MSTW PDFs, Comments on PDF updates and Flavour Schemes

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I will present results on continuing updates in PDFs within the MSTW framework due to some theory improvements and a variety of new data sets. Very much in progress. Partly a combination of individual modifications already presented.

I will also discuss extensions to work previously presented on differences between PDFs in FFNS and GM-VFNS.

Updates in Fits with the MSTW Framework.

Changes in theoretical treatment.

Continue to use extended parameterisation with Chebyshev polynomials, and freedom in deuteron nuclear corrections (and heavy nuclear corrections), as in recent MSTWCPdeut study (Eur.Phys.J. C73 (2013) 2318) – change in $u_V - d_V$ distribution.

Now use "optimal" GM-VFNS choice which is smoother near to heavy flavour transition points (more so at NLO).

Correct dimuon cross-sections for missing small contribution, i.e. where charm is produced away from the interaction point. Previously assumed this was accounted for by acceptance corrections. Previous checks showed correction is a small effect on strange distribution.

Use NMC structure function data with $F_L(x, Q^2)$ correction very close to theoretical $F_L(x, Q^2)$ value. Very little effect.

Changes in data sets.

Replacement of HERA run I neutral current data from HERA and ZEUS with combined data set. Already considered effect of this. Fit to data very good. Slightly better fit at NNLO – 33 units for 553 points.

Inclusion of HERA combined data on $F_2^c(x, Q^2)$. Fit quality $\sim$ 60-65 for 52 points.

Inclusion of run II ZEUS data EPJ C 62 (2009) 625, until recently only run II neutral current set published. Fit quality very similar to HERAPDF fits.

Inclusion of all direct HERA $F_L(x, Q^2)$ measurements. Undershoot data a little at lower Q^2, but χ^2 not much more than one per point.

Inclusion of the D0 electron asymmetry data for $p_T > 25\mathrm{GeV}$ based on 1 fb^{-1} and CDF W-asymmetry data. Keep lower luminosity D0 muon asymmetry data.

Fit quality for two new sets about 2 per point. Due mainly to fluctuations – similar for other groups. However, slight tension between these two sets.

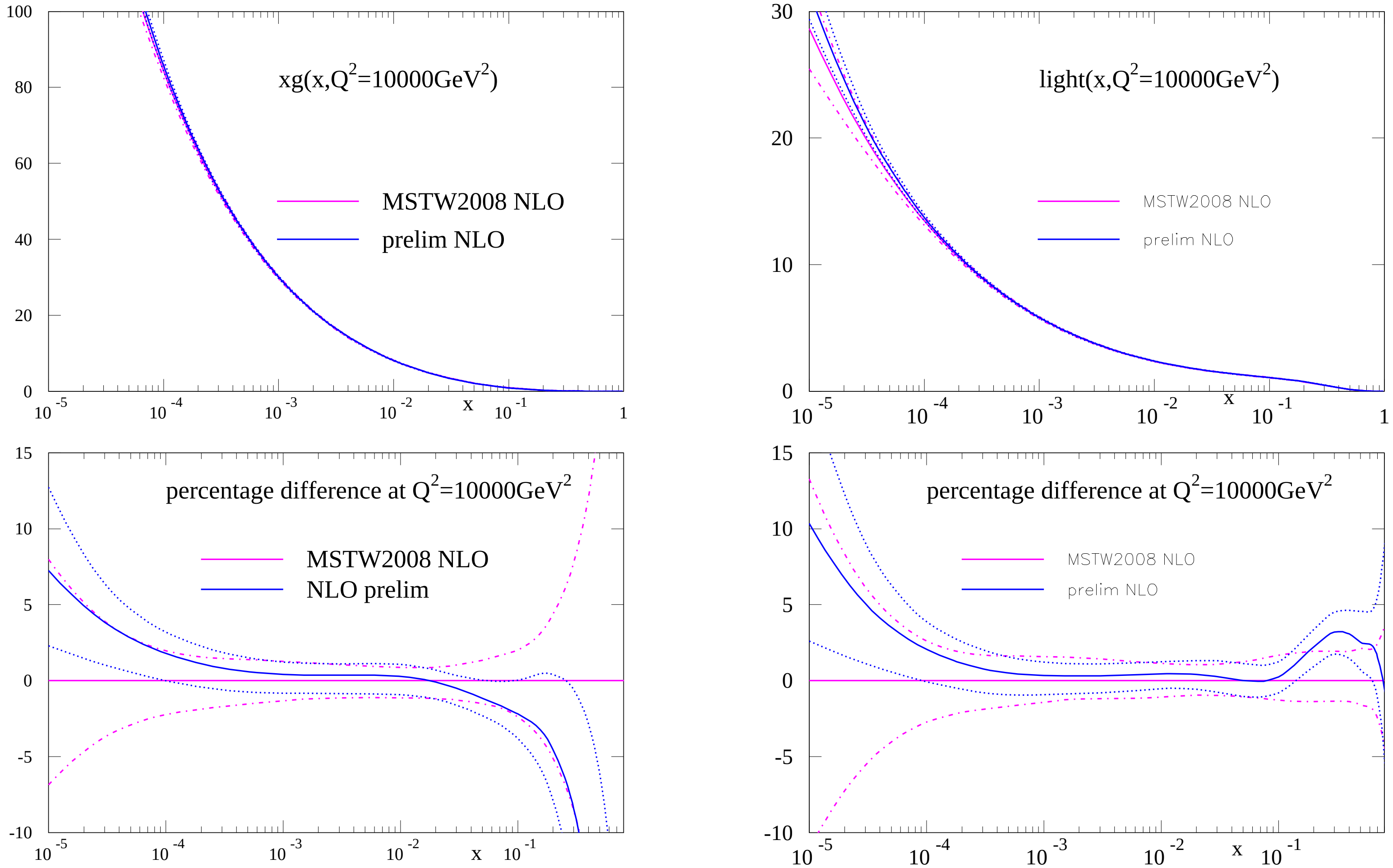
For D0 muon asymmetry data $\chi^2 = 6/10$ as compared to $\chi^2 = 25/10$ for MSTW2008. Due to $u_V - d_V$ change mainly already in MSTWCPdeut.

Include final numbers for CDF Z-rapidity data – final numbers changed after MSTW2008 fit. (Also include very small photon contribution in theory.)

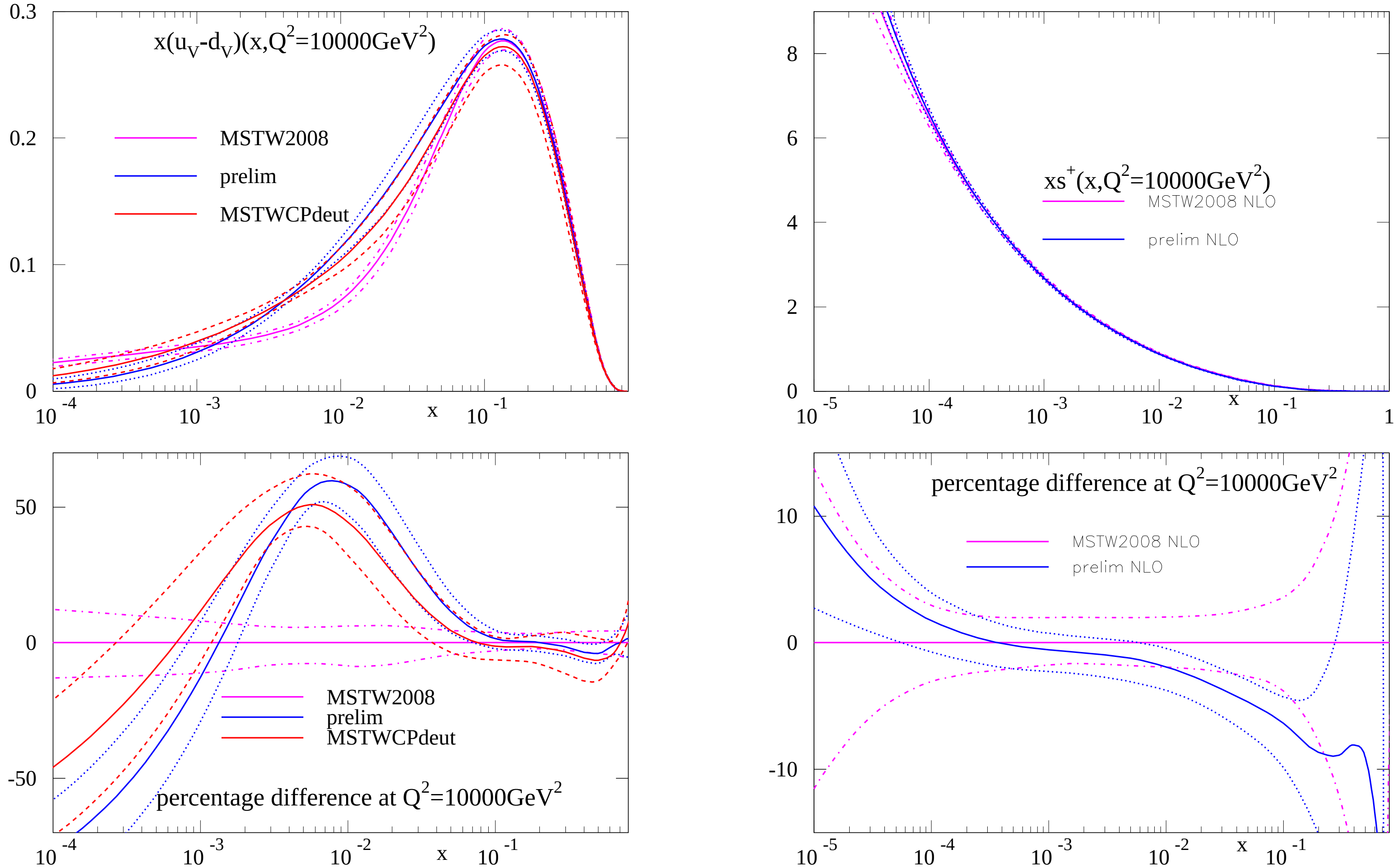
Little change in PDFs. Final data is more consistent with the theory – $\chi^2 \sim 38/28$.

Not much change in PDFs (other than already seen in $u_V - d_V$).

At NLO $\alpha_S(M_Z^2) = 0.1197$ from 0.1202 and at NNLO $\alpha_S(M_Z^2) = 0.1168$ from 0.1171.



Change in NLO PDFs from all updates. Increase in d at high x.

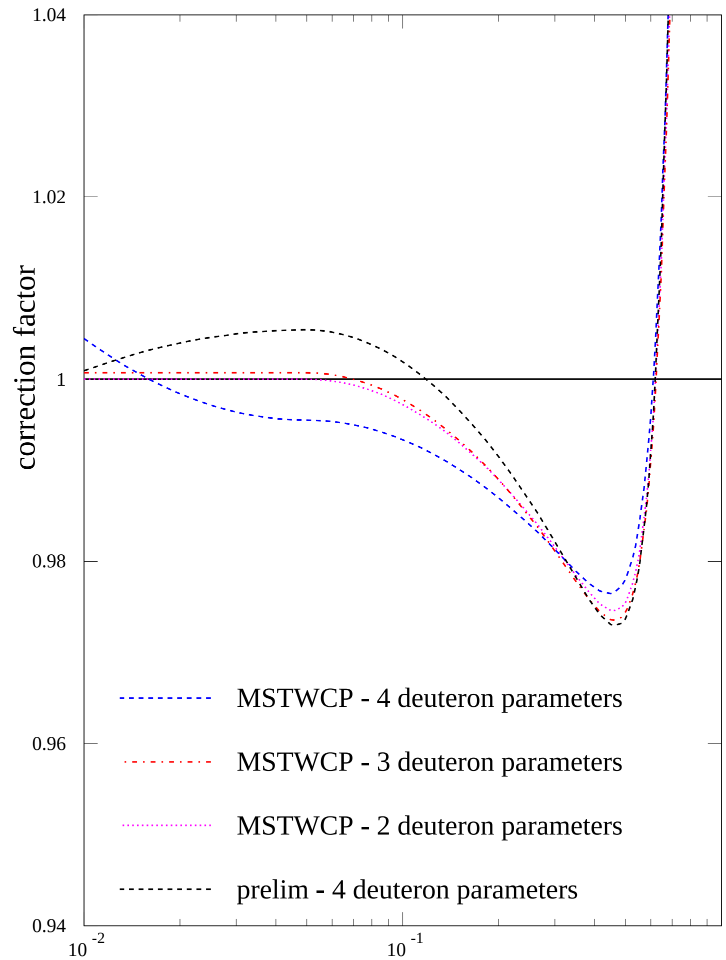


Change in NLO PDFs from all updates.

Result for fitted deuteron correction.

Previously big improvement in fit for MSTWCPdeut, but not exactly as expected at lower x.

Now more like expected for and 4 parameters left free (at NLO). Uncertainty of about $0.5 - 1\%$. Feeds into PDF uncertainty.



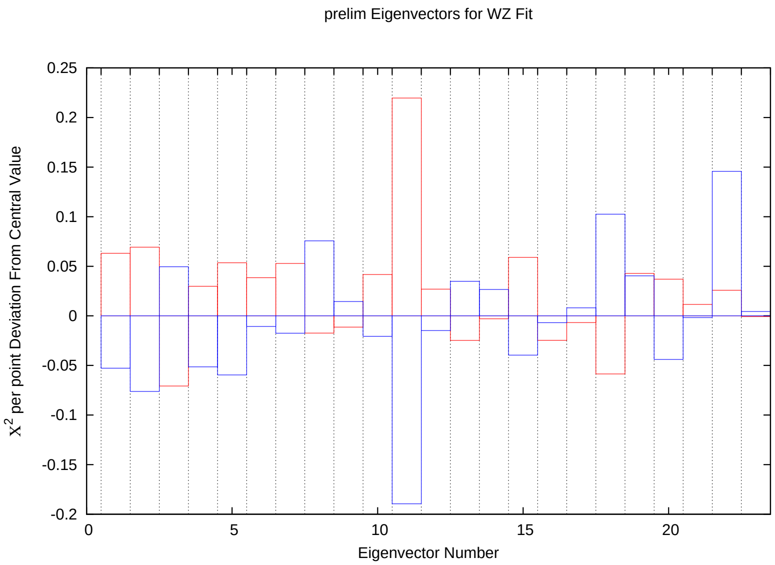
Change in various cross section predictions compared to uncertainty for MSTW2008.

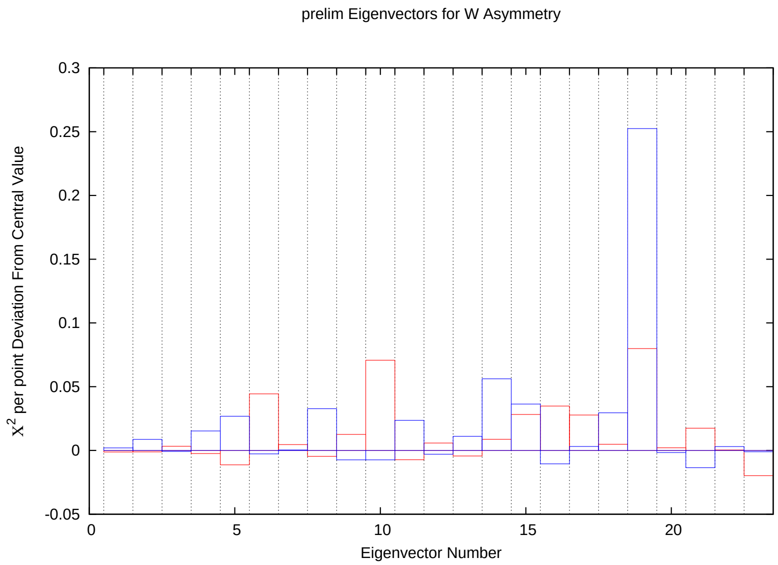
	NLO	NNLO	unc.
W Tevatron (1.96 TeV)	$+2.2$	$+3.3$	1.8
Z Tevatron (1.96 TeV)	$+3.3$	$+2.6$	1.9
W^+ LHC (7 TeV)	$+2.6$	$+0.9$	2.2
W^- LHC (7 TeV)	$+0.5$	$+0.6$	2.2
Z LHC (7 TeV)	$+1.3$	$+0.6$	2.2
W^+ LHC (14 TeV)	$+2.7$	$+0.2$	2.4
W^- LHC (14 TeV)	$+0.8$	-0.3	2.4
Z LHC (14 TeV)	$+1.1$	-0.2	2.4
Higgs Tevatron	-5.0	-3.9	5.1
Higgs LHC (7 TeV)	-2.2	-1.3	3.3
Higgs LHC (14 TeV)	-1.6	-1.3	3.1
$t\bar{t}$ Tevatron	$+1.1$	$+1.6$	3.2
$t\bar{t}$ LHC (7 TeV)	-4.1	-2.3	3.9
$t\bar{t}$ LHC (14 TeV)	-3.0	-1.6	3.1

Some changes of order size of uncertainty - smaller at NNLO. Change in Tevatron W, Z mainly due to combined HERA data.

Comparison to LHC data.

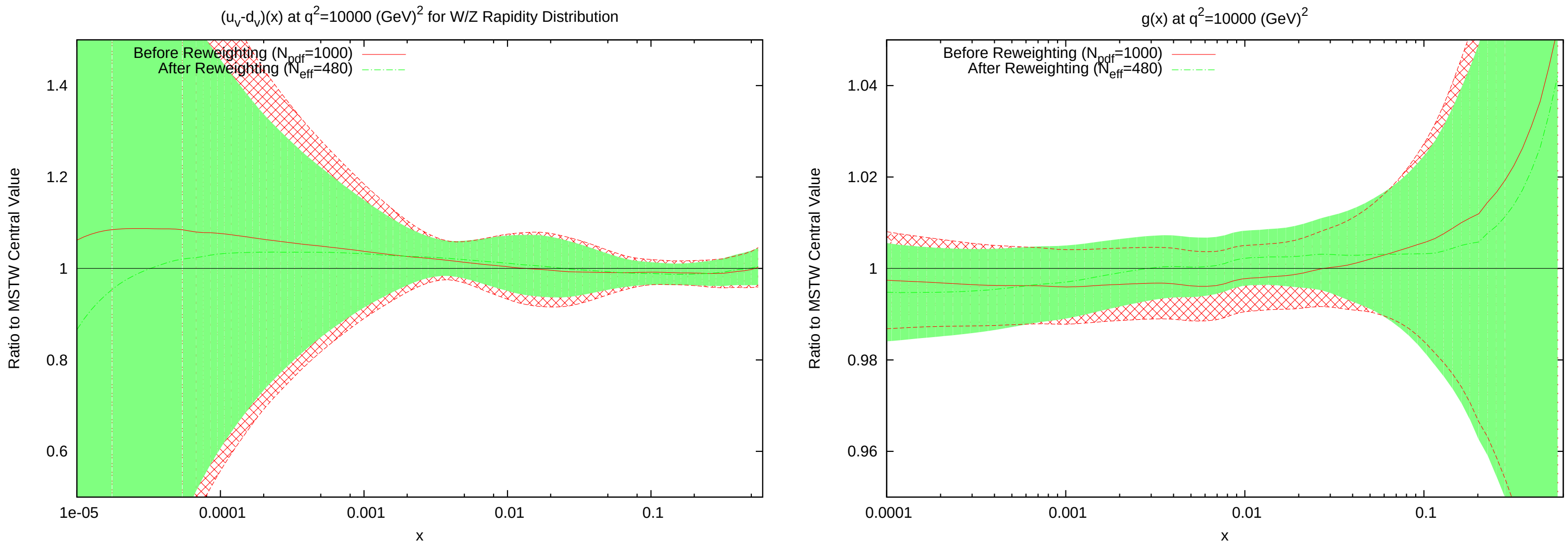
At NLO $\chi^2 = 1.64$ per point for ATLAS W, Z rapidity data, slightly higher at NNLO. Comparable with many other sets and similar to MSTWCPdeut. Asymmetry data alone gives $\chi^2 = 0.4$ per point.





No plausible improvement for asymmetry data. Full rapidity data sensitive to eigenvector 11 (gluon dominated). Inconsistency with ZEUS run II data – seen explicitly. (Plots by B. Watt).

Under PDF reweighting $\chi^2 = 1.44$ per point, a reasonable improvement.



No real change in $u_V - d_V$ – an improvement on MSTWCPdeut

Main change in details of shape of gluon distribution.

For ATLAS jet data $\chi^2 = 0.78 \rightarrow 0.74$ for $R = 0.4$ and practically unchanged at $\chi^2 = 0.79$ for $R = 0.4$. Bit more from Ben Watt.

Dependence on m_c (pole mass) at NLO in prelim fits.

m_c (GeV)	χ^2_{global} 2593 pts	$\chi^2_{F_2^c}$ 52 pts	$\alpha_s(M_Z^2)$
1.15	2638	114	0.1188
1.2	2630	99	0.1190
1.25	2632	87	0.1191
1.3	2635	77	0.1194
1.35	2642	70	0.1196
1.4	2654	65	0.1197
1.45	2668	62	0.1198
1.5	2686	60	0.1201

Some correlation between m_c and $\alpha_S(M_Z^2)$.

Preference for $m_c \sim 1.225\mathrm{GeV}$.

NMC data prefer lower m_c – quicker threshold evolution respectively.

Tension between global fit and charm data.

Dependence on m_c at NNLO in prelim fits.

m_c (GeV)	χ^2_{global} 2465 pts	$\chi^2_{F_2^c}$ 52 pts	$\alpha_s(M_Z^2)$
1.15	2524	86	0.1160
1.2	2516	78	0.1162
1.25	2513	71	0.1163
1.3	2513	67	0.1165
1.35	2516	65	0.1167
1.4	2525	63	0.1168
1.45	2534	63	0.1169
1.5	2551	64	0.1171

Slightly less correlation between m_c and $\alpha_S(M_Z^2)$.

Less variation in fit quality and much less tension.

Preference for $m_c \sim 1.275\text{GeV}$.

Better consistency between NLO and NNLO than before. Previously NLO wanted higher value of m_c. Both same as in Alekhin *et al.* study.

Choices for Heavy Flavours in DIS. (Extension of work in by RT in Phys.Rev. D86 (2012) 074017.)

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state.

Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x, Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Additional problem FFNS known up to NLO (Laenen *et al.*), but are not fully known at NNLO – $\alpha_S^3 C_{2,Hi}^{FF,3}$ unknown.

Approximations based on some or all of threshold, low-x and high-Q^2 limits can be derived, see Kawamura, *et al.*,, and are sometimes used in fits, e.g. ABM11 and MSTW (at low Q^2). Generally not large except at threshold and very low x.

Variable Flavour - at high scales $Q^2 \gg m_H^2$ heavy quarks behave like massless partons. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZM,n_f} \otimes f_j^{n_f}(Q^2).$$

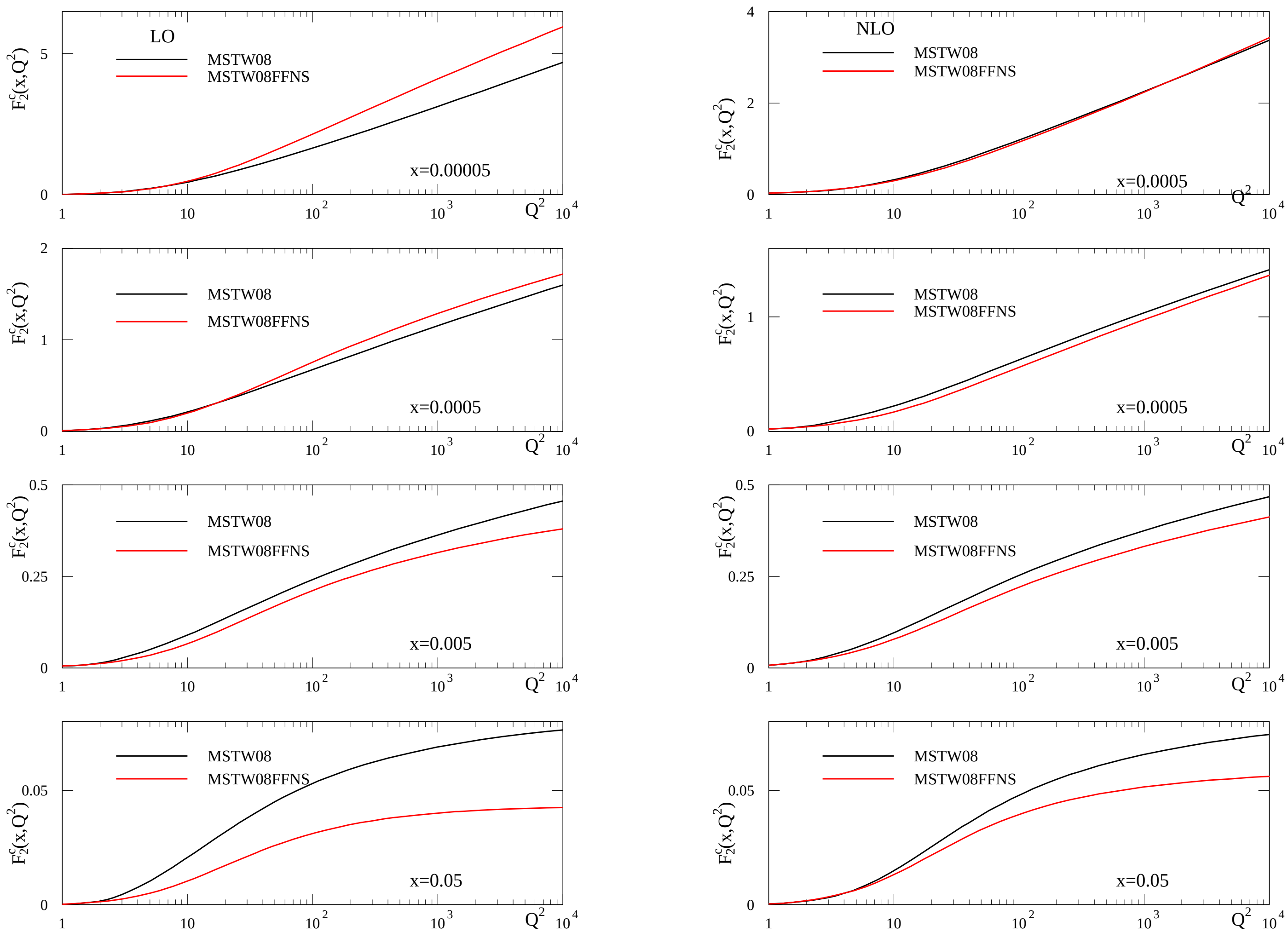
Partons in different number regions related to each other perturbatively.

$$f_j^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

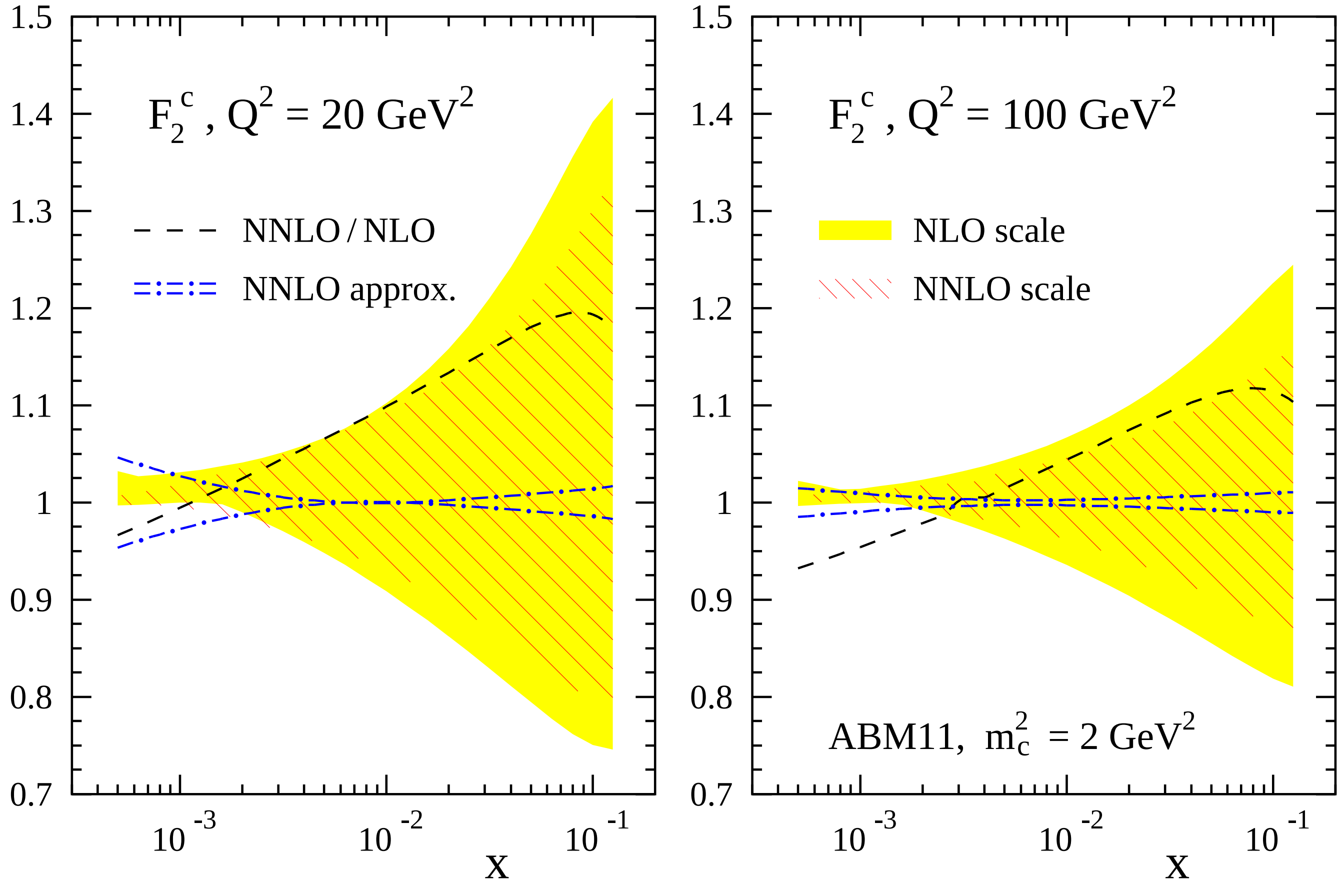
Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ (Buza *et al.*) containing $\ln(Q^2/m_H^2)$ terms relate $f_i^{n_f}(Q^2)$ and $f_i^{n_f+1}(Q^2) \rightarrow$ correct evolution for both.

Want a **General-Mass Variable Flavour Number Scheme** (VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Difference between FFNS and GM-VFNS

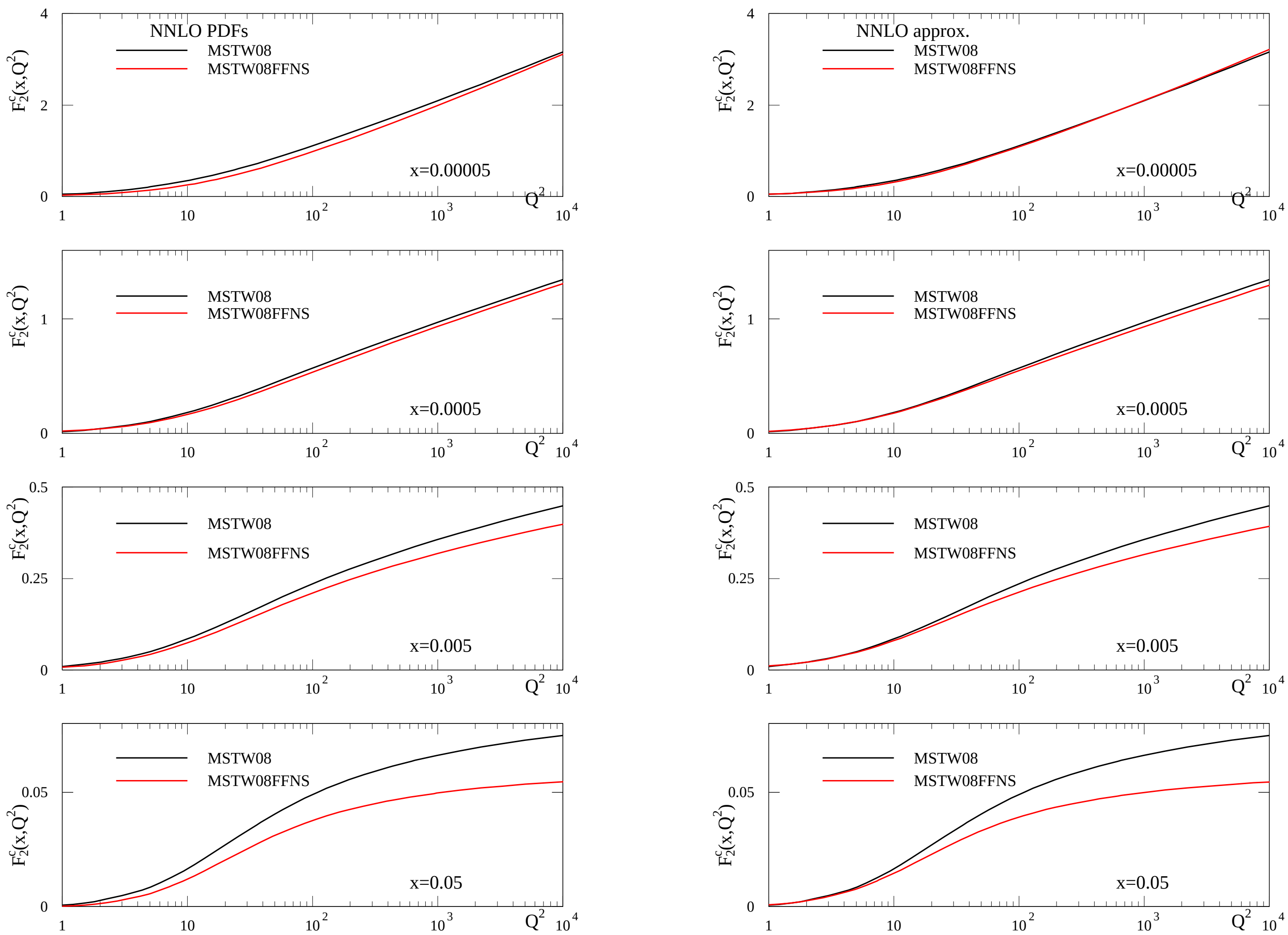


Big difference at LO. At higher Q^2 charm structure function for FFNS nearly always lower than any GM-VFNS at NLO, but mainly at higher x.



Approximate $\mathcal{O}(\alpha_S^3)$ corrections to $F_2^c(x, Q^2)$ by Kawamura *et al.* in Nucl.Phys. B864 (2012) 399-468.

Similar results for $\mathcal{O}(\alpha_S^3)$ approximation used by MSTW at low Q^2 extended to higher Q^2.



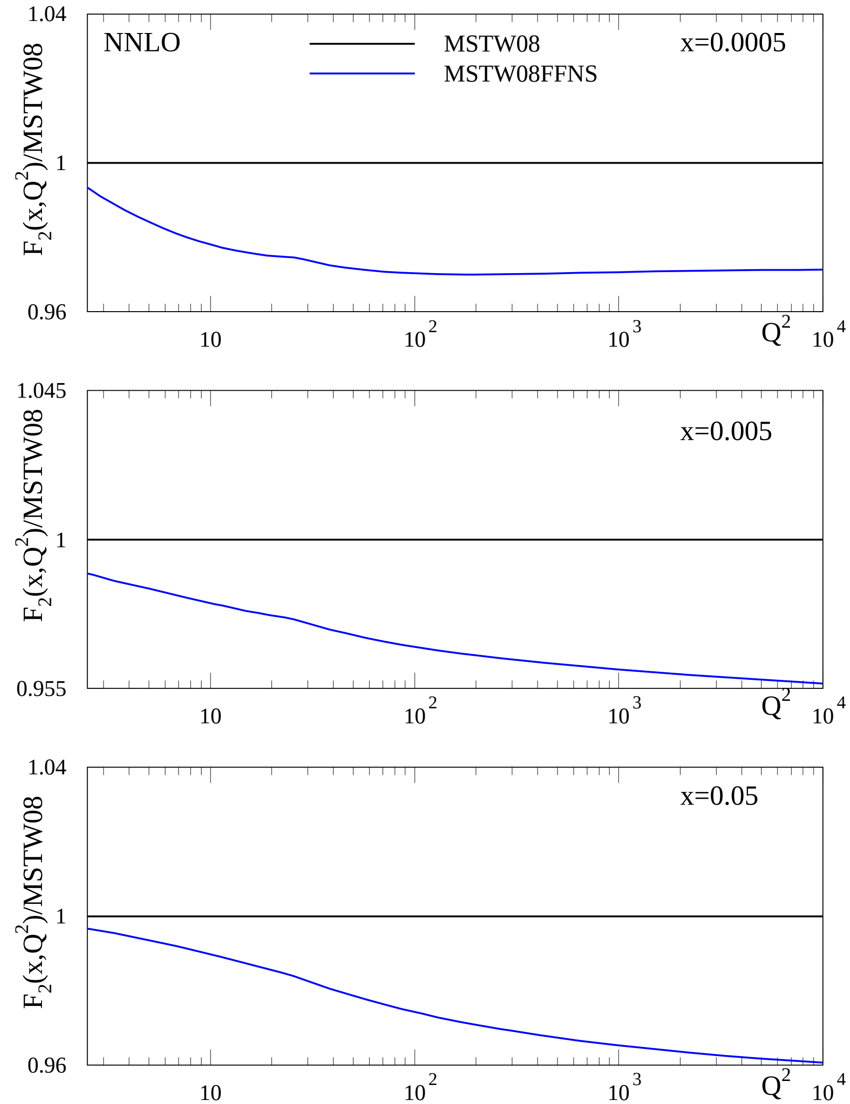
No dramatic change or improvement at NNLO.Left only NNLO PDFs, right uses $\mathcal{O}(\alpha_S^2)$ coefficient functions for $F_2^c(x, Q^2)$. Little difference at high Q^2.

Can lead to over 4% changes in the total $F_2(x,Q^2)$ if the same input PDFs are used in two schemes.

At higher x mainly due to $F_2^c(x,Q^2)$.

At lower x there is a large contribution from light quarks evolving slightly more slowly in FFNS.

At much higher x difference dies away. Charm component becomes very small and light quark evolution not much different. (Light quarks slightly bigger at the highest x.)



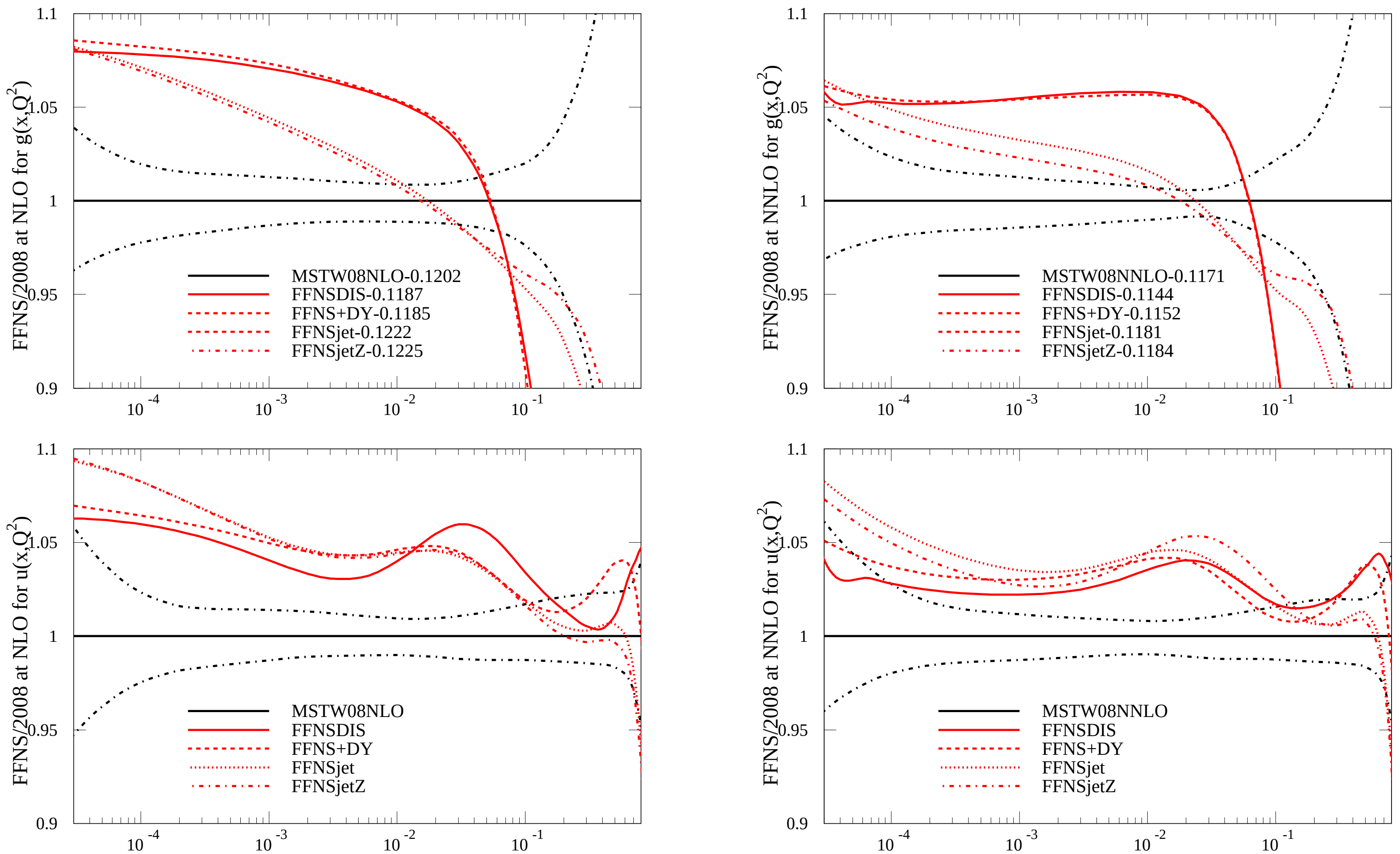
Performed a series of NLO fits using the FFNS scheme and NNLO with up to $\mathcal{O}(\alpha_S^2)$ heavy flavour coefficient functions. (Approximations to the $\mathcal{O}(\alpha_S^3)$ expressions change results very little).

Fit to only DIS and Drell-Yan data but also effectively fit to Tevatron Drell-Yan or Tevatron jet data, if necessary, in 5-flavour scheme as FFNS calculations do not exist.

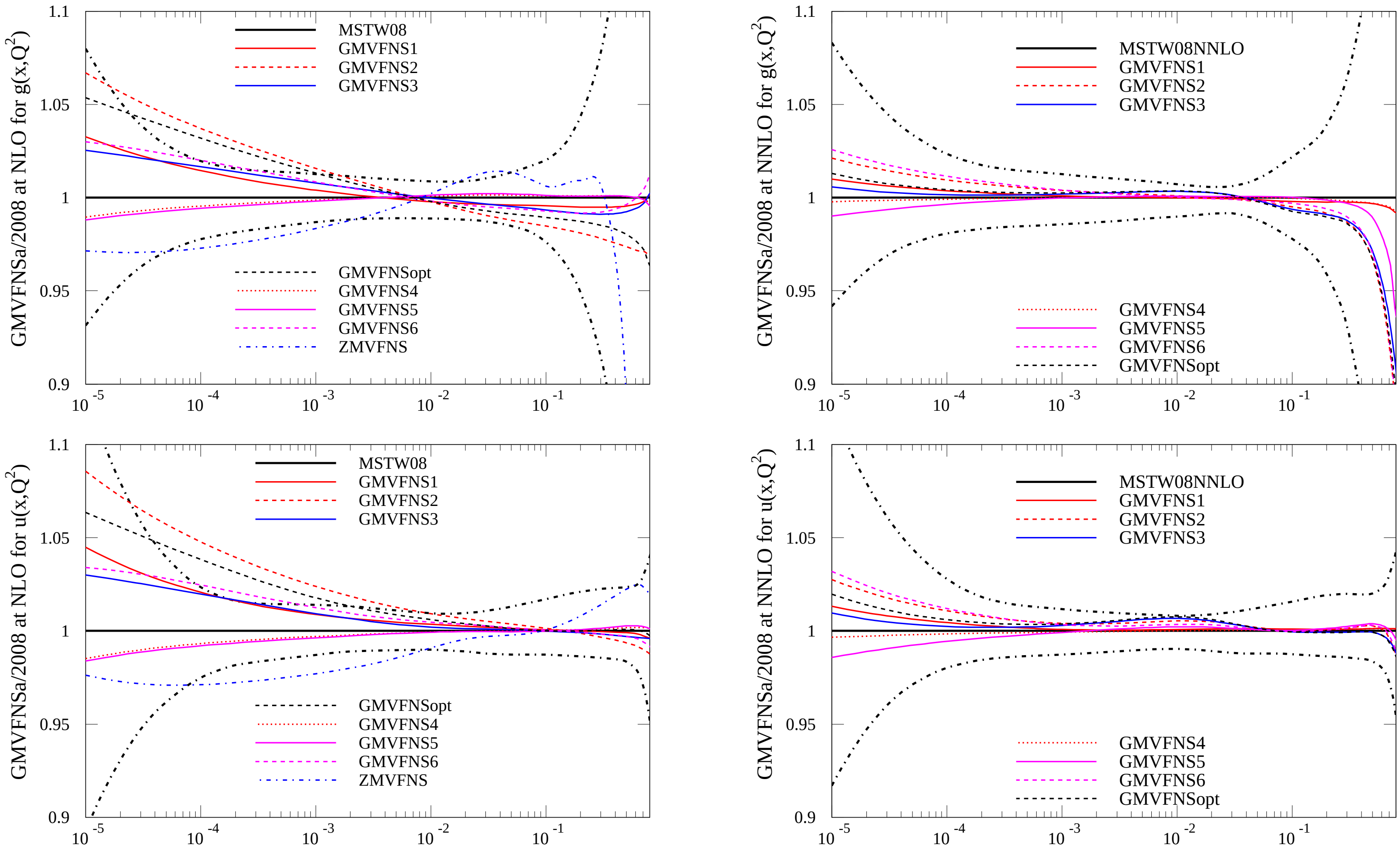
Fits to DIS and Drell-Yan data usually at least a few tens of units worse than MSTW08 to same data (even without refitting MSTW08 to restricted data sets). Often slightly better for $F_2^c(x, Q^2)$, but flatter in Q^2 for $x \sim 0.01$ for inclusive structure function.

As well as (usually) a worse fit to DIS and Drell-Yan data only, in FFNS the fit quality for the DIS and low-energy Drell Yan data deteriorates by in general ~ 50 units when all jet data is included as opposed to < 10 units when using a GM-VFNS.

PDFs evolved up to $Q^2 = 10,000\text{GeV}^2$ (using variable flavour evolution for consistent comparison) different in form to MSTW08. Similar differences found by NNPDF and older ZEUS fits.

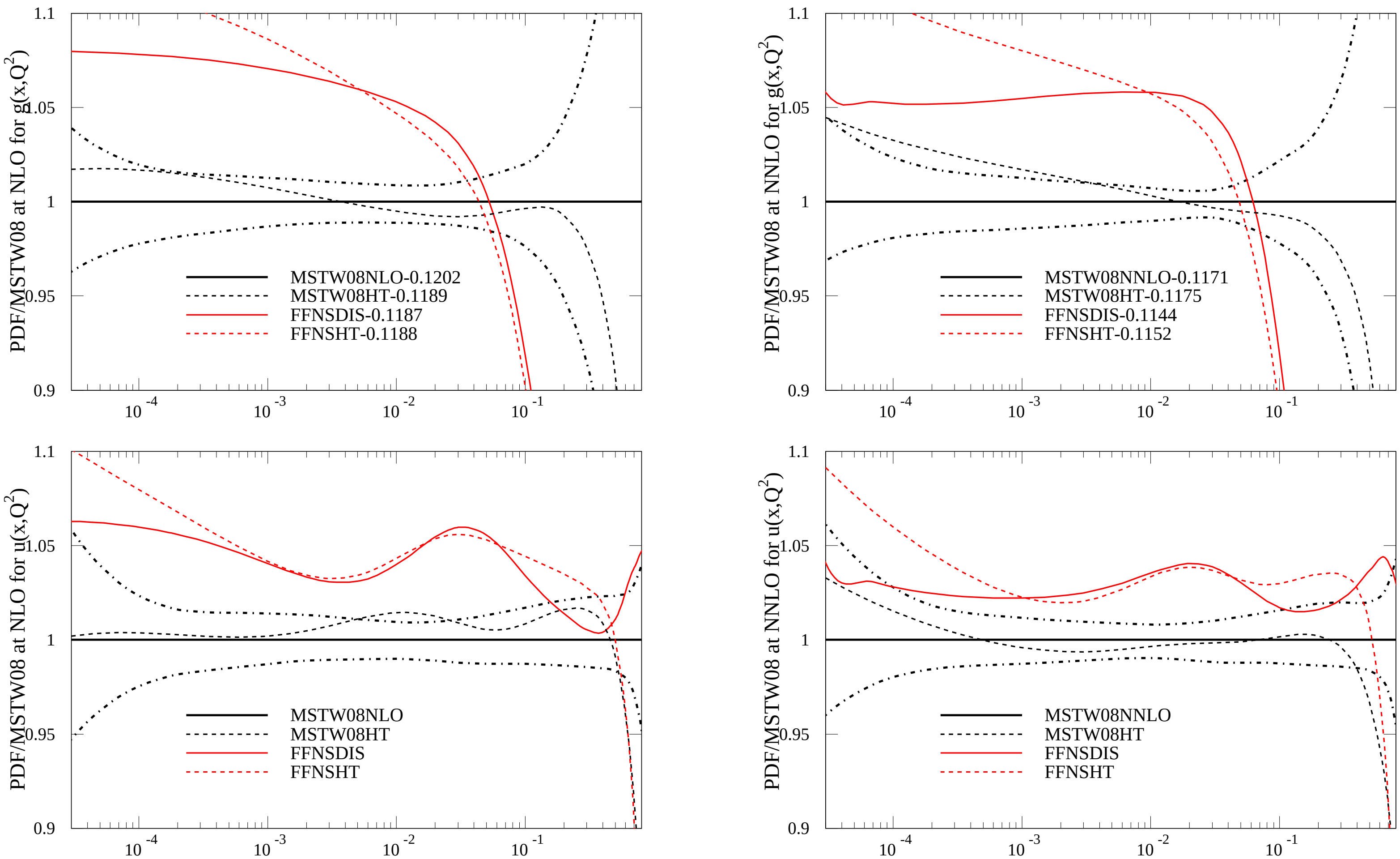


In contrast in standard MSTW2008 fit PDFs usually within uncertainties if Tevatron jet data left out.

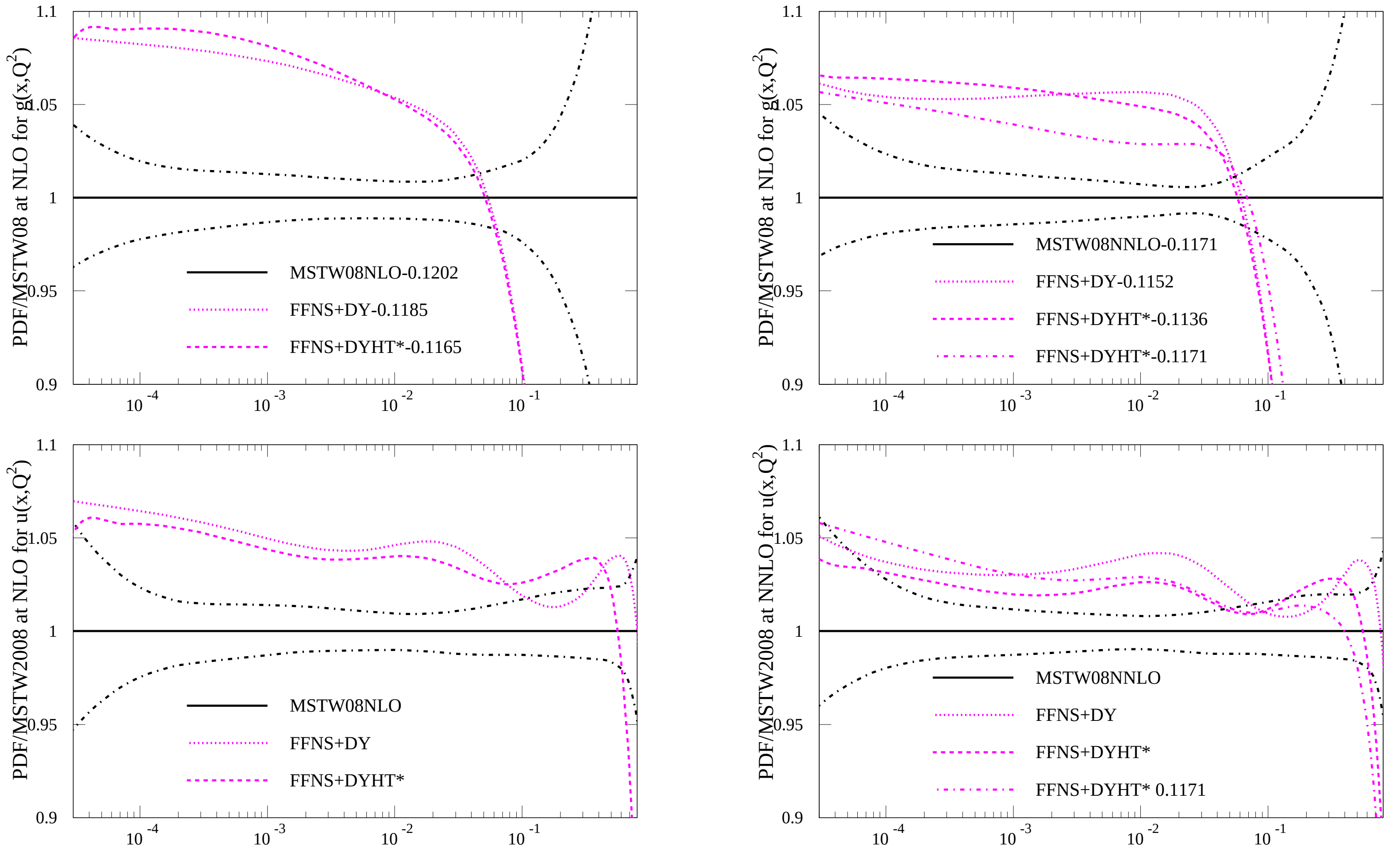


Using FFNS leads to much larger changes than any choice of GM-VFNS mainly due to fitting high-Q^2 DIS data.

Low Q^2 – Higher Twist.

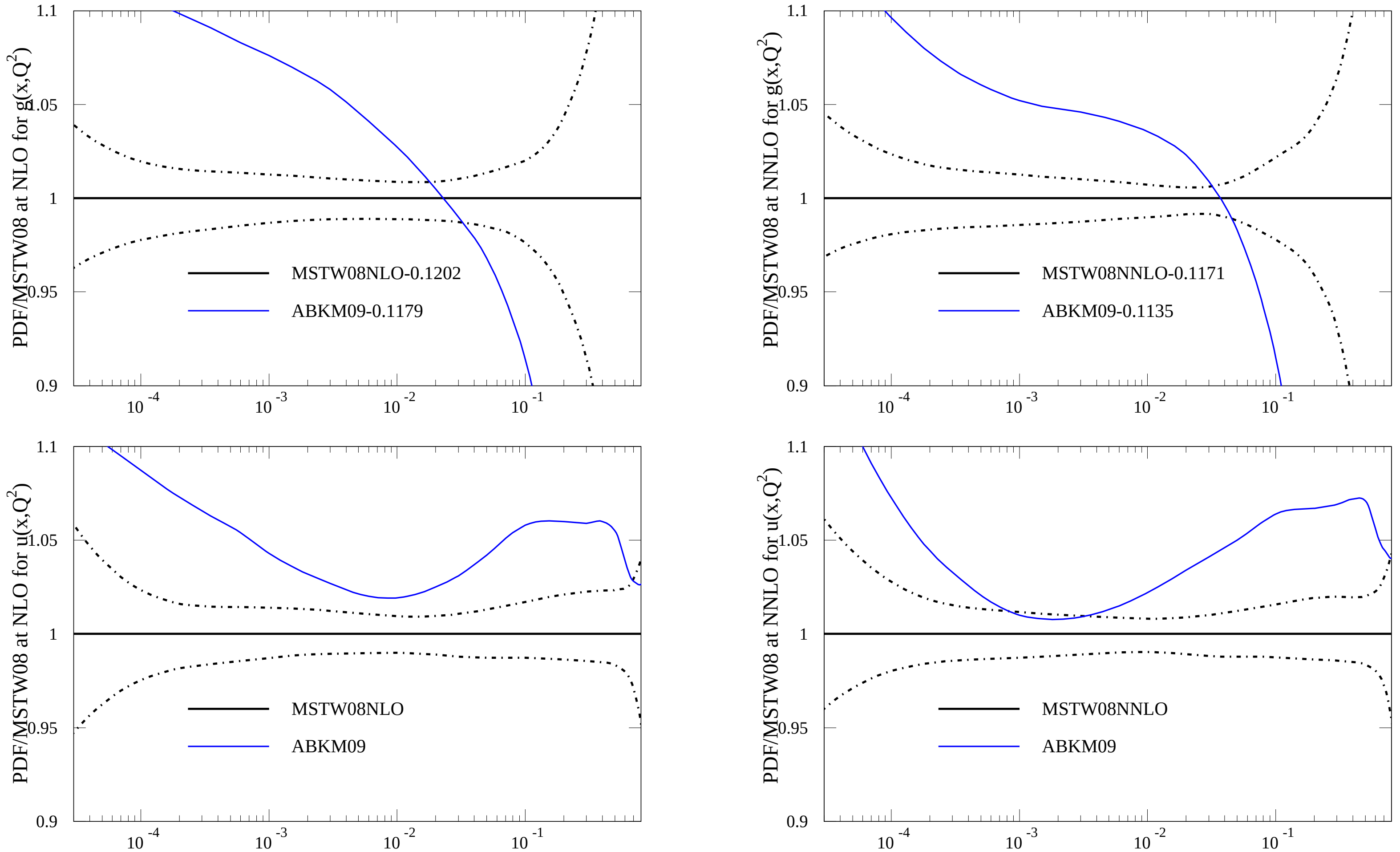


Not a big effect. Largely washes out quickly with Q^2. Similar effect using FFNS as for GM-VFNS.



Restricting higher twist from lowest x value and omitting nuclear target data (except dimuon for strangeness) tends to keep values of α_S lower by ~ 0.02. Fixing α_S reduces effect on gluon. Similar for NNPDF.

Explains some PDF differences? MSTW FFNS ratios and ABKM ratios.



General trend is very similar to fits on previous page.

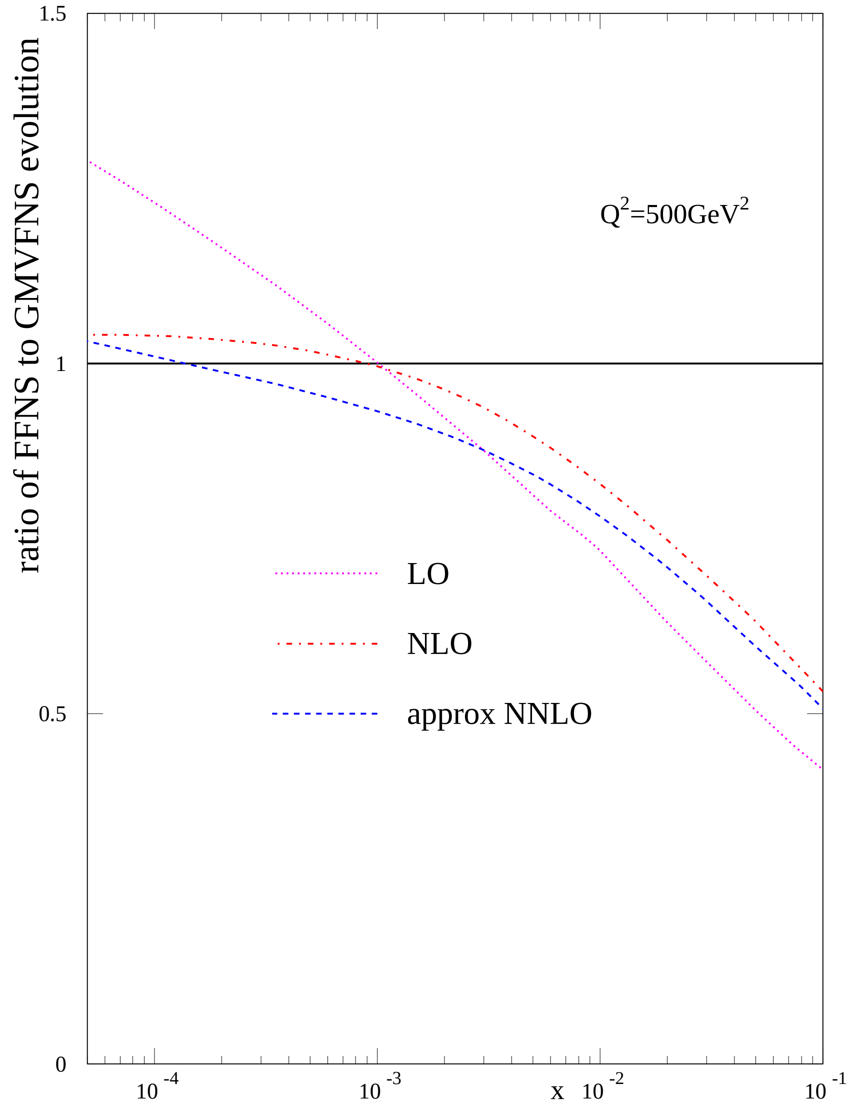
Understanding the differences between FFNS and GM-VFNS

Consider comparison of evolution at high Q^2 where $\mathcal{O}(m_c^2/Q^2)$ contributions negligible.

General form of difference in evolution of F_2^c at $Q^2 = 500\mathrm{GeV}^2$.

Can we understand this?

Look at evolution of F_2^c which to leading $\ln(Q^2/m_c^2)$ is LO PDF evolution in GM-VFNS.



Start at LO where (setting all scales as Q^2)

$$F_2^{c,1,FF} = \alpha_S \ln(\frac{Q^2}{m_c^2}) p^0_{qg} \otimes g + \mathcal{O}(\alpha_S \cdot g) \equiv \alpha_S A^{1,1}_{Hg} \otimes g + \mathcal{O}(\alpha_S \cdot g).$$

Calculating rate of change of evolution

$$\frac{d\, F_2^{c,1,FF}}{d \ln Q^2} = \alpha_S p^0_{qg} \otimes g + \ln(\frac{Q^2}{m_c^2}) \frac{d\,(\alpha_S p^0_{qg} \otimes g)}{d \ln Q^2}.$$

At leading-log in GM-VFNS where $F_2^{c,1,VF} = (c + \bar{c}) = c^+$

$$\frac{d\, c^+}{d \ln Q^2} = \alpha_S\, p^0_{qg} \otimes g + \alpha_S\, p^0_{qq} \otimes c^+$$

where

$$c^+ \equiv \alpha_S \ln(\frac{Q^2}{m_c^2}) p^0_{qg} \otimes g + \cdots \equiv \alpha_S A^{1,1}_{Hg} \otimes g + \cdots$$

so the second term is formally $\mathcal{O}(\alpha_S^2 \ln(\frac{Q^2}{m_c^2}))$.

The first two terms are of the form $\alpha_S \ln(Q^2/m_c^2)$ and are equivalent, but the difference between the two evolutions at LO is

$$\frac{d\,(F_2^{c,1,VF} - F_2^{c,1,FF})}{d\ln Q^2} = \alpha_S^2 \ln(\frac{Q^2}{m_c^2})\left(p^0_{qg}\otimes p^0_{qq}\otimes g - \frac{d\,(\alpha_S p^0_{qg}\otimes g)}{d\ln Q^2}\right) + \cdots$$

$$\equiv \alpha_S^2 \ln(\frac{Q^2}{m_c^2}) p^0_{qg}\otimes(p^0_{qq} + \beta_0 - p^0_{gg})\otimes g + \cdots$$

where $\beta_0 = \frac{9}{4\pi}$ and the effect of p^0_{gg} is negative at high x and positive at small x and that of p^0_{gg} is negative at high x, but smaller than of p^0_{gg}.

Hence the difference is positive and large at high x and large and negative at small x, exactly as observed.

Moreover, this difference can only be eliminated at NLO by defining the leading-log term in the NLO FFNS expression precisely to provide cancellation, i.e.

$$F_2^{c,2,FF} = \alpha_S^2 A^{2,2}_{Hg}\otimes g = \frac{1}{2}\alpha_S^2 \ln^2(\frac{Q^2}{m_c^2}) p^0_{qg}\otimes(p^0_{qq}+\beta_0-p^0_{gg})\otimes g + \mathcal{O}(\alpha_S^2 \ln(\frac{Q^2}{m_c^2})).$$

up to corrections involving quark mixing in evolution and possible sub-dominant scheme-dependent terms.

Looking at evolution at NLO all previous $\mathcal{O}(\alpha_S^2 \ln(\frac{Q^2}{m_c^2}))$ terms cancel between GM-VFNS and FFNS.

However, the derivative of $F_2^{c,2,FF}$ contains a contribution

$$\frac{1}{2}\ln^2(\frac{Q^2}{m_c^2})\frac{d\left(\alpha_S^2 p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes g\right)}{d\,\ln Q^2}$$

which does not cancel. This leads to

$$\frac{1}{2}\alpha_S^3 \ln^2(\frac{Q^2}{m_c^2}) p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes (p_{qq}^0 + 2\beta_0 - p_{gg}^0) \otimes g + \cdots .$$

The additional factor of $(p_{qq}^0 + 2\beta_0 - p_{gg}^0)$ is large, positive at high x and negative at small x, but not until smaller x than previously. Therefore, the term which convolutes the gluon is large and positive at high x, negative for a range of smaller x and positive for extremely small x. Explains behaviour correctly.

Moreover, to cancel this term at NNLO the dominant part of $F_2^{c,2,FF}$ at leading-log is (up to quark-mixing and scheme-dependent terms)

$$\alpha_S^3 A_{Hg}^{3,3} \otimes g = \frac{1}{6}\alpha_S^3 \ln^3(\frac{Q^2}{m_c^2}) p_{qg}^0 \otimes (p_{qq}^0 + \beta_0 - p_{gg}^0) \otimes (p_{qq}^0 + 2\beta_0 - p_{gg}^0) \otimes g.$$

Repeating the argument we find that at NNLO the dominant high-Q^2 uncancelled term between GM-VFNS and FFNS is

$$\frac{1}{6}\alpha_S^4 \ln^3(\frac{Q^2}{m_c^2}) p^0_{qg} \otimes (p^0_{qq} + \beta_0 - p^0_{gg}) \otimes (p^0_{qq} + 2\beta_0 - p^0_{gg}) \otimes (p^0_{qq} + 3\beta_0 - p^0_{gg}) \otimes g.$$

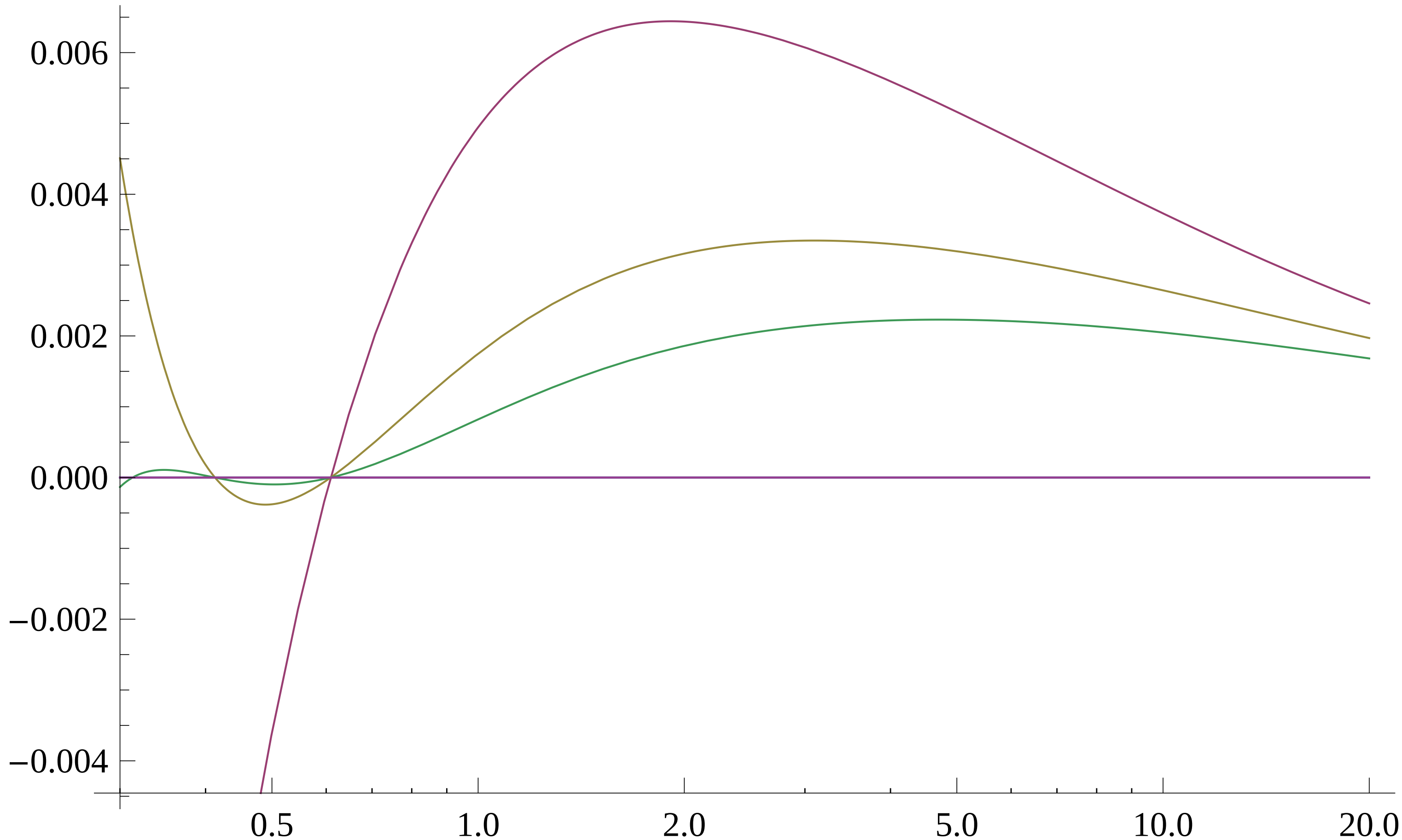
This remains large and positive at high x and changes sign twice but stays small at smaller x until becoming negative at tiny x.

Again explains behaviour correctly.

Can be generalised to higher orders. Similar in some sense to results from expression in Maltoni, Ridolfi and Ubiali, JHEP 1207 (2012) 022 for bottom quark, but this neglected evolution of gluon and hence p^0_{gg} terms – actually the dominant effect at lowish orders.

Can look at the effect of this dominant high-Q^2 difference between GM-VFNS and FFNS in more detail.

Moments of the dominant difference terms at LO, NLO and NNLO. LO in purple, NLO in brown and NNLO in green.

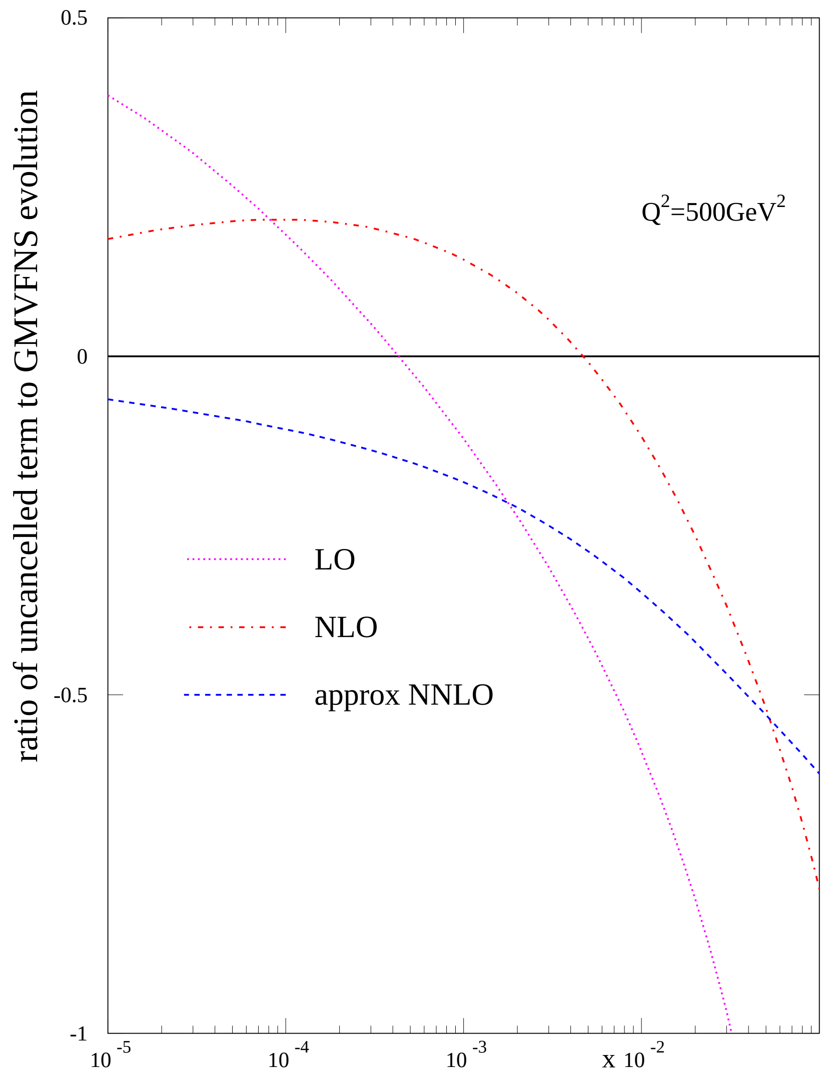


Part of the slow convergence is the decrease in α_S with increasing order.

Fractional effect of dominant difference term between GM-VFNS and FFNS evolution at the various orders.

Precise form of the effect depends on form of gluon. Much steeper at LO than at NLO or NNLO.

Describes the general form of the difference in evolution between GM-VFNS and FFNS very well (though precise details depend on sub-dominant terms.



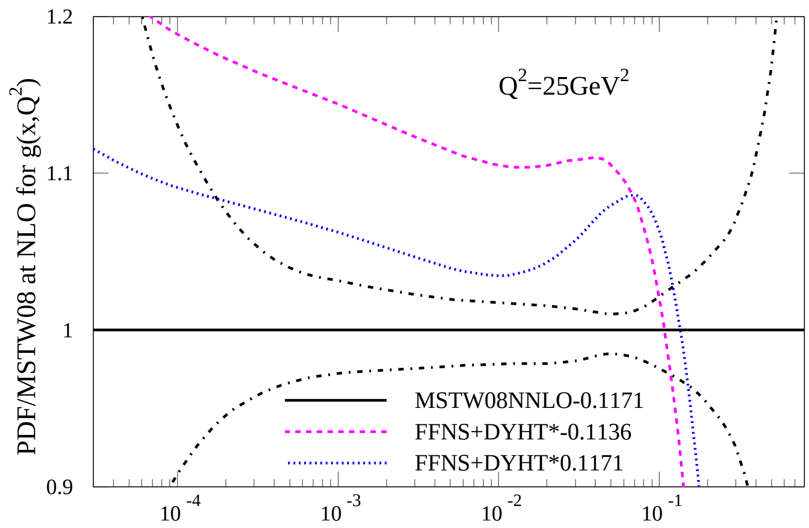
Why is α_S lower in FFNS?

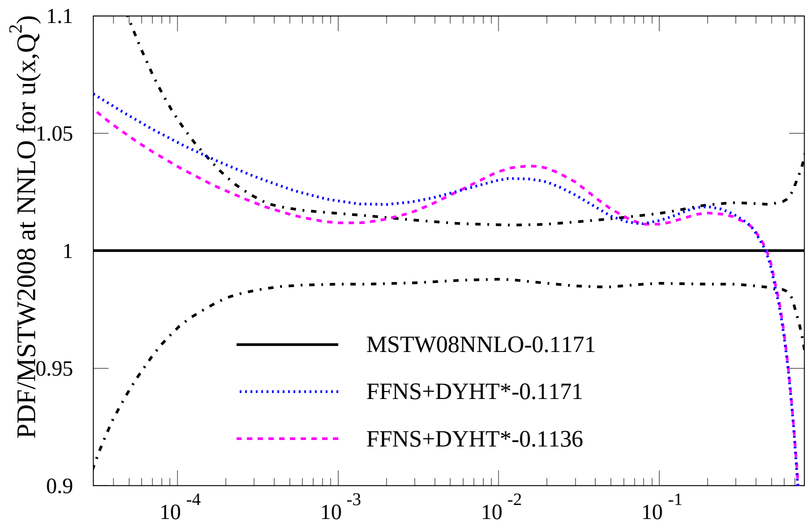
FFNS fit 8 units worse if $\alpha_S(M_Z^2) = 0.1171$. HERA data better, fixed target worse.

Comparing schemes, look at parton ratios at lower Q^2 where evolution must match data, and respective $\alpha_S(M_Z^2)$ values are 0.1171 and 0.1136.

Gluon needs to be bigger at $x \sim 0.01\text{-}0.1$ – smaller at high x – to fit data. Feeds to lower x at higher Q^2.

Inverse correlation between high-x gluon and α_S. Without high-x gluon quark evolution too quick. Need lower α_S.





Conclusions

Ongoing updates on PDFs in MSTW framework. Combination of many previous individual investigations.Combined HERA charm data gives more consistent extractions of m_c, especially at NNLO.

No particularly major changes beyond those in MSTWCPdeut (Eur.Phys.J. C73 (2013) 2318).

Slight improvement in agreement with predictions for LHC data. These data so far have little further effect on PDFs.

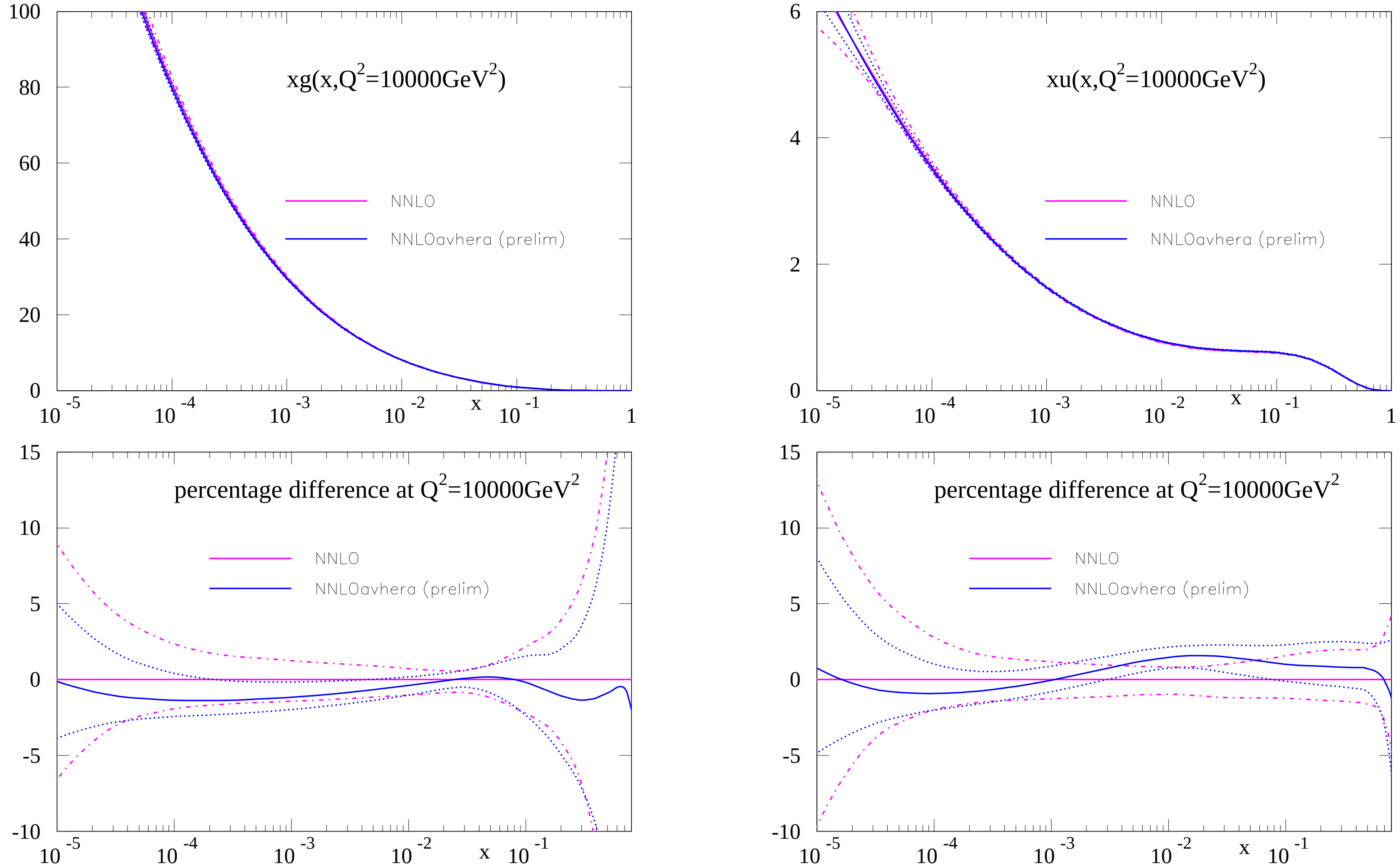
Performing fits using an FFNS leads to worse fits to DIS and low energy Drell-Yan data than GM-VFNS, and much more tension with jet data.

Light quarks (evolved to high Q^2 in variable flavour number scheme) are automatically larger in most regions for FFNS than for GM-VFNS.

The gluon is smaller at high x and larger at small x in FFNS, and $\alpha_S(M_Z^2)$ smaller – e.g. 0.1136 as opposed to 0.1171.

Difference in GM-VFNS and FFNS evolution at high Q^2 slow to converge. Can understand this from behaviour of dominant term.

Back-up



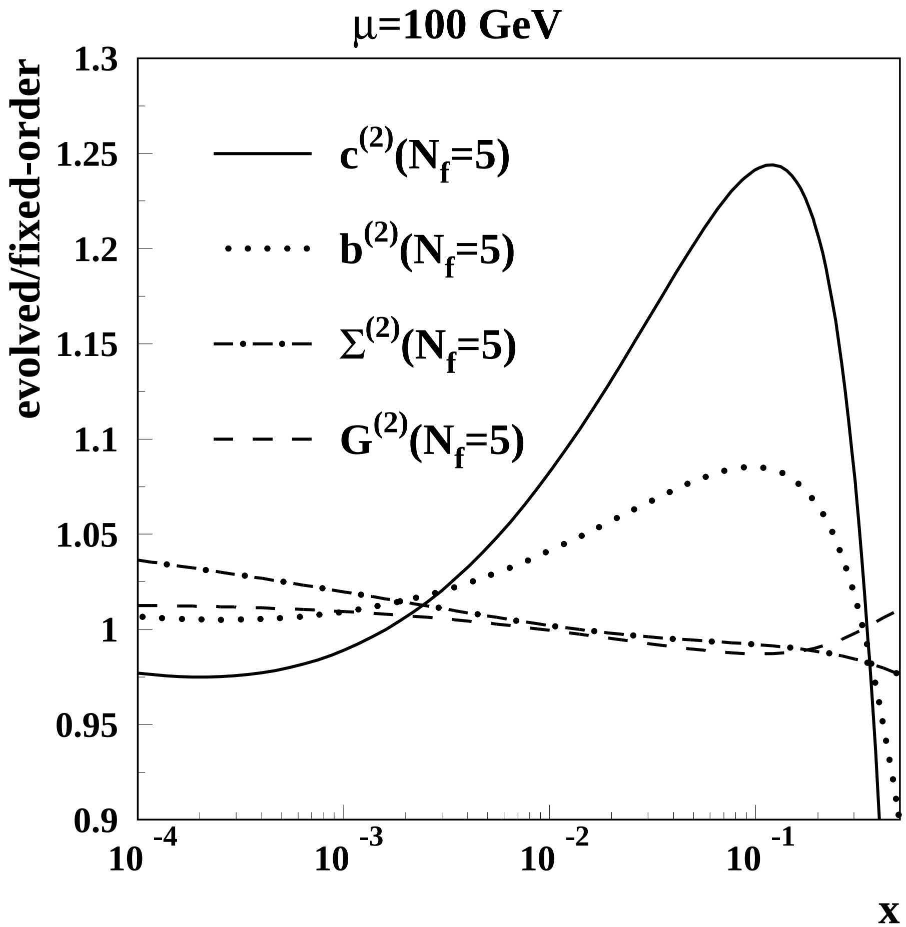
Change in MSTW2008 NNLO PDFs when fitting HERA combined data.

Comparison to LHC data.

Use APPLGrid or FastNLO at NLO (Ben Watt) and correlated errors treated as in the formula,

$$\chi^2 = \sum_{i=1}^{N_{\rm pts.}} \left(\frac{\hat{D}_i - T_i}{\sigma_i^{\rm uncorr.}} \right)^2 + \sum_{k=1}^{N_{\rm corr.}} r_k^2,$$

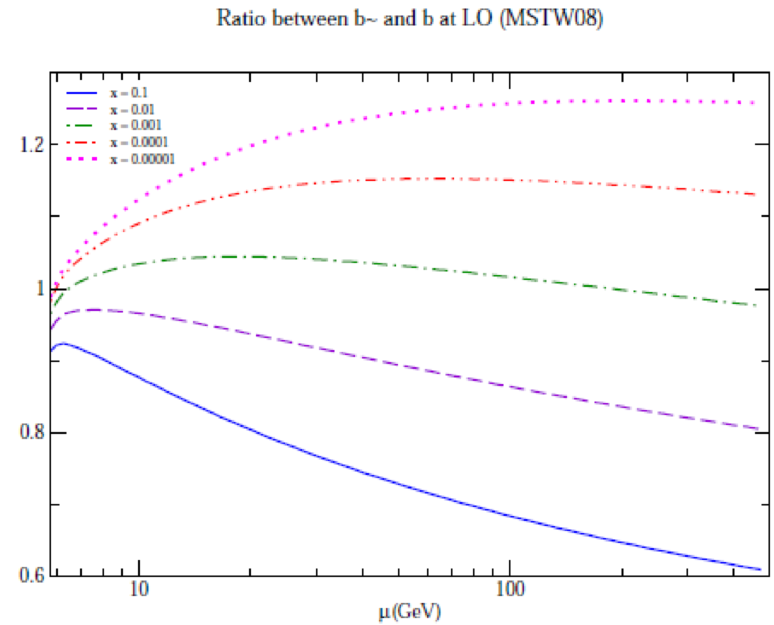
where $\hat{D}_i \equiv D_i - \sum_{k=1}^{N_{\rm corr.}} r_k \, \sigma_{k,i}^{\rm corr.} D_i$ are the data points allowed to shift by the systematic errors in order to give the best fit, and $\sigma_{k,i}^{\rm corr.}$ is a fractional uncertainty. Normalisation is treated as the other correlated uncertainties.

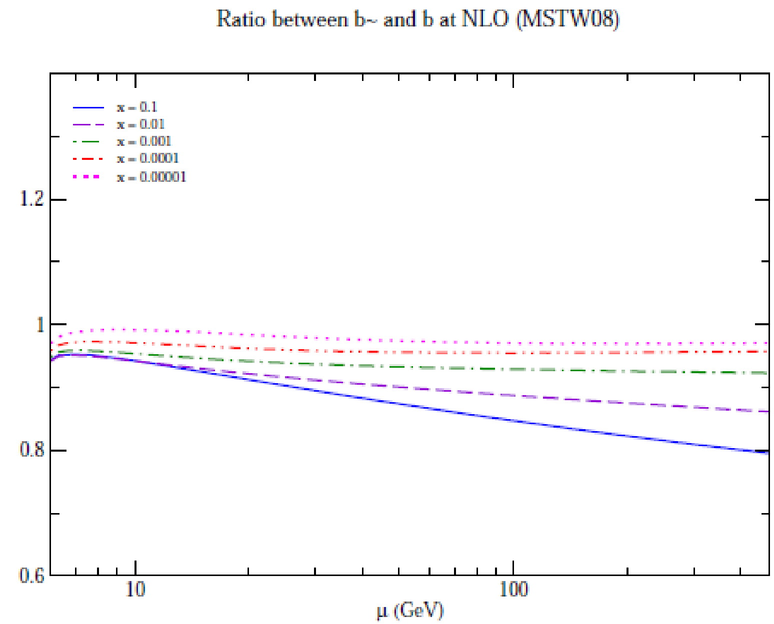


Results for $F_2^c(x, Q^2)$ in GM-VFNS compared to those for FFNS similar to results for PDFs by Alekhin *et al.* in Phys.Rev. D81 (2010) 014032 comparing NNLO evolution to the fixed order result up to $\mathcal{O}(\alpha_S^2)$. Details depend on PDF set and $\alpha_S(M_Z^2)$ value used.

Also verified in evolution of bottom quark (Maltoni, *et al.*, JHEP 1207 (2012) 022).

In this case $\ln(Q^2/m_b^2)$ rather smaller.





Low Q^2 – Higher Twist.

Potentially large corrections at low Q^2 and particularly low W^2. Usual MSTW cuts for

$Q^2_{\rm cut}$ - $2{\rm GeV}^2$

$W^2_{\rm cut}$ - $15{\rm GeV}^2$

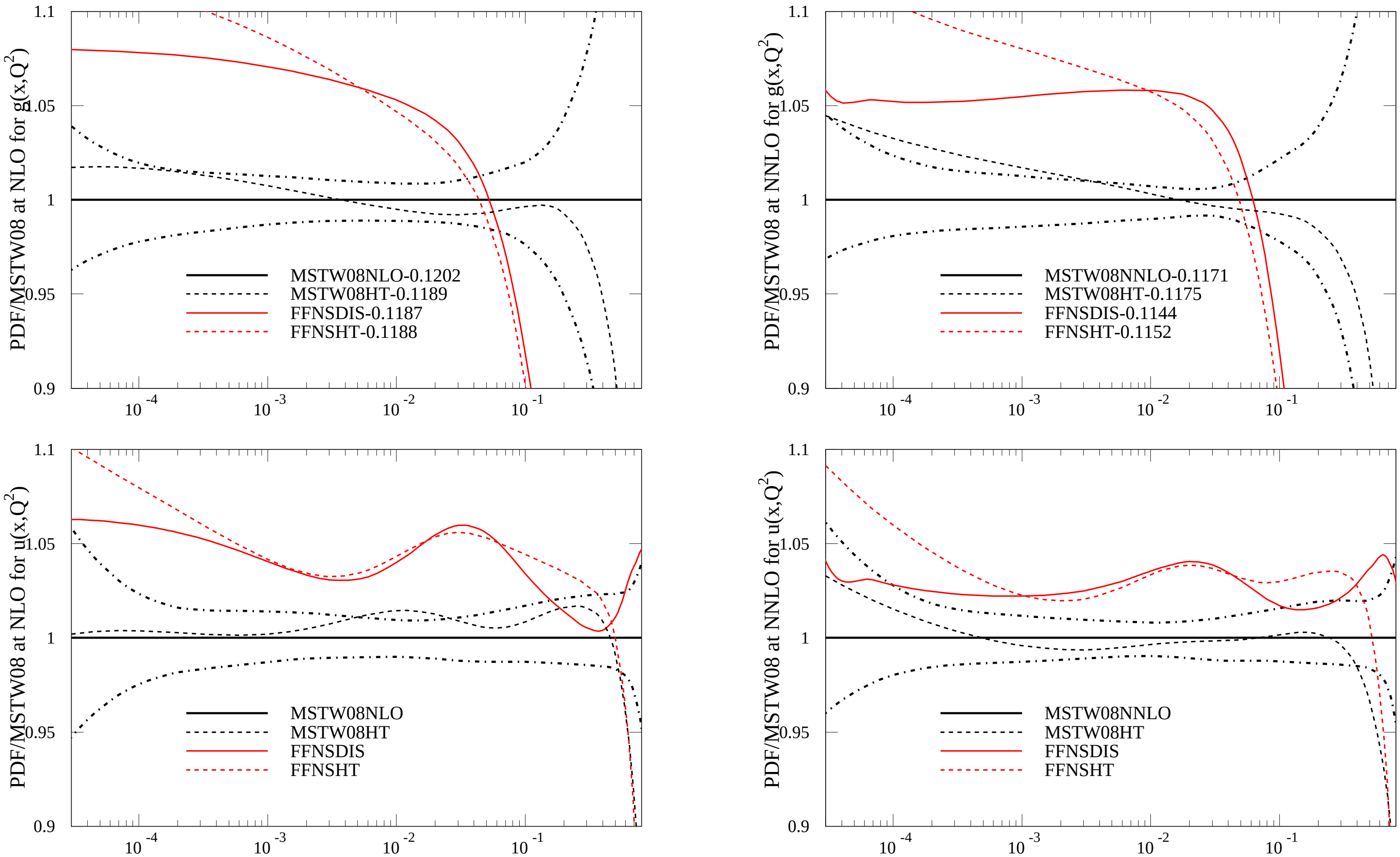
Have tried raising Q^2 cut to $5{\rm GeV}^2$ and $10{\rm GeV}^2$ and W^2 to $20{\rm GeV}^2$. Not much effect on PDFs or α_S.

Can also lower $W^2_{\rm cut}$ to $5{\rm GeV}^2$ and try parameterising higher twist contributions by

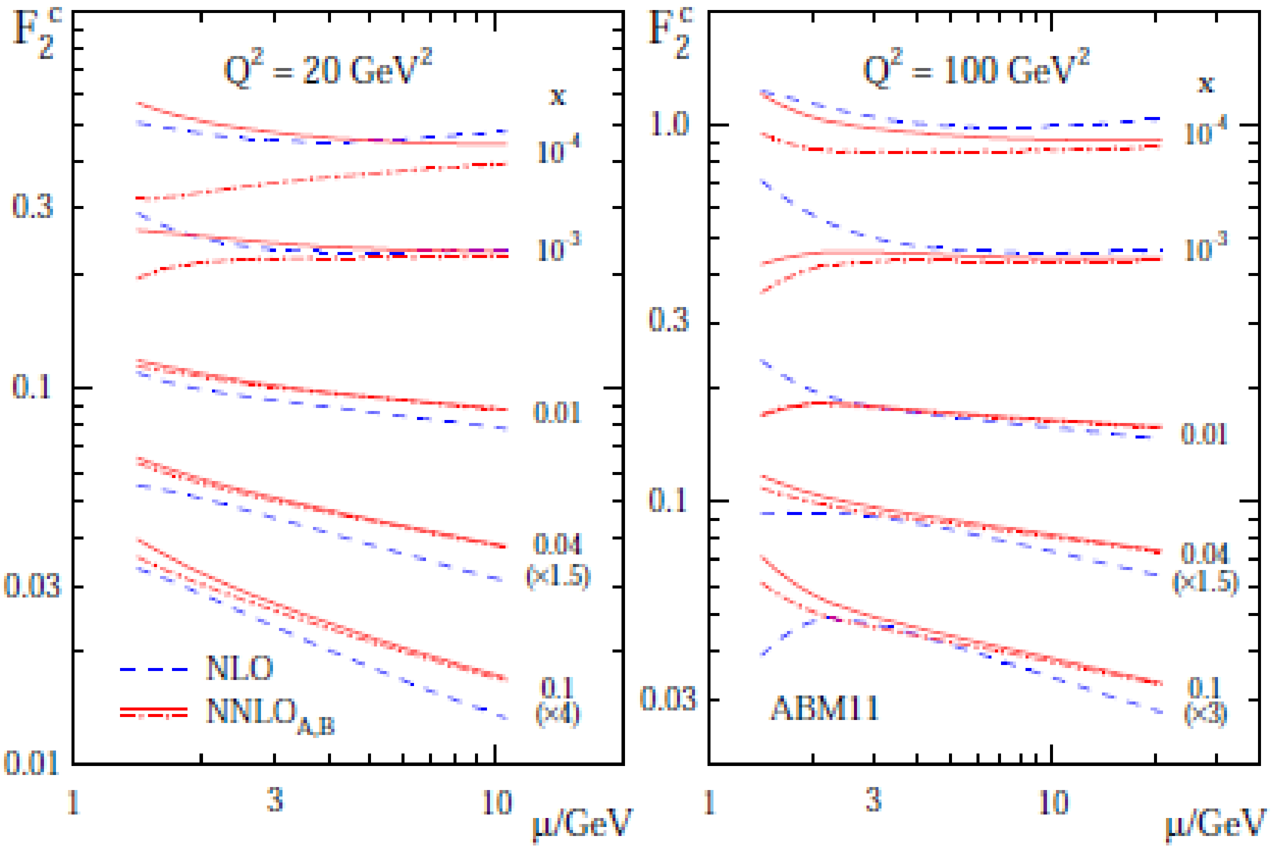
$$F_i^{\rm HT}(x,Q^2) = F_i^{\rm LT}(x,Q^2)\left(1 + \frac{D_i(x)}{Q^2}\right)$$

where i spans bins of x from $x = 0.8 - 0.9$ down to $x = 0 - 0.0005$.

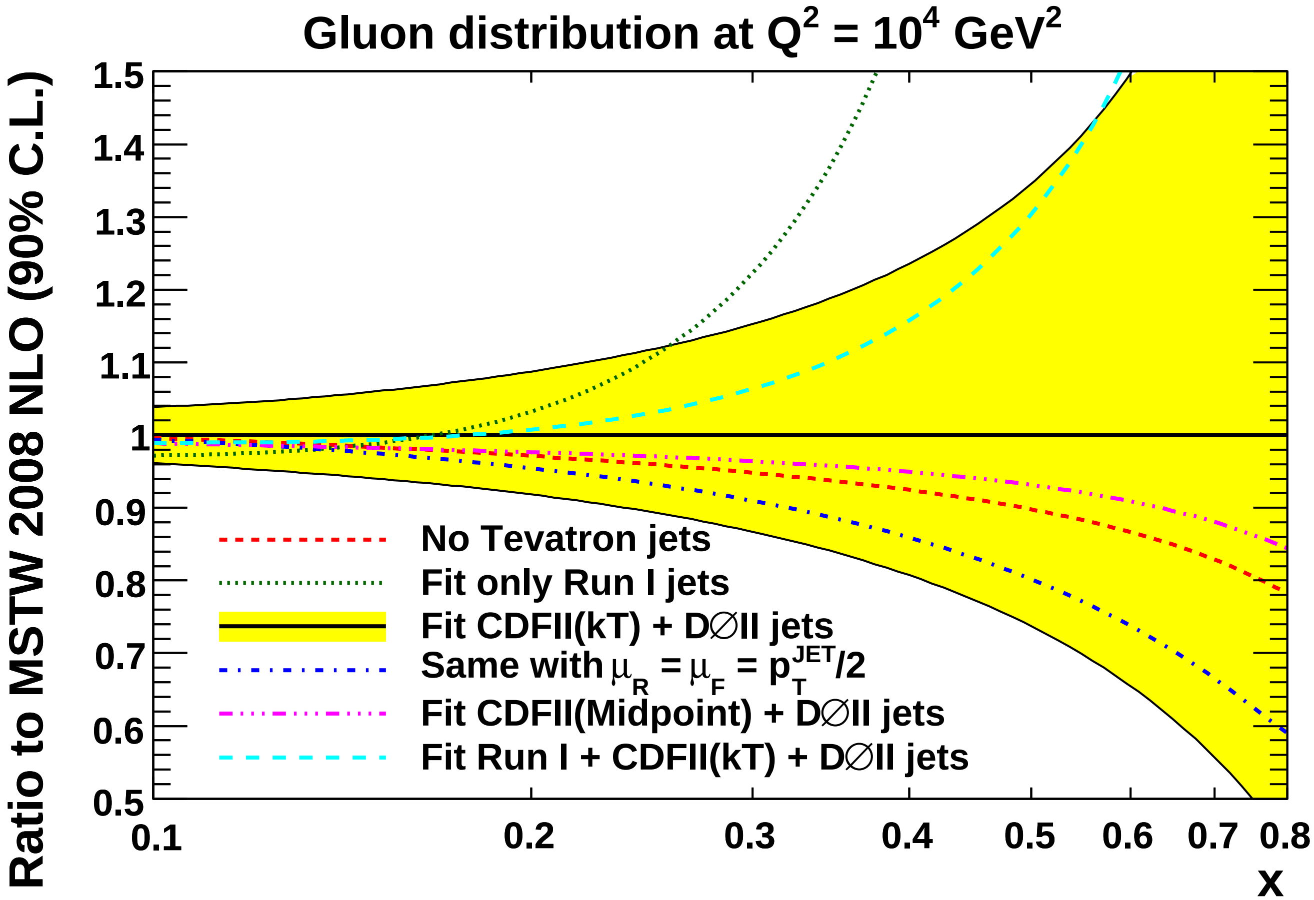
Previously no evidence for much higher twist except at low W^2.



Now more evidence for positive contribution also at very low x. Leads to lower input quarks, more gluon for evolution. Largely washes out quickly with Q^2. Similar effect using FFNS as for GM-VFNS.

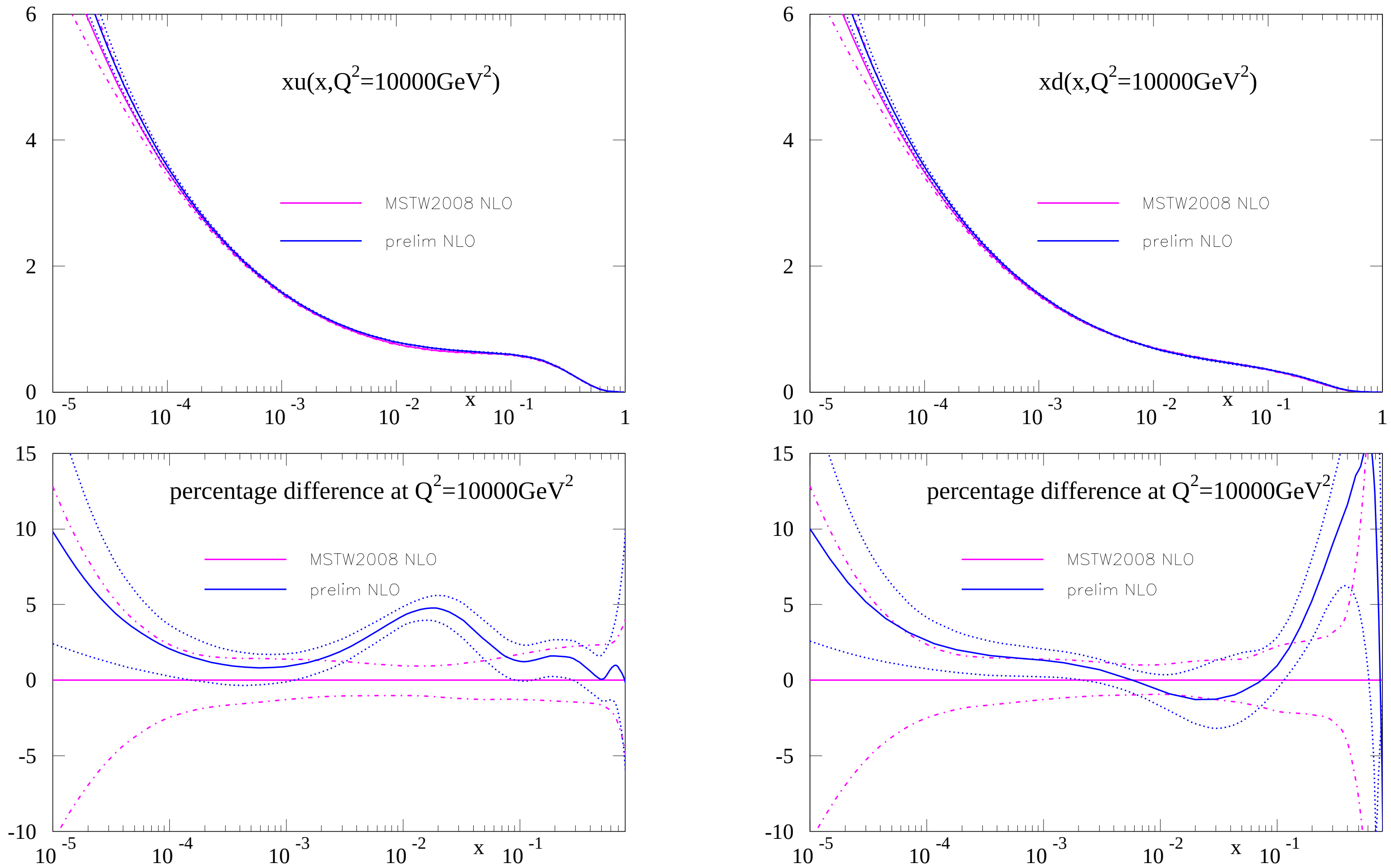


Scale dependence of $F_2^c(x, Q^2)$ using FFNS at NLO and approx. NNLO (Kawamura *et al.*).



In contrast in MSTW2008 fit central gluon hardly changed if Tevatron jet data left out, and only slight further rearrangement of quark flavours if Drell-Yan data left out.

Main effect loss of tight constraint on $\alpha_S(M_Z^2)$. Much the same at NNLO. Similar results from various other groups.



Change in NLO PDFs from all updates.

The GM-VFNS can be defined by demanding equivalence of the n_f light flavour and n_f+1 light flavour descriptions at all orders – above transition point $n_f \rightarrow n_f+1$

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$
$$\equiv C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF,n_f}(Q^2/m_H^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives

$$C_{2,Hg}^{FF,n_f,(1)}(\frac{Q^2}{m_H^2}) = C_{2,HH}^{VF,n_f+1,(0)}(\frac{Q^2}{m_H^2}) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,Hg}^{VF,n_f+1,(1)}(\frac{Q^2}{m_H^2}),$$

The VFNS coefficient functions tend to the $m=0$ limits as $Q^2/m_H^2 \rightarrow \infty$.

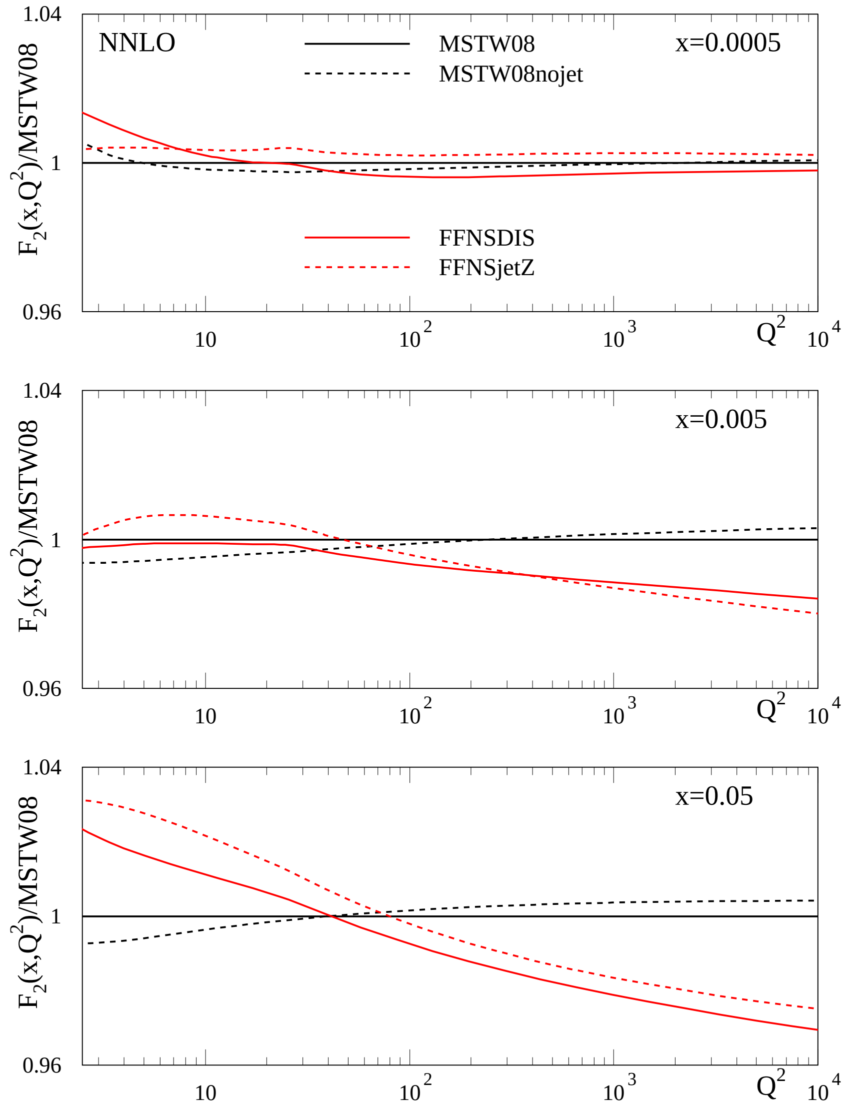
However, $C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in this limit.

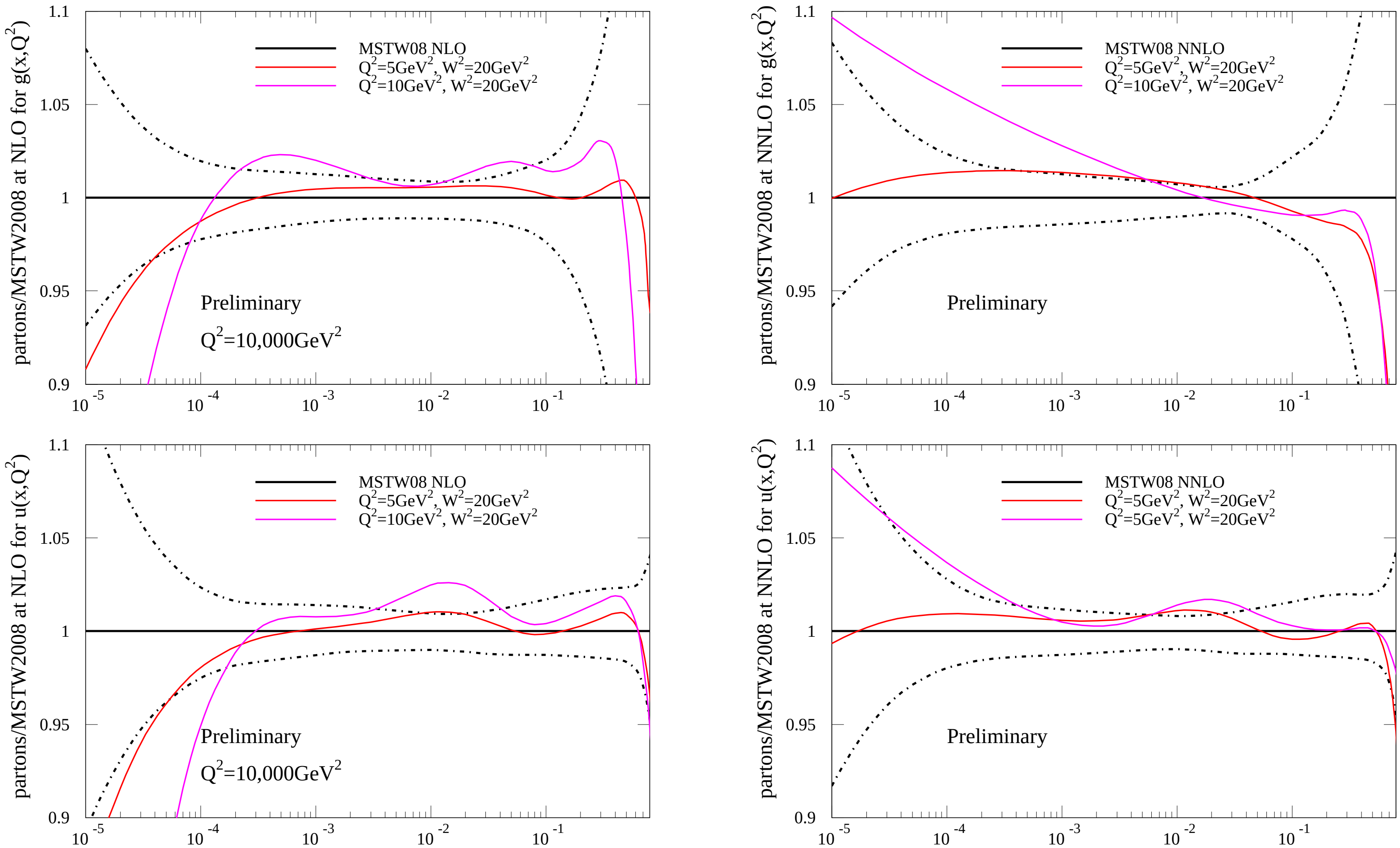
Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

The results for $F_2(x,Q^2)$ when refits are performed.

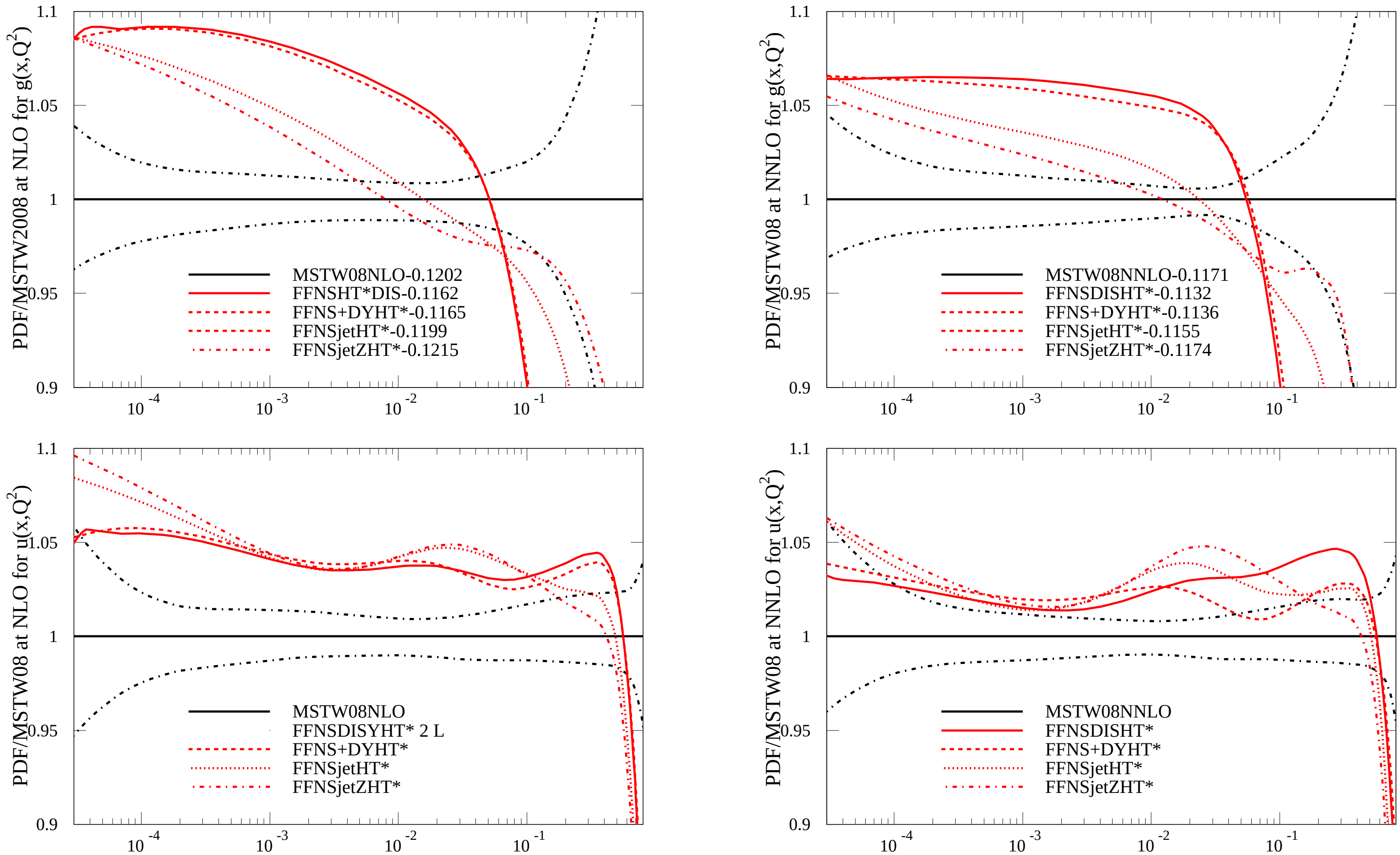
As seen very little change when using GM-VFNS with no jets.

Much more tension and worse fits for FFNS.



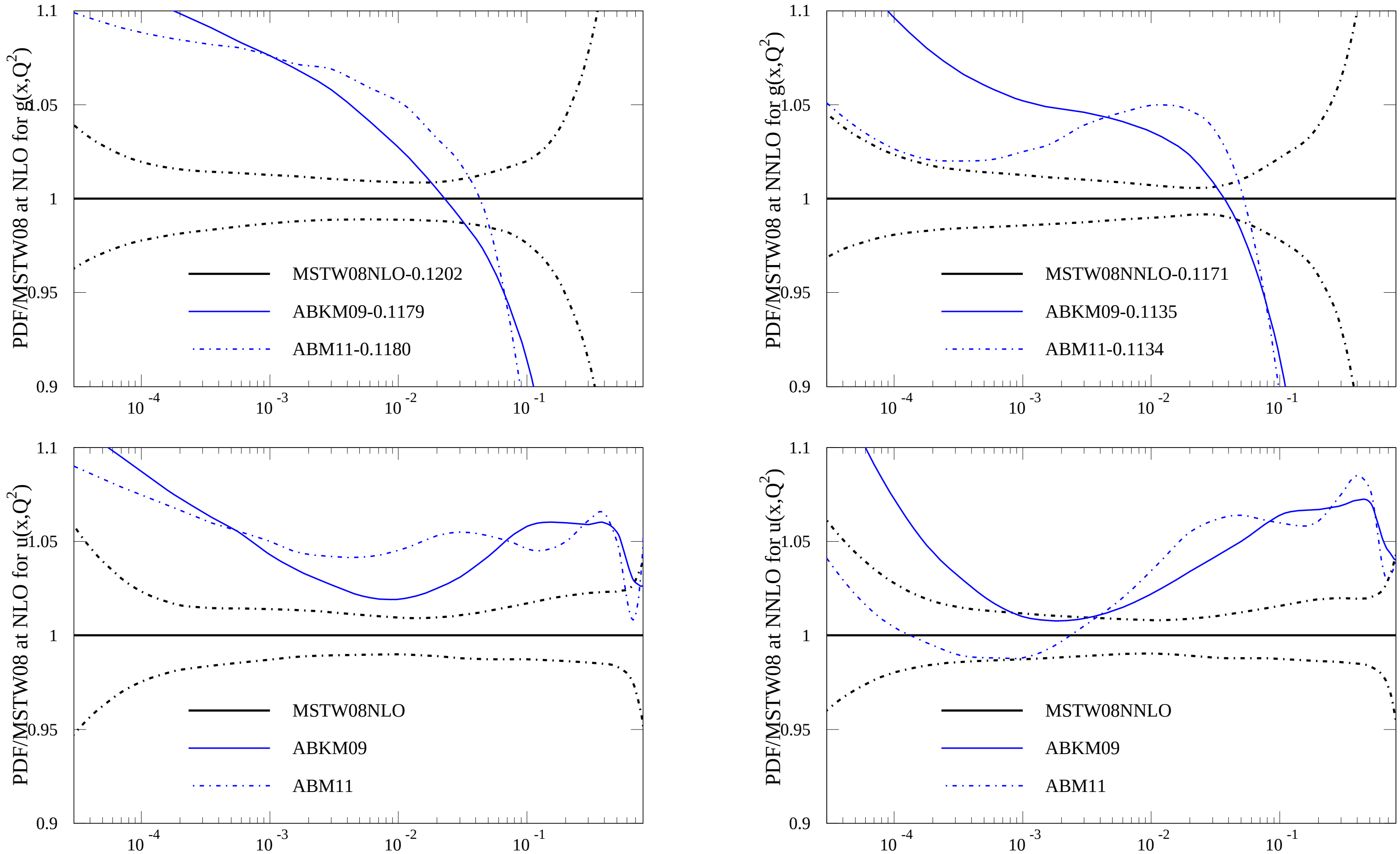


Similar to effect of higher twist, particularly at NNLO. Remember lose data at lowest x.



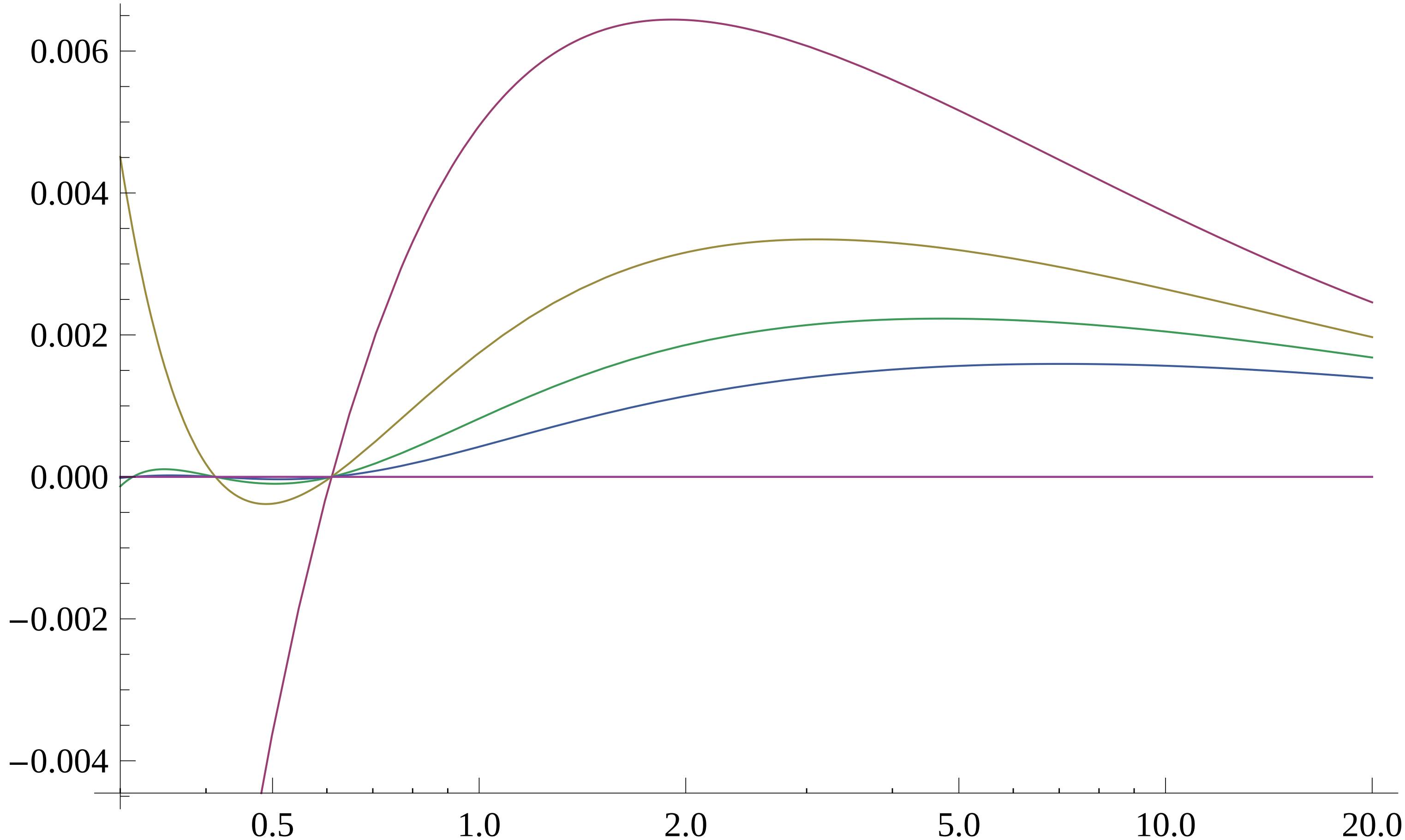
Restricting higher twist from lowest x value and omitting nuclear target data (except dimuon for strangeness). Same trends as for standard fits but slightly lower α_S

Explains some PDF differences? MSTW FFNS ratios and ABKM ratios.



Better to compare to ABKM09 as mass scheme and data fit are more similar.

Moments of the dominant difference terms at LO, NLO and NNLO, and also the term which would be dominant at NNNLO.



LO in purple, NLO in brown, NNLO in green and NNLO in blue.