A model for high energy rho meson leptoproduction based on collinear factorization and dipole models

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in collaboration with

L. Szymanowski, S. Wallon, Nucl. Phys. B **867** (2013) 19-60, ArXiv:1302.1766

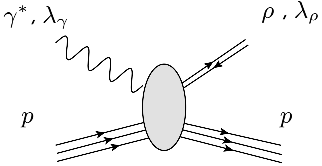
I. V. Anikin, D .Yu. Ivanov, B. Pire, L. Szymanowski, S. Wallon, PhysRevD.84.054004

Introduction

Helicity amplitudes of the diffractive leptoproduction of the ρ meson

- Helicity Amplitudes $T_{\lambda_\rho \lambda_\gamma}$

$$T_{\lambda_\rho \lambda_\gamma} =$$



Examples :

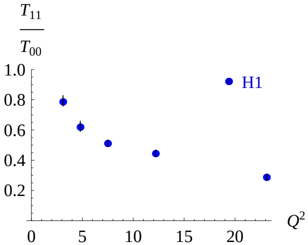
$$\begin{aligned} T_{00} &\iff \gamma_L^* \; p \to \rho_L \; p \\ T_{11} &\iff \gamma_T^* \; p \to \rho_T \; p \end{aligned}$$

- Perturbative Regge Limit :
    - Regge Limit : $s = W^2 >> Q^2, |t|\,, M^2_{\text{hadron}}$
    - Hard scale : $Q >> \Lambda_{QCD}$

Introduction

Experimental data of helicity amplitudes at high energy

- Helicity amplitudes $T_{\lambda_\rho \lambda_\gamma}$: $\gamma^*_{\lambda_\gamma} + p \to \rho_{\lambda_\rho} + p$
- H1 and ZEUS data for Helicity Amplitudes at HERA:



Power counting:

$$\frac{T_{11}}{T_{00}} \propto \frac{1}{Q}$$

$$\frac{T_{01}}{T_{00}} \propto \frac{\sqrt{|t|}}{Q}$$

$$\frac{T_{10}}{T_{00}} \propto \frac{\sqrt{|t|}}{Q^2}$$

S. Chekanov et al. (2007), F.D Aaron et al. (2010)

- Kinematics
  - High energy in the center of mass $30\,\text{GeV} < W < 180\,\text{GeV}$
  - Photon Virtuality $2.5\,\text{GeV}^2 < Q^2 < 60\,\text{GeV}^2$
  - $|t| < 1\,\text{GeV}^2$

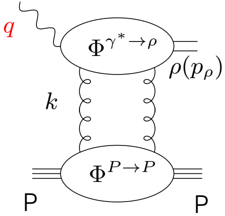
$$\Rightarrow s_{\gamma^* p} = W^2 >> Q^2 >> \Lambda^2_{QCD}$$

Introduction

A Theoretical approach within k_T factorisation

k_T factorisation

- Amplitudes with gluons exchange in $t-$channel dominate at large s ($s = W^2$)



Born order: 2 t-channel gluons

- $T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\underline{k})\ \Phi^{P \to P}(-\underline{k})$

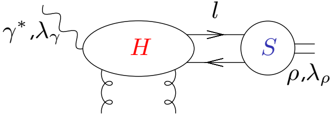
Introduction

A theoretical approach of the $\Phi^{\gamma^* \to \rho}$ impact factor up to twist 3

Impact factors $\Phi^{\gamma^* \to \rho}$

- $\Phi^{\gamma^* \to \rho}$: collinear factorisation

$Q^2 >> \Lambda^2_{QCD}$



- $T_{00} \equiv \gamma^*_L \to \rho_L$ impact factor : Dominant term at twist 2 $\equiv 1/Q$
  Ginzburg, Panfil, Serbo, (1985)

- $T_{11} \equiv \gamma^*_T \to \rho_T$ impact factor : Dominant term at twist 3 $\equiv 1/Q^2$

  Computed at $t = t_{min} \approx 0$

  Anikin, Ivanov, Pire, Szymanowski, Wallon, (2010)

Introduction
Construction of phenomenological models

Phenomenological models to compare to H1 and ZEUS data:

$$T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2(\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k})\ \Phi^{P\to P}(-\underline{k})$$

- First approach:
  (PhysRevD.84.054004 I. V. Anikin, A. B., D .Yu. Ivanov, B. Pire, L. Szymanowski, S. Wallon)
  - Using results for the $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k})$ up to twist 3
  - Using model for the proton impact factor $\Phi^{P\to P}$
- Second approach:
  - $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}$ expressed in coordinate space exhibits the color dipole scattering amplitude with the target.

    Nucl. Phys. B **867** (2013) 19-60. A. B., Szymanowski, Wallon
  - Using a model for the dipole/target scattering amplitude.
    ArXiv:1302.1766, A. B., Szymanowski, Wallon

Collinear factorization

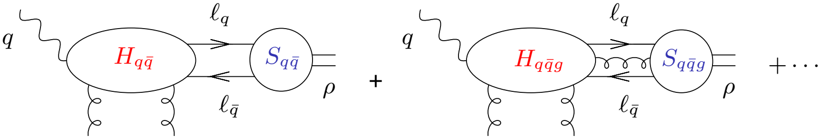
Light-Cone Collinear approach

- The impact factor $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}$ can be written as

$$\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)} = \int d^4\ell \cdots \mathrm{tr}[H^{(\lambda_\gamma)}(\ell\cdots) \quad S^{(\lambda_\rho)}(\ell\cdots)]$$

hard part soft part

(2-parton exchange) (3-parton exchange)



- Soft parts:

$$S_{q\bar{q}}(\ell_q) = \int d^4z\, e^{-i\ell_q\cdot z}\langle\rho(p)|\psi(0)\,\bar{\psi}(z)|0\rangle$$

$$S_{q\bar{q}q}(\ell_q,\ell_g) = \int d^4z_1 \int d^4z_2\, e^{-i(\ell_q\cdot z_1+\ell_g\cdot z_2)}\langle\rho(p)|\psi(0)\,gA_\alpha^\perp(z_2)\bar{\psi}(z_1)|0\rangle$$

Collinear factorization

Light-Cone Collinear approach: (2-parton case)

Collinear factorization 2-parton exchange contribution

- Momentum factorization:

$$\ell_q = y\, p_\rho + \ell^\perp + (\ell_q \cdot p_\rho)\, n \longrightarrow H_{q\bar{q}}(\ell_q) = H_{q\bar{q}}(yp) + \frac{\partial H_{q\bar{q}}(\ell)}{\partial \ell_\alpha}\bigg|_{\ell=yp} \ell_\alpha^\perp + \dots$$

- Spinor (and color) factorisation: $\delta_{ij}\delta_{kl} = \frac{1}{4}\sum_\Gamma (\Gamma^\mu)_{ik}\,(\Gamma_\mu)_{jl}$

$$\Phi_{q\bar{q}}^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)} = \int dy\, \left\{\mathrm{Tr}\left[H_{q\bar{q}}(y\,p)\,\Gamma\right]\, S_{q\bar{q}}^\Gamma(y) + \mathrm{Tr}\left[\partial_\perp H_{q\bar{q}}(y\,p)\,\Gamma\right]\, \partial_\perp S_{q\bar{q}}^\Gamma(y)\right\}$$

- Soft parts parameterization by distribution amplitudes (DAs)

$$S_{q\bar{q}}^\Gamma(y) = \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma_\mu\, \psi(0)|0\rangle \equiv m_\rho f_\rho \left\{\varphi_1(y)(n\cdot e^*)p_\mu,\ \varphi_A(y)\varepsilon_{\mu\, p\, n\, e_\perp^*},\ \varphi_3(y) e^*_{\perp\mu}\right\}$$

$$\partial_\perp S_{q\bar{q}}^\Gamma(y) = \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma_\mu\, i\, \overleftrightarrow{\partial_\perp}\, \psi(0)|0\rangle \equiv m_\rho f_\rho \left\{\varphi_{1T}(y) p_\mu e^*_{\perp\,\alpha},\ \varphi_{AT}(y) p_\mu \varepsilon_{\alpha\, p\, n\, e_\perp^*}\right\}$$

Collinear factorization

Wandzura-Wilczek and Genuine contributions

- Relations between DAs :Equation of motion and n-independence

  $\Rightarrow$ 3 independent DAs $\{\varphi_1, B(y_1, y_2), D(y_1, y_2)\}$
  - φ_1 parameterizes 2-parton correlator ($q\bar{q}$)
  - $B(y_1, y_2), D(y_1, y_2)$ parameterizes 3-parton correlators ($q\bar{q}g$)

- Wandzura-Wilczek approximation (WW):
  $(B(y_1, y_2), D(y_1, y_2) = 0)$

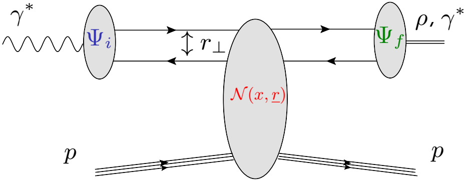
  $$\varphi_1 \Rightarrow \{\varphi_3^{WW}(y), \varphi_A^{WW}(y), \varphi_{1T}^{WW}(y), \varphi_{AT}^{WW}(y)\}$$

- Genuine solutions

  $$\{B(y_1, y_2), D(y_1, y_2)\} \Rightarrow \{\varphi_3^{gen}(y), \varphi_A^{gen}(y), \varphi_{1T}^{gen}(y), \varphi_{AT}^{gen}(y)\}$$

Dipole Models
Dipole model picture

- Factorization of a high energy scattering amplitude into:



  - Initial Ψ_i and final Ψ_f states wave functions.
  - Universal dipole/target scattering amplitude $\mathcal{N}(x, \underline{r})$.
  - In the impact factors "Target" = the two $t-$channel gluons:

$$\mathcal{N}(\underline{r}, \underline{k}) = \frac{4\pi\alpha_s}{N_c}\left(1 - e^{i\underline{k}\cdot\underline{r}}\right)\left(1 - e^{-i\underline{k}\cdot\underline{r}}\right)$$

The 2-parton Impact factor
Fourier transform of the $\gamma^* \to \rho$ impact factor

- Impact factors $\Phi^{\gamma^* \to \rho}_{q\bar{q}} = -\frac{1}{4}\int d^4\ell \; Tr(H_{q\bar{q}}\,\Gamma)(\ell) \; S_{q\bar{q}\Gamma}(\ell)$
- Collinear approximation $\Rightarrow$ expansion around $\ell_\perp = 0$:

$$Tr(H_{q\bar{q}}\,\Gamma)(\ell) = \int \frac{d^2 r_\perp}{2\pi} \tilde{H}^{\Gamma}_{q\bar{q}}(y, r_\perp) e^{-i\ell_\perp \cdot r_\perp}$$

$$= \int \frac{d^2 r_\perp}{2\pi} \underbrace{\tilde{H}^{\Gamma}_{q\bar{q}}(y, r_\perp)}_{\text{factorizes out}} \overbrace{(1 - i\ell_\perp \cdot r_\perp + \cdots)}^{\text{Gives the moments of } S_{q\bar{q}\Gamma}}$$

- 2-parton impact factor

$$\Phi^{\gamma^* \to \rho}_{q\bar{q}} = -\frac{1}{4} m_\rho f_\rho \int dy \int \frac{d^2 r_\perp}{(2\pi)} \Big\{ \tilde{H}^{\gamma,\mu}_{q\bar{q}}(y, \underline{r}) \Big(\varphi_3(y) e^*_{\rho\mu} + i\varphi_{1T}(y) p_{1\mu} (\underline{e}^*_\rho \cdot \underline{r}) \Big)$$
$$+ \tilde{H}^{\gamma_5\gamma,\mu}_{q\bar{q}}(y, \underline{r}) \Big(i\varphi_A(y) \varepsilon_{\mu e^*_\rho p_1 n} + \varphi_{AT}(y) p_{1\mu} \varepsilon_{r_\perp e^*_\rho p_1 n} \Big) \Big\}$$

The 2-parton impact factor
Role of the equation of motion of QCD

- Hard parts Fourier transforms: $\mathcal{N}(\underline{r},\underline{k}) \propto (1-e^{i\underline{k}\cdot\underline{r}})(1-e^{-i\underline{k}\cdot\underline{r}})$

$$\tilde{H}_{q\bar{q}}^{\gamma,\mu}(y,\underline{r}) \propto -y\bar{y}K_0(\mu|\underline{r}|)e_\gamma^\mu + i(y-\bar{y})\mu\frac{e\cdot r}{|\underline{r}|}K_1(\mu|\underline{r}|) \times (\mathcal{N}(\underline{r},\underline{k})-1)\,\frac{p_2^\mu}{s}$$

$$\tilde{H}_{q\bar{q}}^{\gamma\gamma_5,\mu}(y,\underline{r}) \propto \varepsilon^{\mu\nu\rho\sigma}(e_{\gamma\nu}\frac{r_{\perp\rho}}{|\underline{r}|}\frac{p_{2\sigma}}{s})\mu K_1(\mu|\underline{r}|) \times (\mathcal{N}(\underline{r},\underline{k})-1)$$

- 2-parton contribution:

$$\Phi_{q\bar{q}}^{\gamma^*\to\rho} = \int dy \int d\underline{r}\,\psi_{(q\bar{q})}^{\gamma_T^*\to\rho_T} \times \mathcal{N}(\underline{r},\underline{k})$$

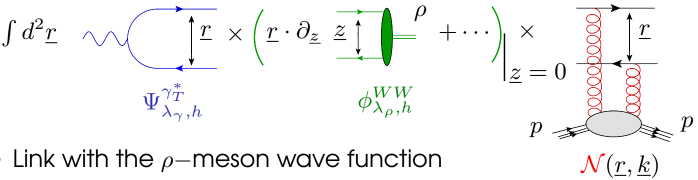
$$+ \quad \text{Hard Terms} \times \underbrace{(2y\bar{y}\varphi_3(y) + (y-\bar{y})\varphi_{1T}(y) + \varphi_{AT}(y))}_{\text{Cancels due to EOM in WW approx.}}$$

Wandzura-Wilczek result

Interpretation

In WW approximation

- Scanning the $\rho-$meson wave function:



- Link with the $\rho-$meson wave function

$$\Psi^{\rho\, q\bar{q}}_{\lambda_\rho, h} = \text{Spinor part} \times \varphi^{(q\bar{q})}_{\lambda_\rho} \quad (1)$$

$$\phi^{WW}_{\lambda_\rho, h}(y, \underline{r}) \propto (\underline{e}^{(\lambda_\rho)} \cdot \underline{r}) \frac{y\delta_{h,\lambda_\rho} + \bar{y}\delta_{h,-\lambda_\rho}}{y\bar{y}} \int^{|\ell_\perp| < \mu_F} d^2\ell_\perp\, \ell_\perp^2\, \varphi^{(q\bar{q})}_{\lambda_\rho}(y, \ell_\perp)$$

The 3-parton impact factor
Expression and kinematics

- The 3-parton amplitude in transverse coordinate space after collinear approximation

$$\Phi_{q\bar{q}g}^{\gamma^*\to\rho} = -\frac{i m_\rho f_\rho}{4} \int dy_1 dy_g \int \frac{d^2 r_{1\perp}}{(2\pi)^2} \frac{d^2 r_{g\perp}}{(2\pi)^2}$$
$$\Big(\zeta_{3\rho}^V B(y_1, y_2) p_\mu e_{\rho\perp\alpha}\ \tilde{H}_{q\bar{q}g}^{\alpha,\gamma^\mu}(y_1, y_g, r_{1\perp}, r_{g\perp})$$
$$+\zeta_{3\rho}^A i D(y_1, y_2) p_\mu \varepsilon_{\alpha e_{\rho\perp} p n}\ \tilde{H}_{q\bar{q}g}^{\alpha,\gamma^\mu\gamma_5}(y_1, y_g, r_{1\perp}, r_{g\perp})\Big)$$

- 3-partons exchanged $\Rightarrow$ Two Colour dipole configurations

The 3-parton impact factor
Results form of the 3-parton impact factor

- 3-partons results:

$$\Phi_{q\bar{q}g}^{\gamma_T^*\to\rho_T} \propto \int dy_1 \int dy_2 \int d^2\underline{r}\,\psi_{(q\bar{q}g)}^{\gamma_T^*\to\rho_T}(y_1,y_2,\underline{r}) \times \mathcal{N}(\underline{r},\underline{k}) + \int dy_1\, dy_2 \,\frac{2\,S(y_1,y_2)}{\bar{y}_1}$$

with $S(y_1,y_2) = \zeta_\rho^V(\mu^2)B(y_1,y_2;\mu^2) + \zeta_\rho^A(\mu^2)D(y_1,y_2;\mu^2)$

- Full twist 3 impact factor:

$$\Phi^{\gamma_T^*\to\rho_T} = \Phi_{q\bar{q}}^{\gamma_T^*\to\rho_T} + \Phi_{q\bar{q}g}^{\gamma_T^*\to\rho_T}$$

$$\propto \int dy_i \int d^2\underline{r}\,\mathcal{N}(\underline{r},\underline{k})\left(\psi_{(q\bar{q})}^{\gamma_T^*\to\rho_T}(y,\underline{r}) + \psi_{(q\bar{q}g)}^{\gamma_T^*\to\rho_T}(y_1,y_2,\underline{r})\right)$$

$$\underbrace{+\int \frac{dy}{y\bar{y}}\left(2y\bar{y}\varphi_3(y) + (y-\bar{y})\varphi_1^T(y) + \varphi_A^T(y)\right) + \int dy_1\,dy_2\,\frac{2\,S(y_1,y_2)}{\bar{y}_1}}_{\text{Cancel due to EOM of QCD}}$$

Helicity amplitudes
Dipole cross-section

- Dipole-target cross-section:

$$\mathcal{N}(\underline{k},\underline{r}) \to \hat{\sigma}(x,\underline{r}) = \frac{N_c^2-1}{4}\int \frac{d^2\underline{k}}{\underline{k}^4}\mathcal{F}(x,\underline{k})\mathcal{N}(\underline{k},\underline{r})$$

- Helicity amplitudes

$$T_{00} = s\int dy \int d\underline{r}\, \psi^{\gamma_L^*\to\rho_L}_{(q\bar{q})}(y,\underline{r};Q,\mu_F)\hat{\sigma}(x,\underline{r})$$

$$T_{11} = s\int dy \int d\underline{r}\, \psi^{\gamma_T^*\to\rho_T}_{(q\bar{q})}(y,\underline{r};Q,\mu_F)\hat{\sigma}(x,\underline{r})$$

$$+\, s\int dy_2 \int dy_1 \int d\underline{r}\, \psi^{\gamma_T^*\to\rho_T}_{(q\bar{q}g)}(y_1,y_2,\underline{r};Q,\mu_F)\hat{\sigma}(x,\underline{r})\,,$$

- Polarized Cross-sections

$$\frac{d\sigma_{L,T}}{dt}(t) = \underbrace{e^{-b(Q^2)t}}_{T_{01}\text{,etc.. encodded}} \frac{d\sigma_{L,T}}{dt}(t=0)$$

$$\sigma_L = \frac{1}{b(Q^2)}\frac{T_{00}(s,t=0)^2}{16\pi s^2}$$

$$\sigma_T = \frac{1}{b(Q^2)}\frac{T_{11}(s,t=0)^2}{16\pi s^2}\,.$$

Helicity amplitudes

A model for the dipole cross-section

Model for the dipole cross-section $\hat{\sigma}(x, r)$

- rc-BK numerical solution
  (Albacete, Armesto, Milhano, Quiroga Arias, Salgado, 2011)
  - fitting DIS data with light quarks u, d, s
  - including heavy quarks c, b contribution to DIS data
  - GBW-like and MV-like initial conditions
- Good description of inclusive and longitudinal structure functions χ^2/dof ≈ 1.2.

Explicit solutions for the Distribution Amplitudes

- Evolution of the DAs P. Ball, V.M Braun, Y. Koike, K. Tanaka

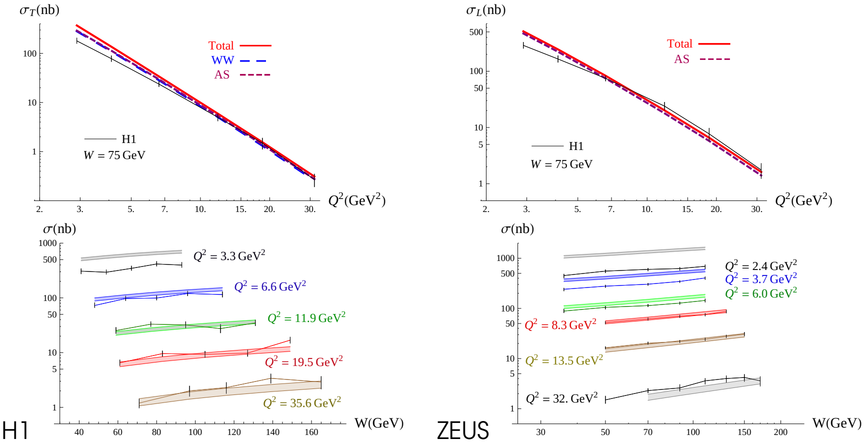
$$\varphi_1(y,\mu_F^2) = 6y\bar{y}\left(1 + a_2(\mu_R^2)\frac{3}{2}(5(y-\bar{y})^2 - 1)\right) \overset{\mu_F^2\to\infty}{\longrightarrow} 6y\bar{y}$$

$$B(y_1,y_2;\mu_F^2) = -5040 y_1\bar{y_2}(y_1-\bar{y_2})(y_2-y_1)$$

$$D(y_1,y_2;\mu_F^2) = -360 y_1\bar{y_2}(y_2-y_1)\left(1 + \frac{\omega^A_{\{1,0\}}(\mu_R^2)}{2}(7(y_2-y_1)-3)\right)$$

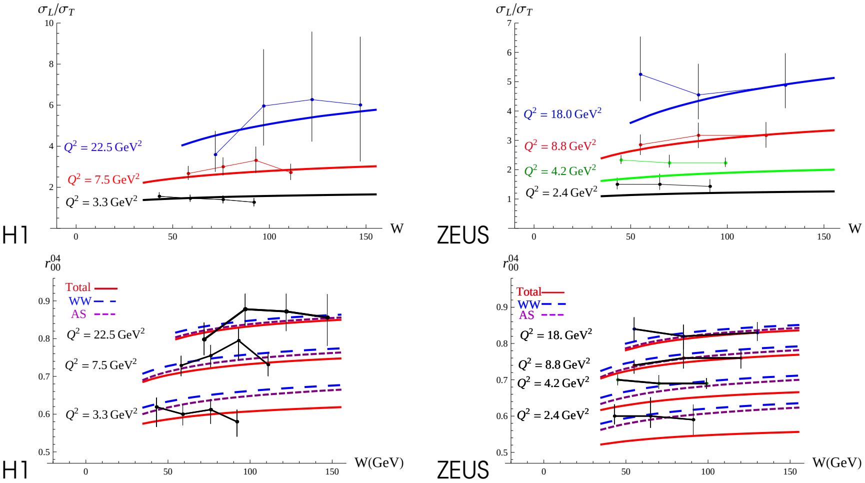
$\mu_R^2 = \mu_F^2 \sim \frac{Q^2+m_\rho^2}{4}$: collinear factorization scale

Results
Comparison with H1 and ZEUS data



Results

Comparison with H1 and ZEUS data



Conclusion

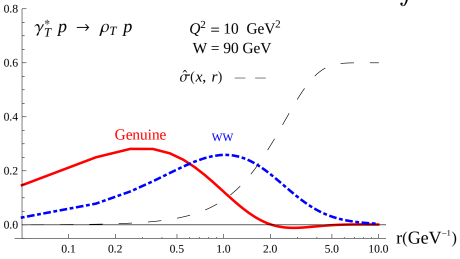
- Results
  - Predictions with normalizations in good agreement with HERA data for Q^2 larger than $\approx 6-8$ GeV^2
  - Predictions not sensitive to the choice of the collinear factorization scale μ_F in the region $Q^2 > 6-8$ GeV^2
  - Discrepancy for $Q^2 < 5$ GeV^2 mostly due to higher twist terms?
- Perspectives
  - genuine saturation regime $\Rightarrow$ Higher twist corrections
  - Implementing $\rho-$meson wave function models through the DAs $\Rightarrow$ how the parameters will change?
  - Extending the kinematics at $t \neq 0 \Rightarrow$ a test for dipole models with impact parameter dependence.

Radial distribution of dipoles

Contributions to the overlap

- Radial distribution of dipoles $\mathcal{P}_{11}(|\underline{r}|) \propto |\underline{r}| \int dy_1 \cdots \psi^{\gamma^*_T \to \rho_T}_{(q\bar{q}(g))}(y_1, \cdots, \underline{r})$

$$T_{11}(x, Q, \mu_F) = \int dr \mathcal{P}_{11}(r, Q, \mu_F)\hat{\sigma}(r, x)$$



- $\mathcal{P}_{11}^{\text{Genuine}} \sim \mathcal{P}_{11}^{\text{WW}}$
- Dipole cross-section $\hat{\sigma}(x, r)$ =filters small dipole $r < R_0(x)$