TMD EVOLUTION OF HELICITY AND TRANSVERSITY

Alexei Prokudin

April 24, 2013

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear case

$$\Phi(x; P, S) = \frac{1}{2}\left\{ f_1(x) \, \not{P} + S_L g_1(x)\gamma_5 \, \not{P} + \frac{1}{2} h_1(x)\gamma_5 [\not{S}_T, \not{P}] \right\}$$

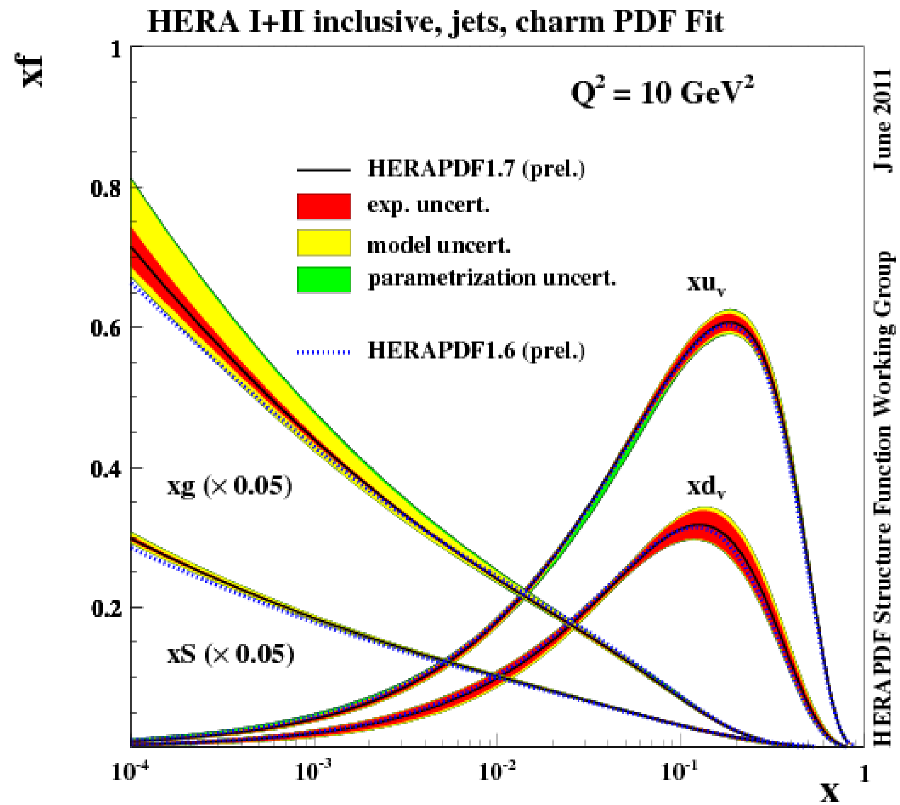
Unpolarised PDF

Helicity distribution

Transversity distribution

Unpolarised PDFs

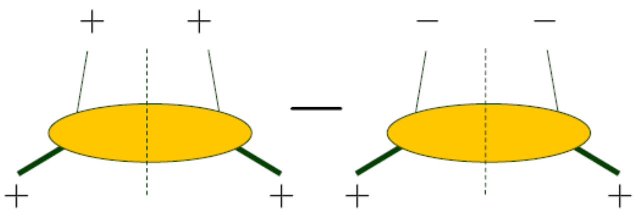
Good knowledge of
unpolarised
Parton
Distribution
Functions
is acquired using
HERA data

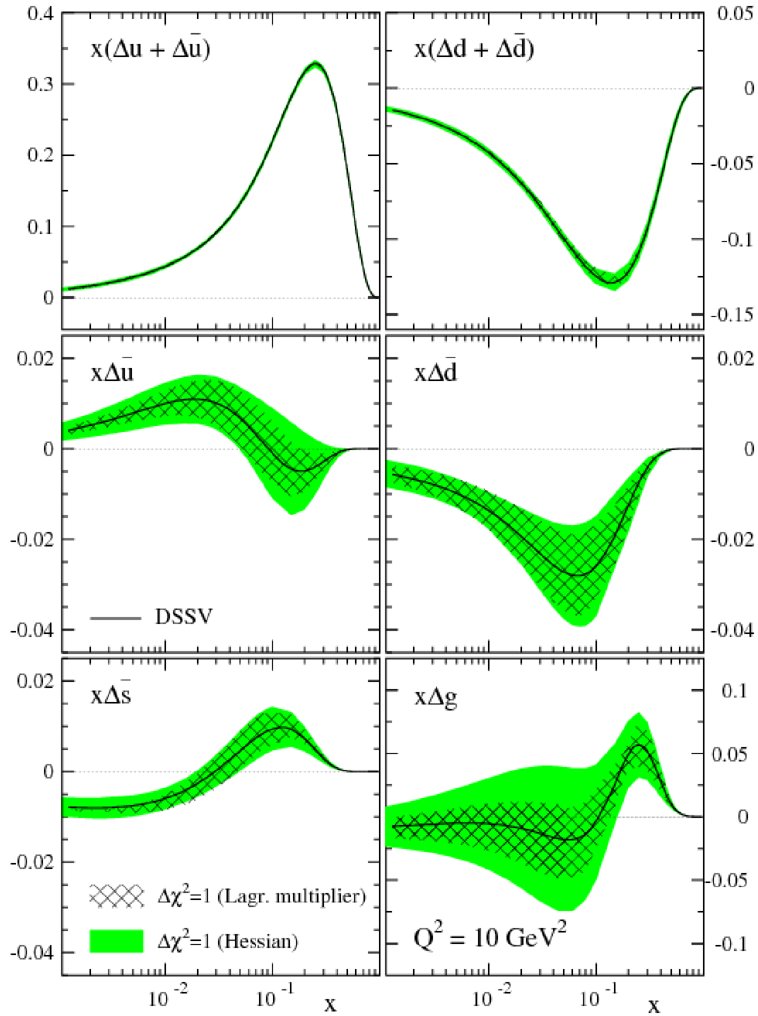


Plot from
Eram Rizvi et.al.,
JHEP 2009 (arXiv:0911.0884)

Helicity distributions

Helicity distributions are relatively well known

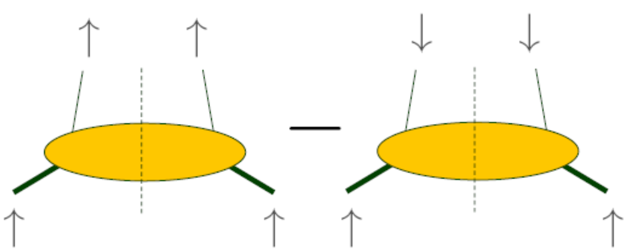




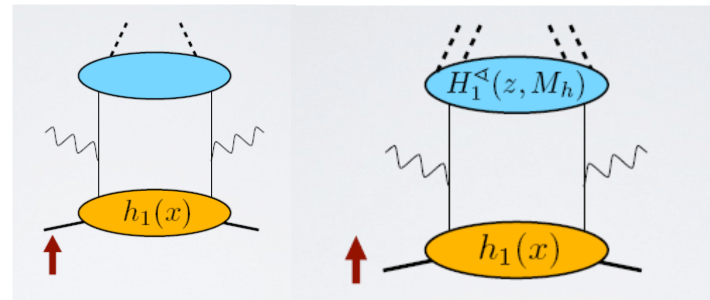
Plot from
DSSV PRD80 (2009) 034030

Transversity distributions

Distribution of transversely polarised quarks inside transversely polarised Nucleon, chiral odd

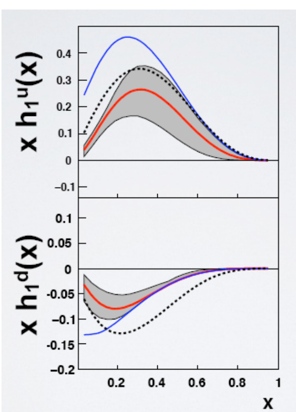


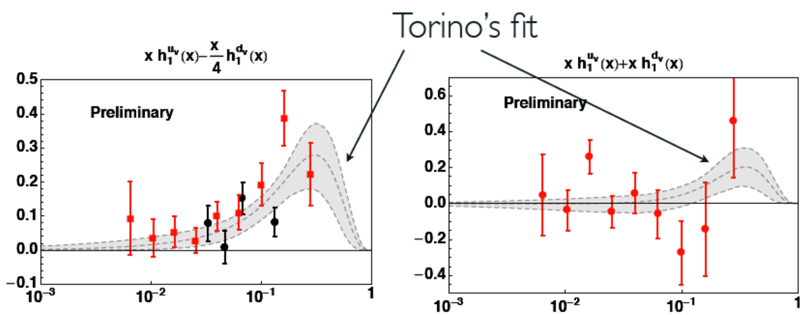
Can be stydied in SIDIS (COMPASS, HERMES, JLAB)



With Collins FF With Dihadron FF

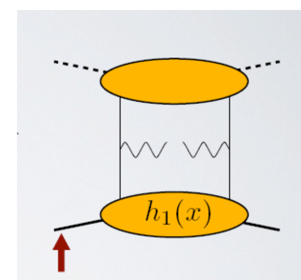
Extractions





Bacchetta et al.,arXiv:1206.1836

Drell-Yan



With transversity or Boer-Mulders

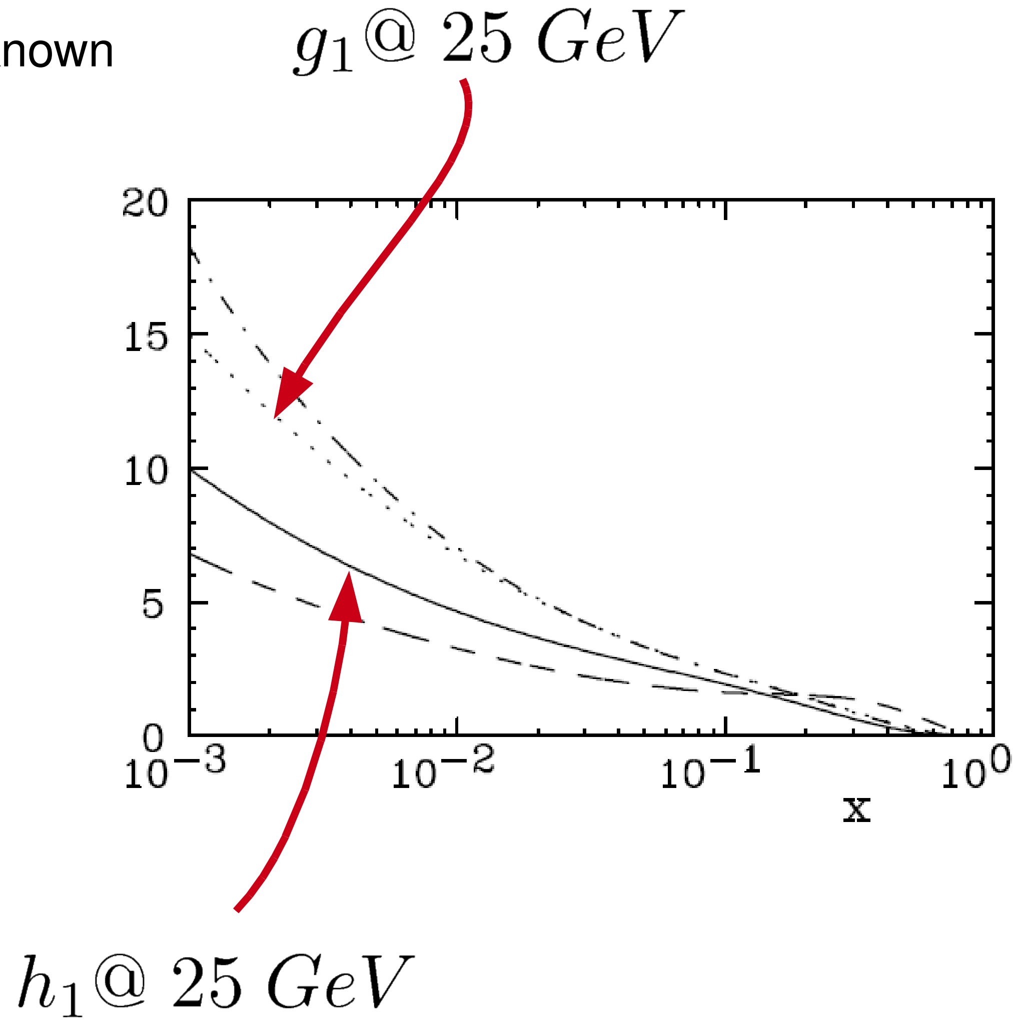
Anselmino et al 09, 13 See talks of Stefano Melis and Aurore Courtoy

Jefferson Lab

Evolution of transversity and helicity

Collinear (DGLAP) case is well known

Barone 1997
Vogelsang 1998



Evolution of Transverse Momentum Dependent transversity and helicity

Alessandro Bacchetta, Alexei Prokudin
arXiv:1303.2129

Definition of TMDs

TMD functions describe processes where quark intrinsic motion is important

Historically TMD factorization is formulated as
Collins-Soper-Sterman resummation

Collins, Soper, Sterman 1985

Proven for polarized case

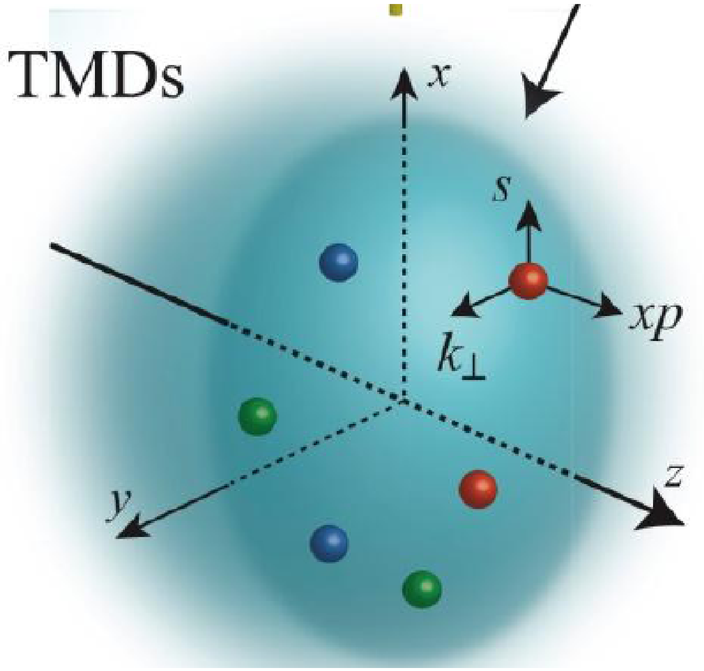
Ji, Ma, Yuan 2004
Collins 2011

Alternative formulations

Cherednikov, Stefanis 2008
Echevarria, Idilbi, Scimemi 2011
Trentadue, Ceccoperi, 2008
Hautman, 2008

Equivalence with some approaches
was shown in

Collins, Rogers 2012



Definition of TMDs

In the following we will use definition by

Collins 2011

TMD is defined in coordinate space

$$\tilde{f}(x, b_\perp; \mu, \zeta)$$

Fourier conjugate to $k_\perp$

Can be studied experimentally

See talks of Mher Aghasyan

Definition of TMDs

In the following we will use definition by

Collins 2011

TMD is defined in coordinate space

$$\tilde{f}(x, b_\perp; \mu, \zeta)$$

RG renormalization

Definition of TMDs

In the following we will use definition by

Collins 2011

TMD is defined in coordinate space

$$\tilde{f}(x, b_\perp; \mu, \zeta)$$

Additional parameter to cancel rapidity divergence

TMD evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins, Soper, Sterman 1985
Collins, 2011

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD evolution:helicity and transversity

Solve evolution equations:

Collins Soper Sterman 1985
Collins 2011

$$\widetilde{f}_1^f(x,b_T;\mu,\zeta_F)=\sum_i(\tilde{C}_{f/i}\otimes f_1^i)(x,b_*;\mu_b)e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)}e^{g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}}\hat{f}^q_{\mathrm{NP}}(x,b_T)$$

Expansion at small bt, Spin dependent

Contains gluon radiation, no spin dependence

TMD evolution:helicity and transversity

Solve evolution equations:

Collins Soper Sterman 1985
Collins 2011

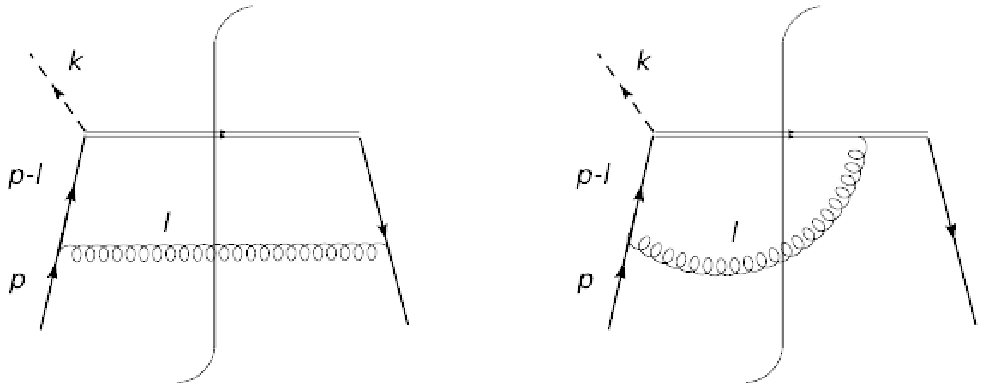
$$\tilde{f}_1^f(x, b_T; \mu, \zeta_F) = \sum_i \left(\tilde{C}_{f/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)} e^{g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}} \hat{f}_{\rm NP}^q(x, b_T)$$

Calculate these in order to derive evolution of helicity and transversity

Bacchetta, Prokudin 2013

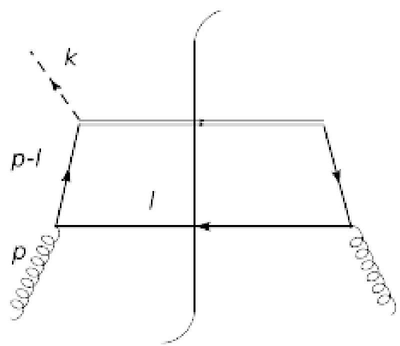
TMD evolution:helicity and transversity

Diagrams:



Quark in quark coefficient function

$$\tilde{C}_{q/q}$$



Quark in a gluon Coefficient function

$$\tilde{C}_{q/g}$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything for quark:

$$\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\frac{1+x^2}{(1-x)_+} + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

$$\Delta\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\frac{1+x^2}{(1-x)_+} + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

$$\delta\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\frac{2x}{(1-x)_+} + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything for gluon:

$$\tilde{C}_{j/g}(x, \boldsymbol{b}_T; \mu, \zeta_F/\mu^2) = \frac{\alpha_s T_f}{\pi} \left\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) \left(x^2 + (1-x)^2\right) + x(1-x) \right\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j/g}(x, \boldsymbol{b}_T; \mu, \zeta_F/\mu^2) = \frac{\alpha_s T_f}{\pi} \left\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) (2x-1) + (1-x) \right\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j/g}(x, \boldsymbol{b}_T; \mu, \zeta_F/\mu^2) = 0\,,$$

True to all orders

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Simplify by choosing : $\mu_b = 2e^{-\gamma_E}/b_*$

$$\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_F}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_F}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \mathcal{O}(\alpha_s^2).$$

$$\tilde{C}_{j/g}(x, b_*; \mu_b) = \frac{\alpha_s T_f}{\pi}\, x(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j/g}(x, b_*; \mu_b) = \frac{\alpha_s T_f}{\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j/g}(x, b_*; \mu_b) = 0\,,$$

Result coincides with CSS:
Koike, Nagashima, Vogelsang 2006

Phenomenology

A. Bacchetta, AP, 2013

Choose initial conditions:

$$xu_0(x) = xd_0(x) \equiv x^{0.5}(1-x)^{0.5}, \qquad x\bar{u}_0(x) = x\bar{d}_0(x) \equiv 0, \qquad xg_0(x) \equiv x^{0.5}(1-x)^{0.5}.$$

Choose initial conditions for evolution:

$$\hat{f}^f_{\rm NP}(x, b_T) = \exp\left(-\frac{b_T^2\,\langle k_T^2\rangle}{4}\right) , \qquad g_K(b_T) = -g\, b_T^2 ,$$

where $\langle k_T^2\rangle = 0.25$ (GeV^2), $g = 0.2$ (GeV^2). We also choose $b_{max} = 1$ (GeV^{-1}).

Phenomenology

A. Bacchetta, AP, 2013

Choose initial conditions, $Q_0 = 1 GeV$:

$$xu_0(x) = xd_0(x) \equiv x^{0.5}(1-x)^{0.5}, \qquad x\bar{u}_0(x) = x\bar{d}_0(x) \equiv 0, \qquad xg_0(x) \equiv x^{0.5}(1-x)^{0.5}.$$

Choose initial conditions for evolution:

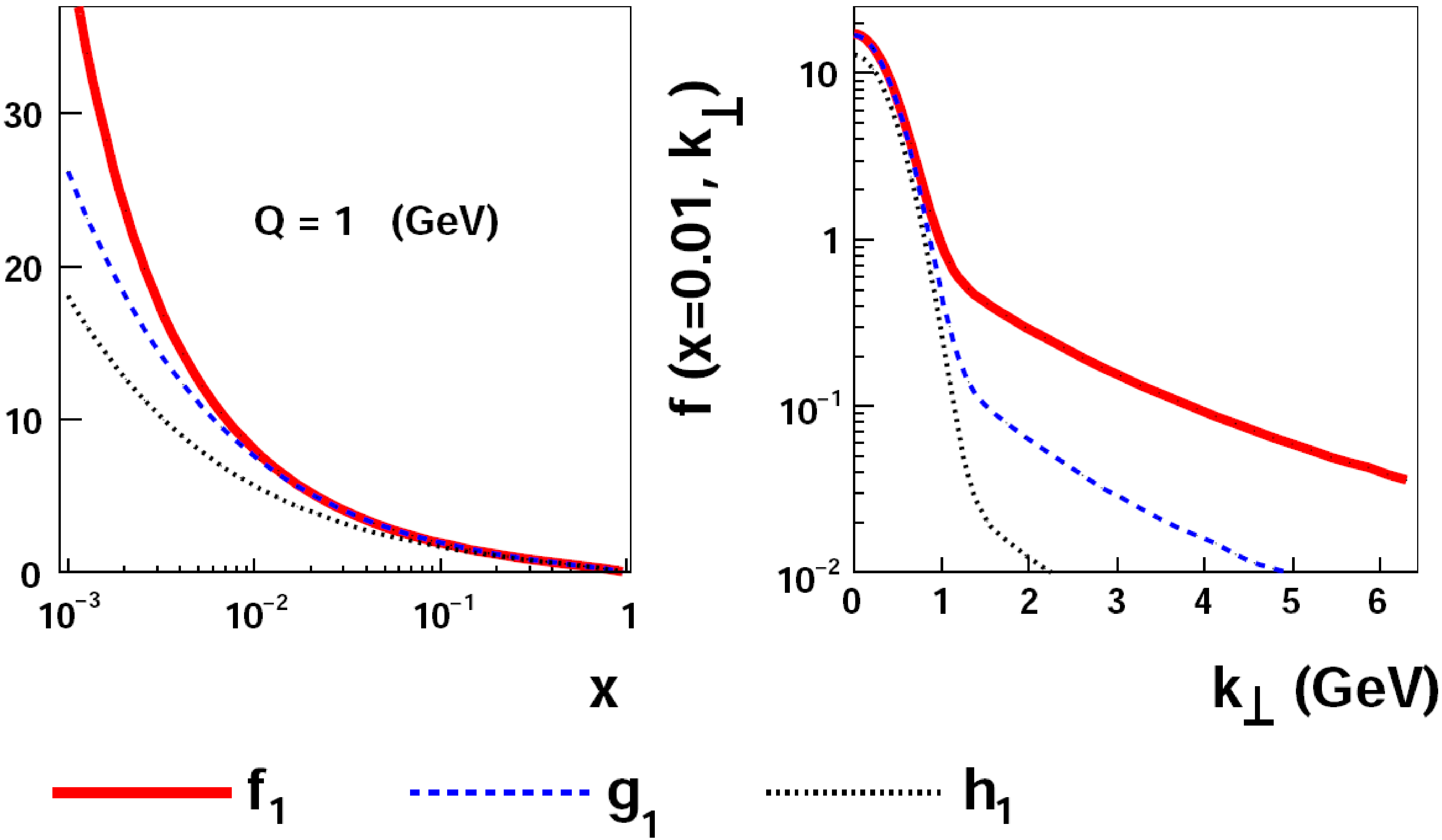
$$\hat{f}^{f}_{\rm NP}(x, b_T) = \exp\left(-\frac{b_T^2 \langle k_T^2 \rangle}{4}\right) \;, \qquad g_K(b_T) = -g\, b_T^2 \;,$$

where $\langle k_T^2 \rangle = 0.25\ ({\rm GeV}^2)$, $g = 0.2\ ({\rm GeV}^2)$. We also choose $b_{max} = 1\ ({\rm GeV}^{-1})$.

Important!

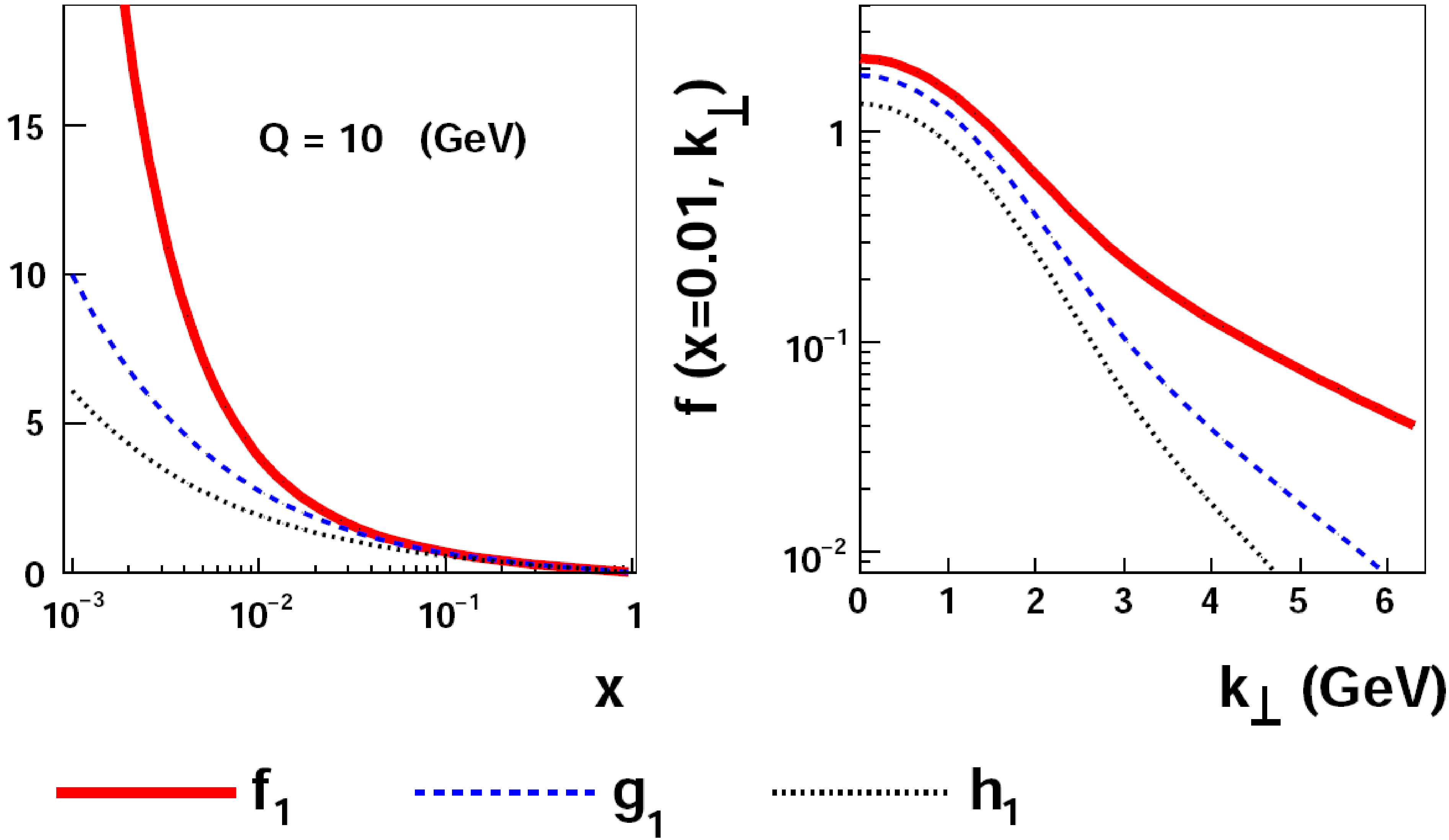
Phenomenology

A. Bacchetta, AP, 2013



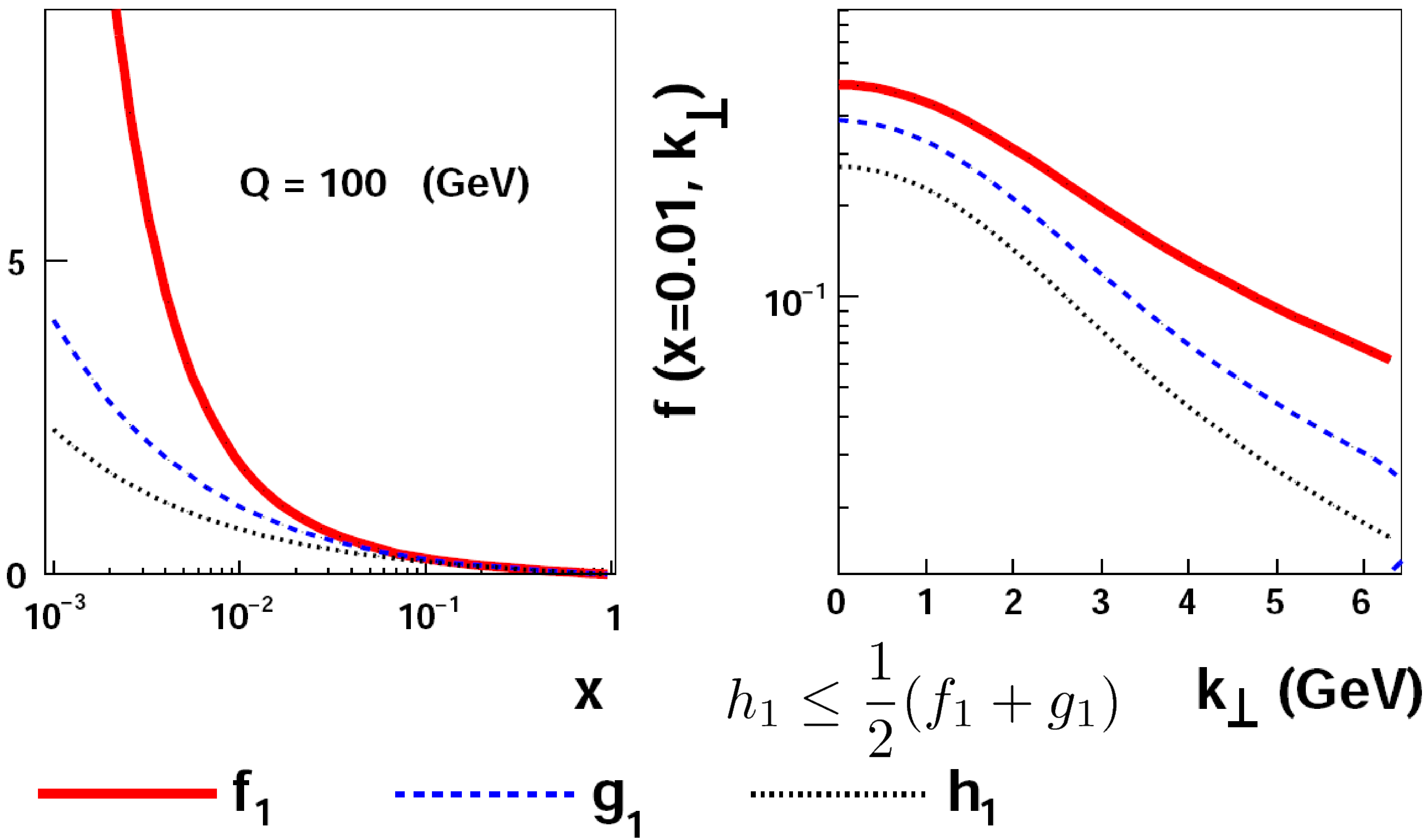
Phenomenology

A. Bacchetta, AP, 2013



Phenomenology

A. Bacchetta, AP, 2013



$$h_1 \leq \frac{1}{2}(f_1 + g_1)$$

Soffer bound on transversity is not violated numerically

Conclusions

- Evolution for TMD transversity and helicity functions is calculated
- Results are checked with CSS formalism results
- Soffer bound on transversity is not violated numerically

QCD Evolution 2013
Jefferson Lab, May 6-10, 2013
http://www.jlab.org/conferences/qcd2013/

QCD Frontier 2013
Exploring QCD with next generation facilities
Jefferson Lab, October 21-22, 2013
http://jlab.org/conferences/qcd-frontier-2013/