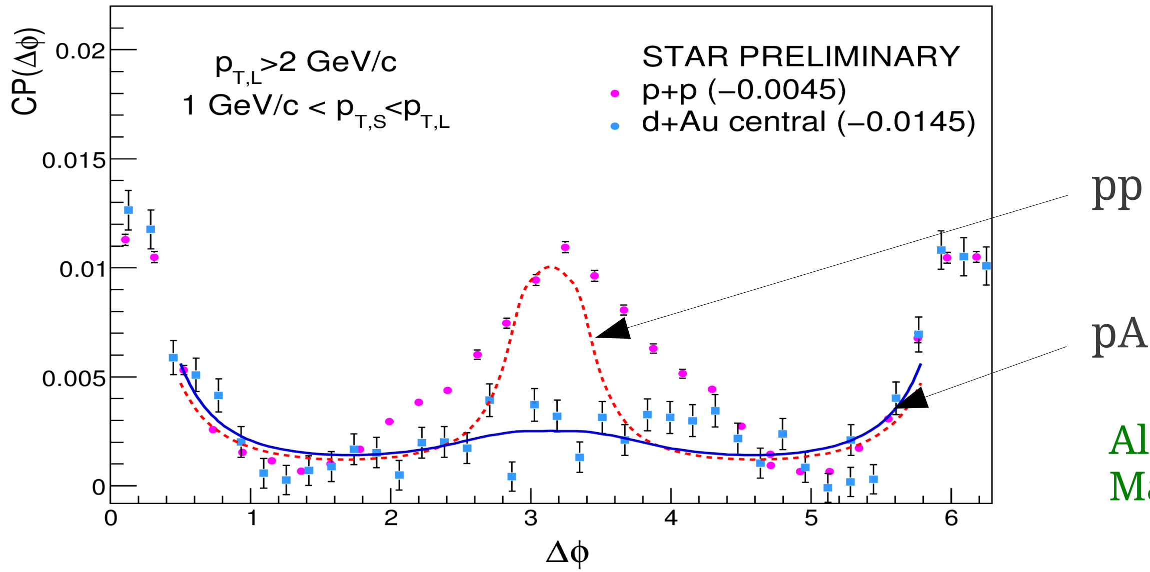

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***This talk:* report on a rigorous link between DIS total cross sections and this kind of semi-inclusive observables in pp and pA collisions.**

A. H. Mueller, S. Munier, Nucl. Phys. A (2012)

Outline

* Formulation of a production process in pA
* Quantum corrections: leading order
* Next-to-leading order

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- Formulation of a production process in pA

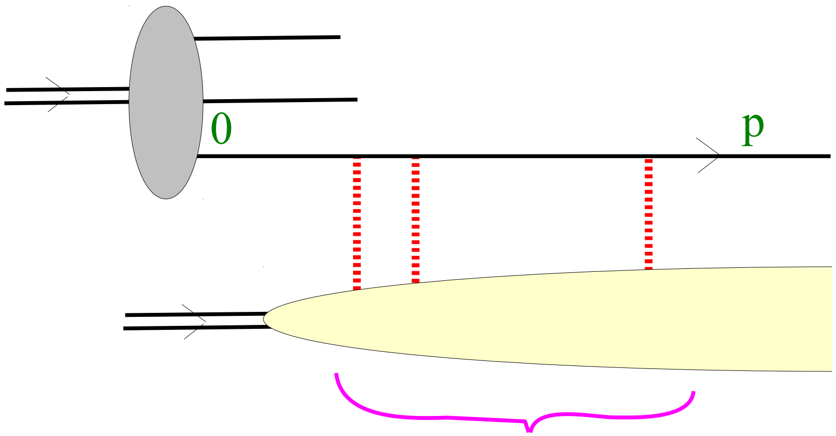
- Quantum corrections: leading order

$$\alpha_s \log s \rightarrow \sum (\alpha_s \log s)^n$$

- Next-to-leading order

$$\alpha_s^2 \log s \rightarrow \sum \alpha_s (\alpha_s \log s)^n$$

Formulation of p_T-broadening



time $t=-\infty$ $t=0$ $t=+\infty$

Initial state *Final state*

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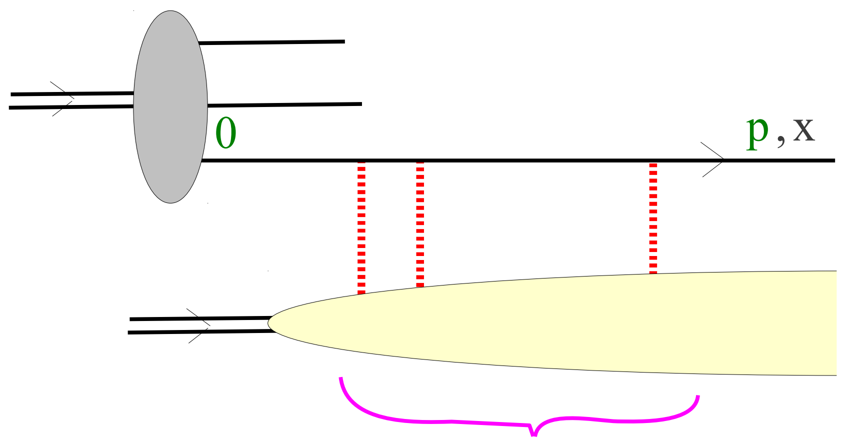

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Initial state *Final state*

$$\int \frac{d^2x}{(2\pi)^2} e^{-ipx}$$

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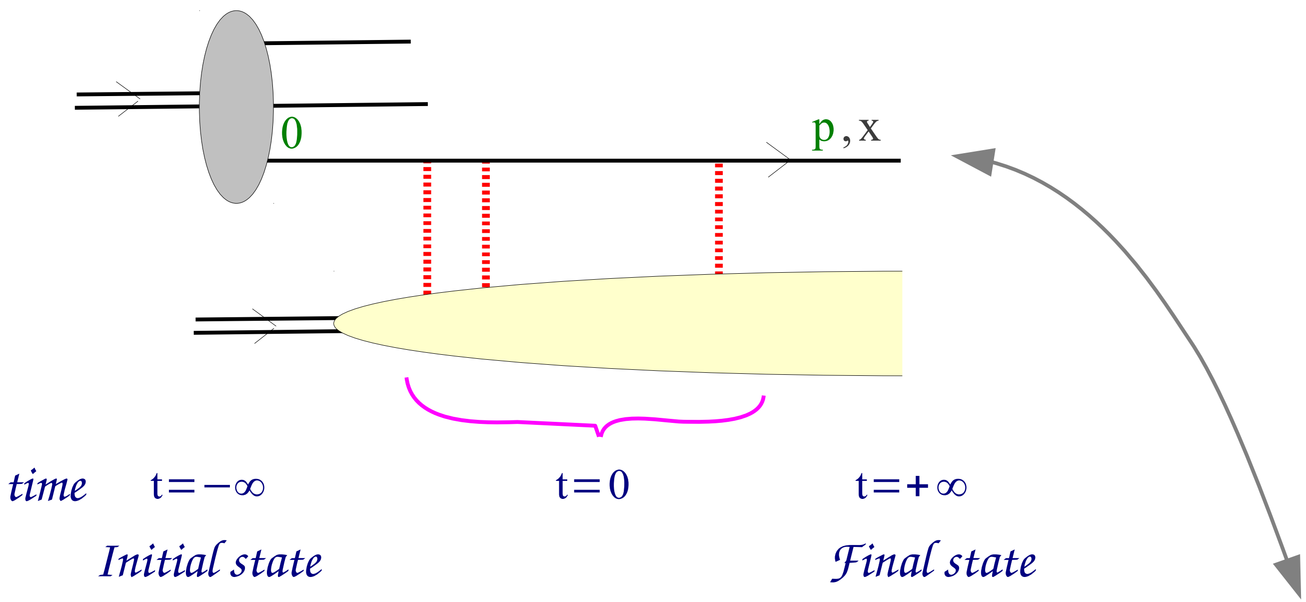
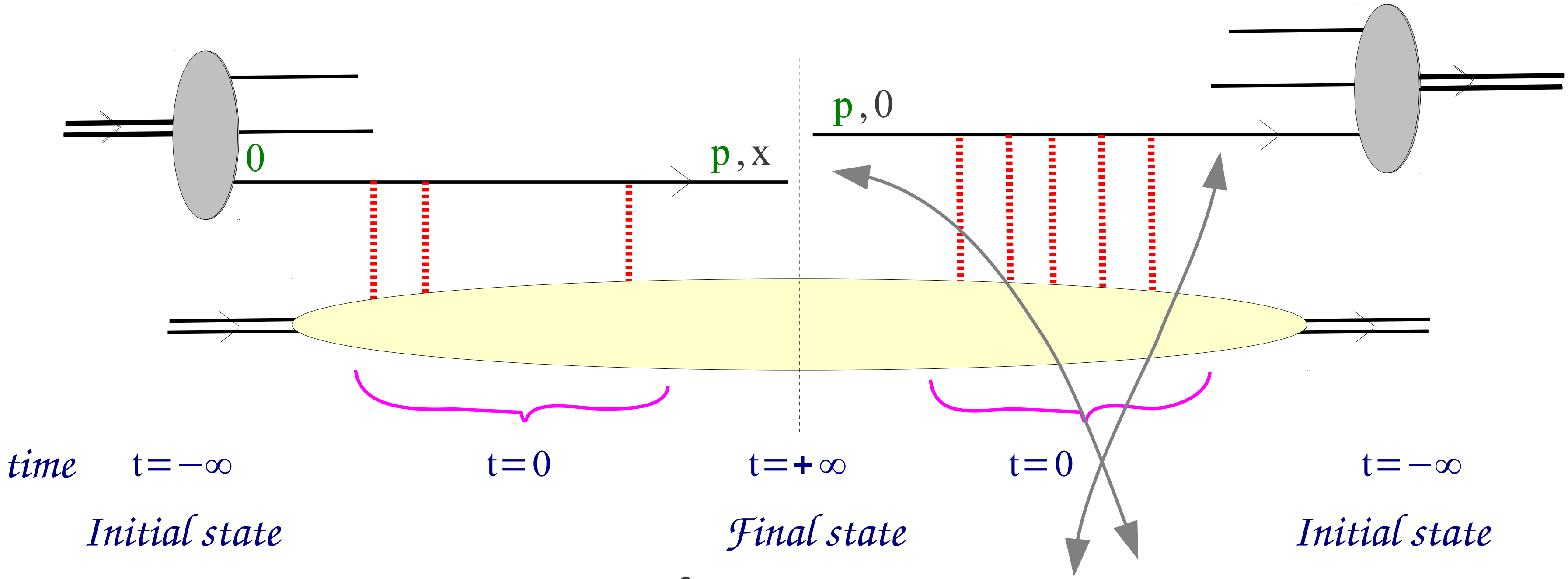


$$\int \frac{d^2x}{(2\pi)^2} e^{-ipx} \qquad V_x$$

Wilson line

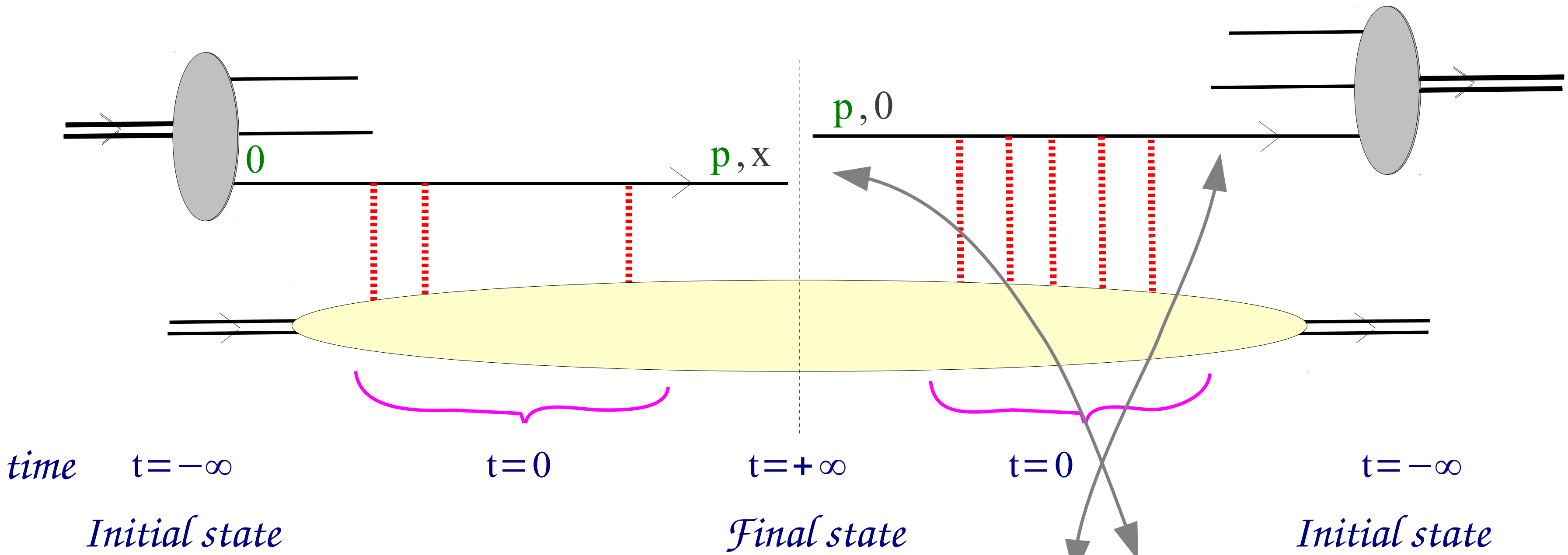
Formulation of p_T-broadening



$$\frac{dN}{d^2 p} \propto \int \frac{d^2 x}{(2\pi)^2} e^{-ipx} V_0^* V_x$$

Wilson lines

Formulation of p_T-broadening



$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-i p x} \left\langle \frac{1}{N_c} \mathrm{Tr}\, V_0^* V_x \right\rangle$$

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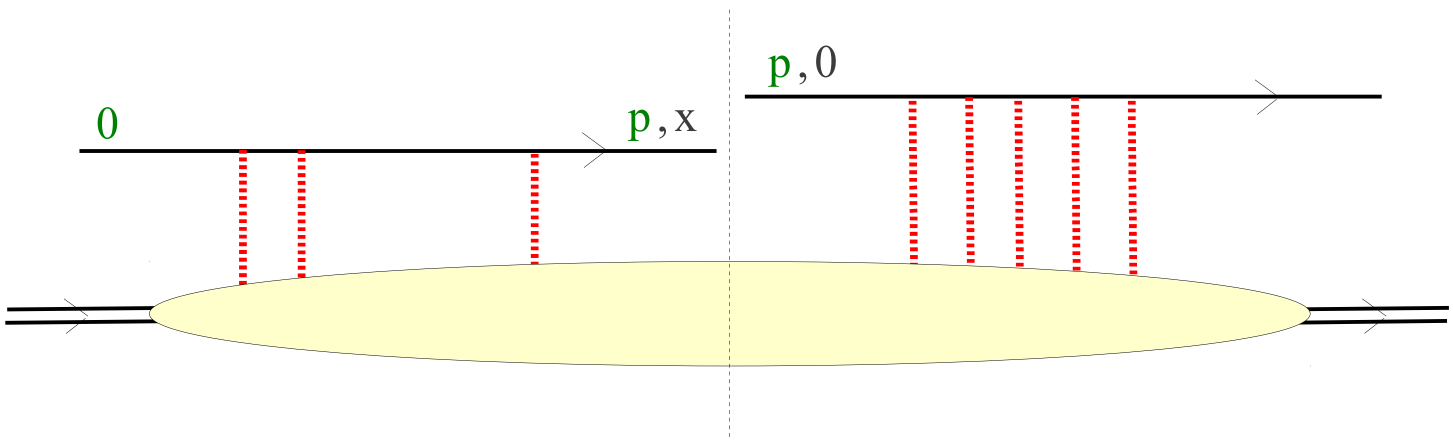

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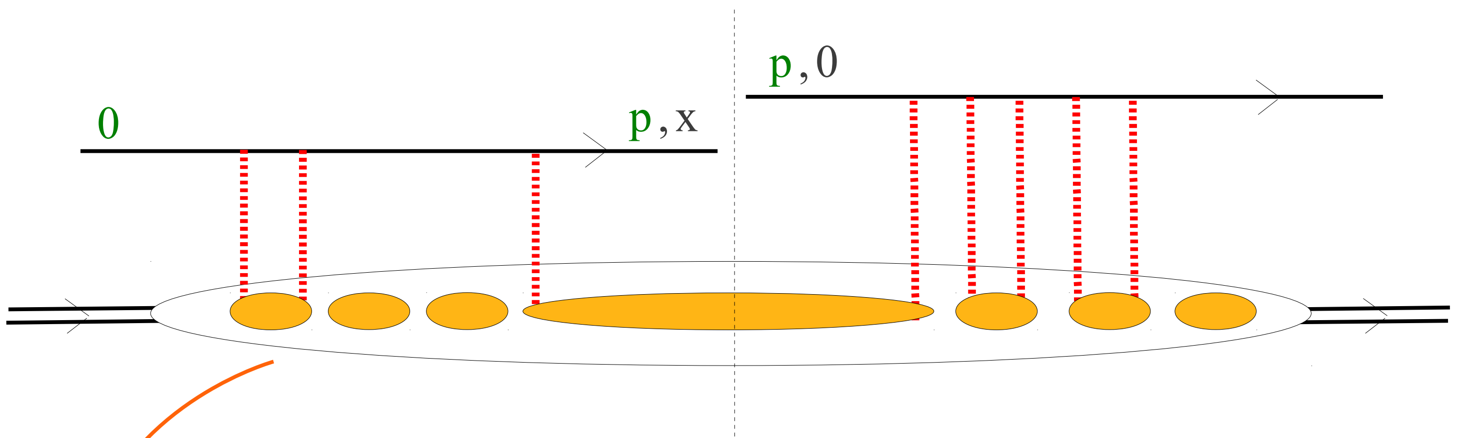

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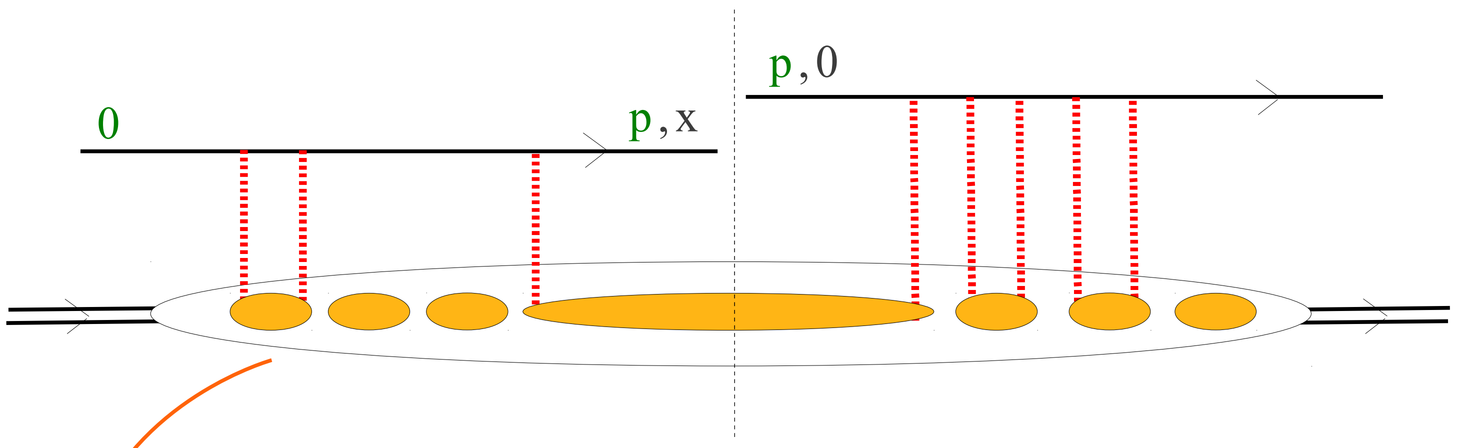
McLerran-Venugopalan model
(assumes 2-gluon exchanges at most)

S-matrix element for the elastic scattering of a color dipole

Formulation of p_T-broadening



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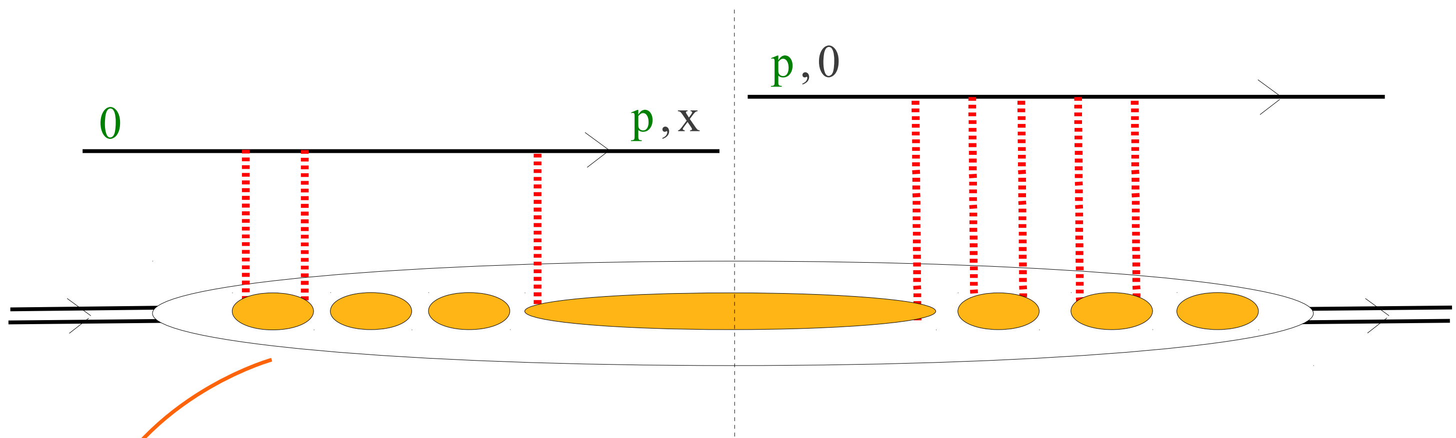

$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-i p x} \quad S_{dipole}(x) \sim e^{-p^2/Q_s^2}$$

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S-matrix element for the elastic scattering of a color dipole

$$S_{dipole}(x) = \exp\left(\frac{-x^2 Q_s^2}{4}\right)$$

Formulation of p_T*-broadening*

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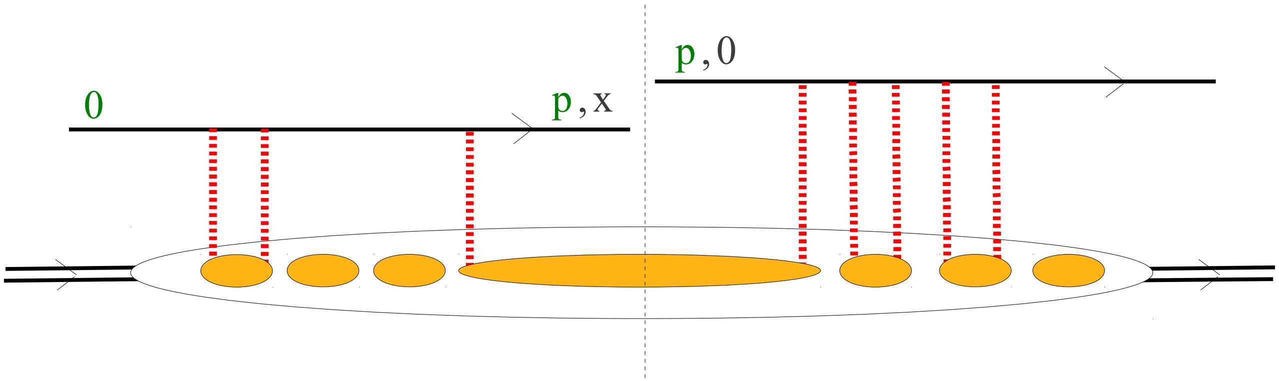

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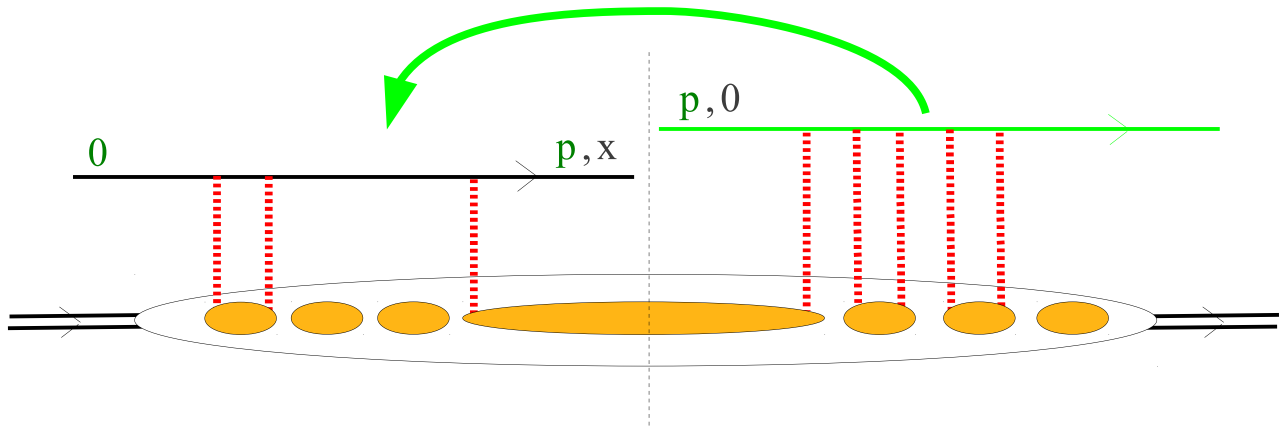

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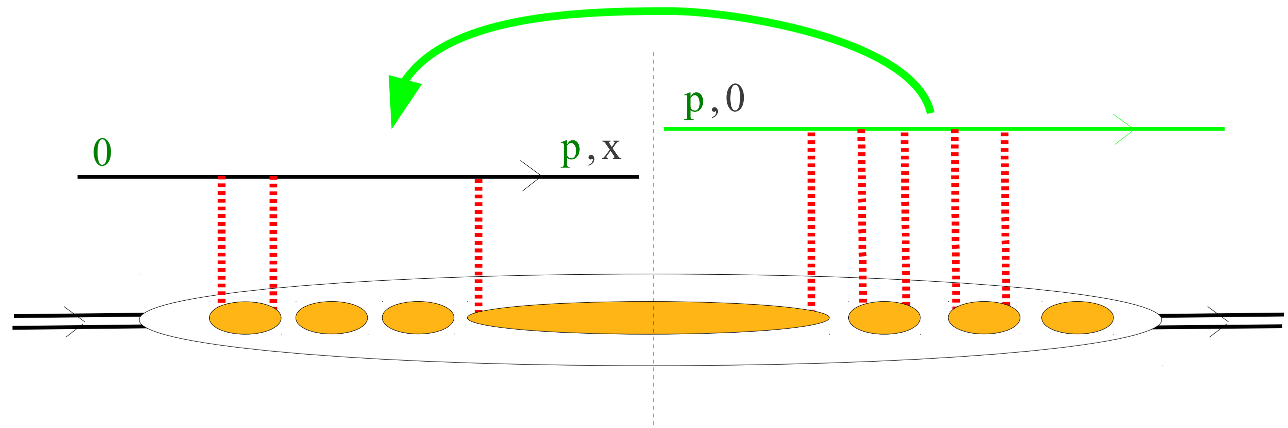

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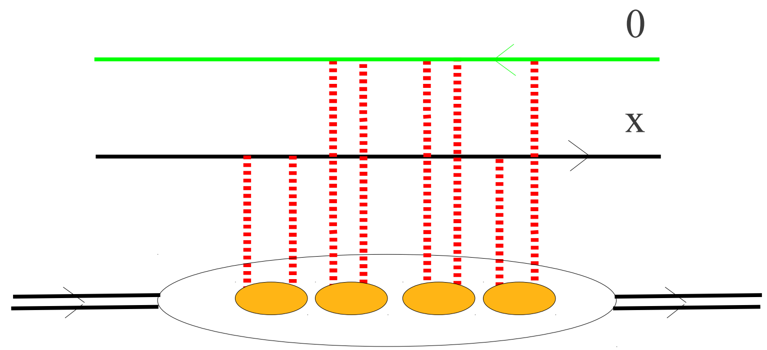

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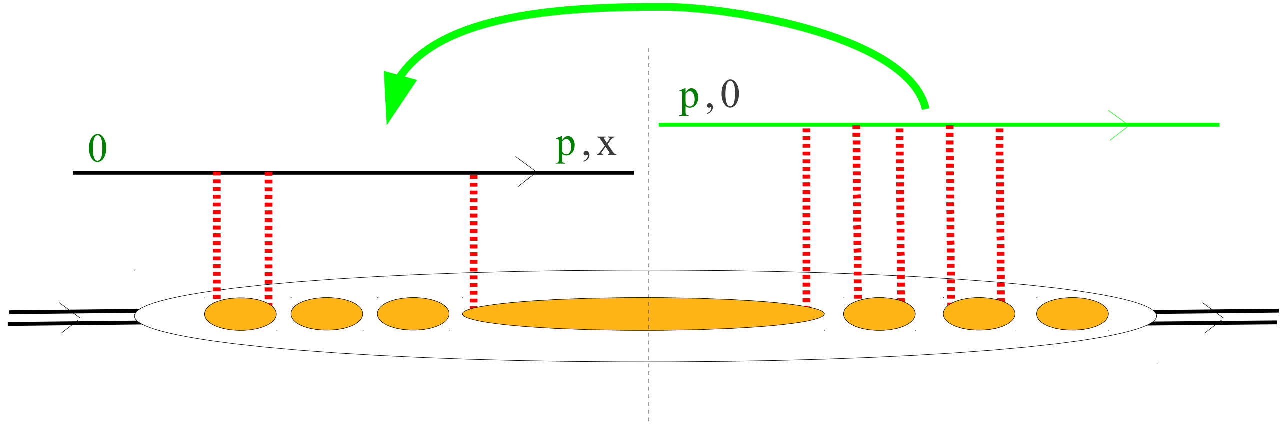

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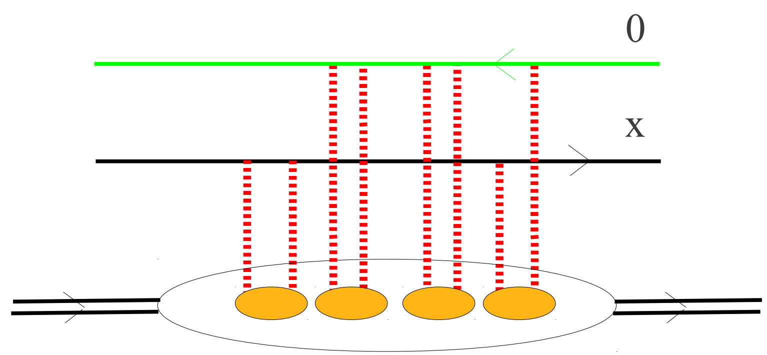
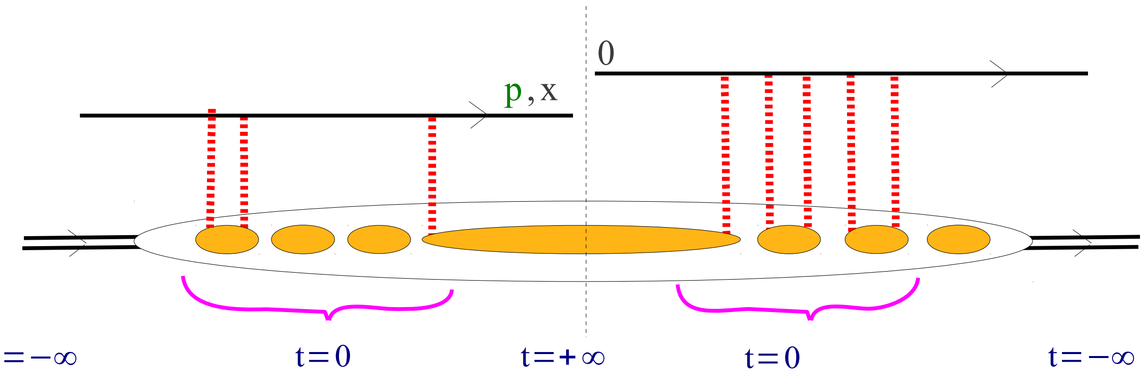


$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-i p x} S_{dipole}(x)$$

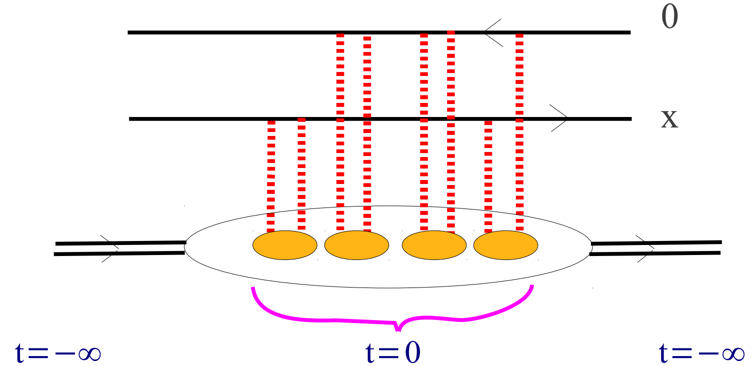
Intuitively: just bend the quark line in the complex conjugate amplitude to an antiquark line to transform it to a dipole amplitude!

Zakharov (1996...)

Quantum corrections



$$\frac{dN}{d^2p}=\int\frac{d^2x}{(2\pi)^2}e^{-ipx}\left\langle\frac{1}{N_c}\mathrm{Tr}\left(\quad V_0^*\quad\right)\left(\quad V_x\quad\right)\right\rangle$$



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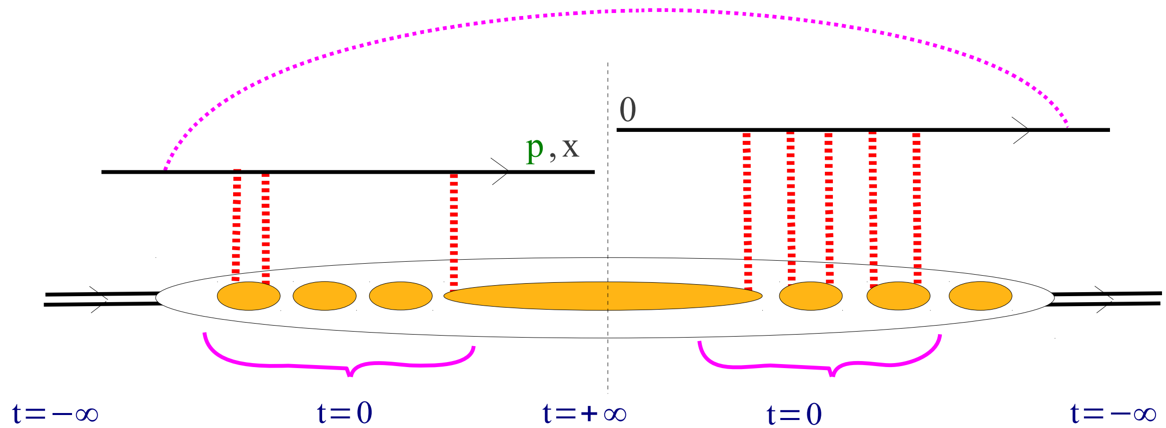

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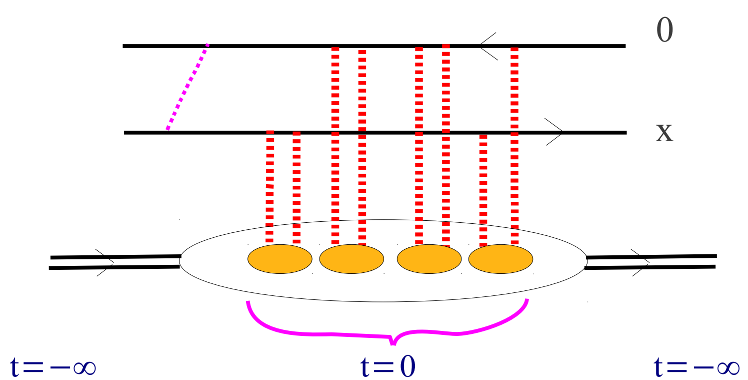

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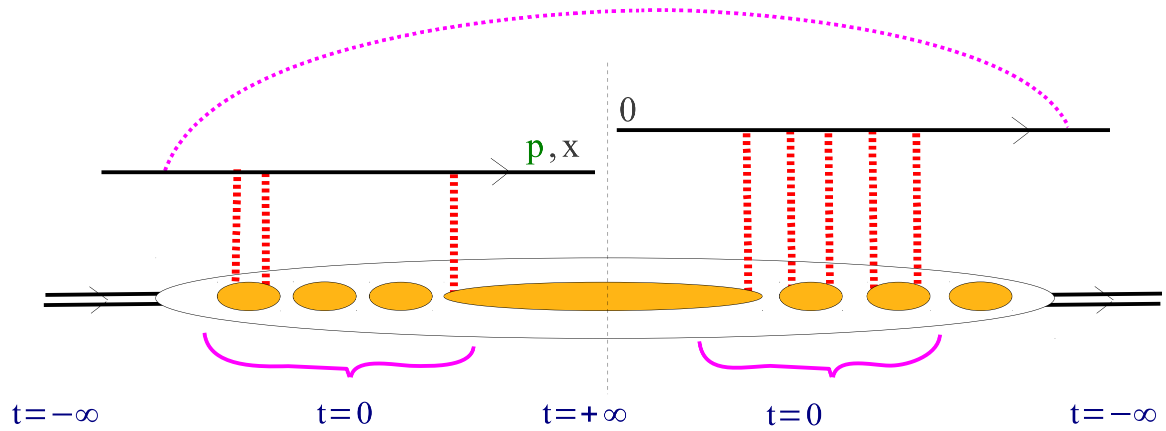

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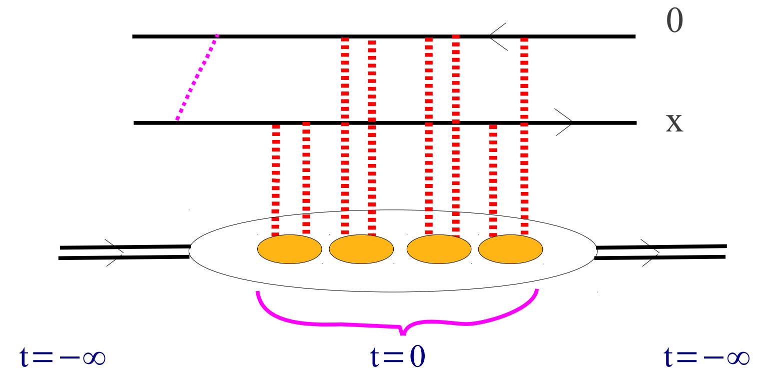

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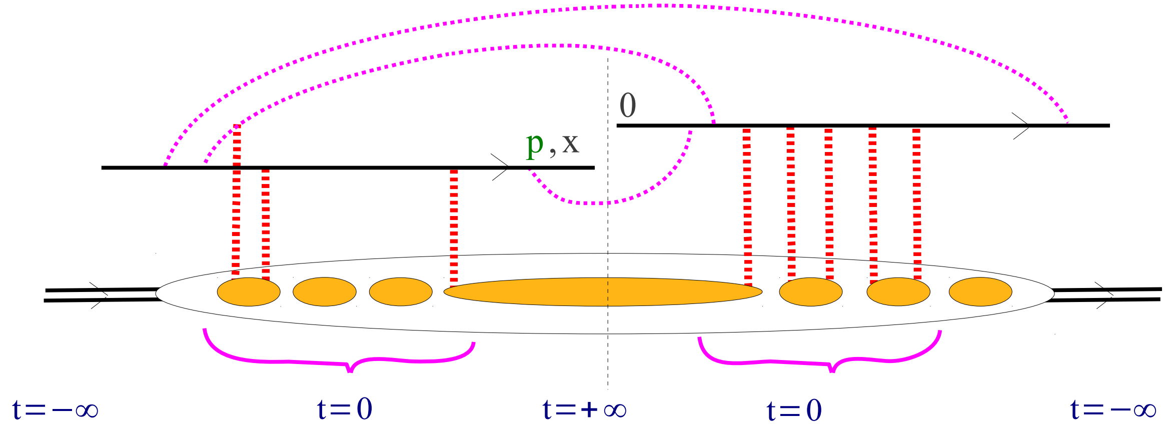

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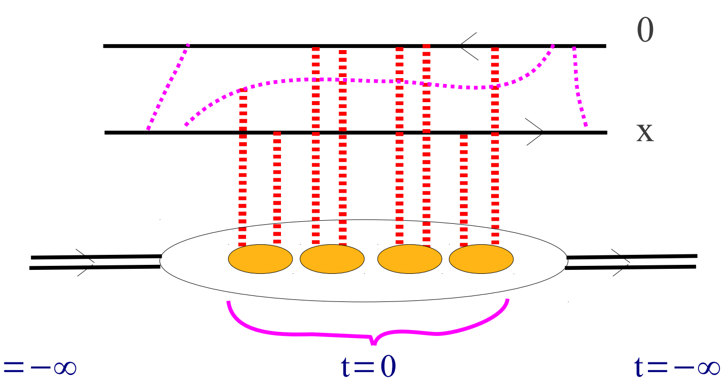

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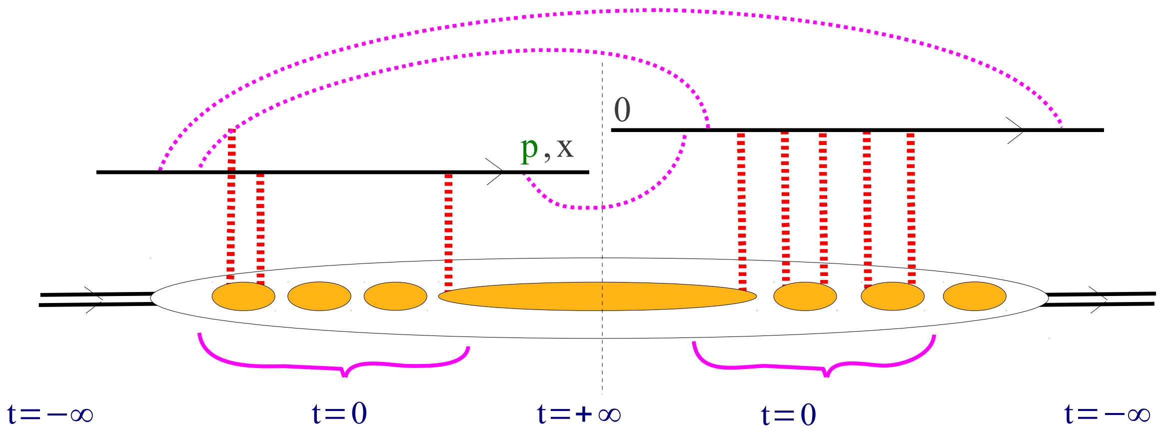

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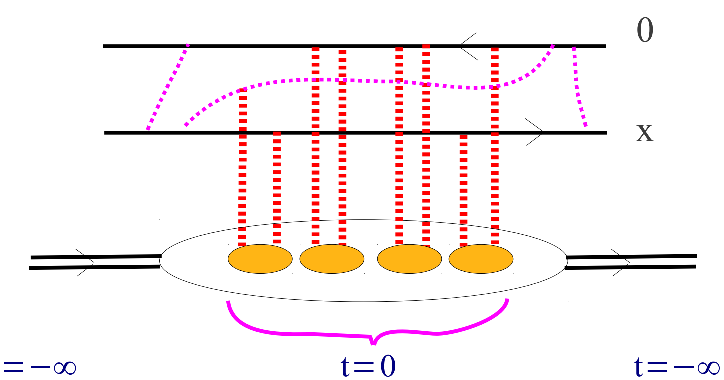

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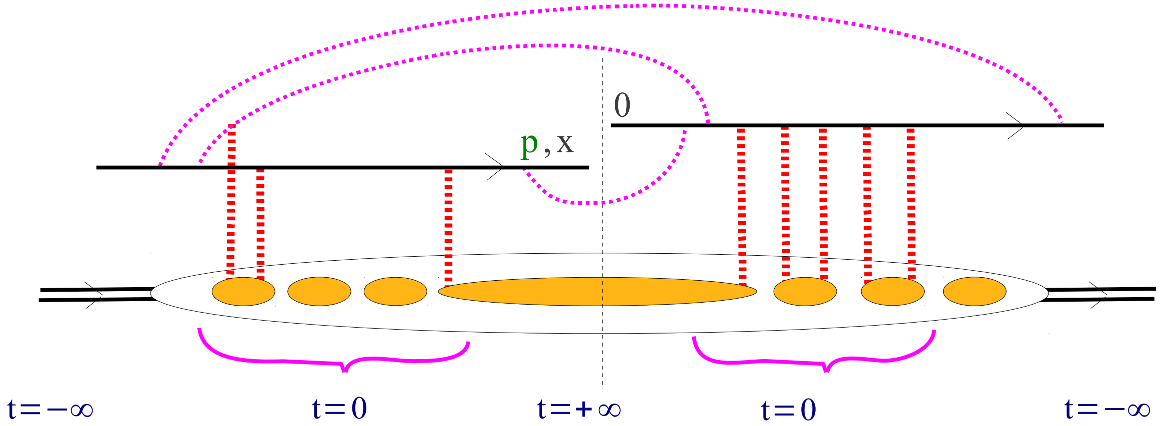

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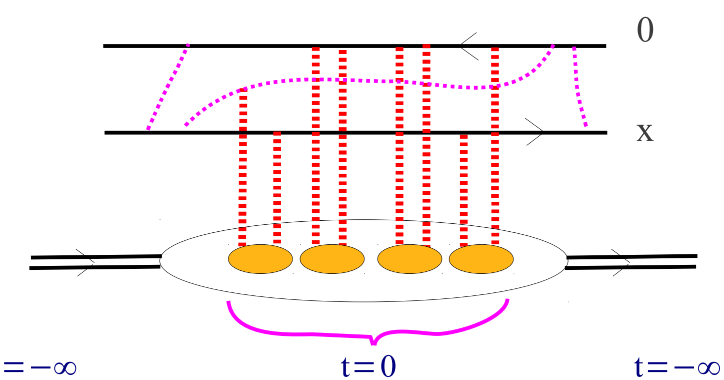
Two times

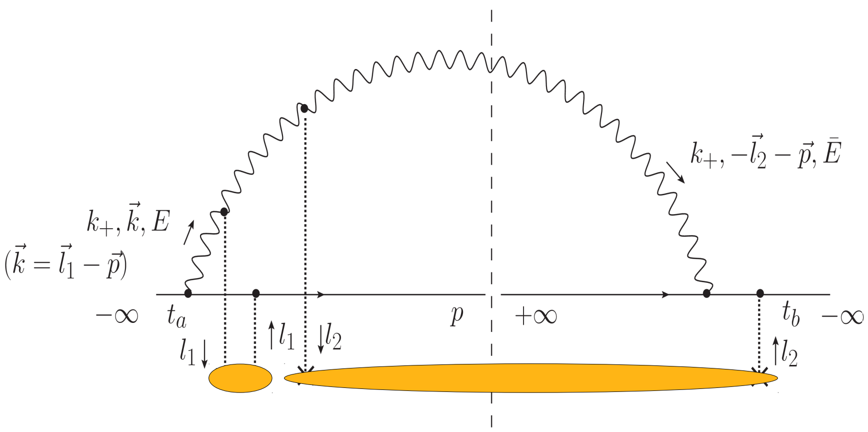


?

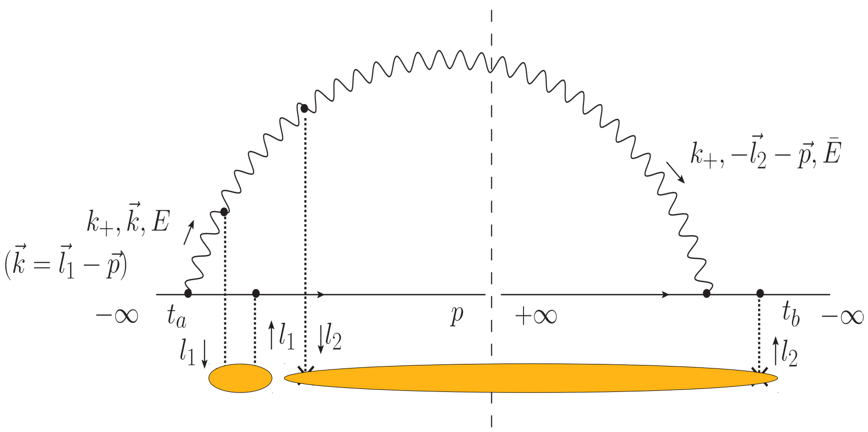
$$S_{dipole}(x)=\left\langle\frac{1}{N_c}\mathrm{Tr}\left(T\,V_0^*\,V_x\,e^{i\int d^4y\,L_I}\right)\right\rangle$$

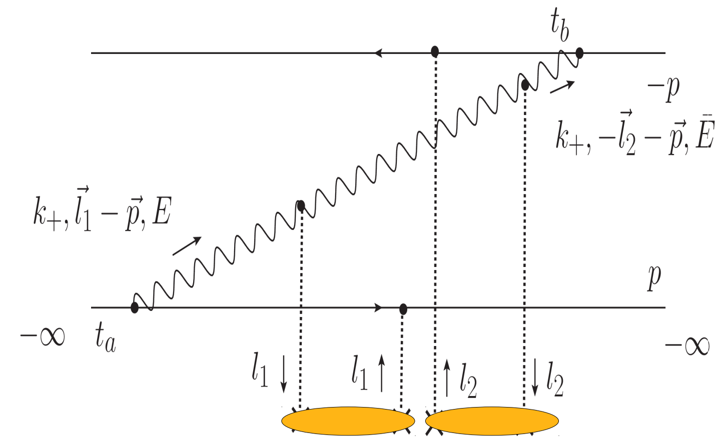
One single time

Quantum corrections: leading order



Quantum corrections: leading order





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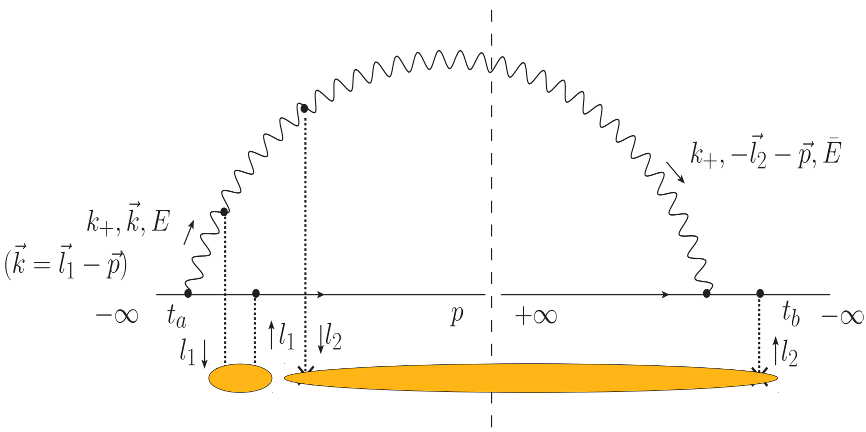
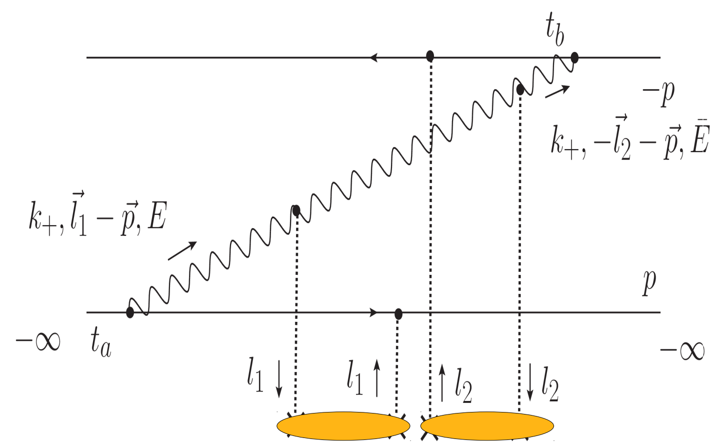


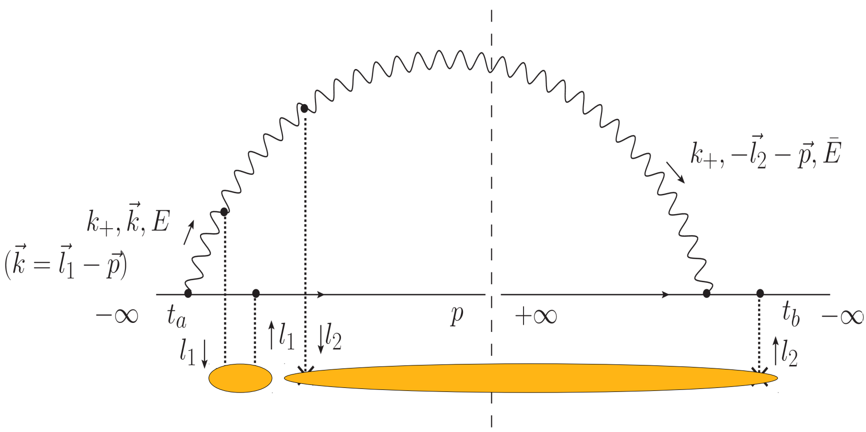


Contribution of this graph to $\frac{dN}{d^2 p}$

$$-\frac{\alpha_s N_c}{N_c^2-1}\int_0^{+\infty}\frac{dk_+}{k_+}\int\frac{d^2 l_1}{l_1^2}\frac{d^2 l_2}{l_2^2}\frac{(p-l_1)(p+l_2)}{(p-l_1)^2(p+l_2)^2}\left(\alpha_s xg(l_1)\right)\left(\alpha_s xg(l_2)\right)$$

Quantum corrections: leading order

Graph-to-graph, momentum-to-momentum correspondence



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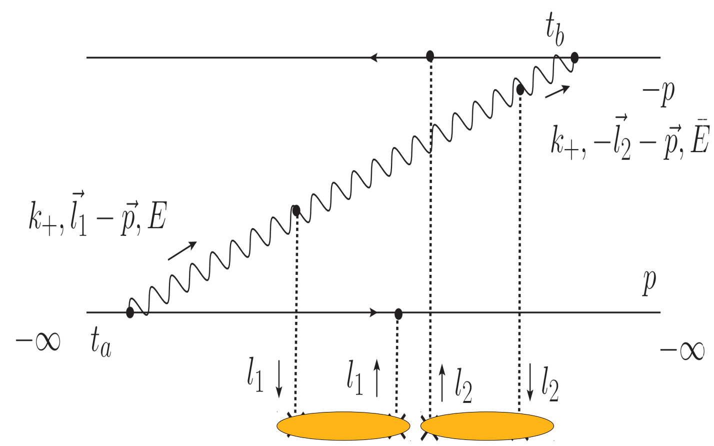
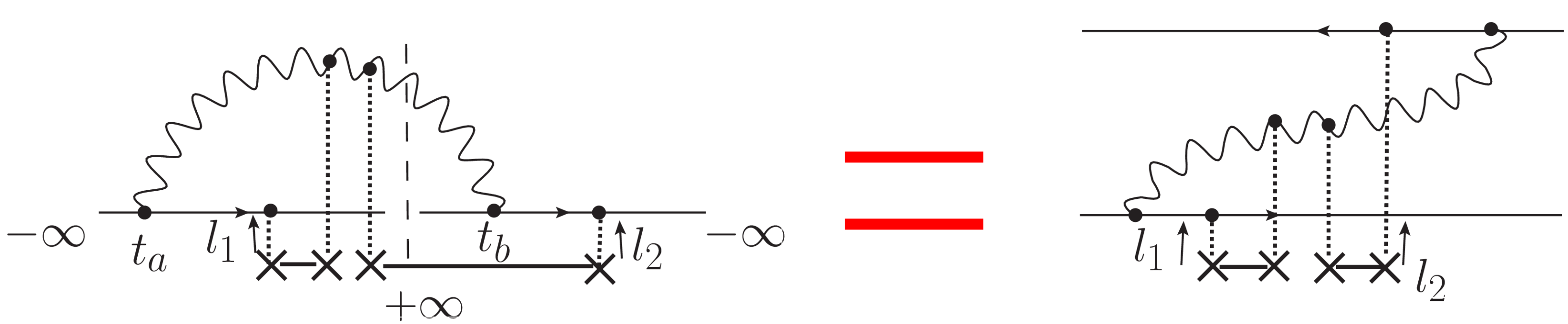


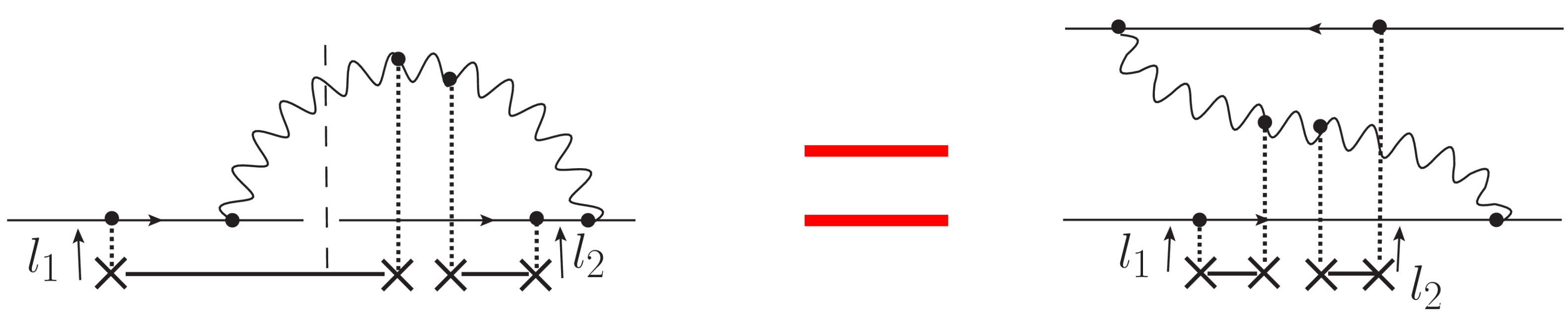
Contribution of this graph to $\frac{dN}{d^2p}$ ***and*** *to the dipole S-matrix element in momentum space:*

$$-\frac{\alpha_s N_c}{N_c^2-1}\int_0^{+\infty}\frac{dk_+}{k_+}\int\frac{d^2l_1}{l_1^2}\frac{d^2l_2}{l_2^2}\frac{(p-l_1)(p+l_2)}{(p-l_1)^2(p+l_2)^2}\big(\alpha_s xg(l_1)\big)\big(\alpha_s xg(l_2)\big)$$

Quantum corrections: leading order

Graph-to-graph, momentum-to-momentum correspondence





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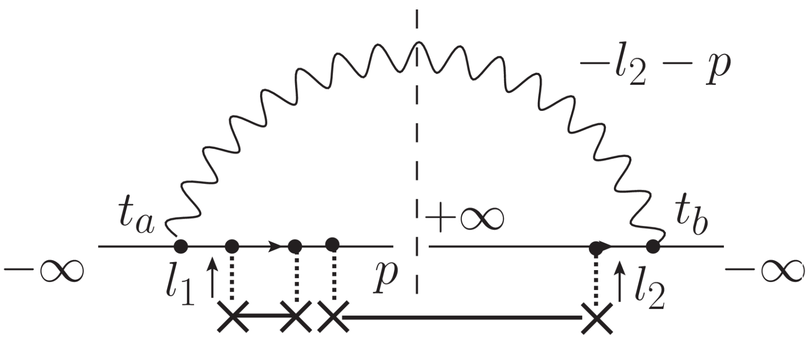
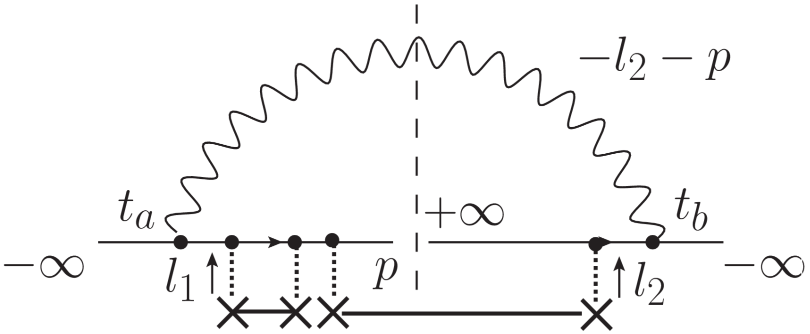


Quantum corrections: leading order

Graph-to-"group of graphs", momentum-to-momentum correspondence



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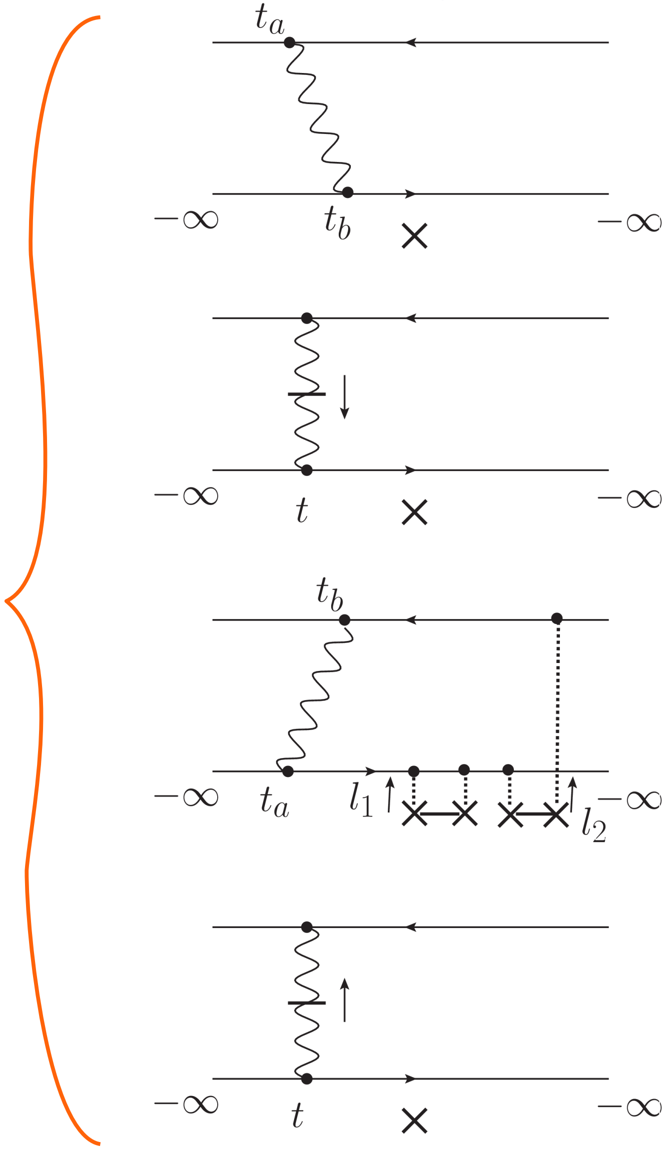

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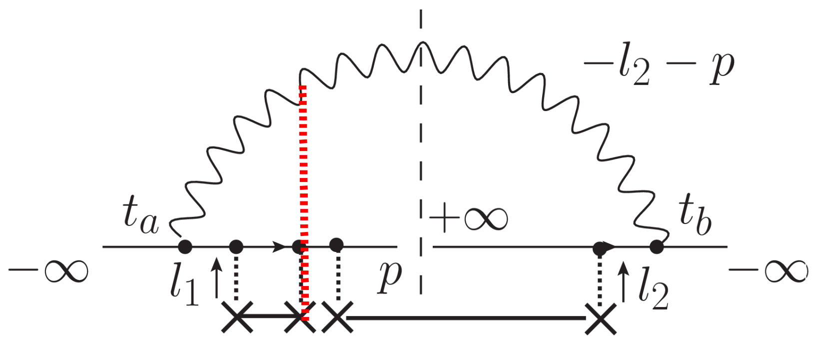

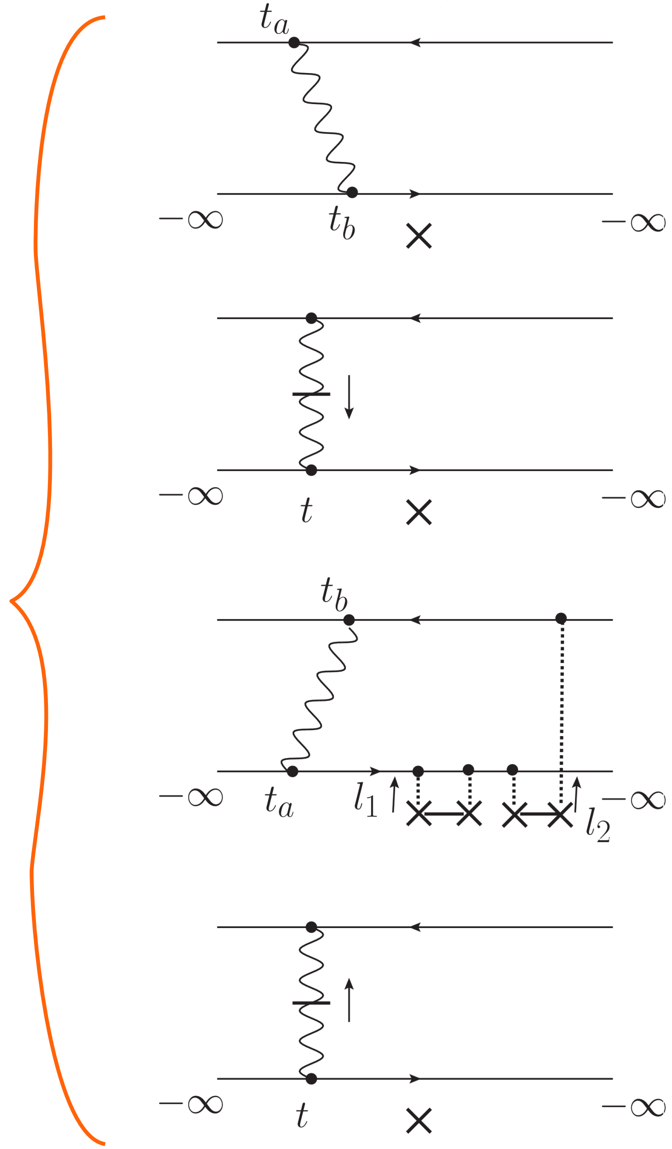

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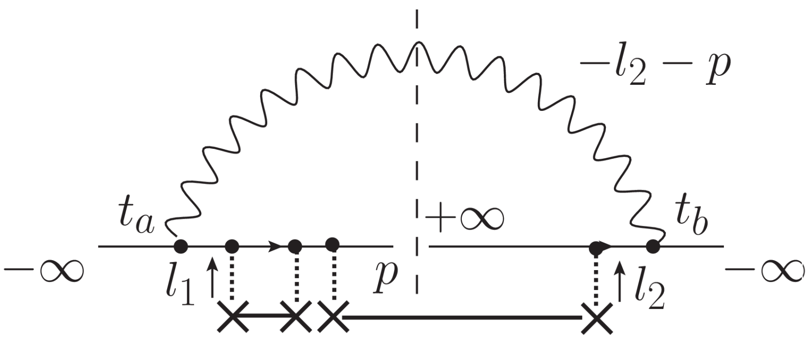

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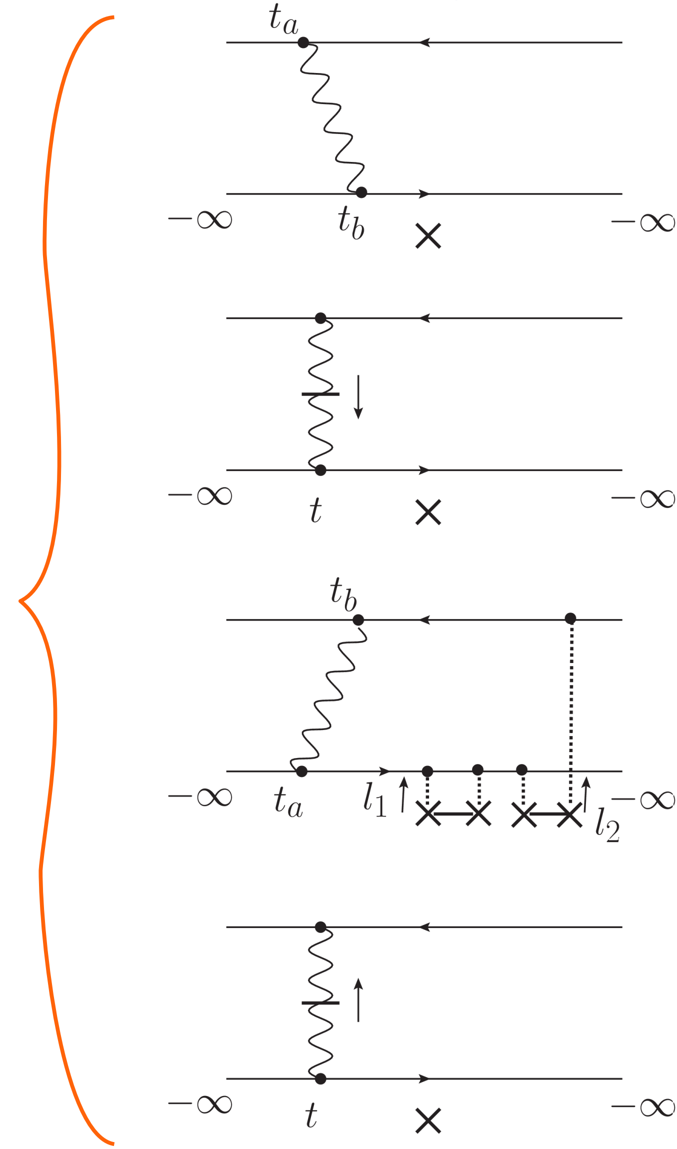

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Leading-order quantum corrections are identical in the cases of broadening and dipole scattering!

Kovchegov et al (2002...)

Quantum corrections: next-to-leading order

Hundreds of graphs on both *(broadening and dipole)* ***sides!***

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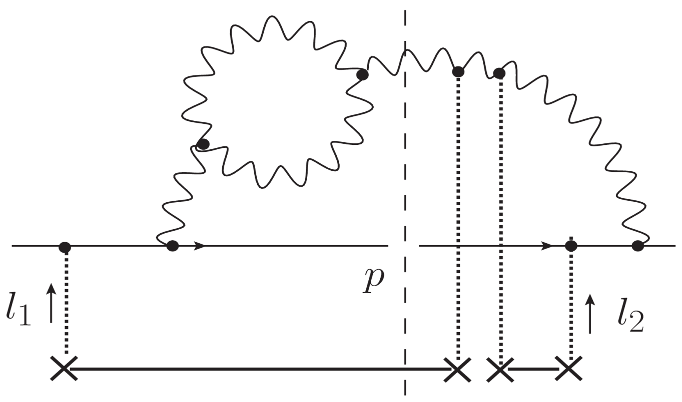

Similar to LO

Momentum-to-momentum correspondence

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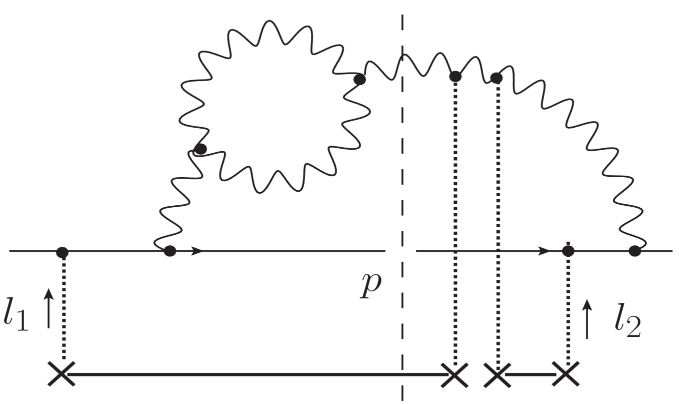

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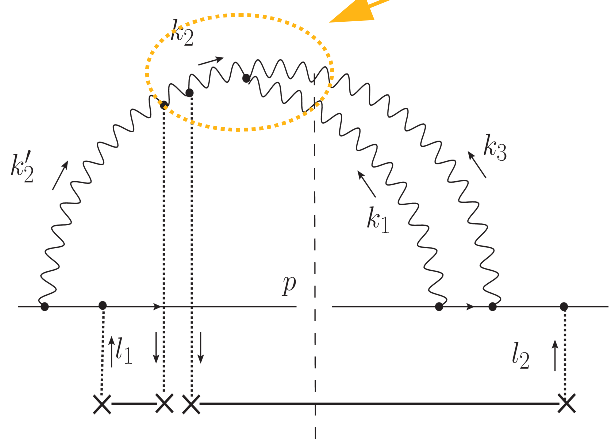

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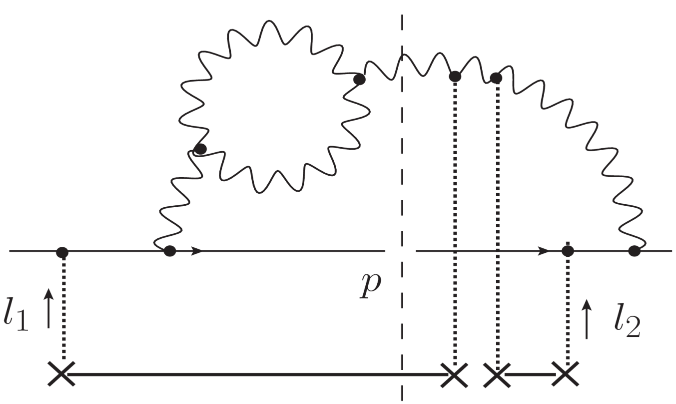

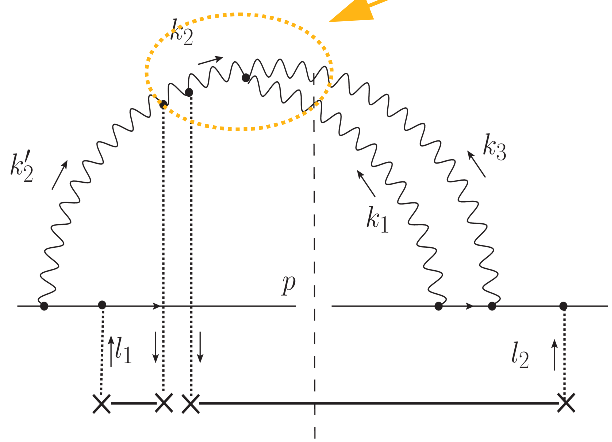

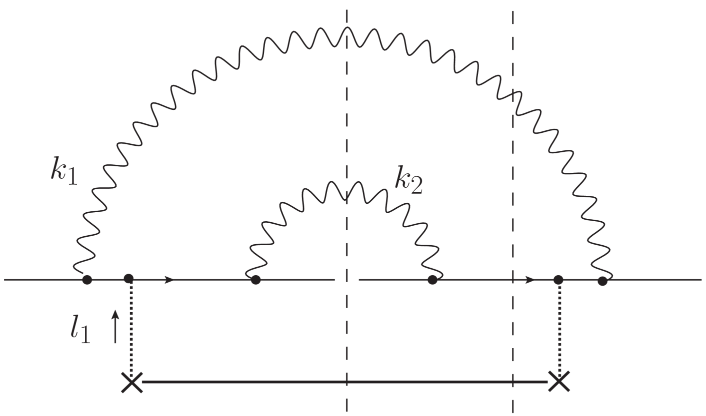

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For these 2 classes, the correspondence is verified only after integration over some of the longitudinal momenta!

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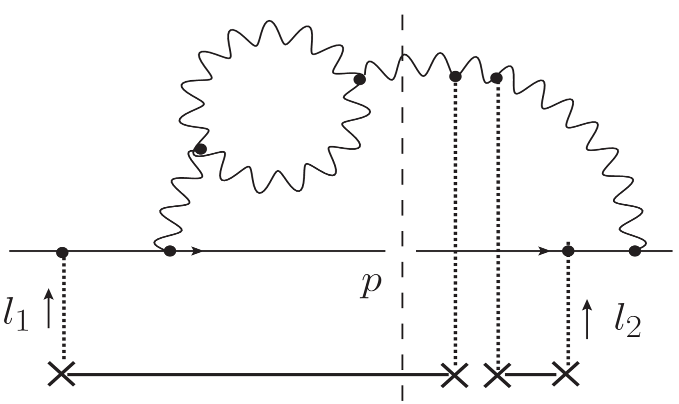

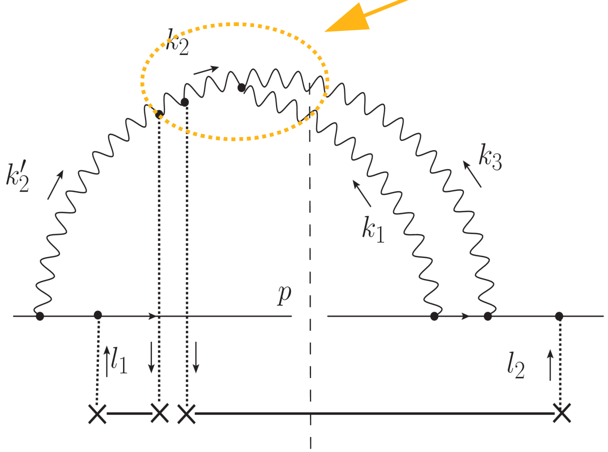

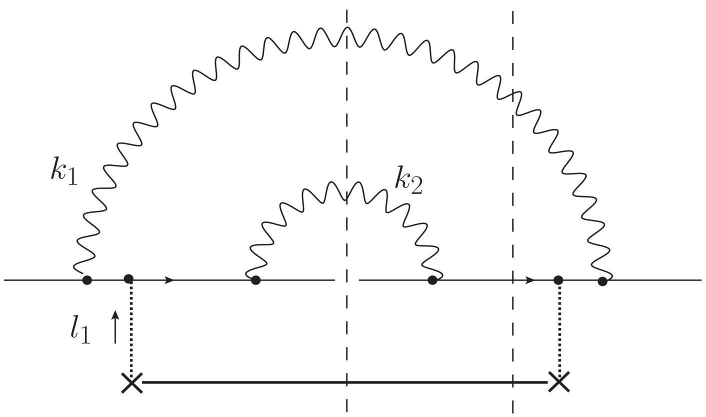

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<u>Our conclusion:</u> the correspondence between broadening and dipole scattering is preserved at NLO!

Summary and outlook

- *We proved that the broadening cross section and the dipole elastic amplitude are indeed related by a simple Fourier transform when quantum corrections are included* ***to next-to-leading order****:*

$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-ipx} S_{dipole}(x)$$

The full BFKL evolution is thus identical for these two processes at NLO.

- *Our conclusion is valid for other final-state observables, such as angular correlations of jets (related to dipole and quadrupole amplitudes).*

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- *Our conclusion is valid for other final-state observables, such as angular correlations of jets (related to dipole and quadrupole amplitudes).*

- *The method we used was "brute force" inspection of all relevant graphs. Is there a deeper way to understand the identity? Too many "miracles" happen to believe that this correspondence is an accident...*

- *Is the identity valid beyond NLO? (We actually think that it is not).*