Studies of TMDs from SIDIS

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DIS 2013

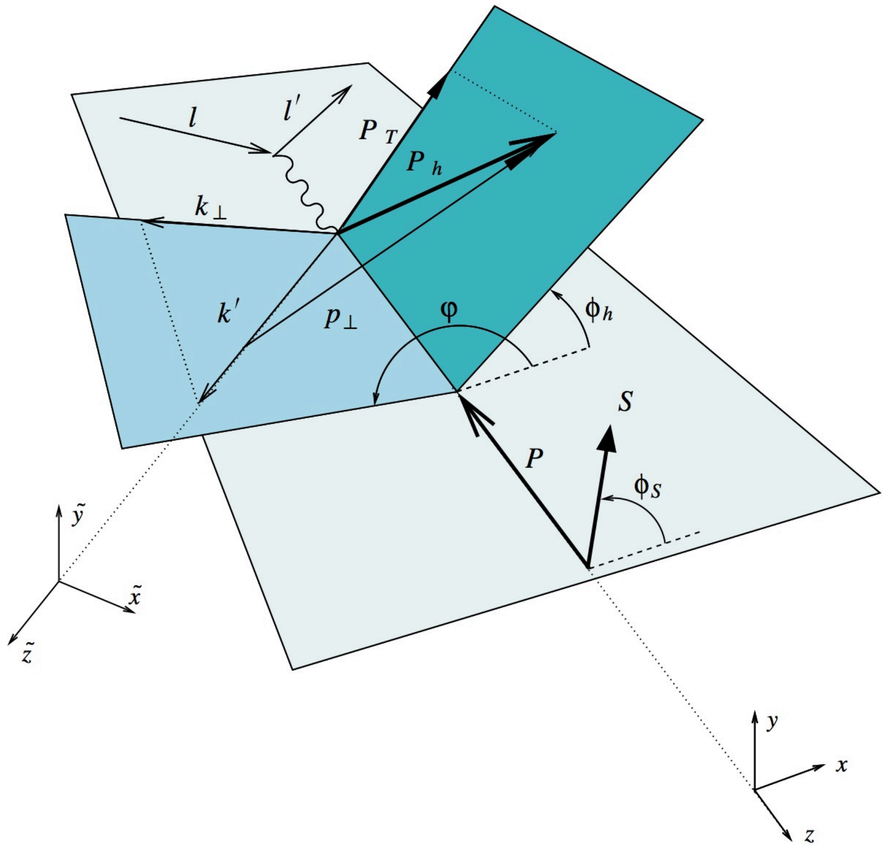
Outline

- SIDIS cross section
- Experimental measurements
- Bessel weighting
  - ➢ Fully differential MC with
    - ✓ Factorized DF and FF
    - ✓ Non factorized DF and FF
- Results

SIDIS cross-section

$$eP \to e'hX$$

Figure from **PRD 71, 074006 (2005).**



$$\frac{d\sigma}{dxdydzd^2\mathbf{p}_\perp d^2\mathbf{k}_\perp d\phi_{l'}} =$$

Assuming single photon exchange, after integration, the lepton-hadron cross section can be expressed in a model-independent way:

$$\frac{d\sigma}{dxdydzdP^2_{h,T}d\phi_s d\phi_h} = \ldots$$

SIDIS cross-section

Bacchetta: arXiv:hep-ph/0611265

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{h,T}^2\,d\phi_s\,d\phi_h} = K(x,y) \times [F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos(\phi_h) F_{UU}^{\cos(\phi_h)} +$$

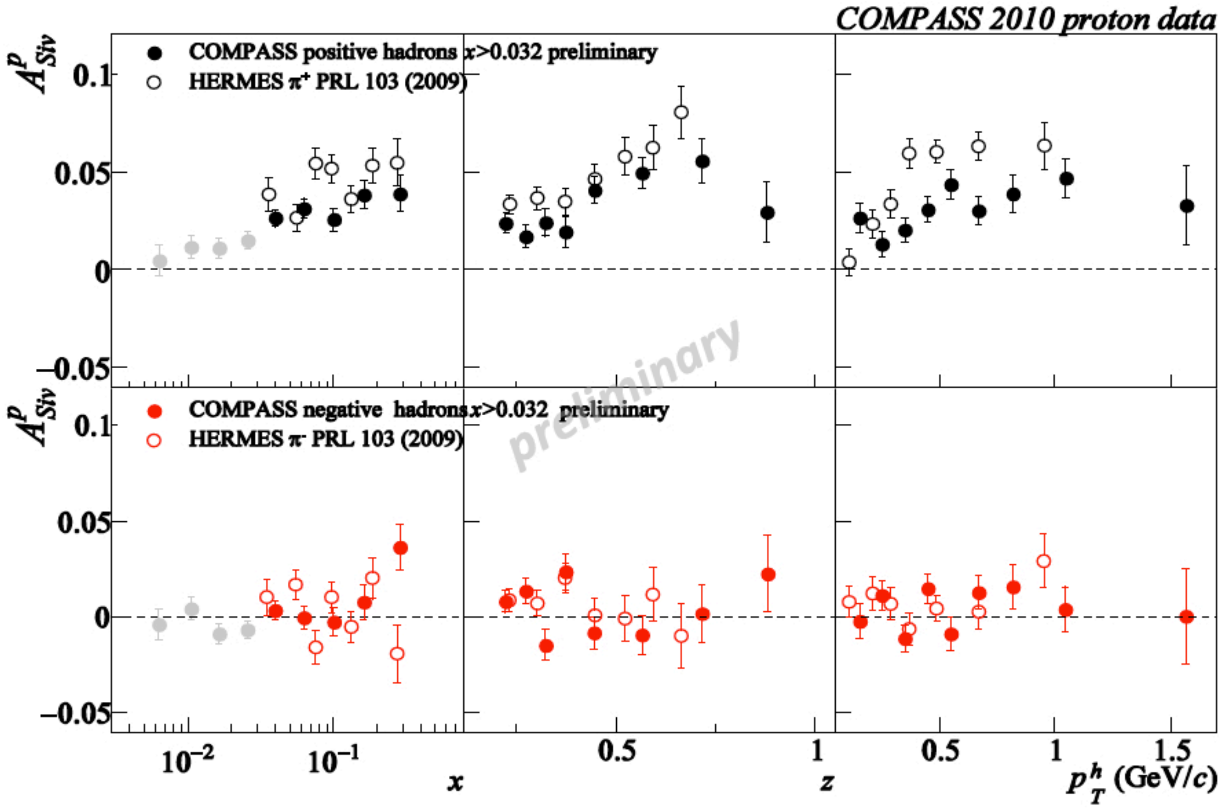
$$+\varepsilon\cos(2\phi_h) F_{UU}^{\cos(2\phi_h)} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin(\phi_h) F_{LU}^{\sin(\phi_h)} +$$

$$+S_{||}\lambda_e\sqrt{1-\varepsilon^2} F_{LL} + |\mathbf{S}_\perp|\sin(\phi_h-\phi_s)(F_{UT,T}^{\sin(\phi_h-\phi_s)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_s)}) + \ldots]$$

$$F_{UU,T} = \mathcal{C}\left[f_1(x,k_\perp) D_1(z,p_\perp)\right] = f_1(x,k_\perp) \otimes D_1(z,p_\perp)$$

Each structure function is convolution of the DFs and FFs.

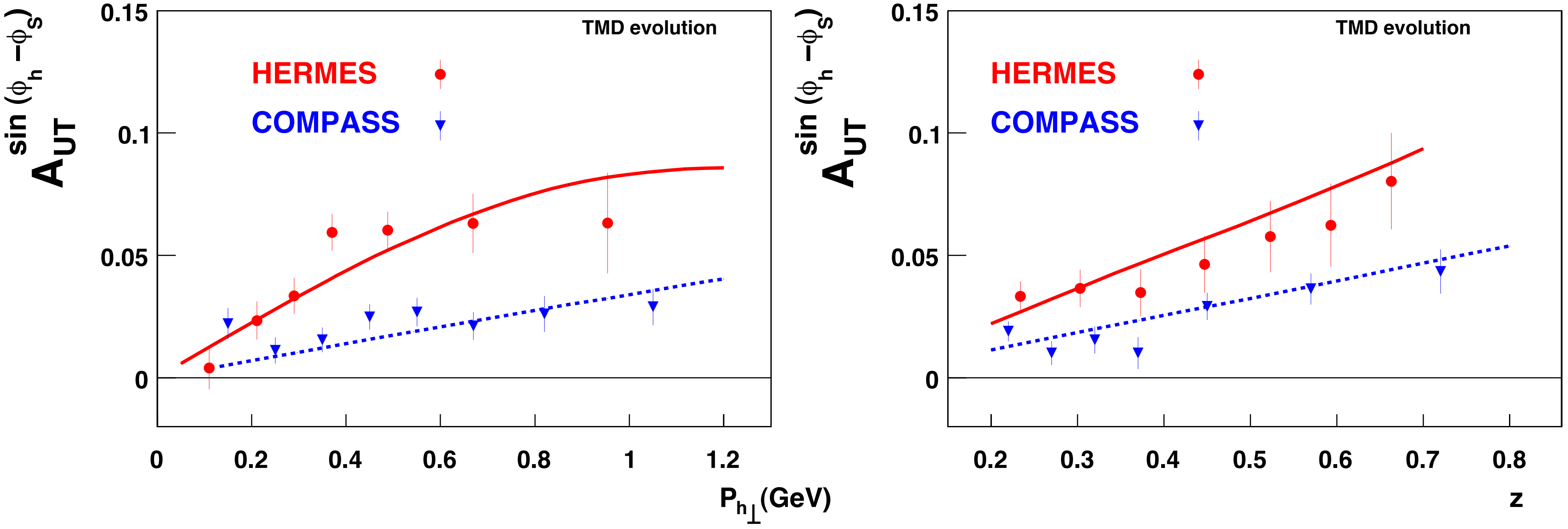
Sivers asymmetry



Data suggests Q^2 evolution of Sivers function may be significant.

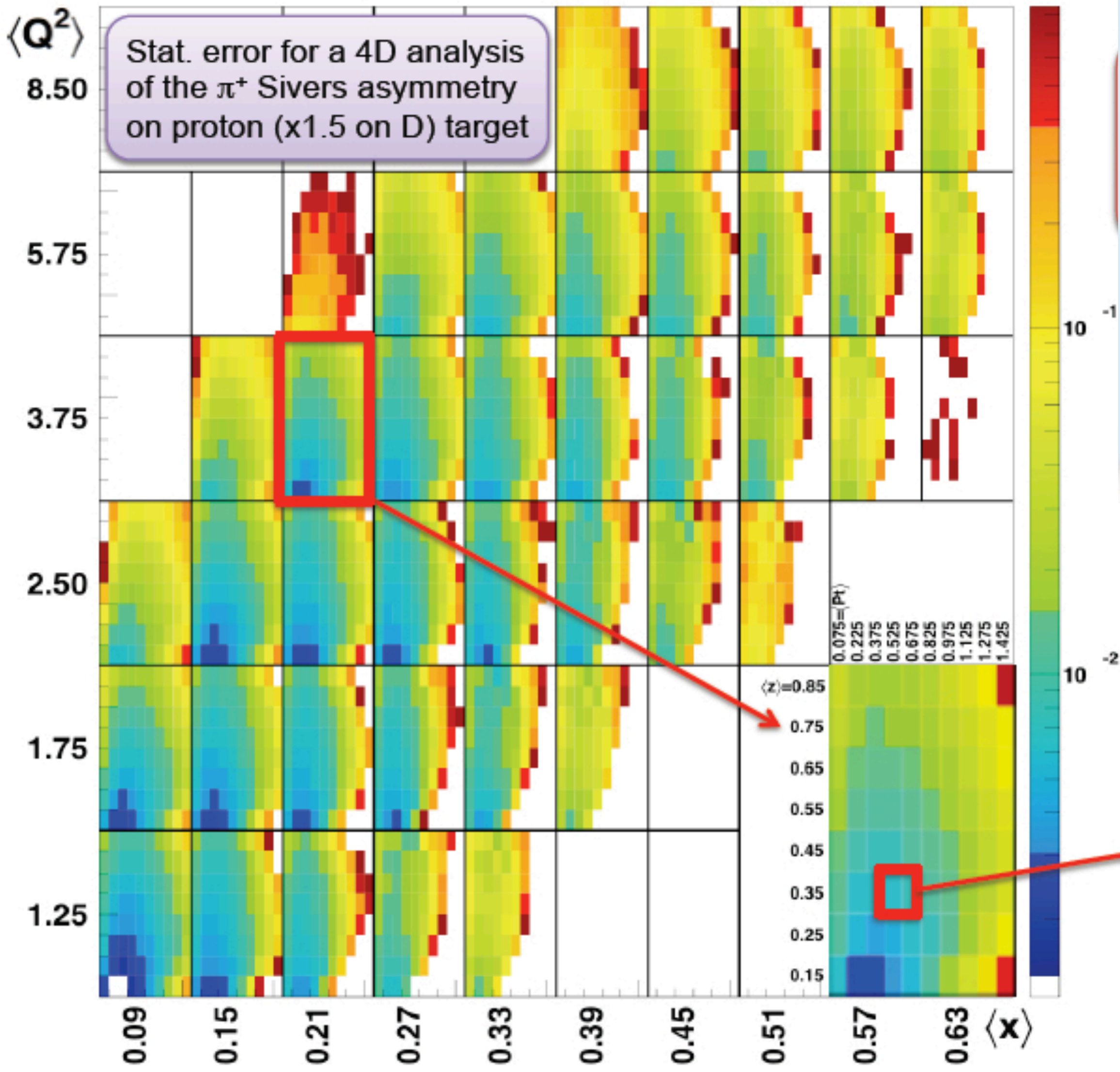
Sivers asymmetry with phenomenological fit

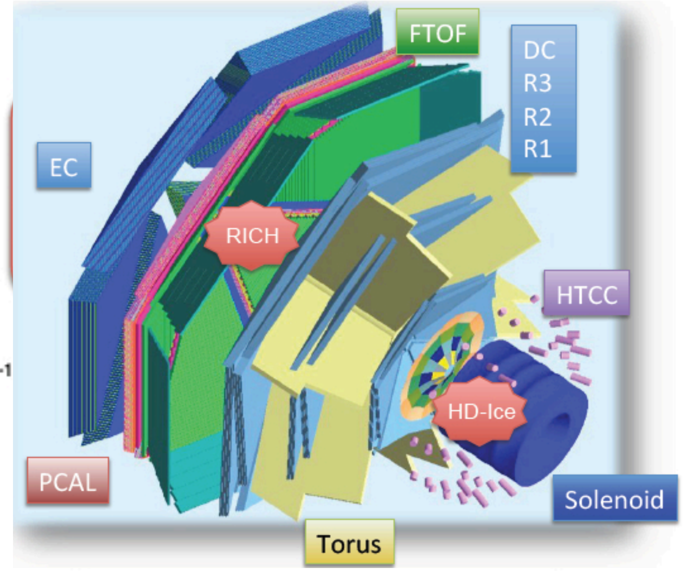
Mert Aybat, Alexei Prokudin, Ted Rogers, PRL 108 (2012) 242003

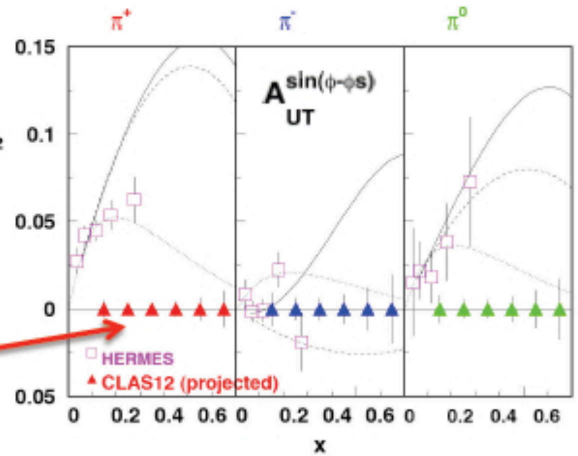


TMD evolution taken into account for Sivers function, and it describes data better!

CLAS12 A_{UT} with transverse proton target

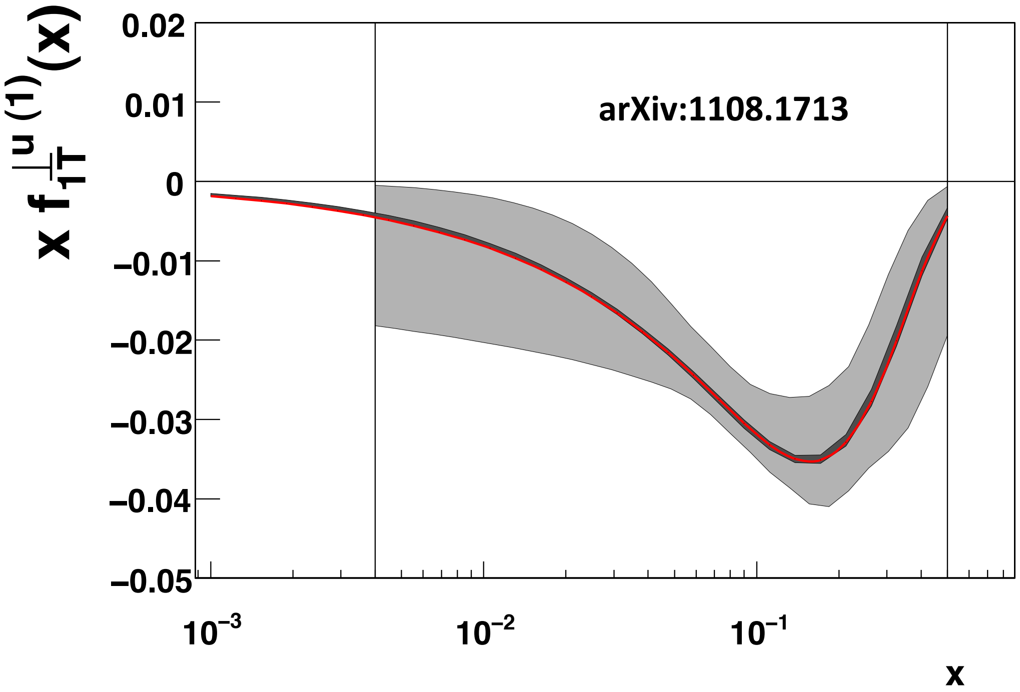






Curves from hep-ph/0507266 and hep-ph/0507181

Extraction from EIC



Systematic error from model dependence in this approach is hard to control.
Does not allow extraction of underlying $k_\perp$ dependence in model independent way.

Comparison of the of extractions of the Sivers function for u quarks from pseudo-data generated for the EIC with energy setting of √s= 45 GeV
The uncertainty estimates are for the specifically chosen (fixed $k_\perp$ dependence) underlying functional form.

Need model independent extraction!

Bessel-weighted extraction

Model independent extraction of flavor decomposition of $k_\perp$ dependent PDFs. Boer:JHEP10(2011)021

Complicated convolution

$$\sigma^h(P_{hT}) \sim \sum_q f^q(x, k_\perp) \otimes D^{q\to h}(z, p_\perp)$$

Experimental Fourier transform

$$\tilde{\sigma}^h(b_T) \sim \sum_q \tilde{f}^q(x, b_T) \cdot \tilde{D}^{q\to h}(z, b_T)$$

product

Fully differential MC

$$\frac{d\sigma}{dxdydzd^2\mathbf{p}_\perp d^2\mathbf{k}_\perp d\phi_{l'}} = K(x,y)J(x,Q^2,k_\perp)\times$$

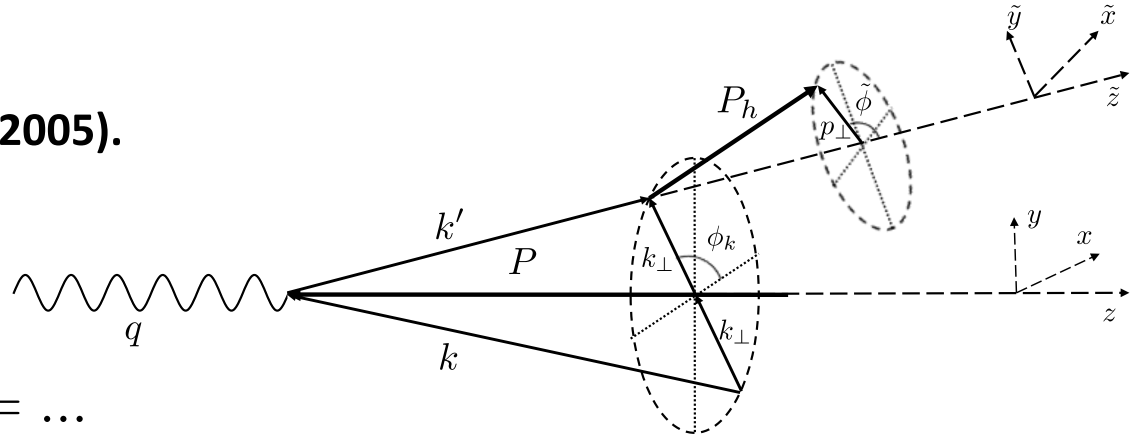
$$\times \sum_q e_q^2 \left[f_{1,q}(x,k_\perp)D_{1,q}(z,p_\perp) + \lambda\sqrt{1-\varepsilon^2} g_{1L,q}(x,k_\perp)D_{1,q}(z,p_\perp) \right]$$

Detected hadron transverse momentum is constructed from quark intrinsic transverse momentum after the convolution.

Is necessary to understand the extraction of quarks transverse momentum dependence!

Model for fully differential MC

Anselmino: PRD 71, 074006 (2005).



$$\frac{d\sigma}{dxdydzd^2\mathbf{p}_\perp d^2\mathbf{k}_\perp d\phi_{l'}} = ...$$

Quark intrinsic motion with Torino model: $M_p = 0$ $x_{LC} = k^-/P^-$

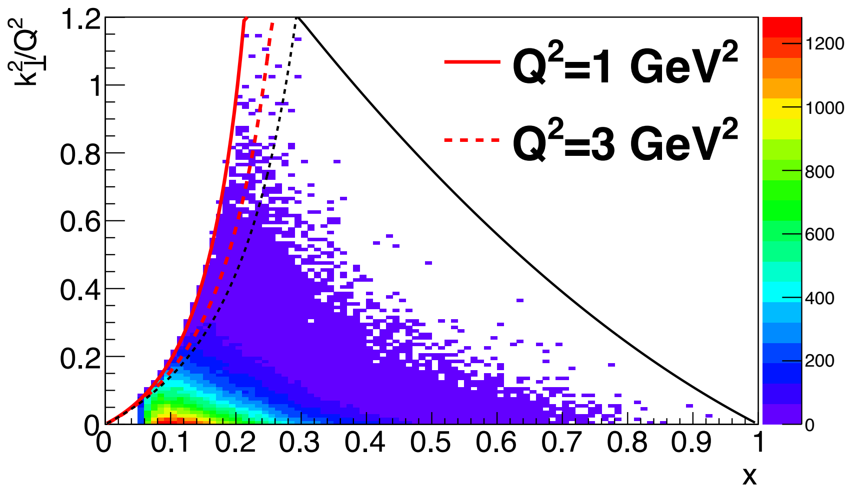
Quark inside the proton have the momentum:

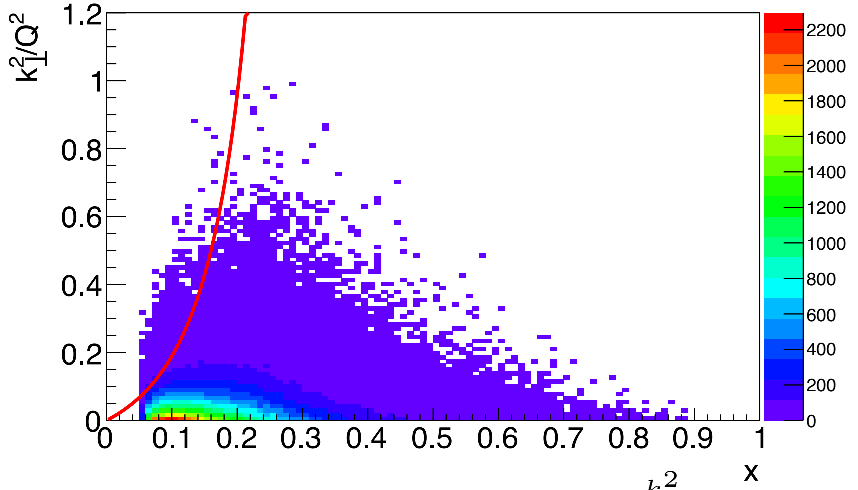
$$k = \left(x_{LC}P_0 + \frac{k_\perp^2}{4x_{LC}P_0}, \mathbf{k}_\perp, -x_{LC}P_0 + \frac{k_\perp^2}{4x_{LC}P_0} \right)$$

Where $x_{LC} = \frac{1}{2}x\left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}}\right)$, and P_0 is the proton energy.

Phase space in MC

$k_z < 0$ **Boglione: PRD 84,034033 (2011)**





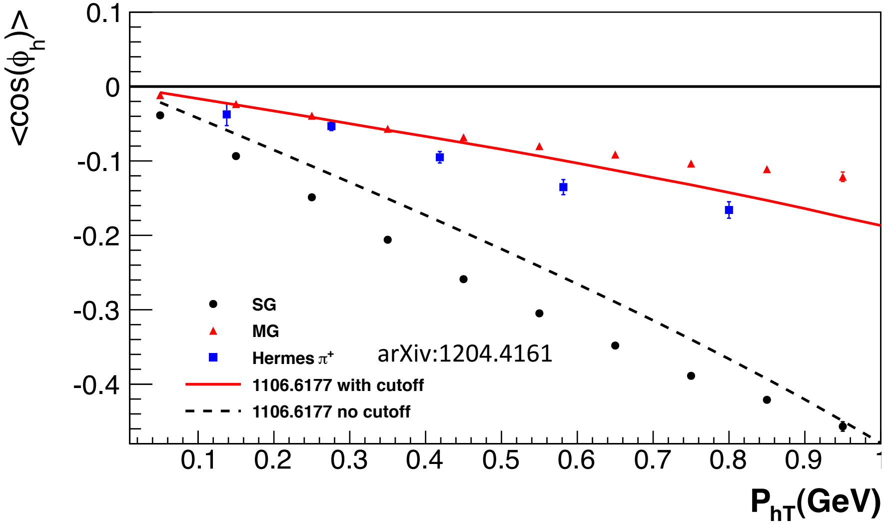
$$f_1(x,k_\perp) = f_1(x)\frac{e^{-\frac{k_\perp^2}{<k_\perp^2>_{f_1}}}}{<k_\perp^2>_{f_1}}$$

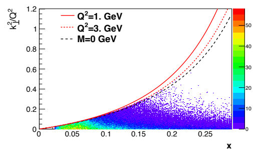
$$f_1(x,k_\perp) = f_1(x)\frac{e^{-\frac{k_\perp^2}{<k_\perp^2(x)>}}}{<k_\perp^2(x)>}$$

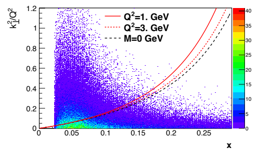
$$<k_\perp^2(x)>= Cx(1-x)$$

x and $k_\perp$ are not factorized even in Gaussian approach.

MC with models and Measurements Cahn effect

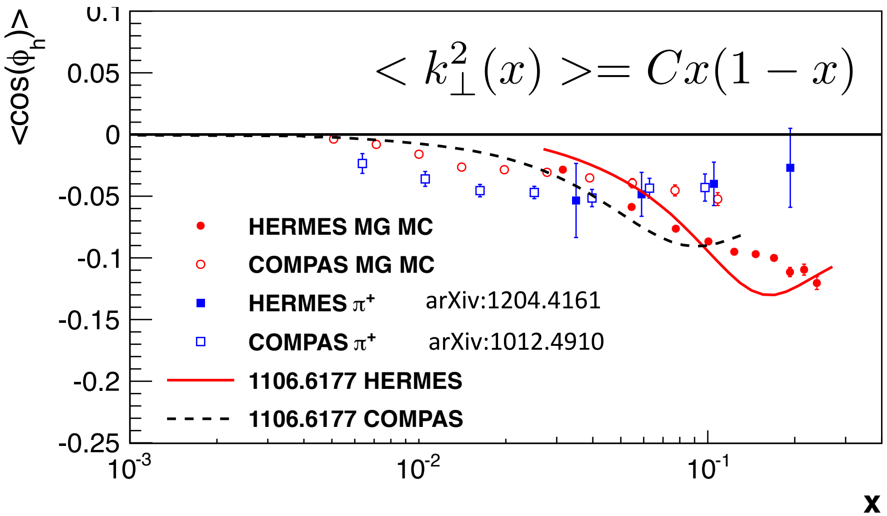






And MG, SG with kinematic cutoff describes HERMES data better.

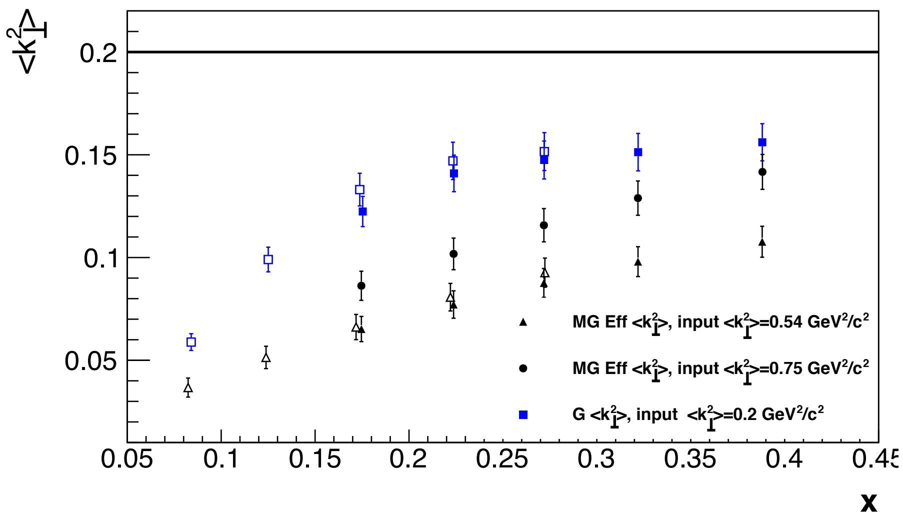
$$f_1(x, k_\perp) = f_1(x)\frac{e^{-\frac{k_\perp^2}{<k_\perp^2(x)>}}}{< k_\perp^2(x) >}$$

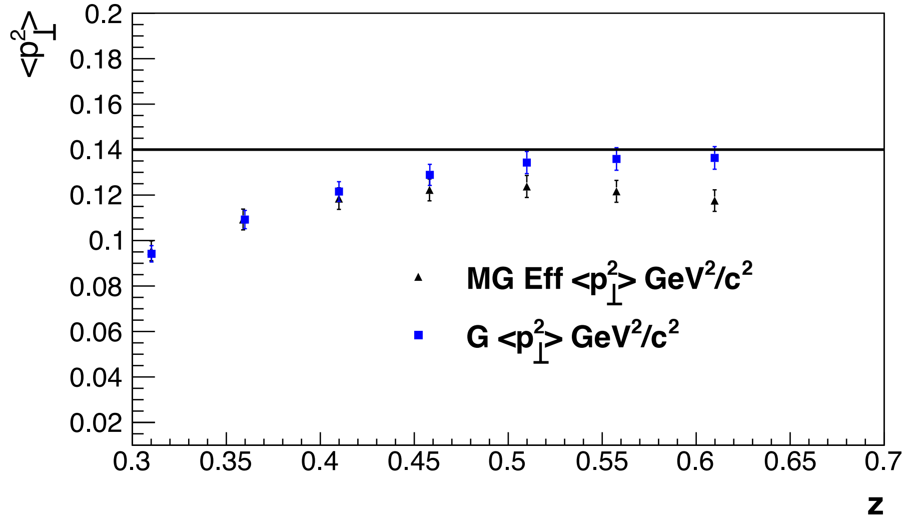


Non factorized DF and FF describe preliminary COMPASS data better without parameter tuning.

$\langle k_\perp^2 \rangle$ vs x and $\langle p_\perp^2 \rangle$ vs z

Blue: simple Gaussian DF with restrictions





Modified Gaussian DF (and FF) allows to use one fixed parameter for different x!

Bessel-weighted extraction of the double spin asymmetry A_{LL}

Boer: JHEP10(2011)021

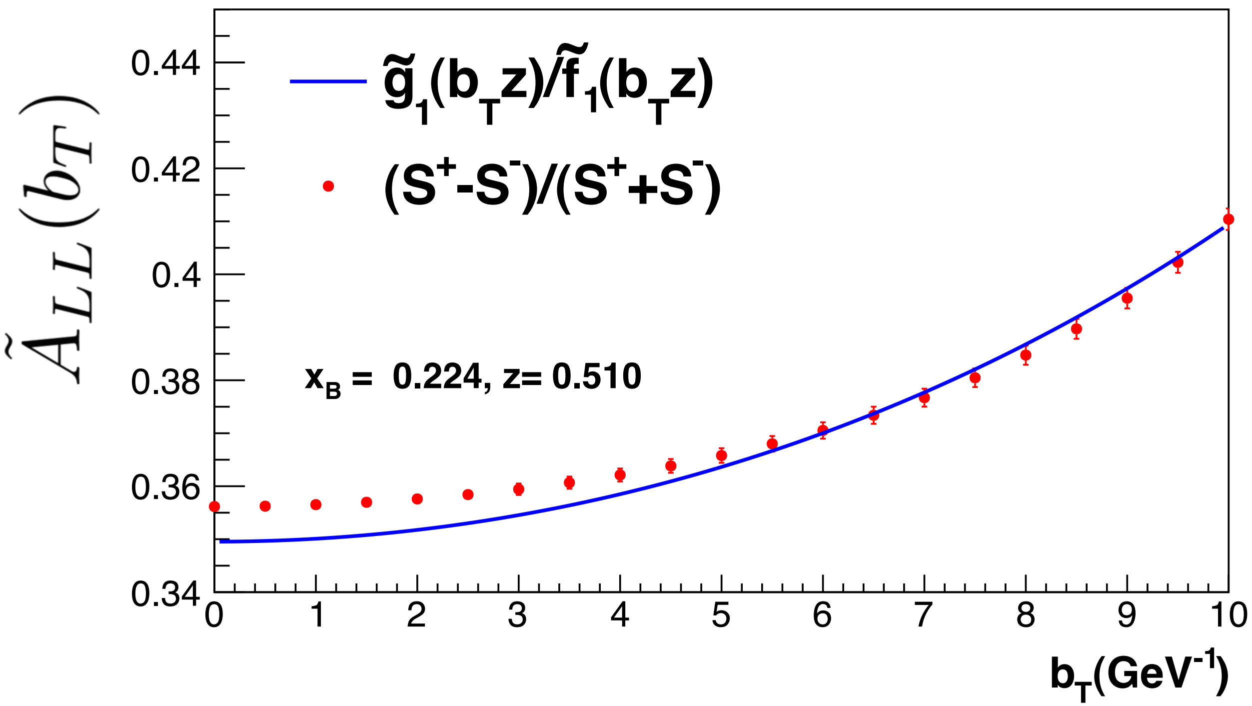
$$A_{LL}^{J_0(b_T P_{h,T})}(b_T) = \frac{\tilde{\sigma}^+(b_T) - \tilde{\sigma}^-(b_T)}{\tilde{\sigma}^+(b_T) + \tilde{\sigma}^-(b_T)} = \frac{\tilde{\sigma}_{LL}(b_T)}{\tilde{\sigma}_{UU}(b_T)} = \sqrt{1-\varepsilon^2}\,\frac{\sum_q \tilde{g}_1^q(x, z^2 b_T^2)\tilde{D}_1^q(z, b_T^2)}{\sum_q \tilde{f}_1^q(x, z^2 b_T^2)\tilde{D}_1^q(z, b_T^2)}$$

where
$$\tilde{\sigma}^{\pm}(b_T) = \int \frac{d\sigma^{\pm}}{dP_{h,T}} J_0(b_T P_{h,T}) P_{h,T}\, dP_{h,T}$$

Or for MC events
$$\tilde{\sigma}^{\pm}(b_T) \simeq S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0(b_T P_{h,T,i})$$

In Fourier space convolution of transverse momentum dependent parton DF and FF become simple products!

Bessel-weighted A_{LL}



$$points = \frac{1}{\sqrt{1-\varepsilon^2}} \frac{S^+ - S^-}{S^+ + S^-}$$ where $$S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0(P_{h,T} b_T)$$

Curve calculated: $\tilde{f}_1(x, b_T) = f_1(x) e^{\frac{-<k_\perp^2>_{f_1} b_T^2}{4}}$ $\tilde{g}_1(x, b_T) = g_1(x) e^{\frac{-<k_\perp^2>_{g_1} b_T^2}{4}}$

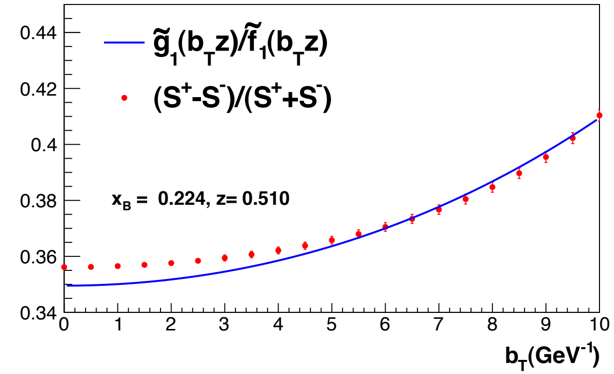
$\langle k_\perp^2 \rangle$ were obtained using fits on $k_\perp^2$ distributions for given bin of the MC sample!

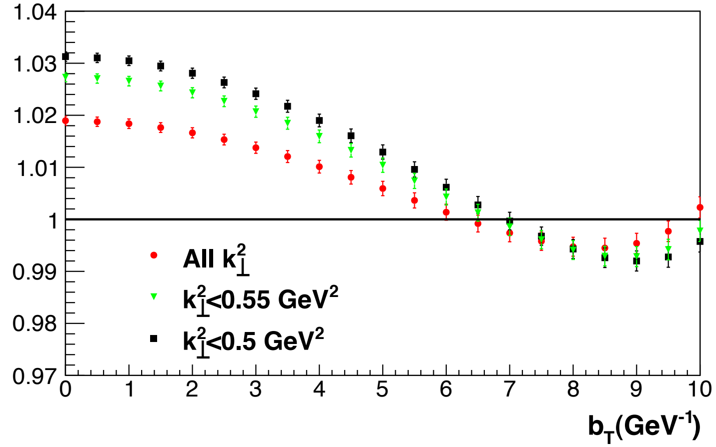
The reason of the systematic discrepancy

$$\tilde{T}(b_T) = 2\int_0^\infty J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt = e^{-\frac{a^2 b_T^2}{4}} > 0$$

$$2\int_{t_{min}}^{t_{max}} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt \gtreqless 0$$

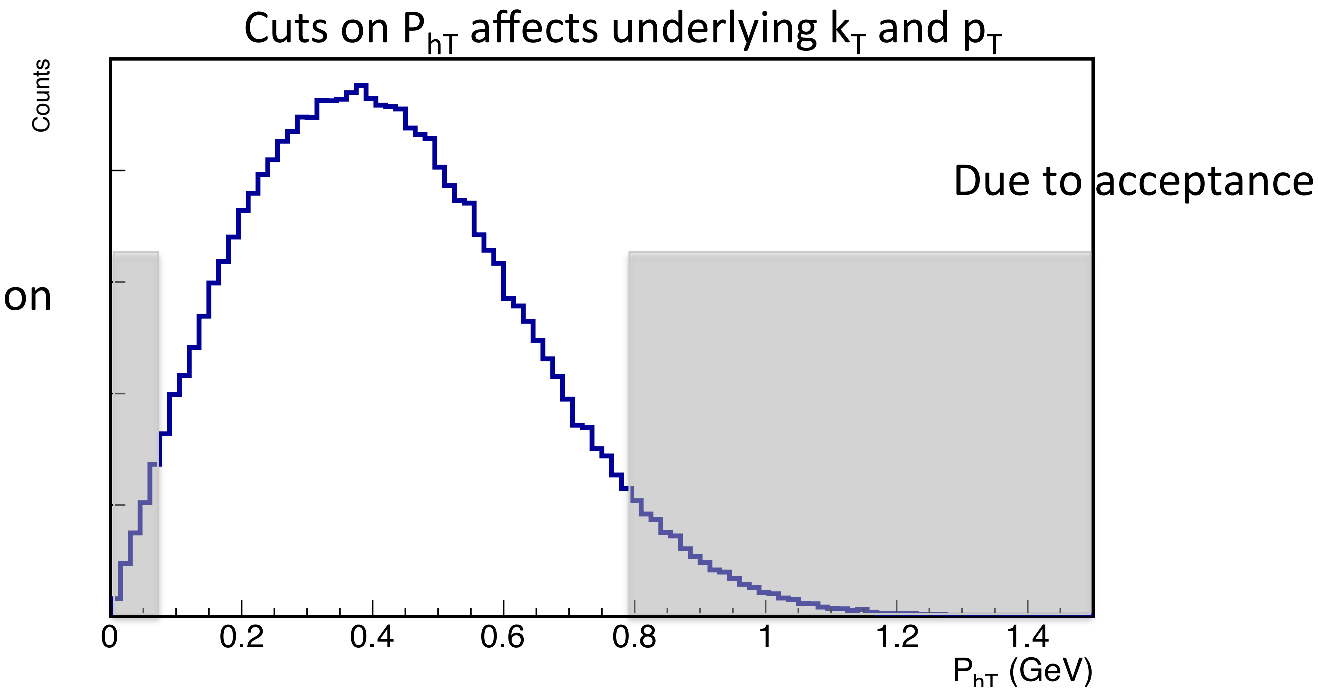
$$t \rightarrow P_{hT}, k_\perp, p_\perp$$





The ratio of the BW extraction to the curve (curve is integrated from zero to infinity) has systematic shift, which is increasing with decrease of $k_\perp^2$ range.

Correction or Calculation

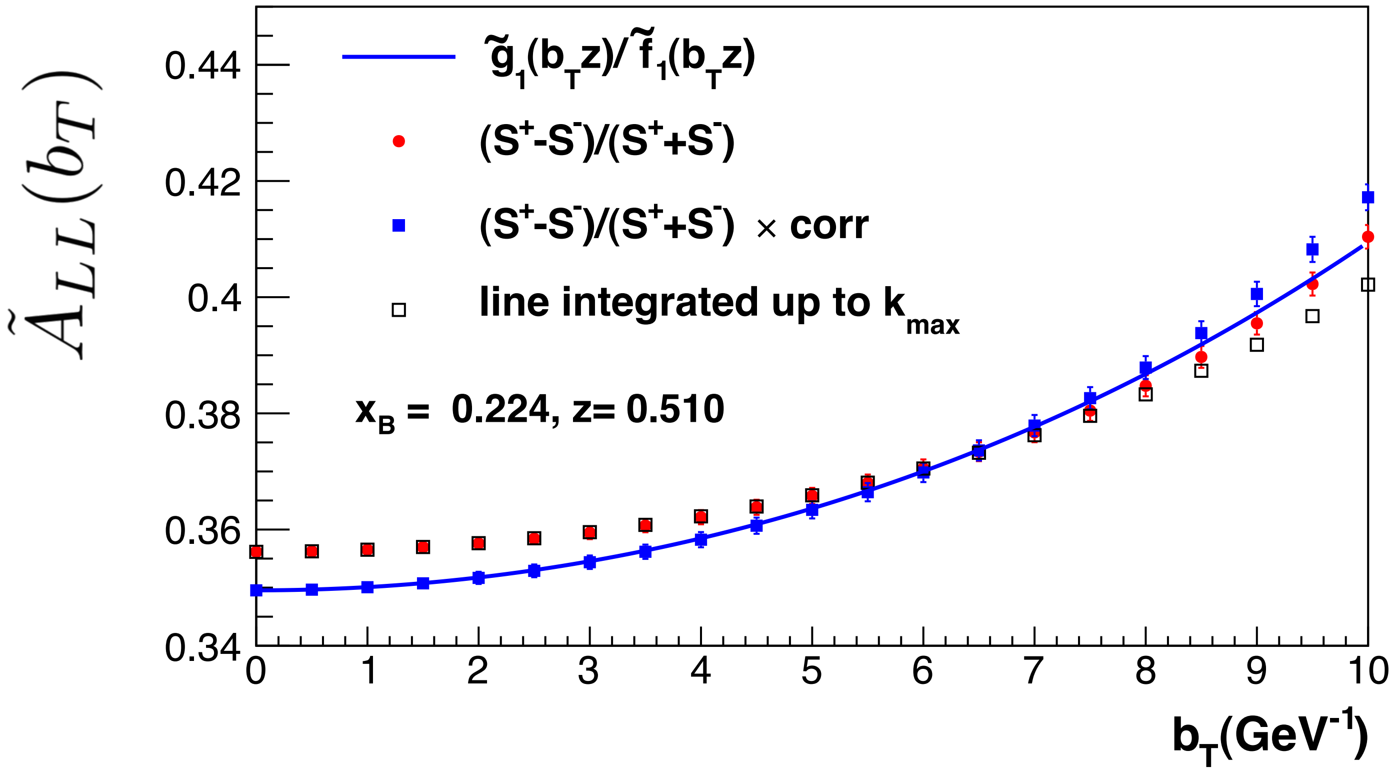


$$\int_0^{\infty} ...P_{hT} dP_{hT} = \int_0^{P_{min}} ...P_{hT} dP_{hT} + \int_{P_{min}}^{P_{max}} J_0(b_T P_{hT}) \frac{e^{-P_{hT}^2/a^2}}{a^2} P_{hT} dP_{hT} + \int_{P_{max}}^{\infty} ...P_{hT} dP_{hT}$$

One can correct data using model dependent approach (for example Gaussian)

Integral within experimental range could be calculated for different models, w/o modifying data with model assumptions.

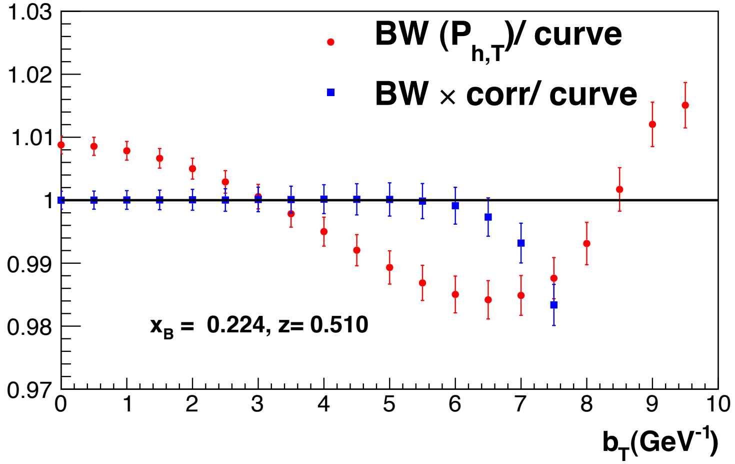
Correction and calculation



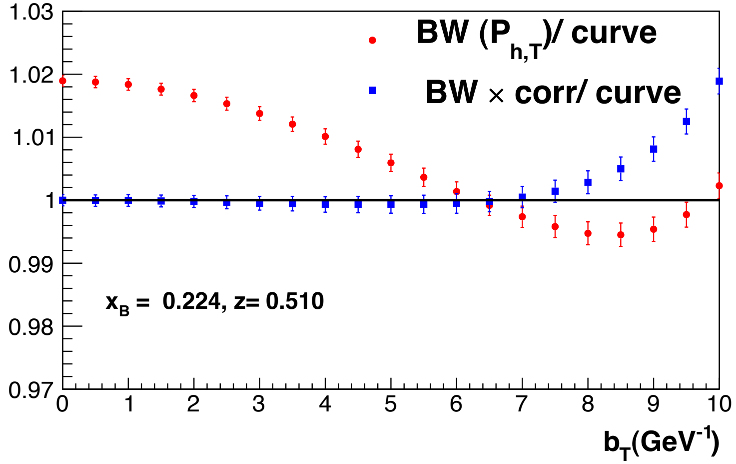
One can correct red points using series of assumptions (Gaussian was used in this example), which is presented with blue points.
Or, **more precise, one can do calculations for the exact bin using integration (from minimum) up to the maximum value given from the experiment!**

Model dependence

Factorized DF and FF



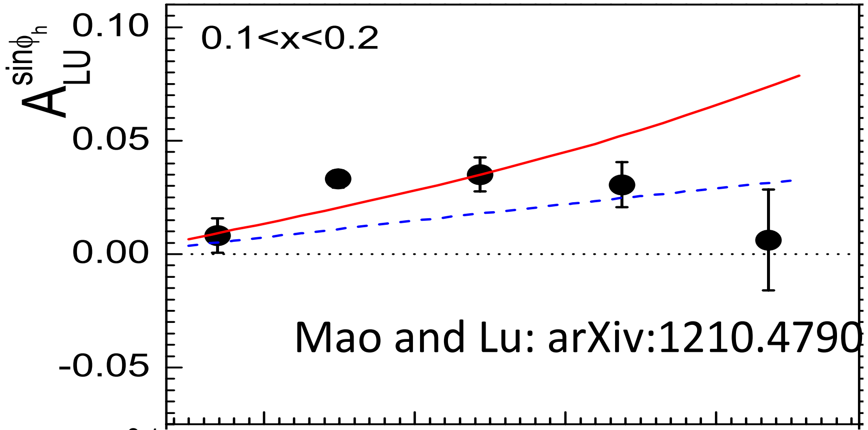
Non factorized DF and FF



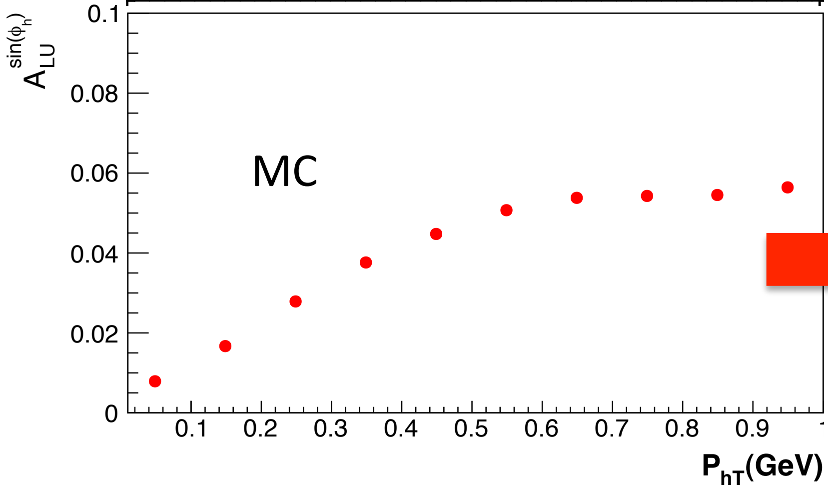
If we consider b_T range up to 6-7 GeV^{-1}, $b_T < 1.2\ fm$
Bessel weighted extraction has accuracy bellow 2%

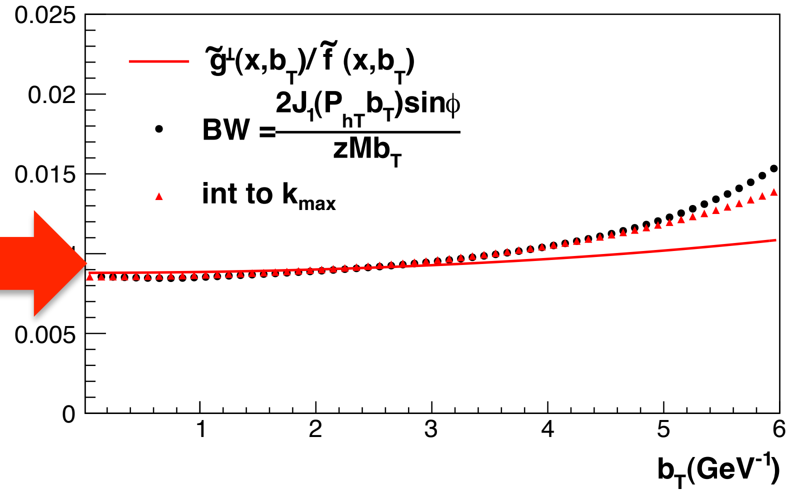
Beam Spin asymmetry

$$F_{LU}^{\sin\phi_h} = \frac{2M_p}{Q}\,\mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{p}_\perp}{M}\left(xeH_1^\perp + \frac{M_h}{M_p}f_1\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{k}_\perp}{M}\left(\underline{xg^\perp D_1} + \frac{M_h}{M_p}h_1^\perp\frac{\tilde{E}^\perp}{z}\right)\right]$$

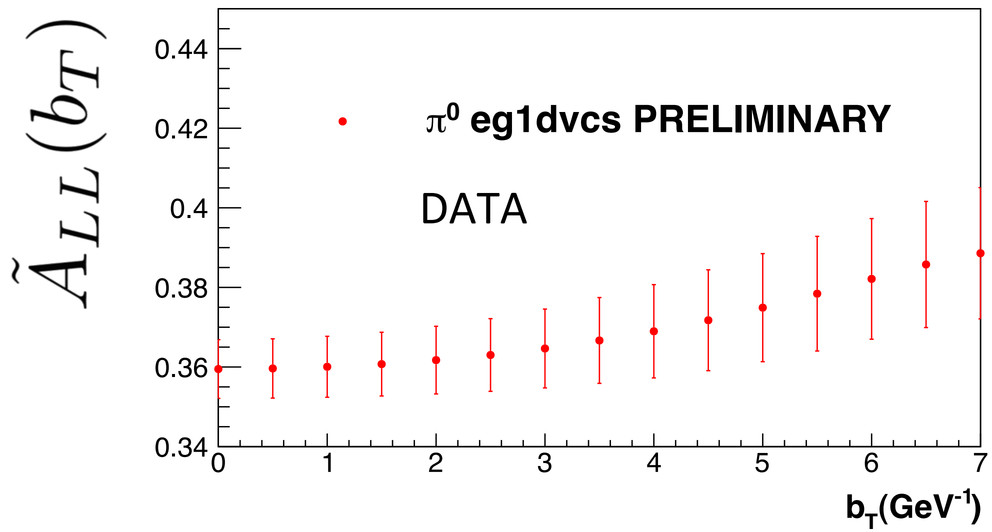


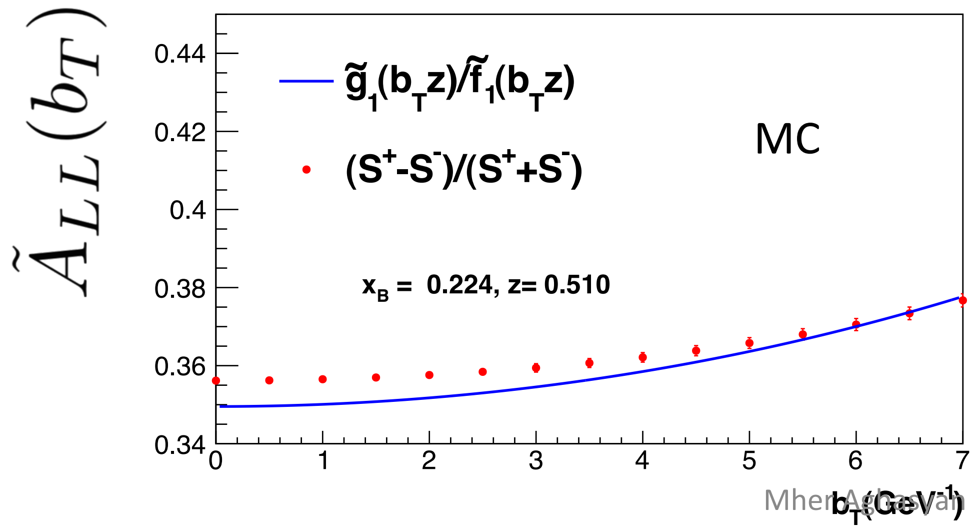
$$\tilde{A}_{LU} = \frac{\pm\sum_{i=1}^{N^\pm}\frac{2J_1(P_{h,T}b_T)}{zMb_T}\sin(\phi_h)}{\sum_{i=1}^{N}J_0(P_{h,T}b_T)}$$





BW from Data





First time: BW of data

$$points = \frac{1}{\sqrt{1-\varepsilon^2}} \frac{S^+ - S^-}{S^+ + S^-}$$

$$S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0(P_{h,T} b_T)$$

Data can be reproduced with MC!
More precise estimates of nuclear effects under investigation.

Summary

- New technique for flavor decomposition of Transverse Momentum Dependent Distributions (TMD) of partons, based on the Bessel weighting formalism developed.
- The procedure is applied to study:
  - ✓ double longitudinal spin asymmetries in SIDIS,
  - ✓ beam spin asymmetries,

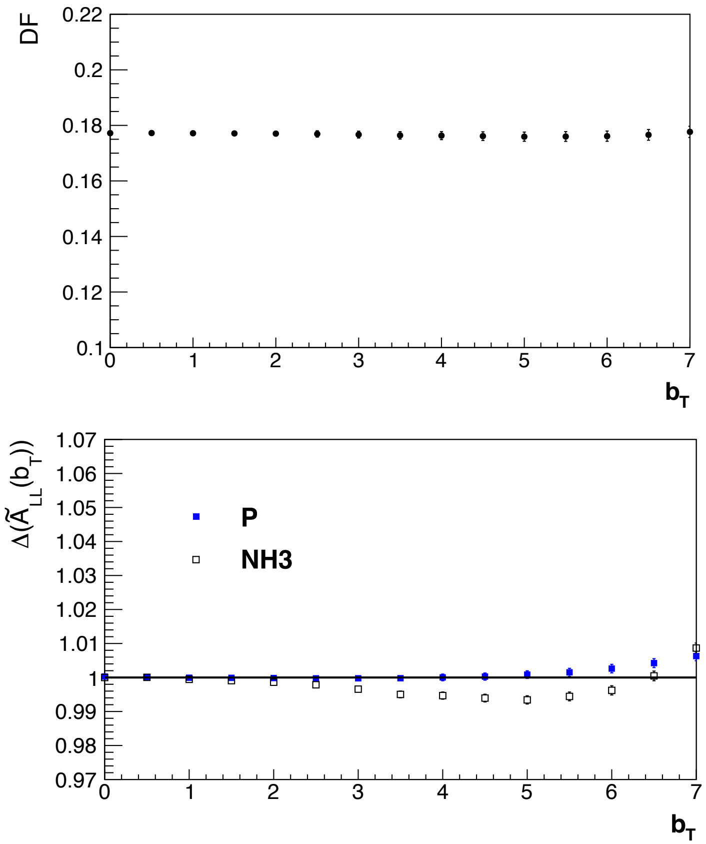
  using a new dedicated Monte Carlo generator, which includes quark intrinsic transverse momentum within the generalized parton model based on the fully differential cross section for the process.
- Systematic effects on TMD extraction due to Model dependence has been studied using $k_\perp - x$ factorized and non-factorized models for TMDs.
- First time BW of experimental data is presented.

Thank you!

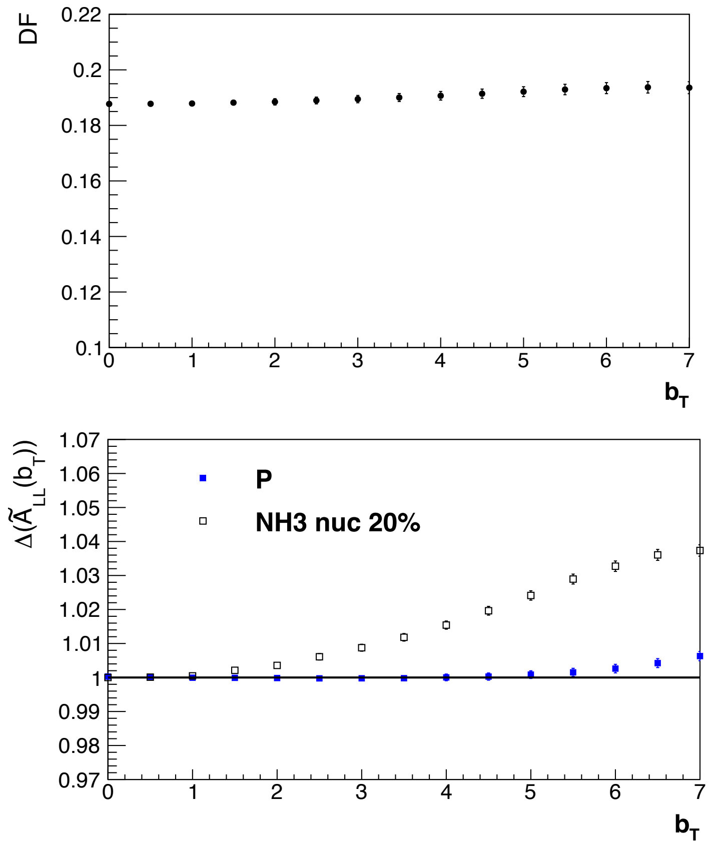
Support

DF and nuclear effects in MC

No nuclear effects



Nuclear effects: smear unpol. widths +20%



Nuclear broadening and/or smearing affects b_T distribution at few % level up to b_T~5-6GeV^{-1}.

Correction

$$\int_0^{t_{max}} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2}tdt = \int_0^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2}tdt - \int_{t_{max}}^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2}tdt = \frac{1}{2}e^{-\frac{a^2 b_T^2}{4}} \times (1-\epsilon)$$

where

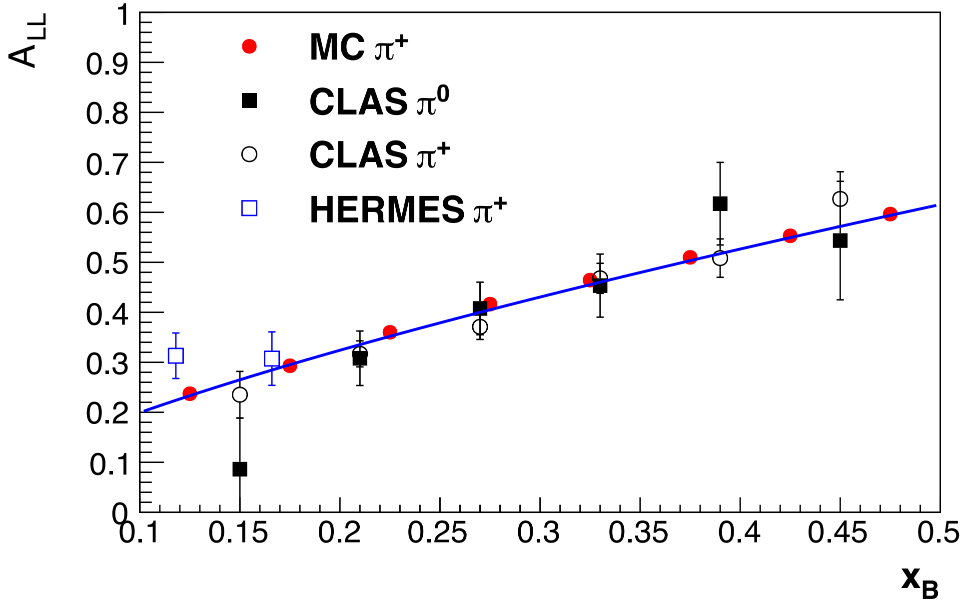
$$\epsilon = \frac{\int_{t_{max}}^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2}tdt}{\int_0^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2}tdt} = 2e^{\frac{a^2 b_T^2}{4}}\int_{t_{max}}^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2}tdt$$

$$A_{LL,measured}^{J_0(b_T P_{h,T})}(b_T) = \sqrt{1-\varepsilon^2}\frac{\tilde{g}_1(x, z^2 b_T^2)}{\tilde{f}_1(x, z^2 b_T^2)} \times \frac{1-\epsilon_{g_1}}{1-\epsilon_{f_1}} = A_{LL}^{J_0(b_T P_{h,T})}(b_T) \times \frac{1-\epsilon_{g_1}}{1-\epsilon_{f_1}}$$

One can use for example Gaussian approach to correct extracted points!
Correction can be done and at the quark intrinsic transverse momentum level and at the level of final detected hadron transfers momentum.

A_{LL} in MC

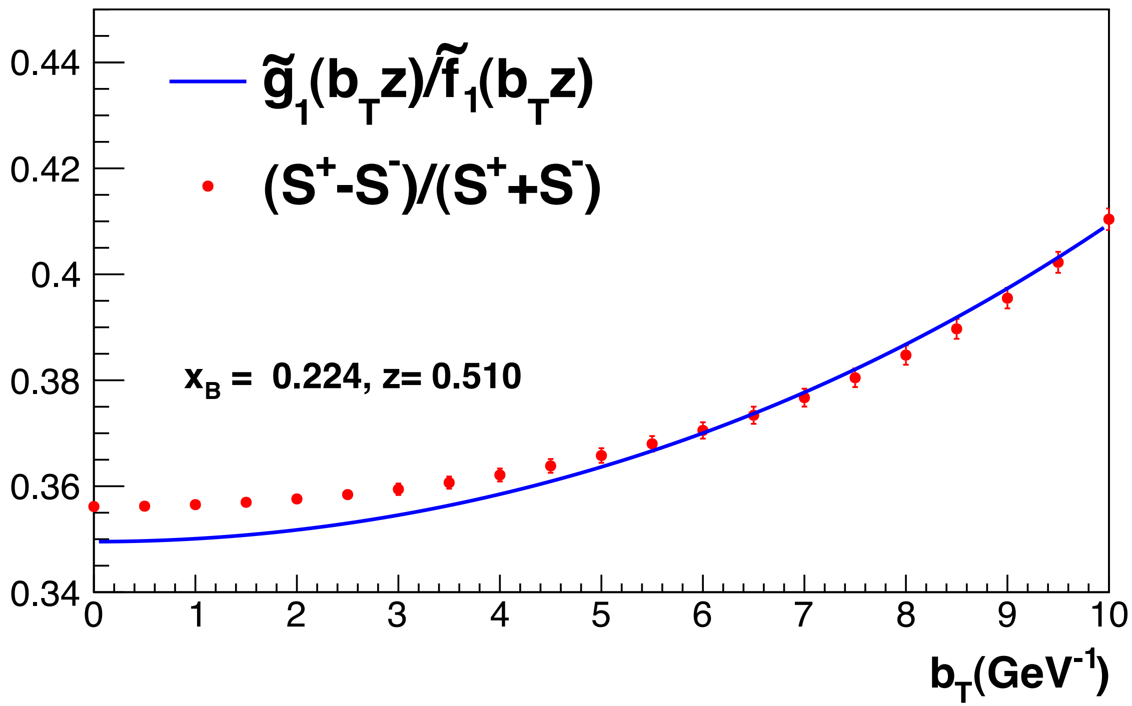
Avakian: arXiv:1003.4549



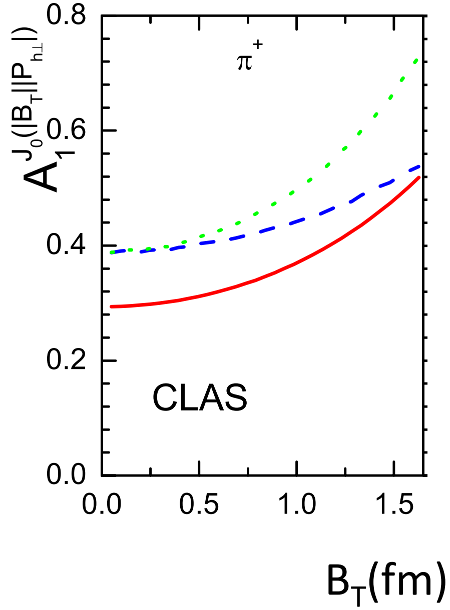
$$g_1(x) = x^{0.7} f_1(x)$$

Double-spin asymmetry from MC is consistent with CLAS and HERMES.

Bessel-weighted A_{LL}



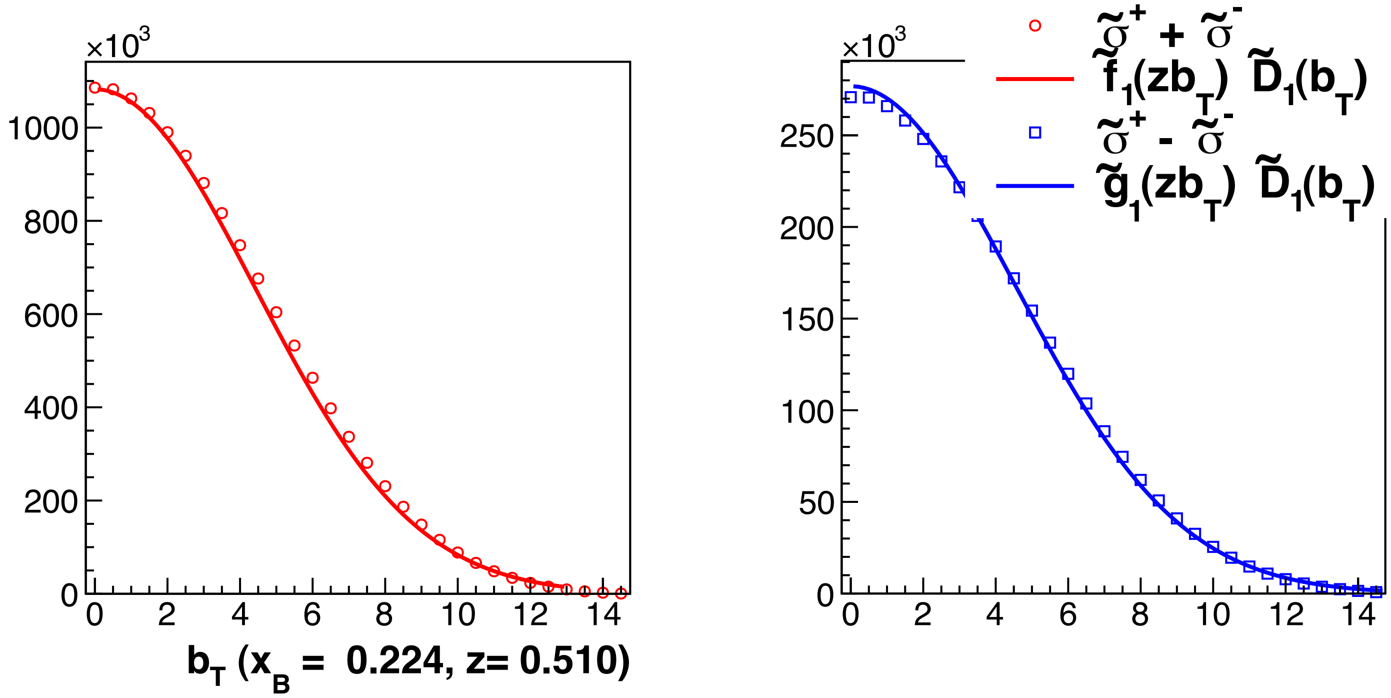
Lu and Ma:
arXiv:1212.6864



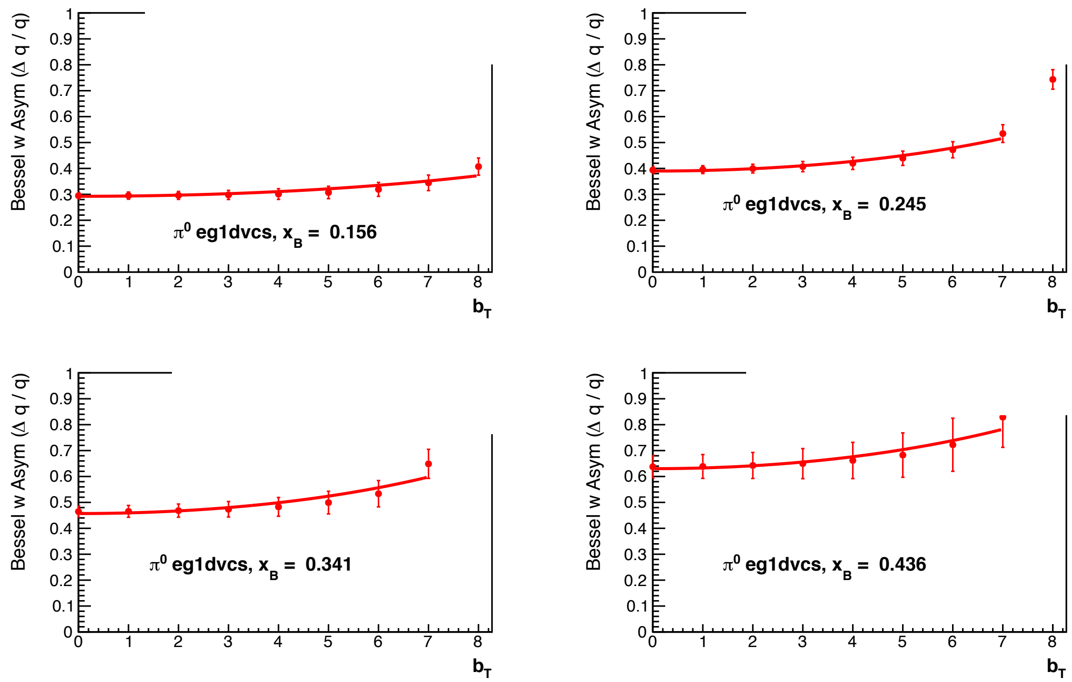
Curve calculated: $\tilde{f}_1(x, b_T) = f_1(x) e^{\frac{-<k_\perp^2>_{f_1} b_T^2}{4}}$ $\tilde{g}_1(x, b_T) = g_1(x) e^{\frac{-<k_\perp^2>_{g_1} b_T^2}{4}}$

Different colors correspond to different widths for $\langle k_\perp^2 \rangle$

Cross section vs b_T



BW of A_{LL} from eg1dvcs



Very preliminary A_{LL} extraction vs b_T from eg1dvcs

Generalization to non zero proton mass

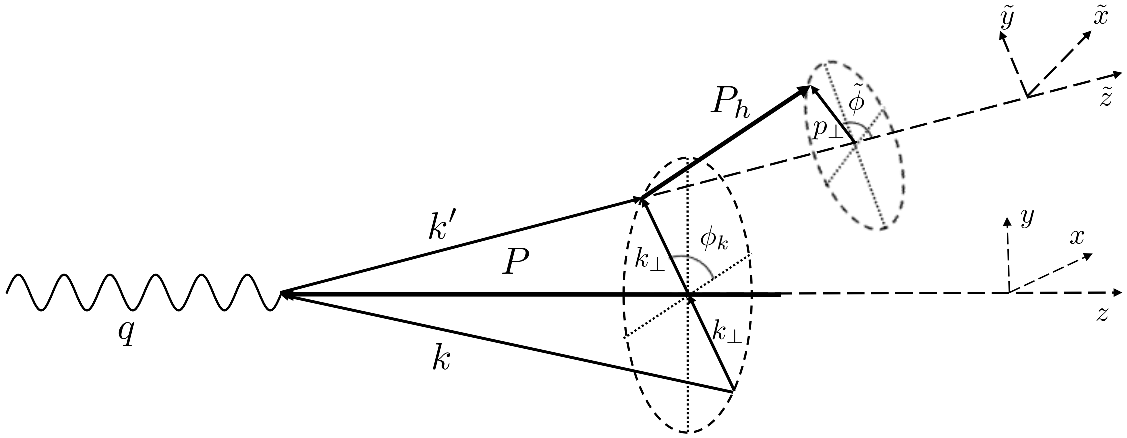
Assume that quark inside the proton have the momentum:

$$k = \left(x_{LC}P' + \frac{k_\perp^2}{4x_{LC}P'}, \mathbf{k}_\perp, -x_{LC}P' + \frac{k_\perp^2}{4x_{LC}P'} \right)$$

$$x_{LC} = \frac{x}{x_N}\left(1+\sqrt{1+\frac{4k_\perp^2}{Q^2}}\right), \quad x_N = 1+\sqrt{1+\frac{4M_p^2x^2}{Q^2}},$$

Where $P' = 0.5(E_p + |P_{pz}|)$ is the proton energy with non zero proton mass.

Fragmentation



Scattered quark 4 momentum calculated: $k' = k + q$

Final hadron generated with the momentum:

$$P_{\tilde{x},h} = p_\perp \cos(\tilde{\phi}) \qquad P_{\tilde{y},h} = p_\perp \sin(\tilde{\phi}) \qquad P_{\tilde{z},h} = z_{LC} E_{k'} - \frac{p_\perp^2 + M_h^2}{4 z_{LC} E_{k'}}$$

To account and understand all the assumptions, integrations, correlations and more, fully differential SIDIS cross-section should be studied.

Bessel weighting: simple example

Let assume we can present:

$$\sigma_{LL}(P_{h,T}) = C_{LL} e^{-\frac{P_{h,T}^2}{<P_{h,T}^2>_{LL}}}$$

$$\tilde{\sigma}_{LL}(b_T) = \int_0^\infty \sigma_{LL}(dP_{h,T}) J_0(b_T P_{h,T}) P_{h,T} dP_{h,T} = C_{LL}\frac{1}{2} e^{-\frac{<P_{h,T}^2>_{LL} b_T^2}{4}}$$

Assuming: $\langle P_{h,T}^2 \rangle = \langle k_\perp^2 \rangle z^2 + \langle p_\perp^2 \rangle$ $\quad \tilde{\sigma}_{LL}(b_T) = C\frac{1}{2}\tilde{g}_1(x, z b_T) \times \tilde{D}_1(z, b_T)$

Where:

$$\tilde{g}_1(b_T) = 2\int_0^\infty J_0(bk_\perp) e^{-\frac{k_\perp^2}{<k_\perp^2>_{g_1}}} k_\perp dk_\perp = e^{\frac{-<k_\perp^2>_{g_1} b_T^2}{4}}$$

$$\tilde{D}_1(z, b_T) = \int_0^\infty J_0(bp_\perp) e^{-\frac{p_\perp^2}{<p_\perp^2>}} p_\perp dp_\perp = e^{\frac{-<p_\perp^2> b_T^2}{4}}$$

This is just very simple presentation based on chain of assumption…

Bessel-weighting strategy does not depend on a Gaussian approach at all!

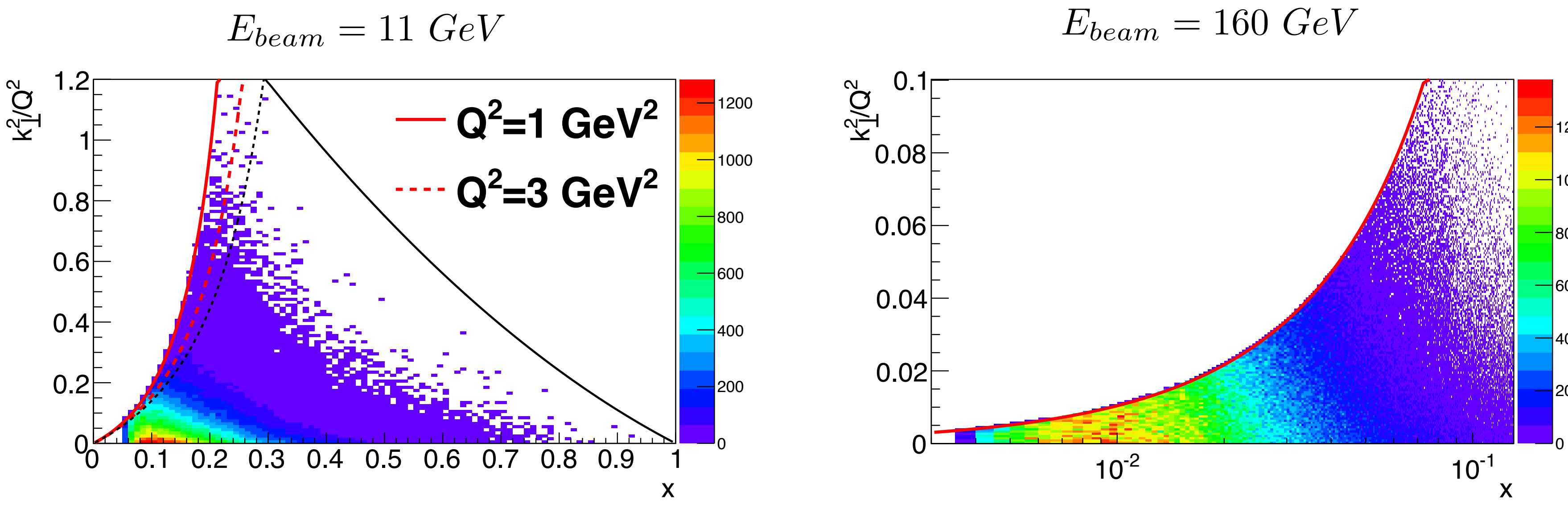
BW error calculation

$$\Delta \tilde{A}_{LL}^{J_0(b_T P_{hT})}(b_T) = \sqrt{\frac{1 - \tilde{A}_{LL}^2(b_T)}{\Delta S^+ + \Delta S^-}}$$

where

$$\Delta \tilde{\sigma}^{\pm}(b_T) \simeq \Delta S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0^2(b_T P_{hT,i})$$

Phase space in MC: simple Gaussian DF and FF



$$< k_\perp^2 >_{f_1} = 0.2GeV^2, < k_\perp^2 >_{g_1} = 0.16GeV^2, < p_\perp^2 >= 0.14GeV^2$$

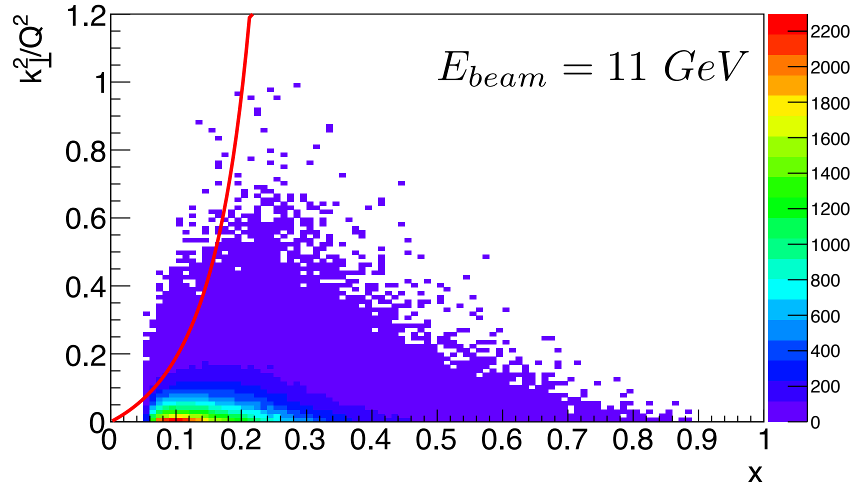
Kinematic cut-off is sharper at higher beam energy and smaller x

Modified Gaussian FF and DF

Stan Brodsky, "Novel Features of Hadron Dynamics and Light-Front Holography"
Warsaw - July 3 - 6, 2012

$$f_1(x, k_\perp) = f_1(x) \frac{e^{-\frac{k_\perp^2}{<k_\perp^2>_{f_1} x(1-x)}}}{<k_\perp^2>_{f_1} x(1-x)} \qquad D_1(z, p_\perp) = D_1(z) \frac{e^{-\frac{p_\perp^2}{<p_\perp^2> z(1-z)}}}{<p_\perp^2> z(1-z)}$$

No cuts with red lines.
Only energy and momentum conservation.



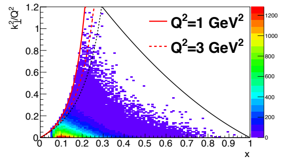
$$<k_\perp^2>= 0.75GeV^2, <p_\perp^2>= 0.5GeV$$

$k_z \leq 0$ requirement is satisfied automatically for 95-99% of events.

$k_\perp^2$ dependence for fixed x bins

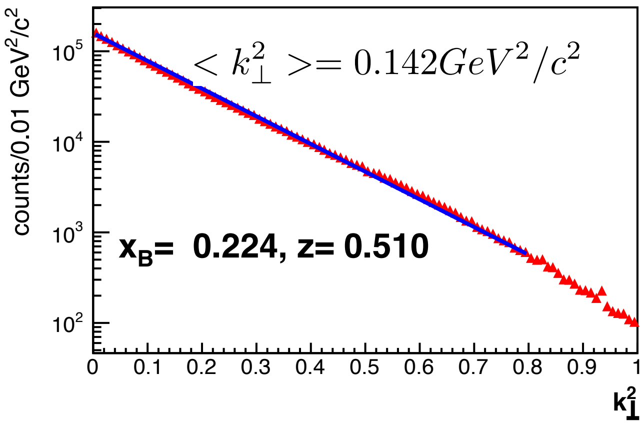
$E_{Beam} = 6\ GeV$

$< k_\perp^2 >= 0.2 GeV^2/c^2$



$e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}}$

$k_z \leq 0$

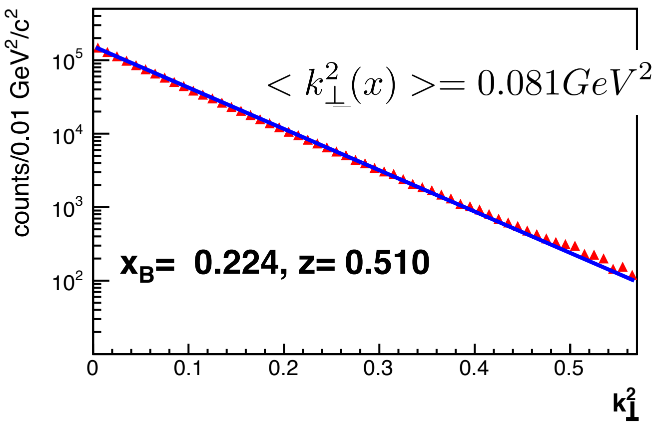


"Implemented" width changed due to:

a) Energy and momentum conservation
b) Binning
c) kinematic cutoff

$e^{-\frac{k_\perp^2}{< k_\perp^2 (x) >}}$

~~$k_z \leq 0$~~



$< k_\perp^2 (x = 0.224) >= 0.099 GeV/c^2$

"Implemented" width changed due to:

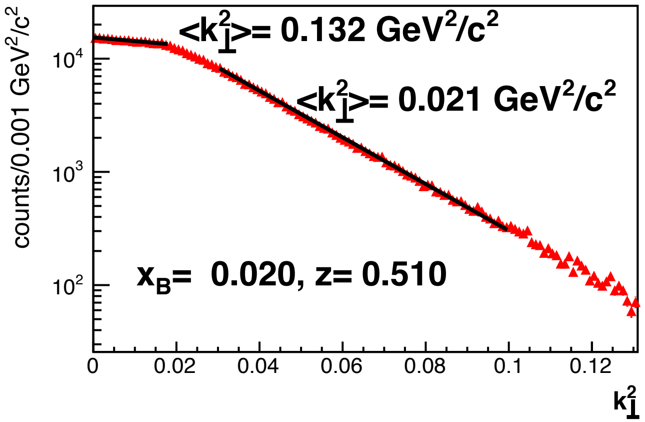
1) Energy and momentum conservation
2) Binning

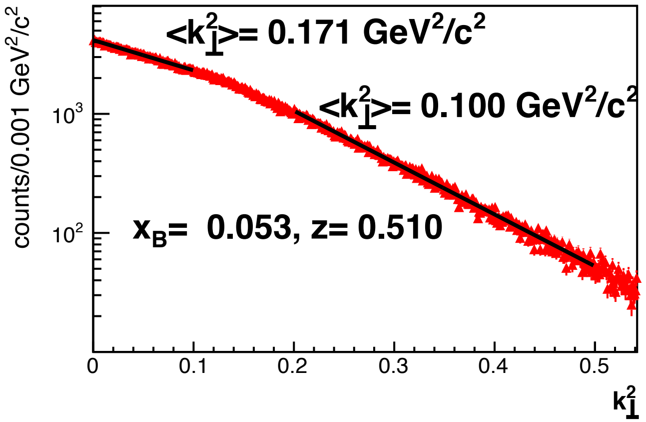
Widths of quark transverse momentum PDF-s obtained from MC events after energy and momentum conservation and kinematic restrictions

$k_\perp^2$ dependence for different x bins simple Gaussian DF and FF

$$< k_\perp^2 >_{f_1} = 0.2 GeV^2, < k_\perp^2 >_{g_1} = 0.16 GeV^2, < p_\perp^2 > = 0.14 GeV^2$$

$E_{beam} = 160\ GeV$





At low $k_\perp^2$ and higher x the outcome is close to implemented value for small $k_\perp^2$