Heavy-quark production in DIS

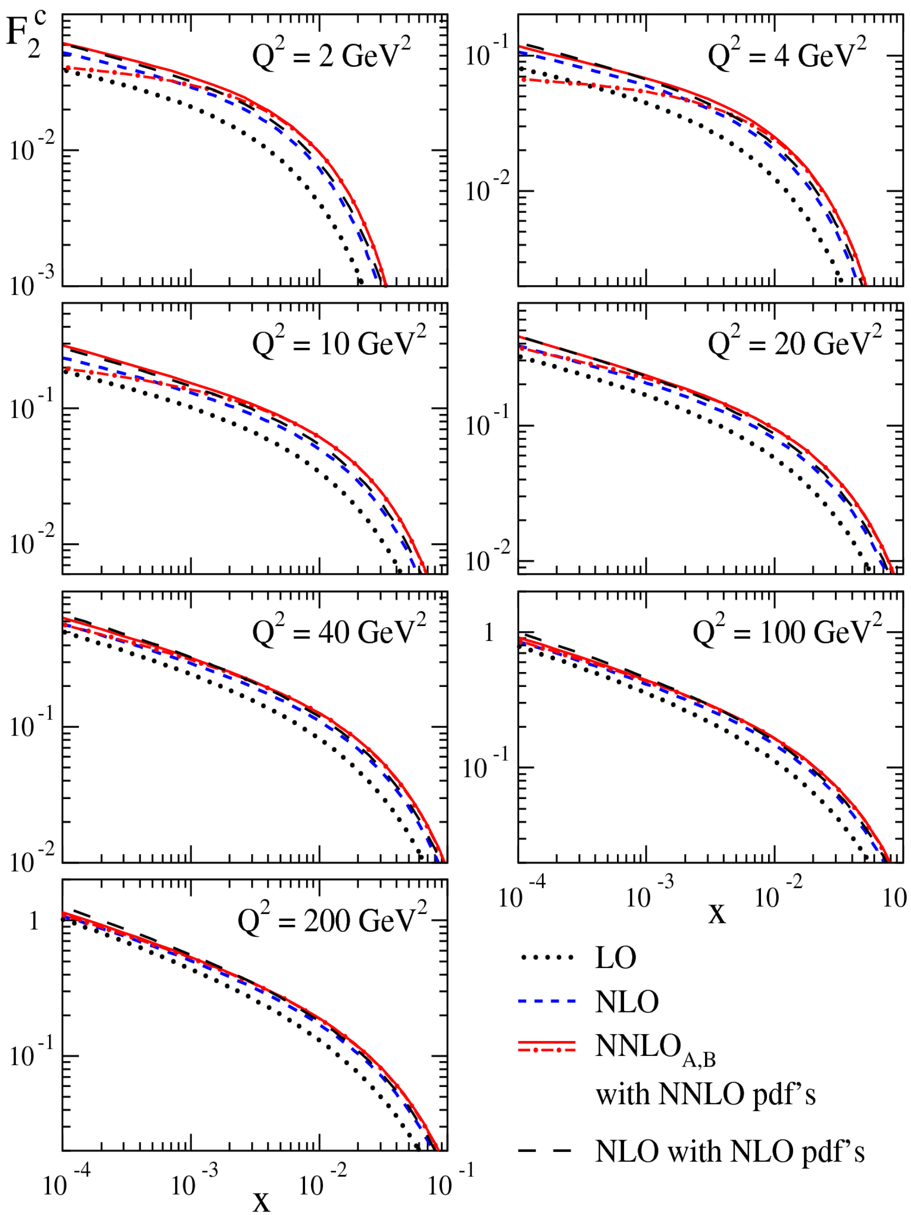
S.Alekhin (*IHEP Protvino & DESY-Zeuthen)*

- Theoretical update of FFN scheme
  - massive NNLO Wilson coefficients
  - running-mass definition
- The c-quark mass determination
- Theoretical errors in the VFN and FFN schemes

sa, Blümlein, Daum, Lipka, Moch PLB 720, 172 (2013)

DIS2013, Marseille, 25 Apr 2013

Massive NNLO coefficients updated



- The NNLO log terms are known due to the recursive relations
- The constant NNLO term stem from:
  - the threshold resummation terms including the Coulomb one
  - high-energy asymptotics obtained with the small-x resummation technique

Catani, Ciafaloni, Hautmann NPB 366, 135 (1991)

  - available NNLO Mellin moments for the massive OMEs

Ablinger at al. NPB 844, 26 (2011)

Bierenbaum, Blümlein, Klein NPB 829, 417 (2009)

- The uncertainty in the NNLO coefficients is due to matching of the threshold corrections with the high-energy limit → two options for the coefficients are provided
- Further improvement should come from additional Mellin moments

Blümlein at al. in progress

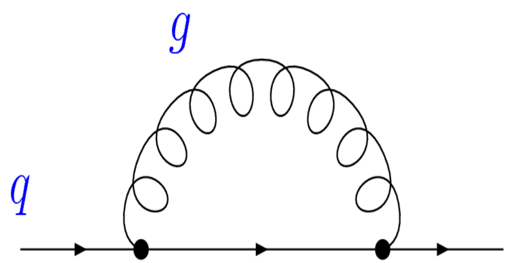
Kawamura, Lo Presti, Moch, Vogt NPB 864, 399 (2012)

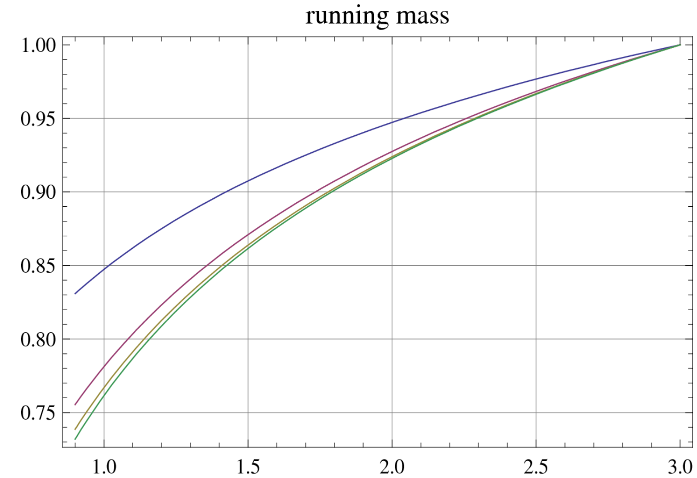
Running mass in DIS

The pole mass is defined for the free (*unobserved*) quarks as a the QCD Lagrangian parameter and is commonly used in the QCD calculations

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q}\,(\mathrm{i}\not{D} - m_q)\,q$$

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2 = m_q^2}$$

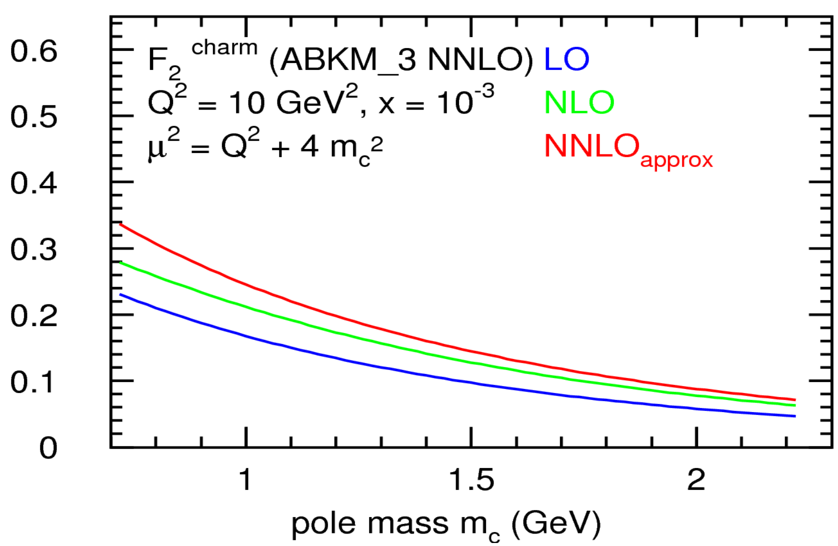


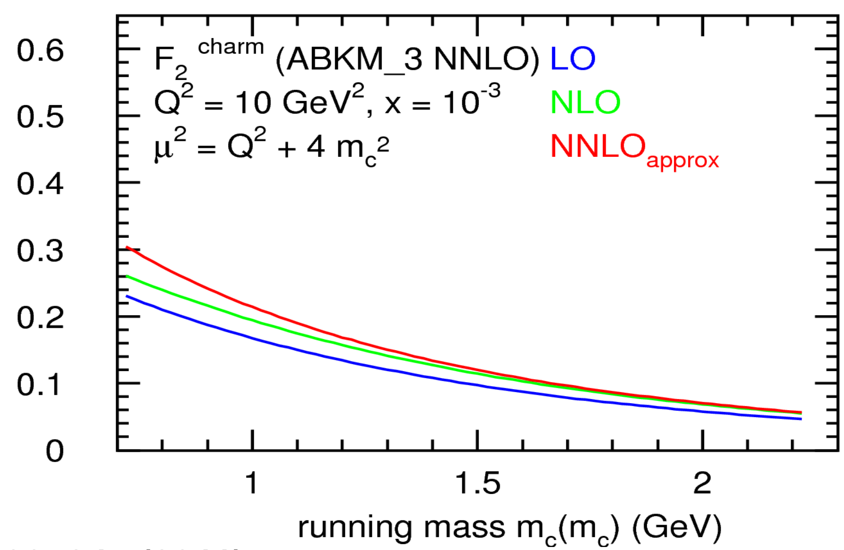


The quantum corrections due to the self-energy loop Integrals receive contribution down to scale of $O(\Lambda_{QCD})$

→ sensitivity to the high order corrections, particularly at the production threshold

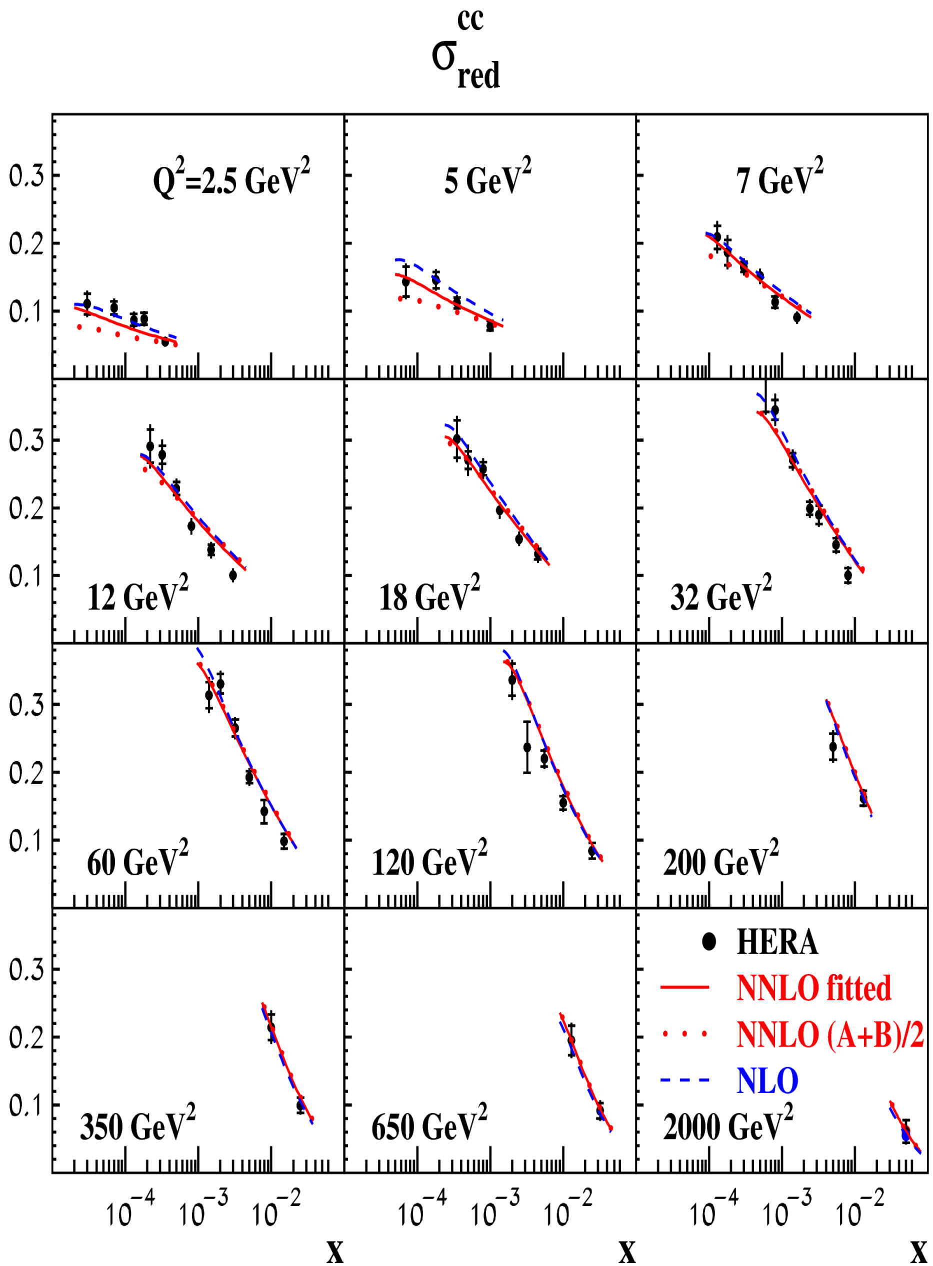
$$\mu^2 \frac{d}{d\mu^2} m(\mu) = \gamma(\alpha_s) m(\mu)$$





sa, Moch PLB 699, 345 (2011)

c-quark mass from the ABM11 fit



sa, Blümlein, Daum, Lipka, Moch PLB 720, 172 (2013)

From the variant of ABM11 fit including the HERA charm data: **H1/ZEUS PLB 718, 550 (2012)**

$m_c(m_c)$=1.15±0.04(exp.) GeV NLO

$m_c(m_c)$=1.24±0.03(exp.),+0.-0.07(th) GeV NNLO

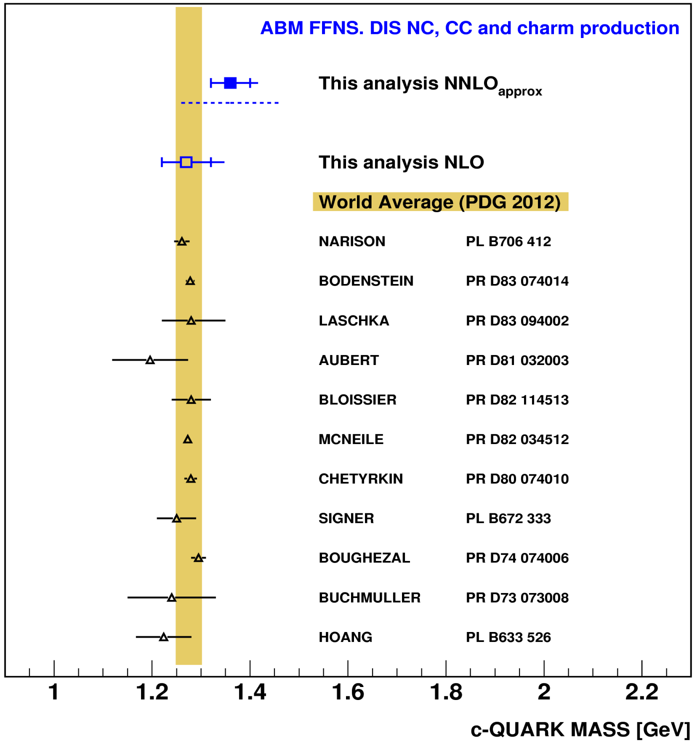
The constant term in the massive NNLO Wilson coefficients is modeled as a linear combination of the options A and B provided by KIPMV

The data prefer option A, the option B is clearly disfavored. The dominant uncertainty in $m_c(m_c)$ at NNLO is due to variation of the massive Wilson coefficients between options A and (A+B)/2

From the HERA fit:
$m_c(m_c)$=1.26±0.05(exp.) GeV NLO
(cut on Q^2, impact of the dimuon νN data, PDFs).

	ABM11	JR	MSTW08	NN21
NLO	1.21	1.21	1.12	1.01
NNLO	1.28	1.27	1.29	-

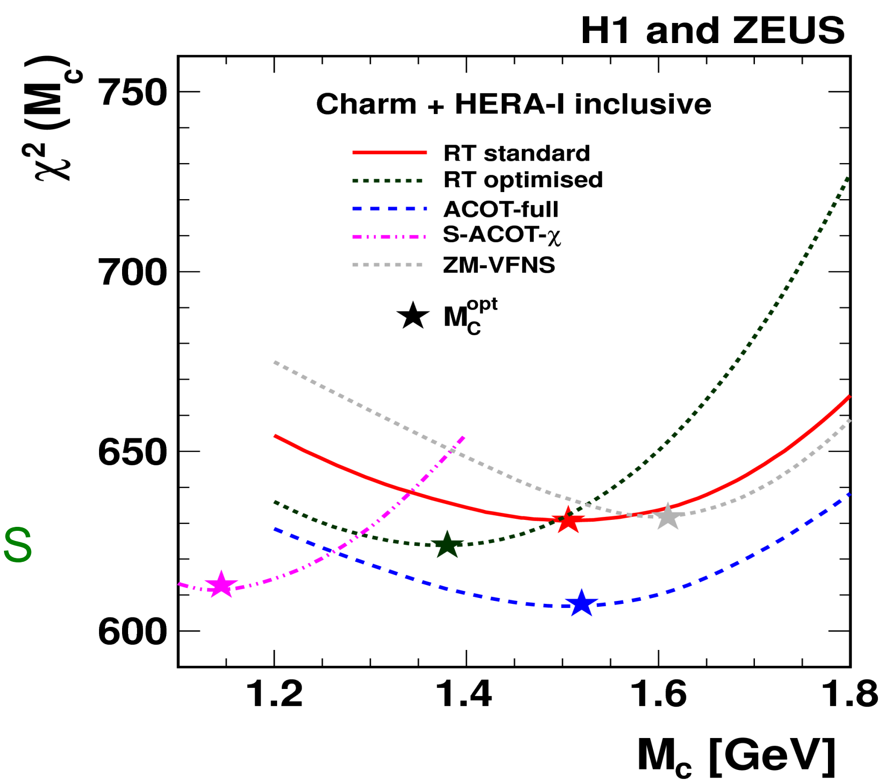
c-quark mass in different schemes



sa, Daum, Lipka, Moch hep-ph/1209.0436

In contrast, the values of pole mass m_c used by different groups and preferred by the PDF fits are systematically lower than the PDG value

	MSTW	NNPDF	JR	CTEQ	PDG
m_c(GeV)	1.40	√2	1.3	1.3	1.66



H1/ZEUS PLB 718, 550 (2012)

Good agreement of $m_c(m_c)$ obtained from DIS in the FFN scheme with the e+e- results

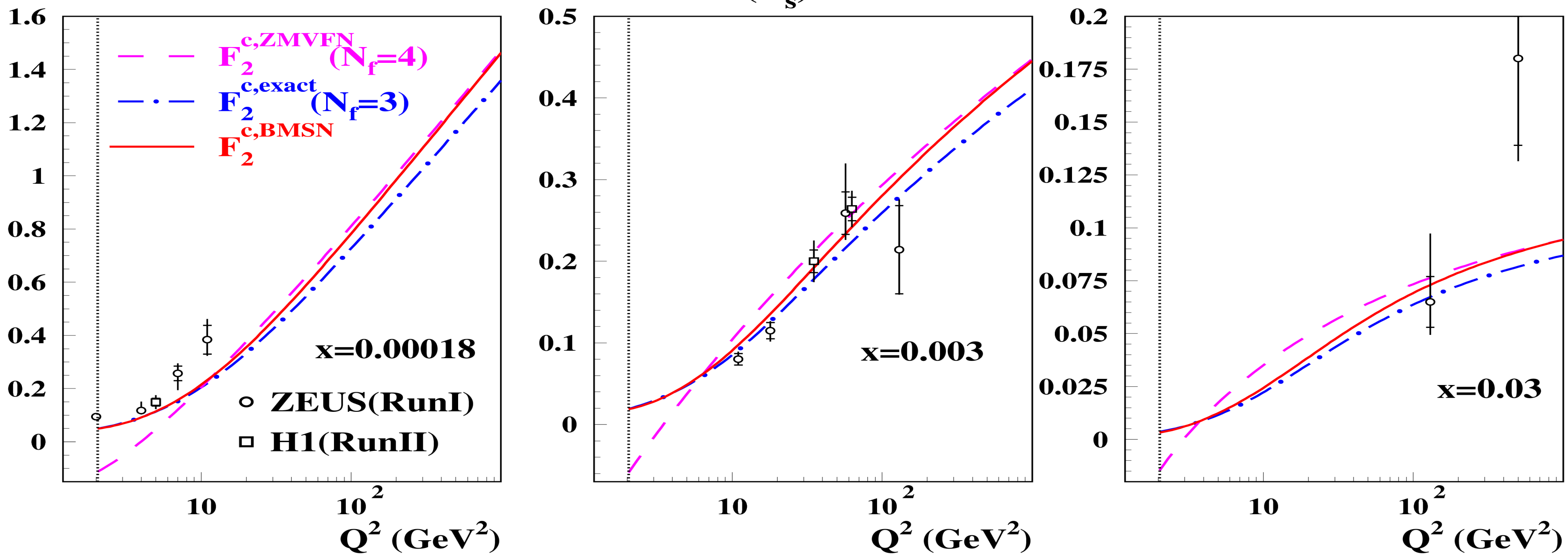
Wide spread of the m_c obtained in different version of the GMVFN schemes → quantitative illustration of the GMVFNS uncertainties

BMSN prescription of GMVFNS

Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2)=F_2^{h,\mathrm{exact}}(N_f,x,Q^2)+F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2)-F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)$$

$O(\alpha_s^2)$ Cacciari, Greco, Nason JHEP 9805, 007 (1998)

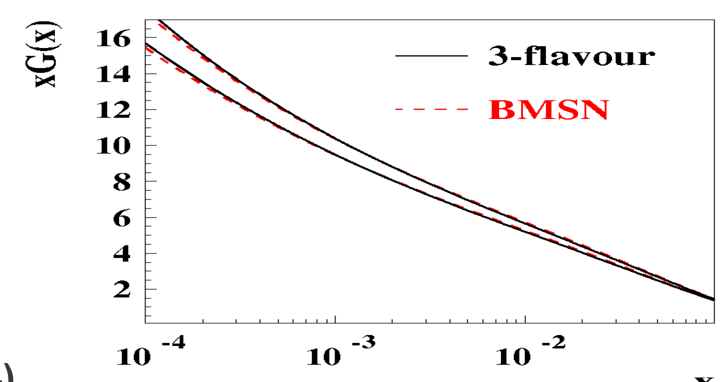


sa, Blümlein, Klein, Moch PRD 81, 014032 (2010)

- Very smooth matching with the FFNS at $Q \rightarrow m_h$
- Renormgroup invariance is conserved; the PDFs in MSbar scheme

In the $O(\alpha_s^2)$ the FFNS and GMVFNS are comparable at large scales since the big logs appear in the high order corrections to the massive coefficient functions

Glück, Reya, Stratmann NPB 422, 37 (1994)



The big-log resummation is important NNPDF

The value of $\alpha_s(M_z)$ is reduced in FFN MSTW

$\alpha_s(M_z)$=0.1135±0.0014 FFN

$\alpha_s(M_z)$=0.1129±0.0014 BMSN

FOPT PDFs and QCD evolution

$$c^{(1)}(x,\mu^2) = a_s(\mu^2)\int_x^1 \frac{dz}{z} A_{hg}^{(1)}(\frac{\mu^2}{m_c^2},z)g\left(\frac{x}{z},\mu^2\right)$$ LO c-quark PDF (FOPT)

$$A_{hg}^{(1)}\left(\frac{\mu^2}{m_c^2},z\right) = \ln\left(\frac{\mu^2}{m_c^2}\right)P_{qg}^{(0)}(z)$$ LO massive OME

$$\dot{c}^{(1)}(x,\mu^2) \equiv \frac{dc^{(1)}(x,\mu^2)}{d\ln\mu^2} = a_s(\mu^2)\int_x^1 \frac{dz}{z}P_{qg}^{(0)}(z)g\left(\frac{x}{z},\mu^2\right)$$ c-quark evolution in LO, FOPT boundary condition at $\mu_0 \approx m_c$

$$\delta\dot{c}^{(1)}(x,\mu^2) = \frac{da_s}{d\mu^2}\frac{c^{(1)}(x,\mu^2)}{a_s}$$ (FOPT – evolved) in LO: 0 $\mu = m_c$

$$A_{hg,hq}^{(2)} = a_{hg,hq}^{(2,0)} + a_{hg,hq}^{(2,1)}\ln\left(\frac{\mu^2}{m_c}\right) + a_{hg,hq}^{(2,2)}\ln^2\left(\frac{\mu^2}{m_c}\right)$$ NLO massive OME

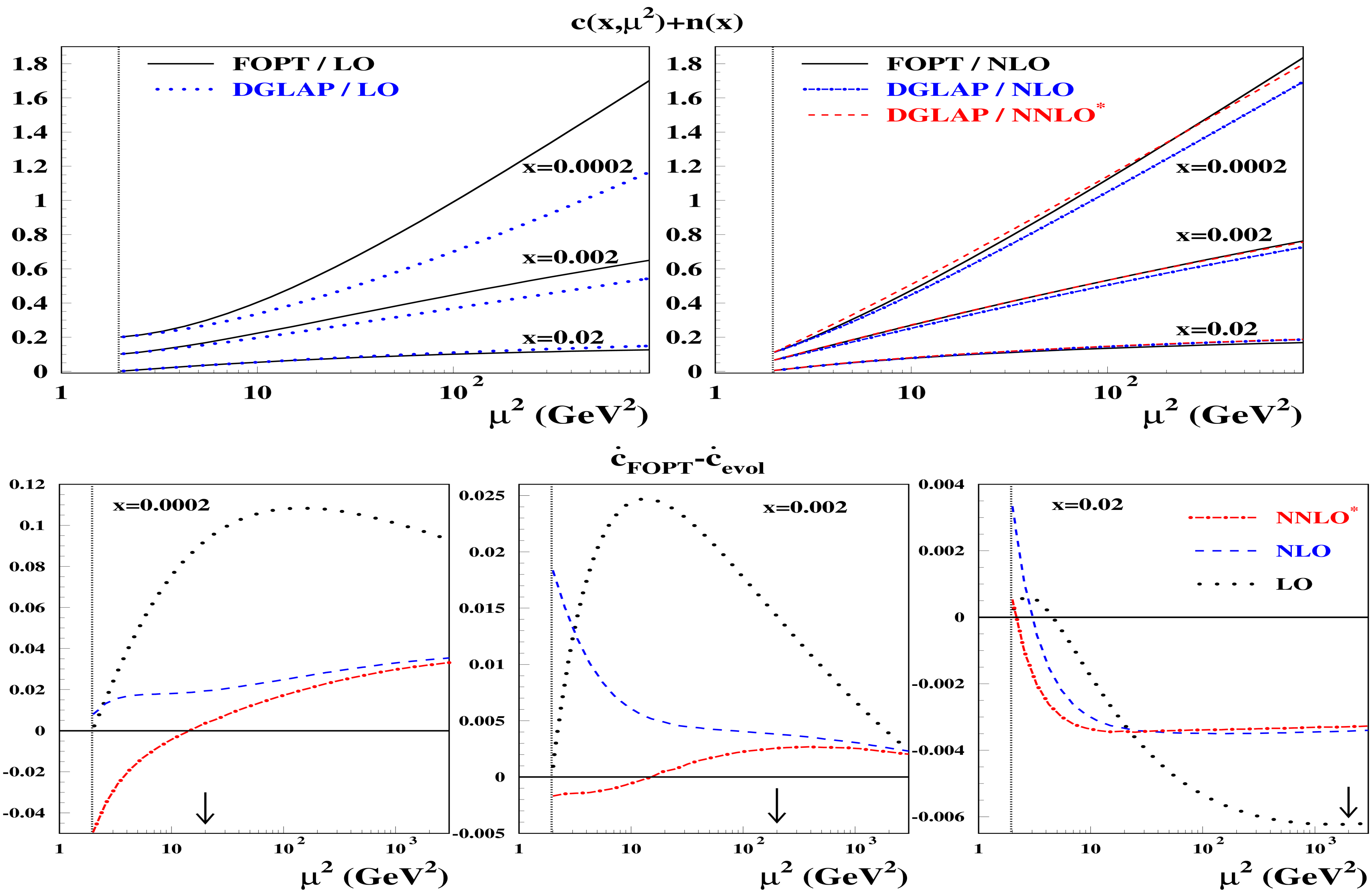
$$\delta\dot{c}^{(2)}(x,\mu^2) \sim a_s\frac{da_s}{d\mu^2}a_{hg}^{(2,0)}$$ (FOPT – evolved) in NLO: ≠0 $\mu = m_c$

NLO: NLO evoultion with the FOPT boundary conditions in NLO

NNLO*: NNLO evoultion with the FOPT boundary conditions in NLO

Blümlein, Riemersma, Botje, Pascaud, Zomer, van Neerven, Vogt hep-ph/9609400

Comparison of the FOPT and evolved c-quark PDFs



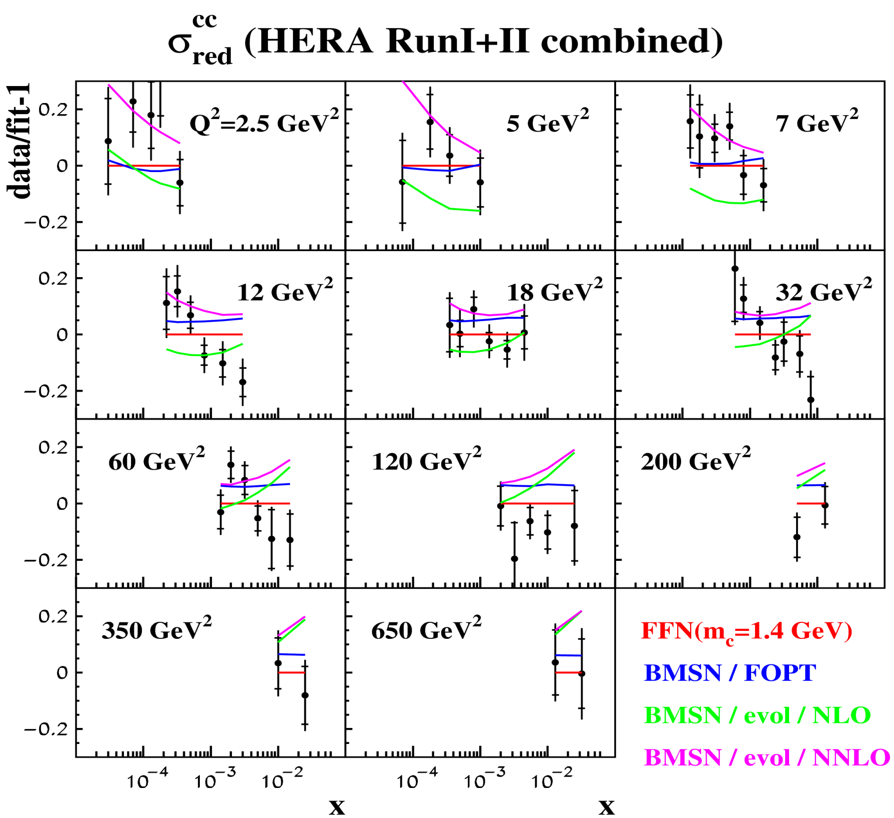
The difference between FOPT and evolved PDFs is localized at small scales: uncertainties due to missing high-orders rather than impact of the big-log resummation

BMSN with the evolved PDFs

- Combined HERA charm production data

H1/ZEUS PLB 718, 550 (2012)

- PDFs from variant of ABM11 fit with m_c=1.4 GeV (pole mass definition), option A of NNLO W.coef.

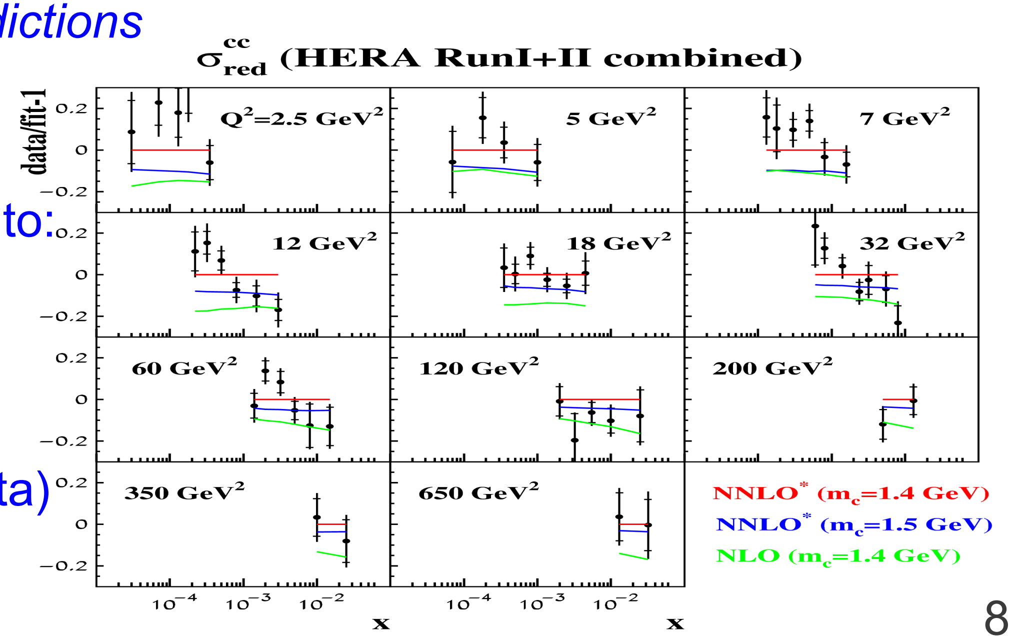


- Two variants of 4-flavor PDF evolution
  NNLO (consistent with the light PDF evolution, inconsistent with the NLO matching) **
  NLO (inconsistent with the light PDF evolution, consistent with the NLO matching)
  ** commonly used in the VFN fits
- Substantial difference between NLO and NNLO versions
- The evolved predictions demonstrate strong x-dependence and weak Q^2-dependence

The difference with FOPT appears rather due to inconsistent evolution than due to big-logs → should be considered as a theoretical uncertainty in the VFN predictions

For the FFN scheme basic uncertainties are due to:

- incomplete NNLO terms in the massive Wilson coefficients
- the c-quark mass variation (marginal if fitted to the data and/or constrained from e+e- data)



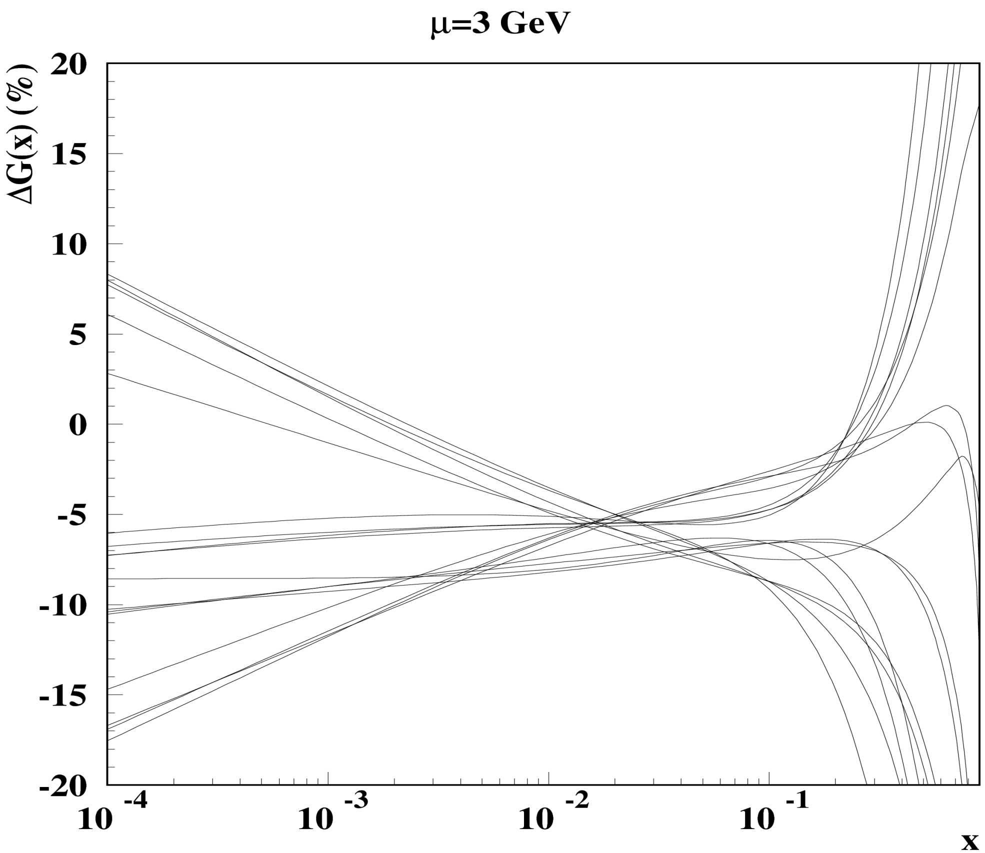
Uncertainties due to m_c and matching point

NLO

m_c (Gev) \ μ_0 (GeV)	1.2	1.3	1.4	1.5
1.2	−0.0006	−0.0007	−0.0007	−0.0010
1.5	+0.0009	+0.0009	+0.0010	+0.0009

NNLO*

m_c (Gev) \ μ_0 (GeV)	1.2	1.3	1.4	1.5
1.2	−0.0007	−0.0006	−0.0006	−0.0005
1.5	−0.0004	−0.0001	−0.0002	+0.0001



Change in $\alpha_s(M_z)$ due to PDF evolution is -0.0015 ± 0.0010

The uncertainties due to PDF evolution are comparable to experimental ones

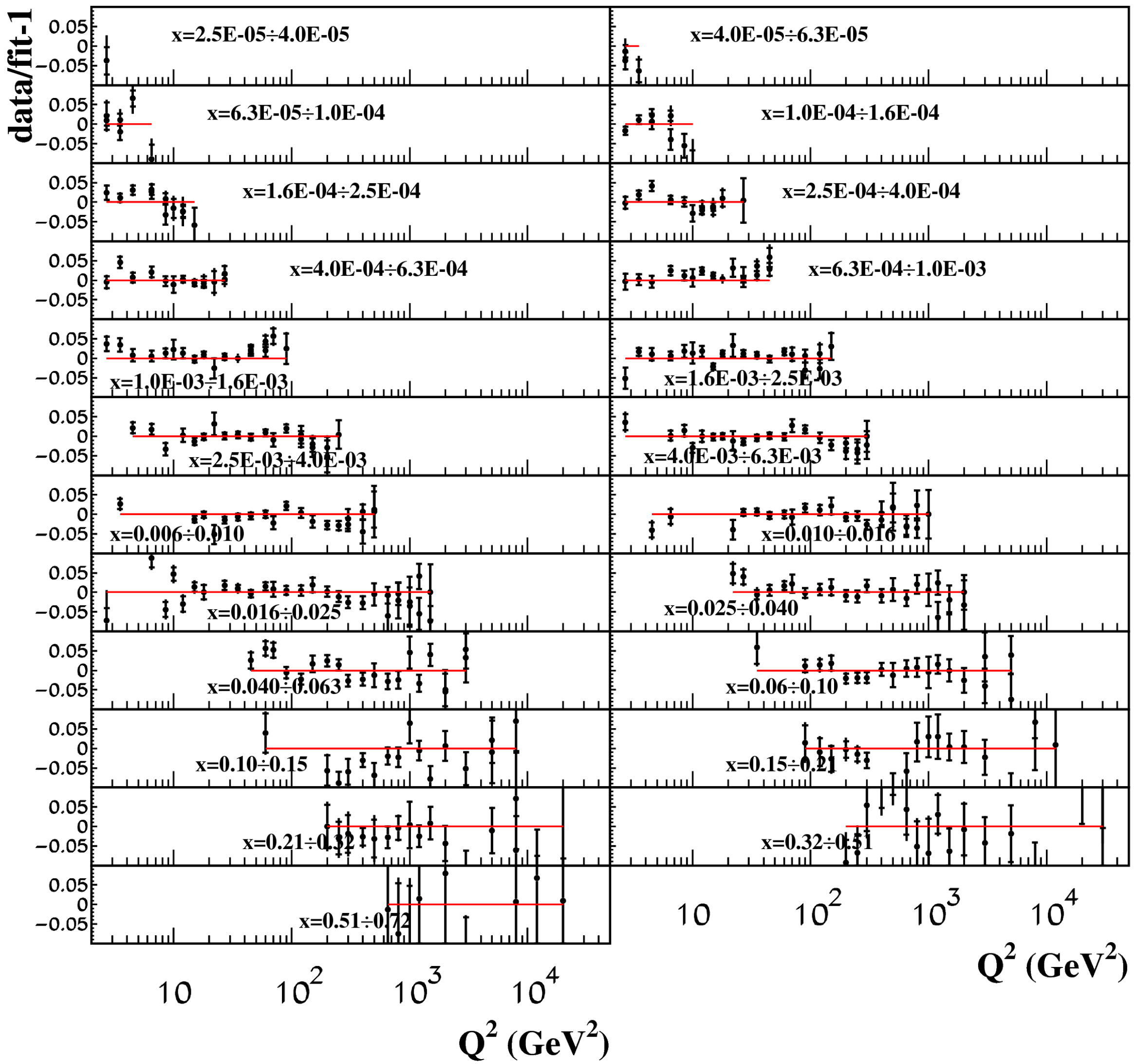
"We conclude that the FFN fit is actually based on a less precise theory, in that it does not include full resummation of the contribution of heavy quarks to perturbative PDF evolution, and thus provides a less accurate description of the data." **NNPDF 13013.1189**

The NNPDF conclusion is wrong: the theoretical uncertainties have not been considered

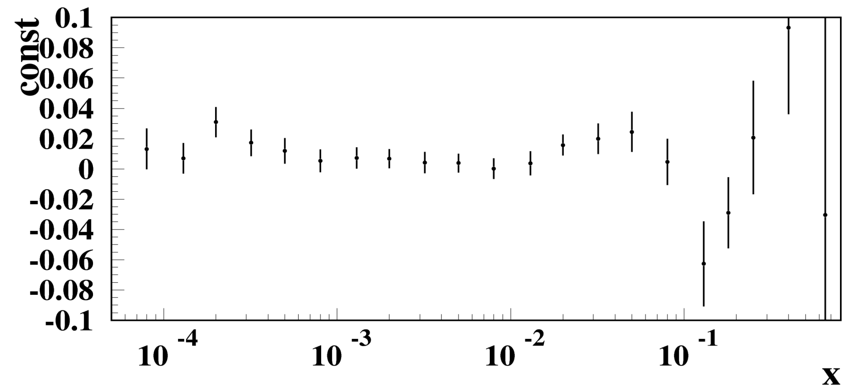
Gao, Guzzi, Nadolsky hep-ph/1304.3494

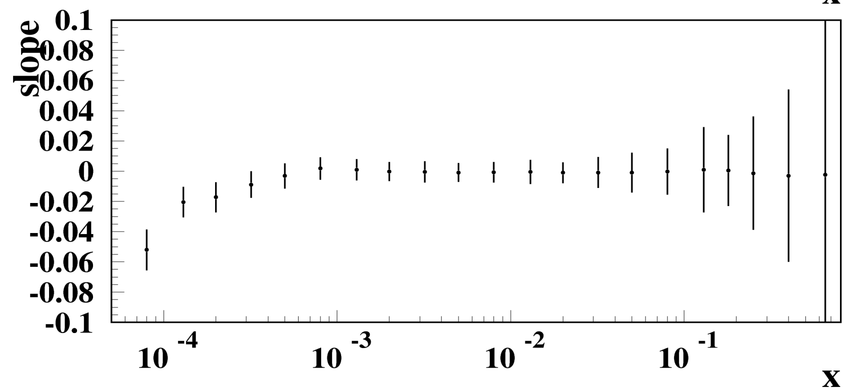
Statistical check of the big-log impact

HERA-I e^+p



Q^2_{min} (GeV^2)	χ^2/NDP
10	366 / 324
100	193 / 201
1000	95 / 83





pulls=const + slope * $\log(Q^2/Q^2_0)$

No traces of big logs

Summary

- The FFN scheme with the NNLO massive coefficients and running mass definition provides good description of the existing data
  - no impact of big logs at large Q^2 up to 10000 GeV^2
  - the MSbar values

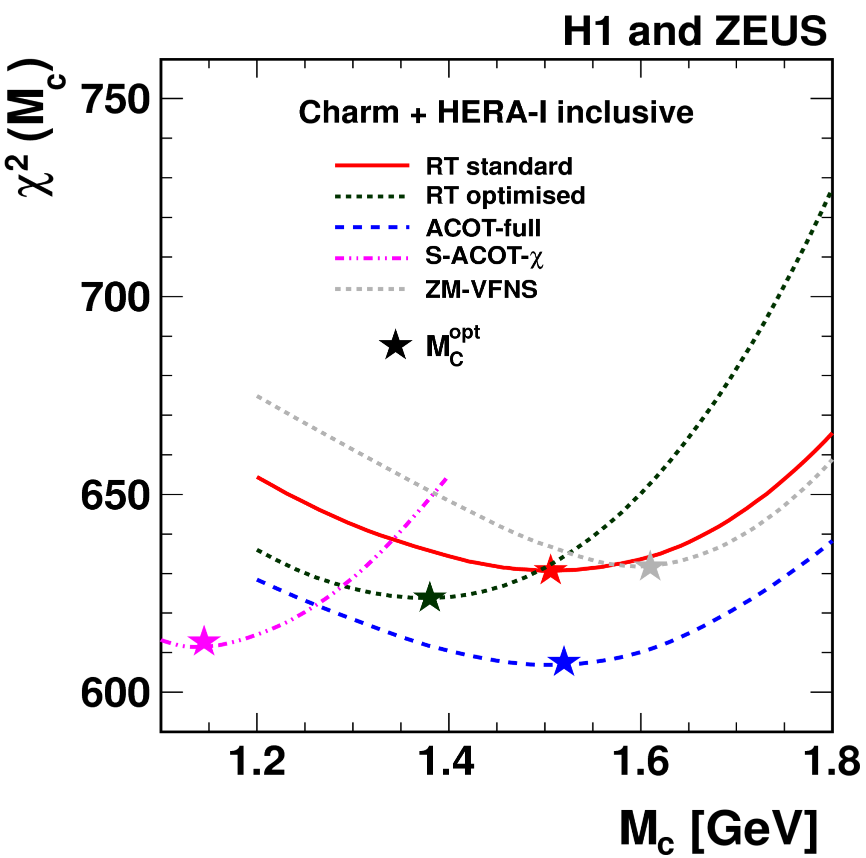
    $m_c(m_c)$=1.15±0.04(exp.),+0.04,-0(scale) GeV NLO

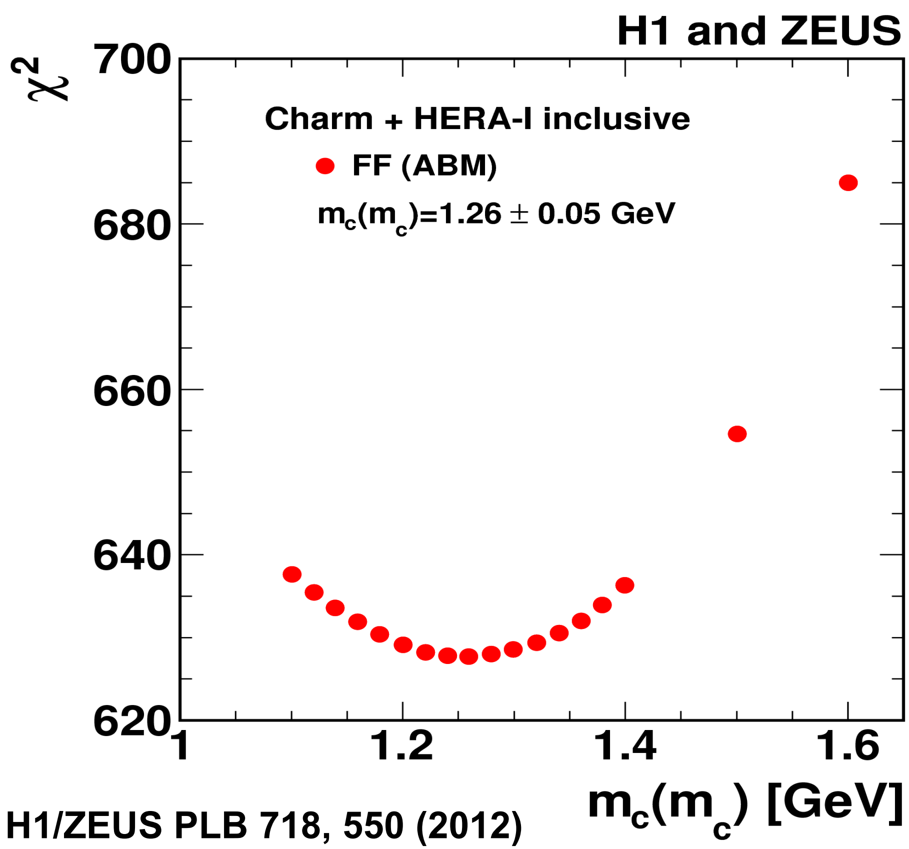
    $m_c(m_c)$=1.24±0.03(exp.),+0.03,-0.02(scale),+0,-0.07(th) GeV NNLO

    are in good agreement with the e+e- results
- The theoretical uncertainties related to the PDF evolution in the VFN schemes are comparable to the experimental ones
  - the value of $\alpha_s(M_z)$ obtained in the VFN version of the ABM11 fit (BMSN with PDF evolution) gains additional theoretical uncertainty of 0.0010

Extras

Statistical check of the FFNS and VFNS





H1/ZEUS PLB 718, 550 (2012)

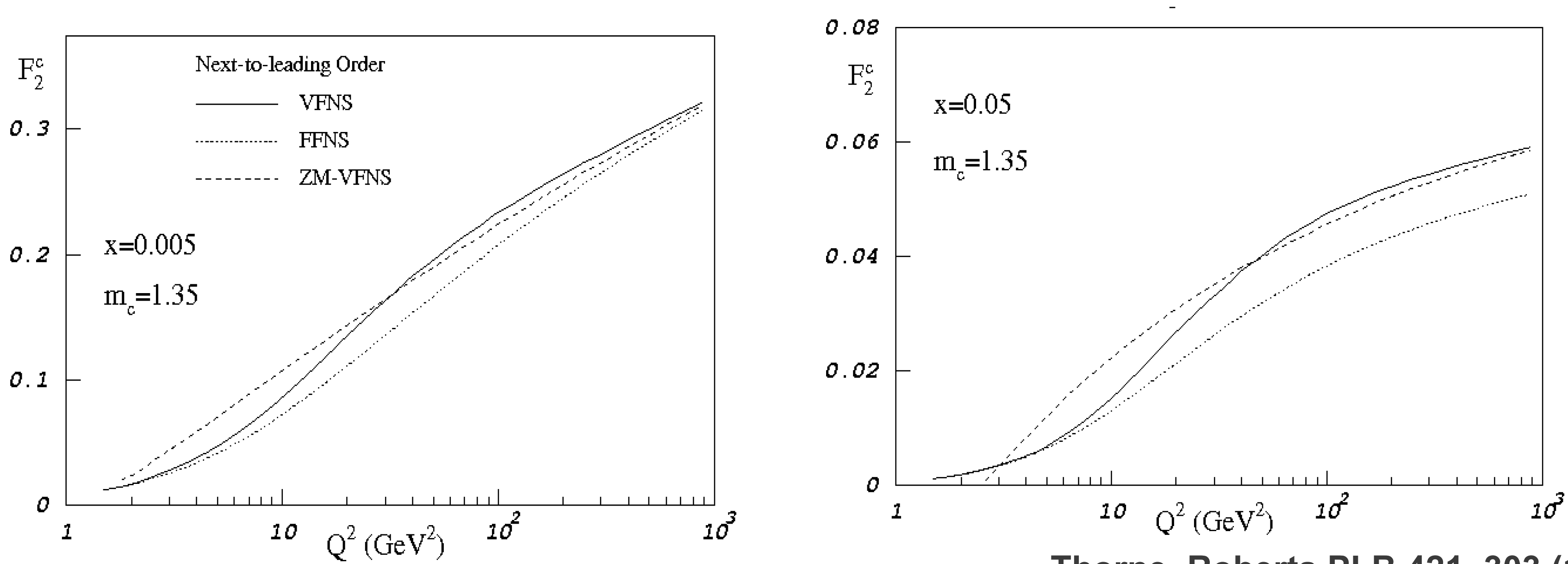
- *In the NNPDF fit the FFNS value of χ^2 for the FFNS is bigger than VFNS one by 77/592 for the HERA-I inclusive data (combined HERA charm data are not considered)*

- No significant difference in the description quality between VFNS and FFNS is observed in the HERAPDF analysis
- In the variants of ABM fit with different versions of BMSN the value of χ^2 is worse by some 20/608 for the HERA-I inclusive data
- A detailed benchmarking is difficult since the NNPDF code is not publicly available

ZMVFN and GMVFN schemes

ZMVFN (zero-mass variable-flavor-number) scheme

- The PDFs, including the the heavy-quark one are convoluted with the massless coefficient functions
- The corrections up to N^3LO are available
- The big logs $\sim ln^n(Q/m_c)$ can be in a natural way resummed in the massless QCD evolution
- Irrelevant outside the asymptotic region $Q>>m_h$



Thorne, Roberts PLB 421, 303 (1998)

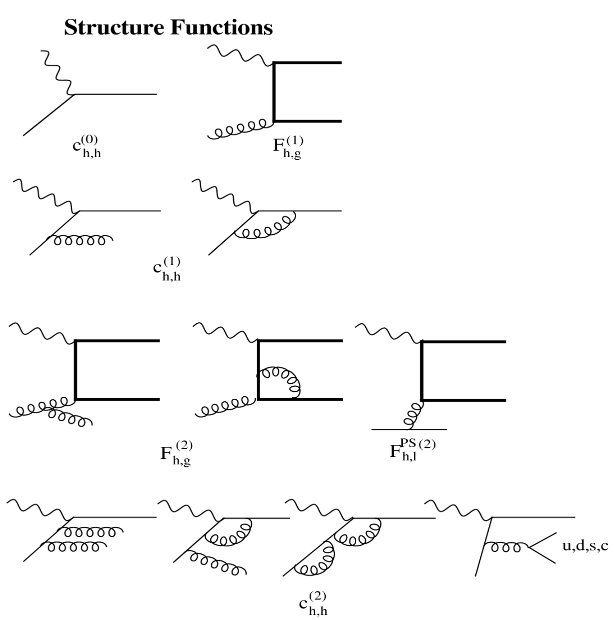
GMVFN (general-mass variable-flavor-number) scheme

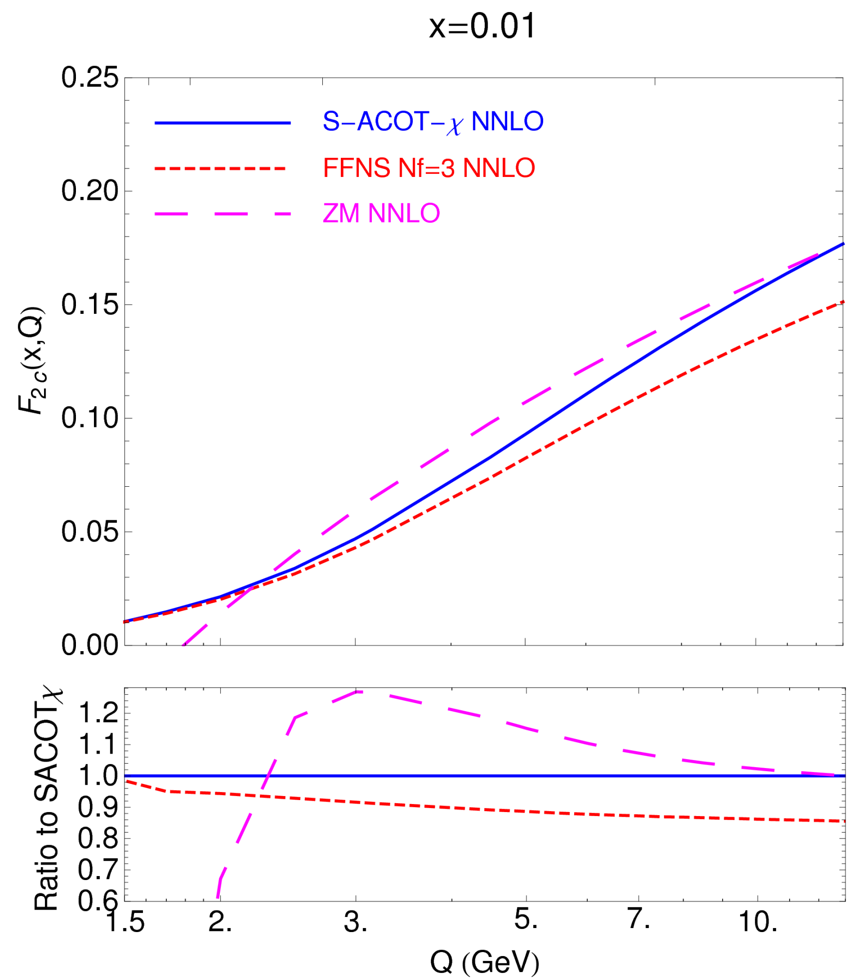
- Provides matching with the FFNS in the limit of $Q \rightarrow m_h$
- Modeling at small Q cannot be based on the solid footing; many prescriptions available that causes theoretical uncertainty

ACOT prescription

Guzzi, Nadolsky, Lai, Yuan PRD 86, 053005 (2012)

The prescription is based on the subtractions, similarly to the BMSN one





Extrapolation to $Q = m_h$ is based on the assumption for the coefficient function of heavy-quark initiated processes

$$C^{(k)}_{h,h}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{m_h}{Q}\right)=c^{(k)}_{h,h}\left(\frac{\chi}{\xi},\frac{Q}{\mu},m_h=0\right)$$

$$\chi = x\left(1+\frac{(\sum_{f_s} m_h)^2}{Q^2}\right)$$

$$x=\frac{\zeta}{1+\zeta^{\lambda}\cdot(4m_c^2)/Q^2},$$

- The “slow-rescaling” is consistent with the QCD factorization
- A variety of rescaling forms gives different prescription: SACOT, ACOT-χ, …..
- Matching with FFNS $Q = m_h$ is not very smooth

Thorne's prescription

Thorne hep-ph/1201.6180

Based on the ACOT *(different from the Thorne-Roberts prescription)*

Thorne, Roberts PLB 421, 303 (1998)

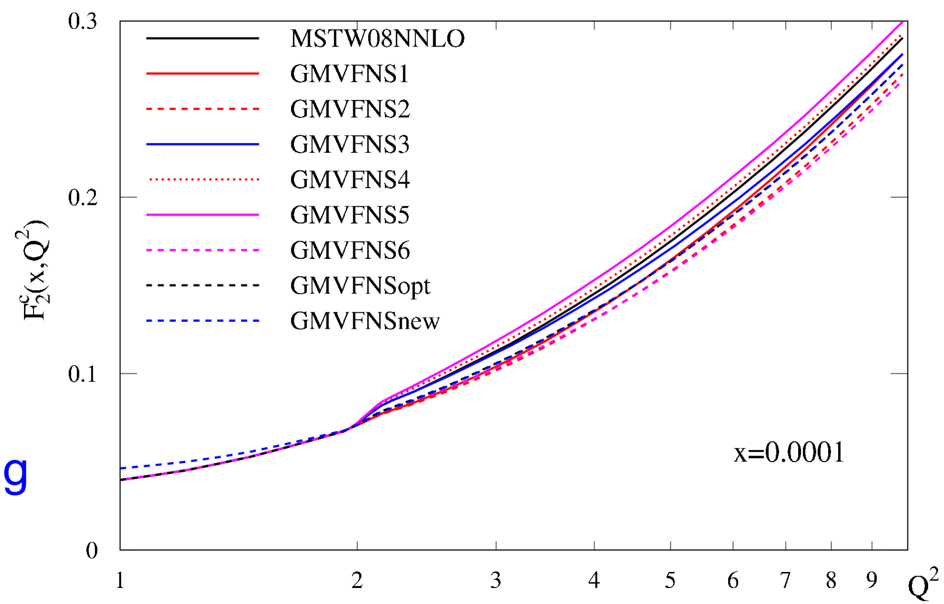
$$C^{\mathrm{GMVF},(0)}_{2,h\bar{h}}(Q^2/m_h^2, z) \rightarrow (1+b(m_h^2/Q^2)^c)\delta(z-x_{\max})$$

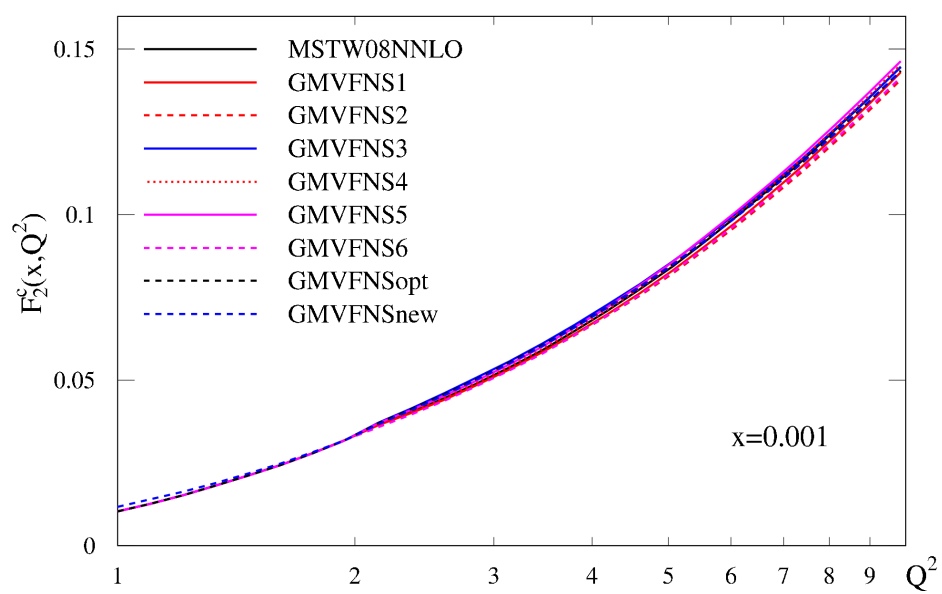
$$\xi = x/x_{\max} \rightarrow x(1+(x(1+4m_h^2/Q^2))^d 4m_h^2/Q^2)$$

Additional parameters b and c improved matching with FFNS and the NNLO term stemming from the threshold resummation added

$$A(Q^2/m_h^2)(1-z/x_{\max})^{\tilde{a}}(\ln(1/z)-\tilde{b})/z,$$

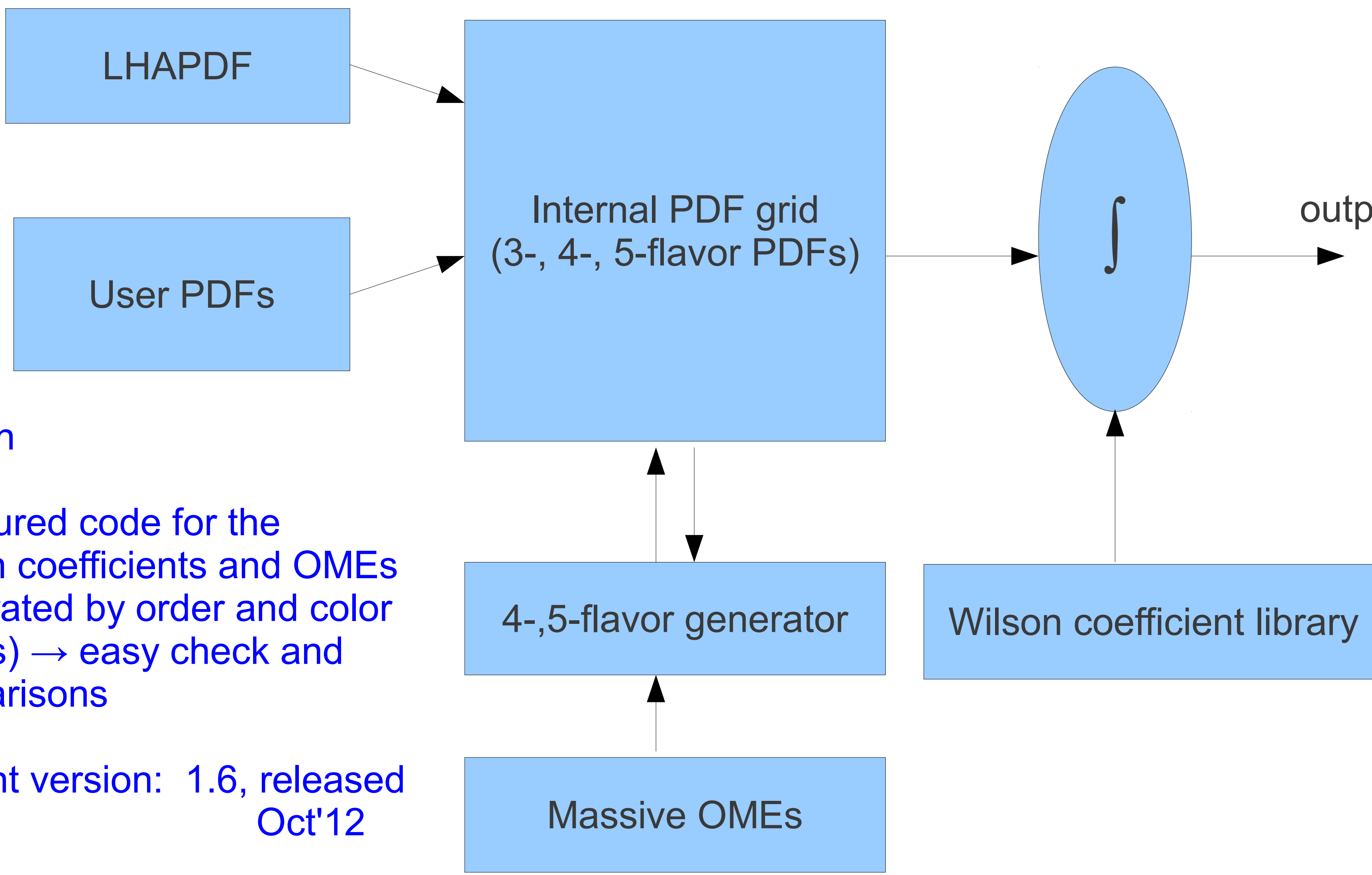
- With the variety of parameters smooth matching is achieved
- Does the MSbar scheme persist?
- With a smooth matching to FFNS provided at $Q = m_h$ the Thorne's prescription in NNLO does not differ very much from FFNS elsewhere





OPENQCDRAD

www-zeuthen.desy.de/~alekhin/OPENQCDRAD



Fortran

Structured code for the Wilson coefficients and OMEs (separated by order and color factors) → easy check and comparisons

Current version: 1.6, released Oct'12

- Updated massive NNLO Wilson coefficients — **Kawamura, Lo Presti, Moch, Vogt NPB 864, 399 (2012)**
- Z-exchange term up to NNLO — **Klein, Rieman ZPC 24, 151 (1984)**; **Zijlstra van Neerven NPB 383, 525 (1992)**