Valence transversities: the collinear extraction

DIS 2013
Marseille

Aurore Courtoy
IFPA-Université de Liège (Belgium)

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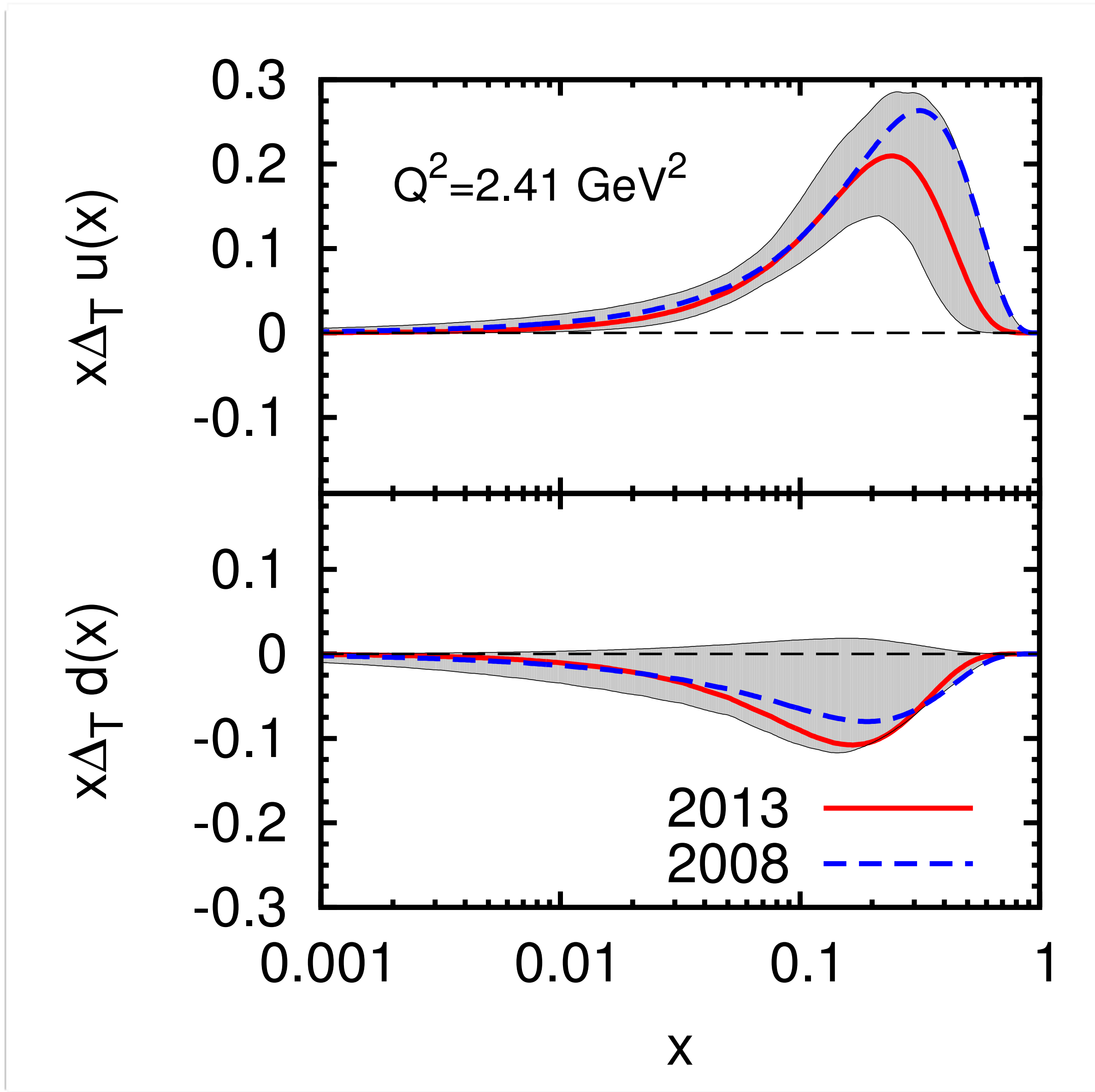
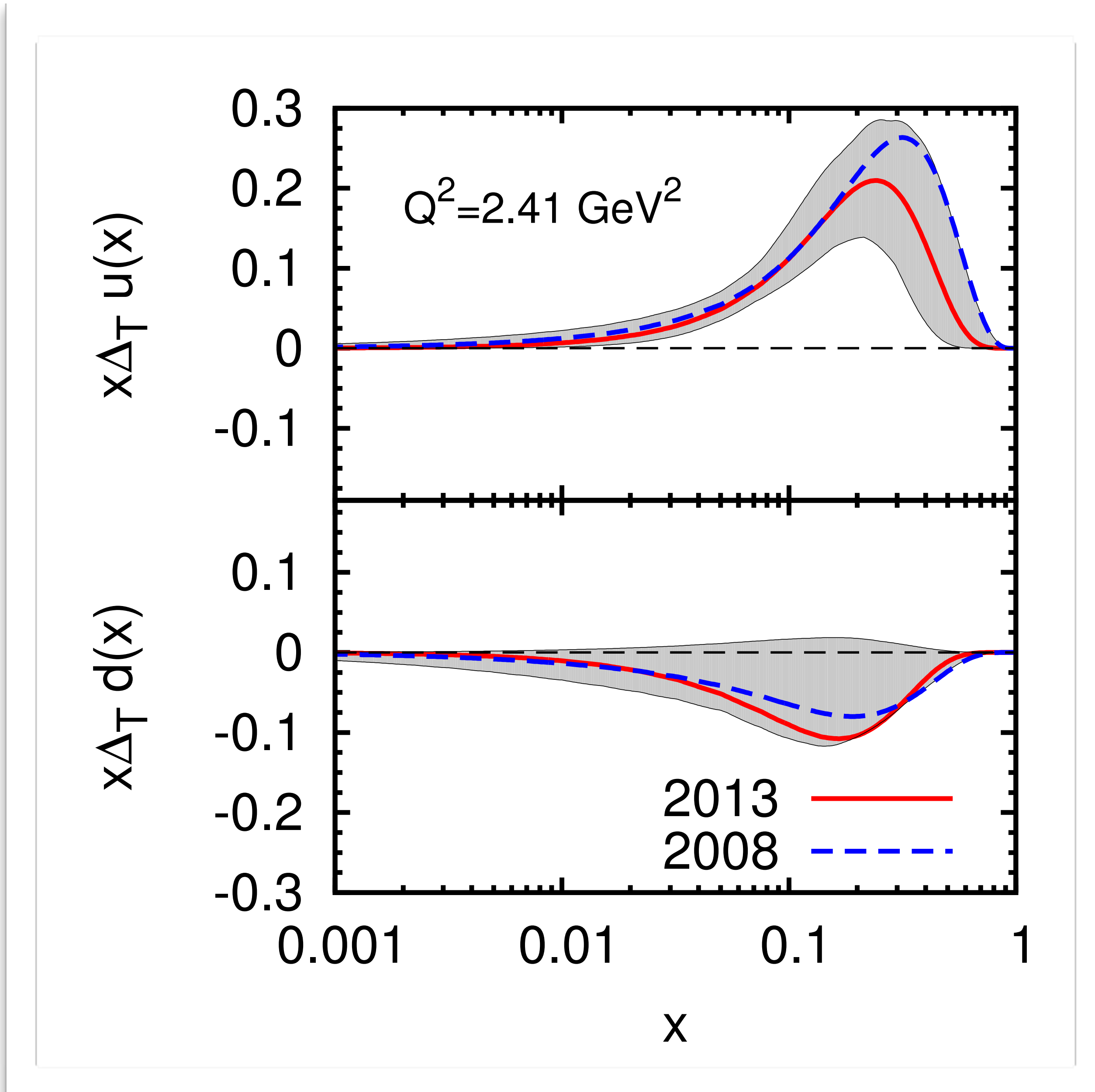
in collaboration with Alessandro Bacchetta and Marco Radici in Pavia

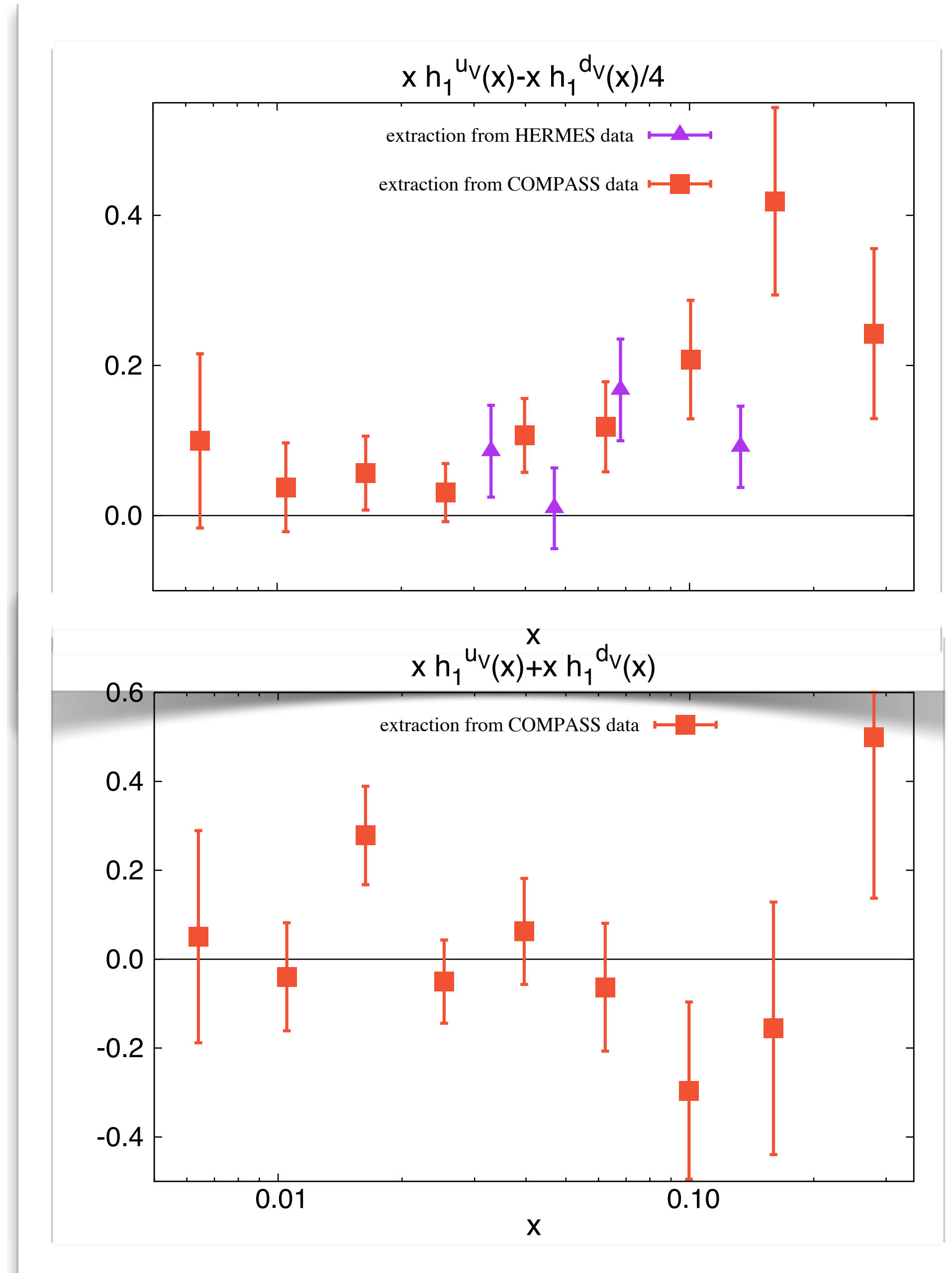
State-of-the-art:
Extractions of transversity



"TMD extraction"

State-of-the-art:
Extractions of transversity

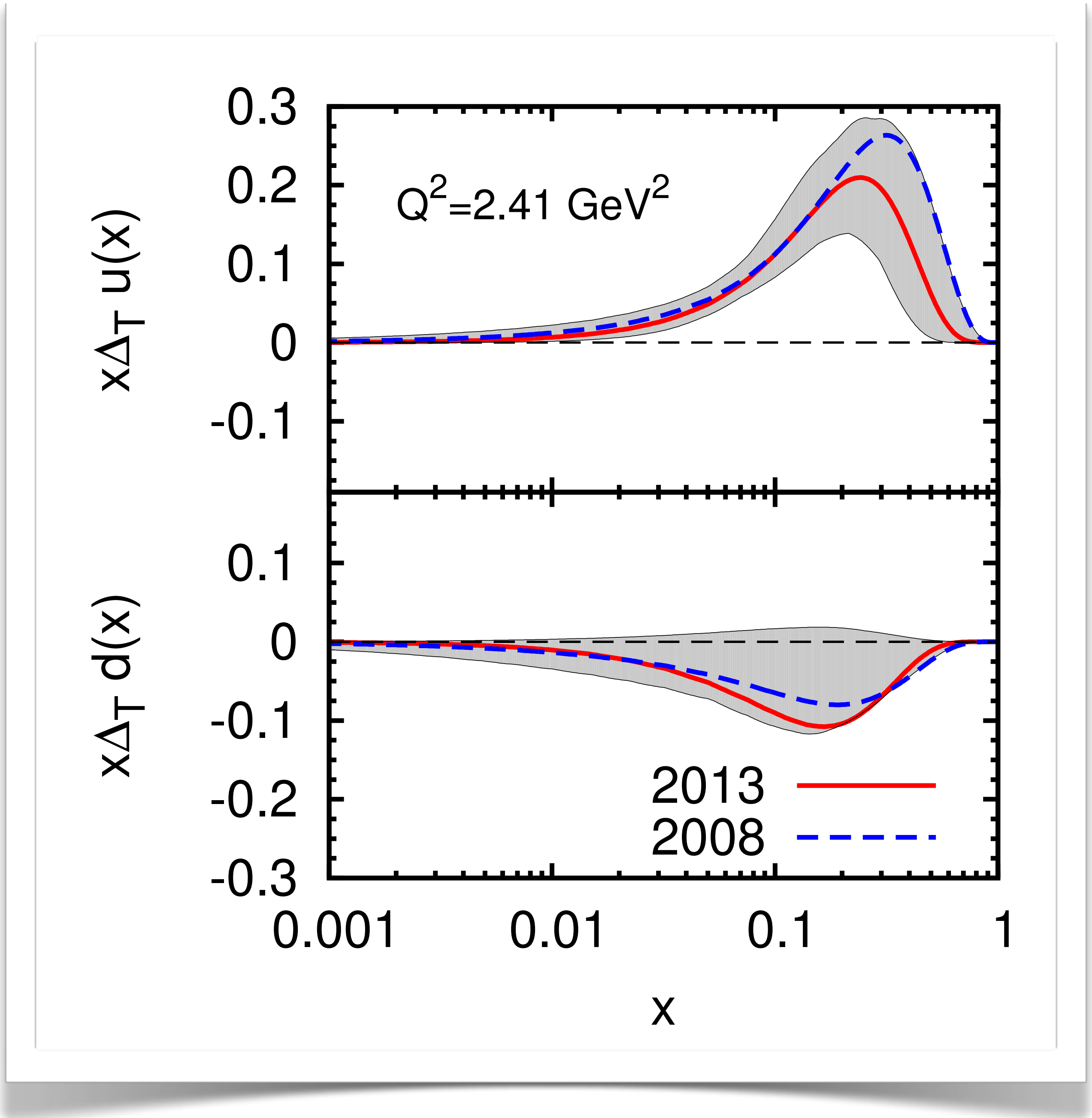


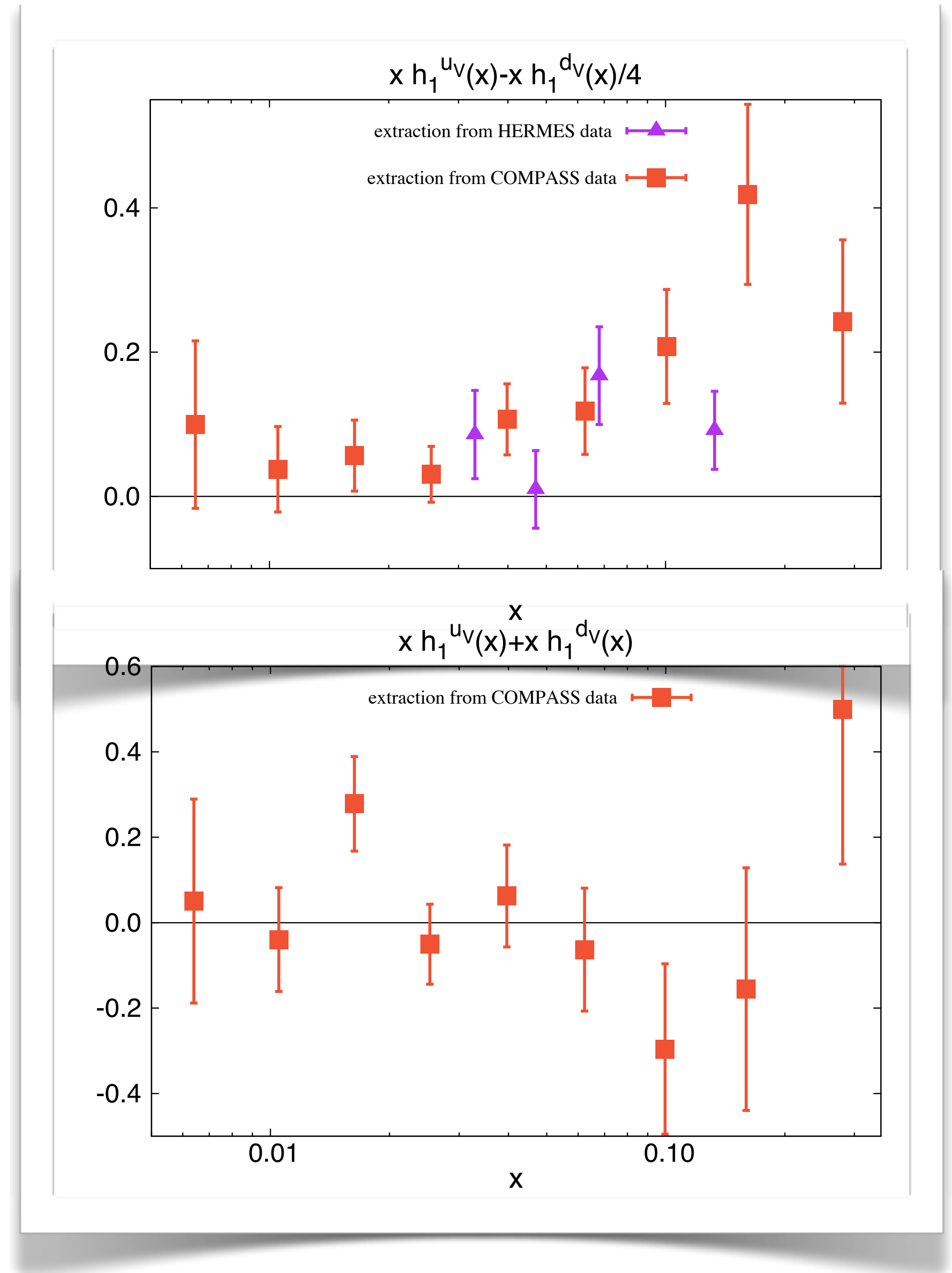


“TMD extraction”

“Collinear extraction”
Pavia 11-12

State-of-the-art:
Extractions of transversity



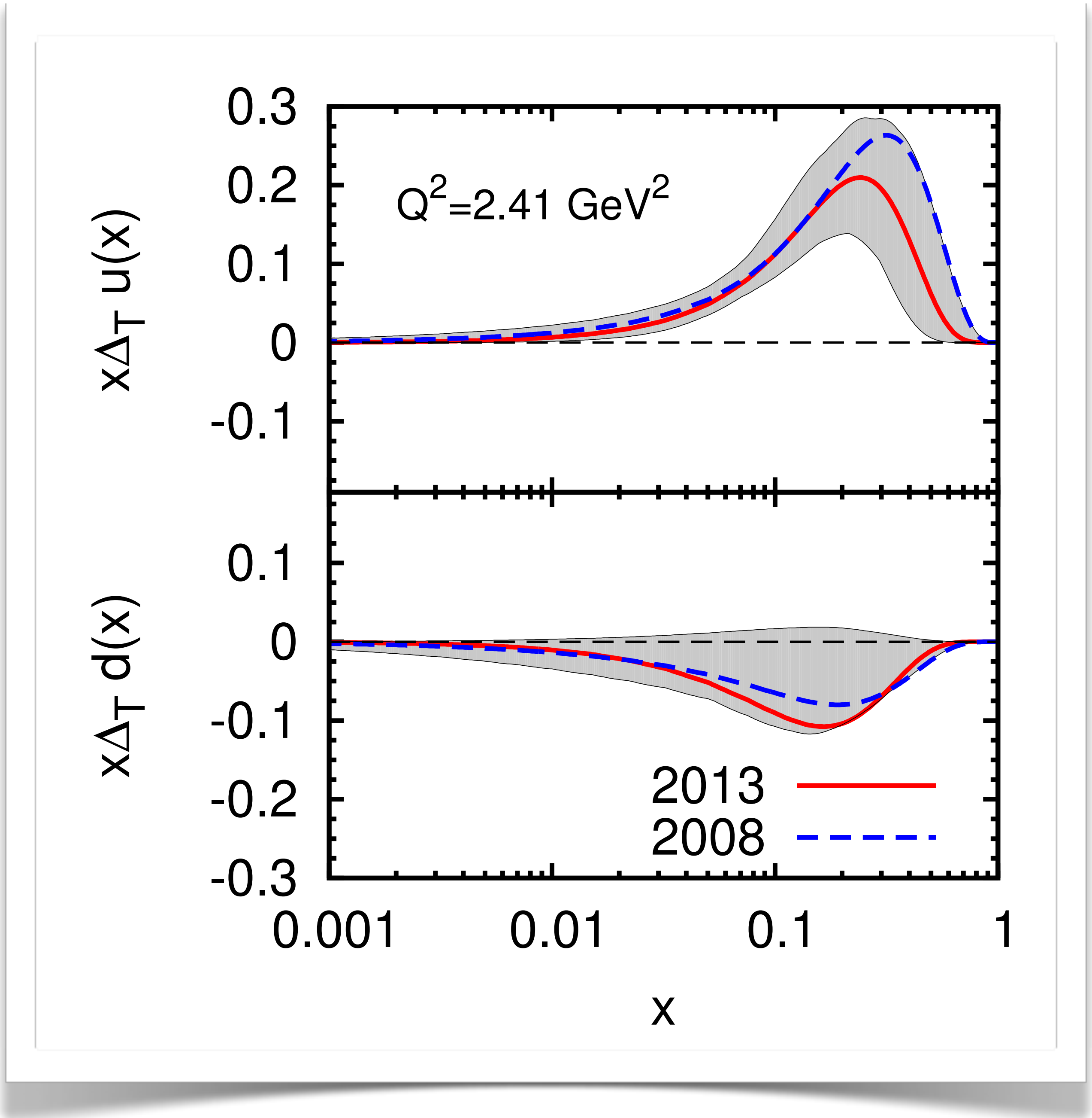


"TMD extraction"

"Collinear extraction"
Pavia 11-12

"Torino-Cagliari-JLab extraction"
Torino 09 &1303.3822

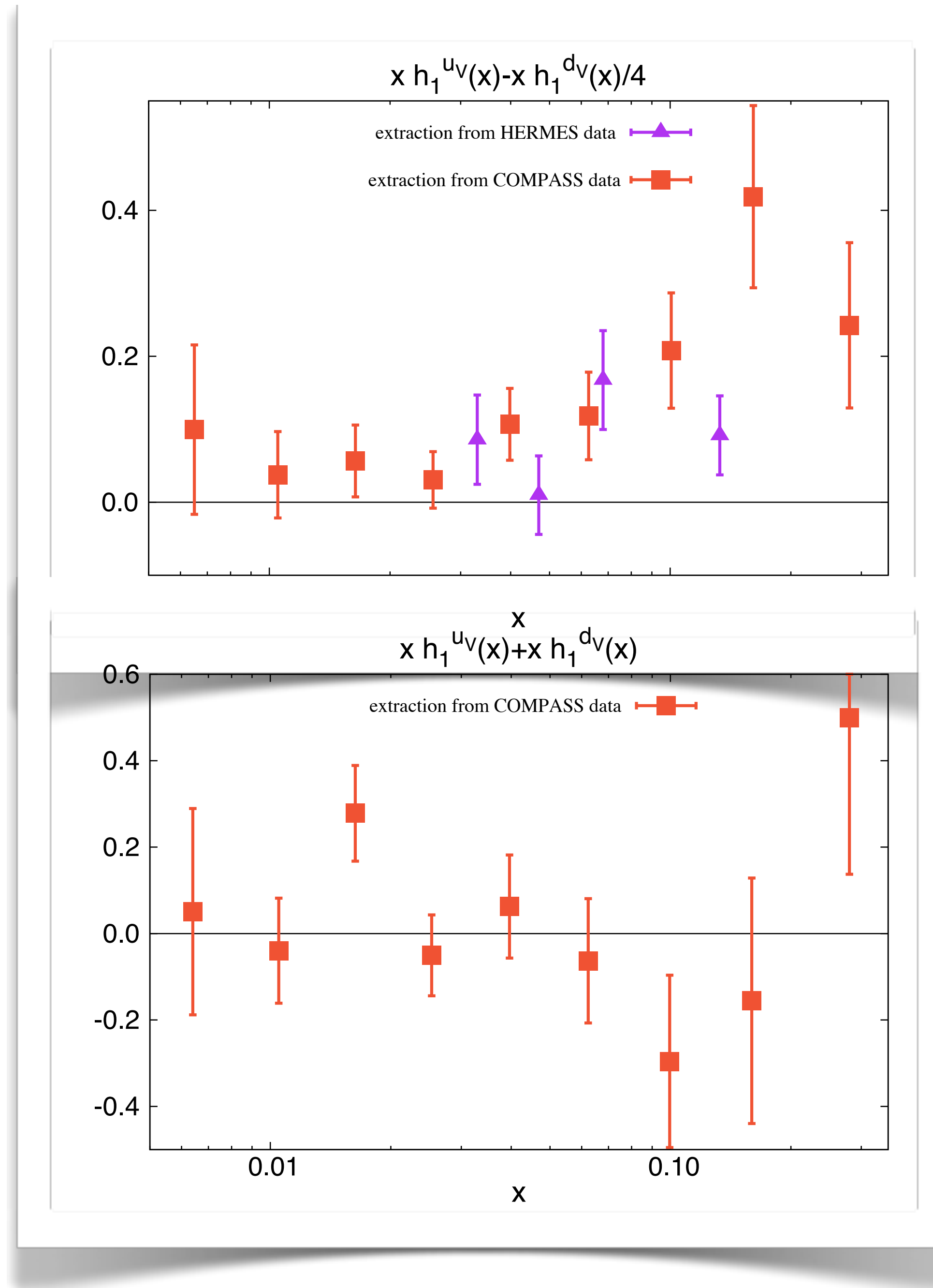
State-of-the-art:
Extractions of transversity



“TMD extraction”

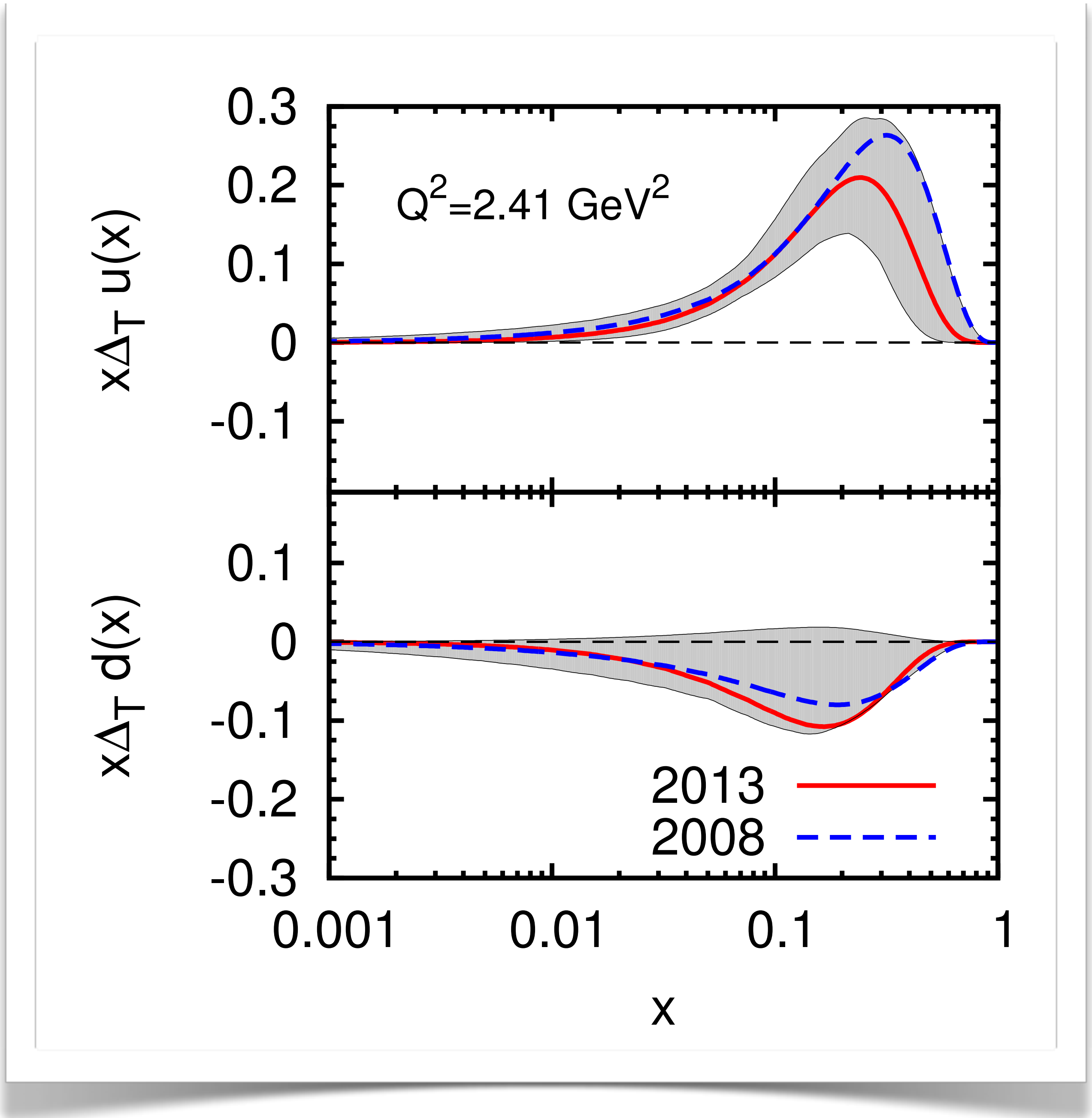
Talk by S. Melis

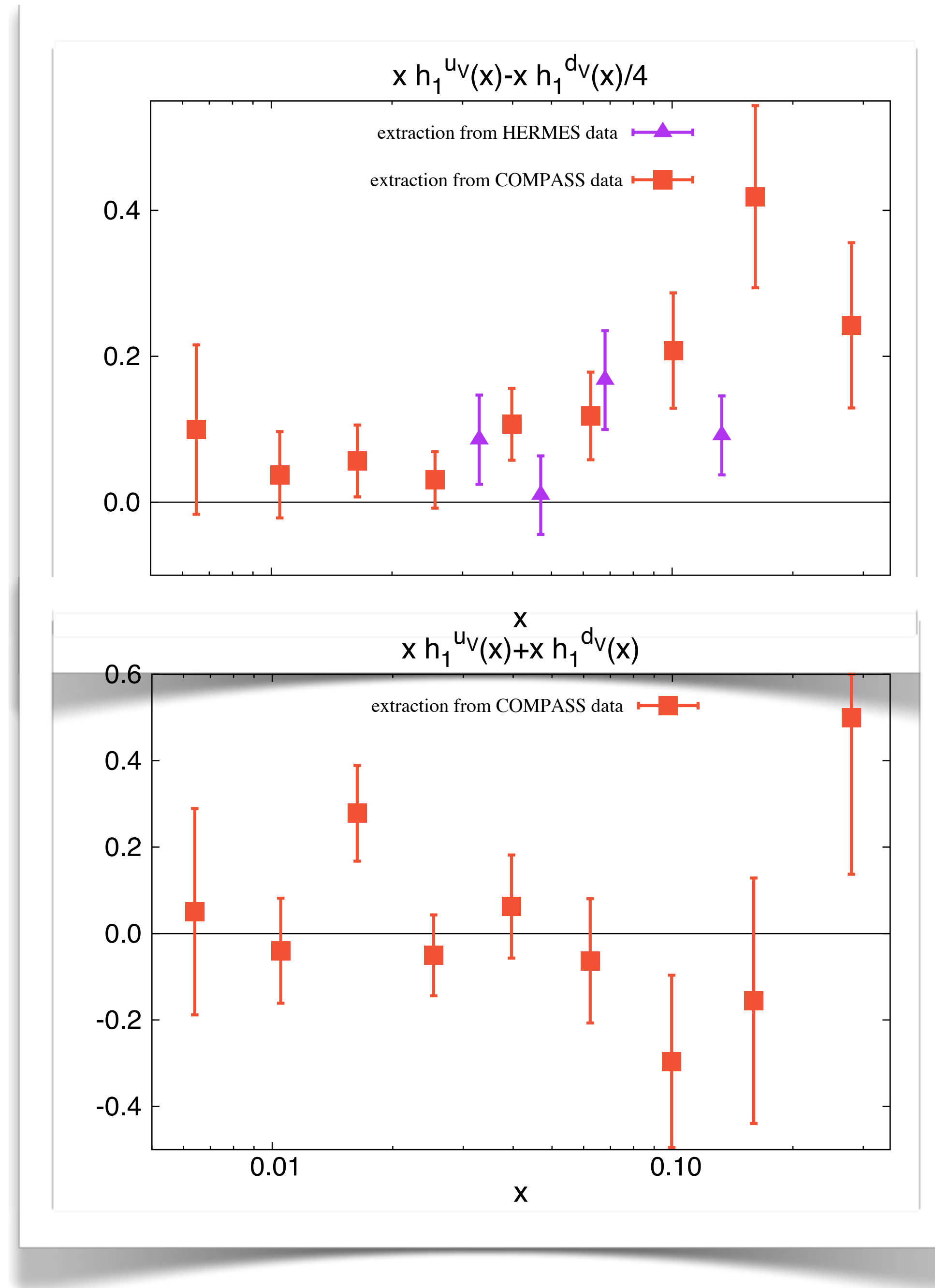
“Torino-Cagliari-JLab extraction”
Torino 09 &1303.3822



“Collinear extraction”
Pavia 11-12

State-of-the-art:
Extractions of transversity





"TMD extraction"

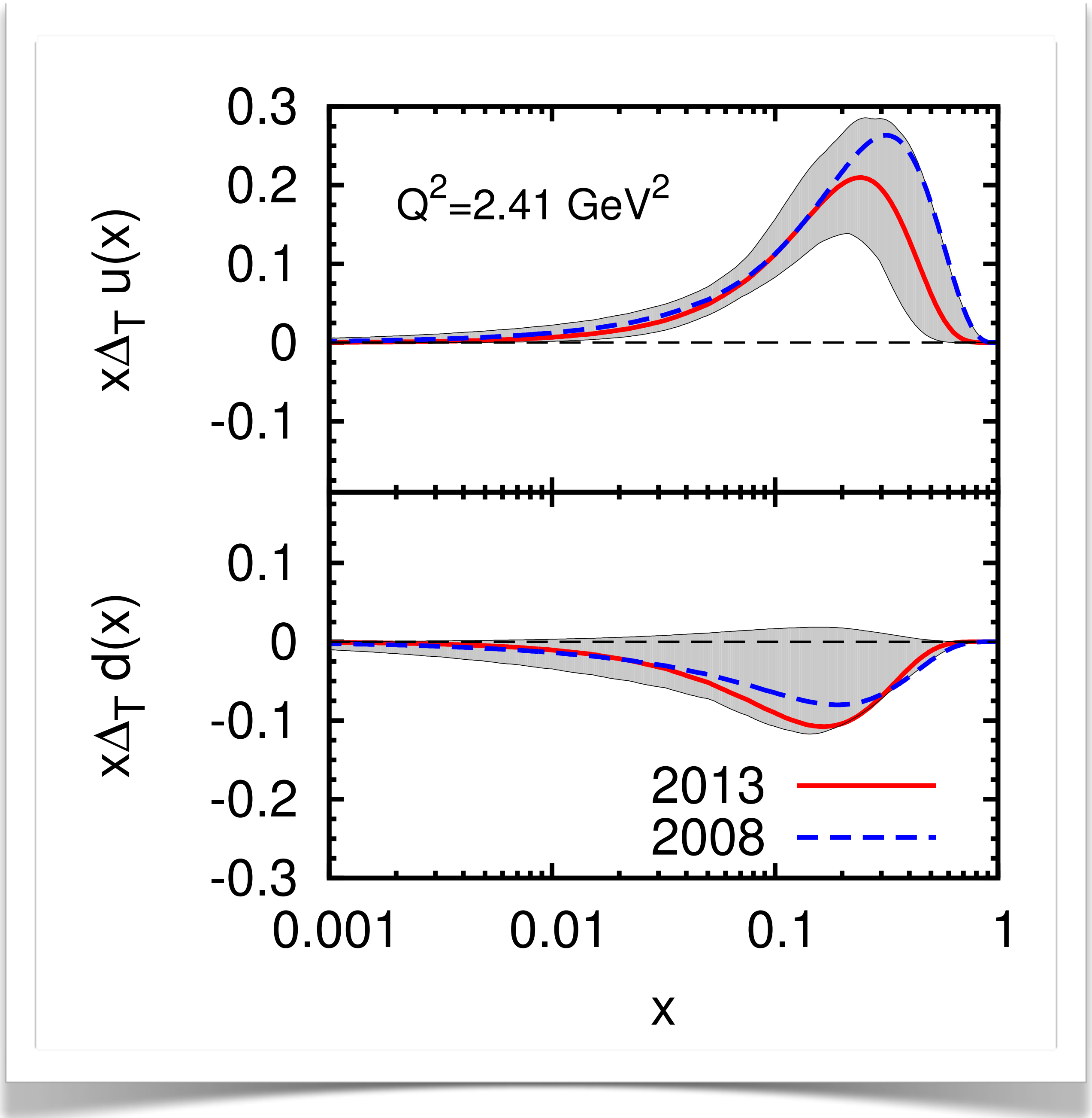
"Collinear extraction"
Pavia 11-12

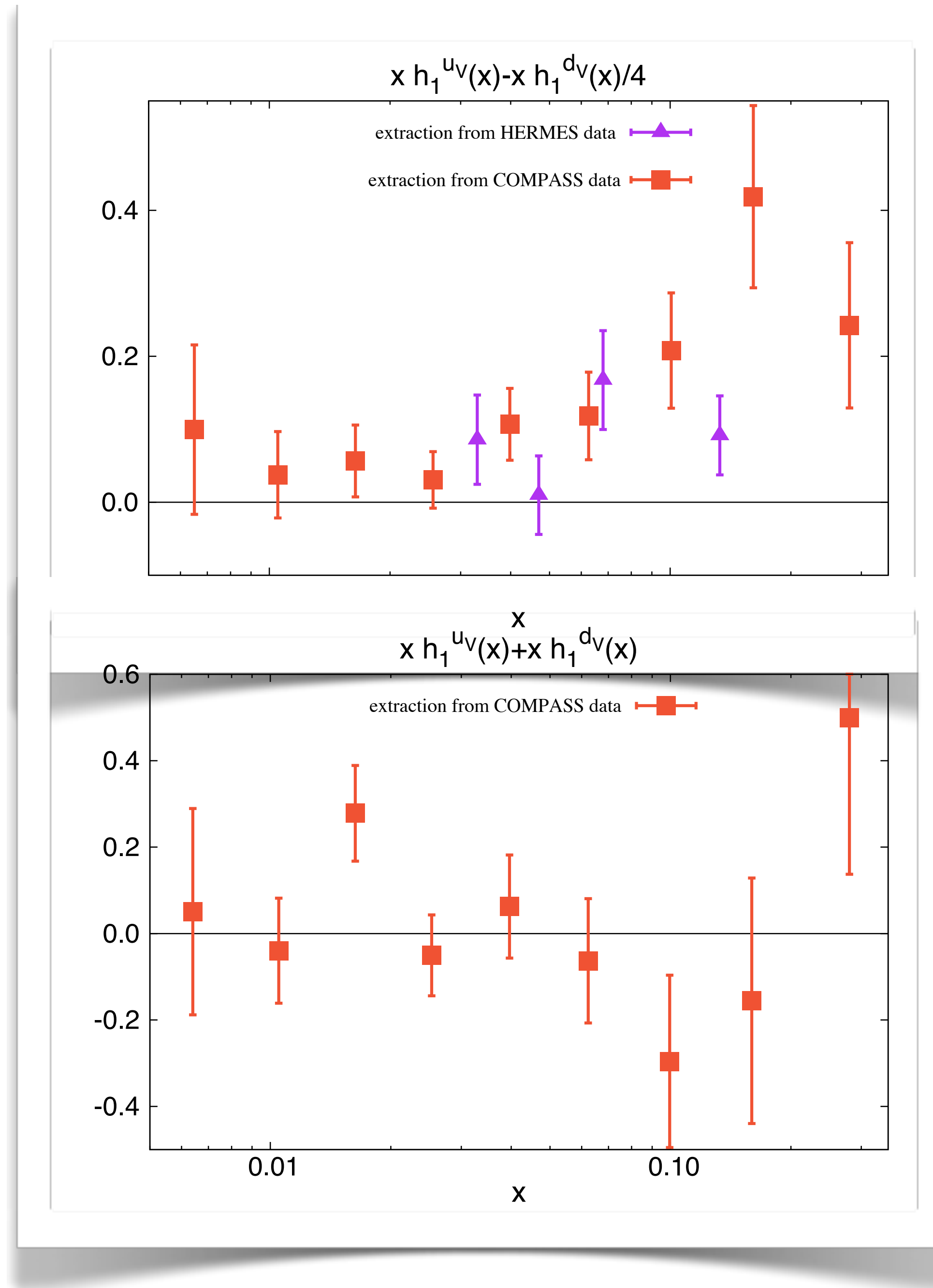
Talk by S. Melis

This talk

"Torino-Cagliari-JLab extraction"
Torino 09 &1303.3822

State-of-the-art:
Extractions of transversity





"TMD extraction"

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Pavia 11-12

This talk

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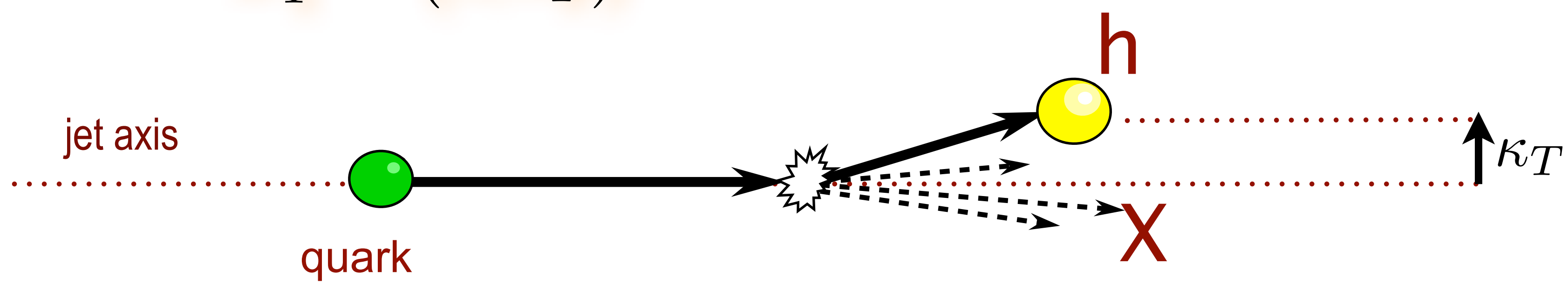
Torino 09 &1303.3822

State-of-the-art: Extractions of transversity

UPDATE "Collinear extraction"
Pavia 13
JHEP 1303 (2013) 119

Dihadron Fragmentation Functions in a nutshell

✦ **TMD FF** $$D_1^{q\to h}\left(z,\kappa_T^2\right)$$



TMD factorization

Dihadron Fragmentation Functions in a nutshell

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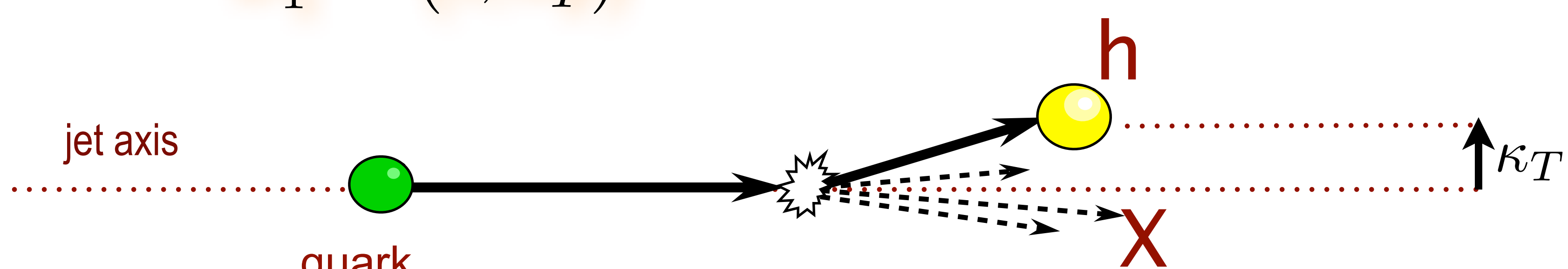
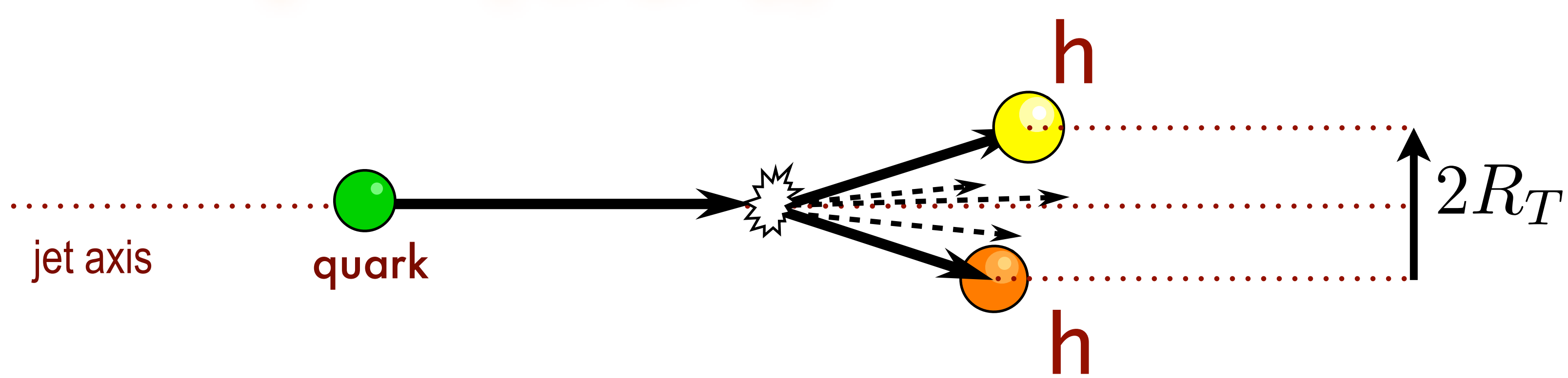


TMD factorization

✦ **DiFF** $D_1^{q\to h_1 h_2}\left(z_1, z_2, R_T^2\right)$



Collinear factorization

Here: $D_1^{q\to\pi^+\pi^-}(z, M_h)$ $\quad z = z_1 + z_2 \qquad 2\,|\mathbf{R}| = \sqrt{M_h^2 - 4m_\pi^2}$

Two complementary approaches

- partner of Collins FF
- convolution

$$\boxed{\int d^2\mathbf{p}_T d^2\mathbf{k}_T \, \delta^2(\mathbf{k}_T + \mathbf{q}_T - \mathbf{p}_T) \, h_1(x, k_T) \, H_1^\perp(z, p_T)}$$

- QCD evolution: TMD evolution
- ongoing progresses

[Rogers, Aybat, Prokudin, Bacchetta,...]

- need input Functional Form of the transversity

- partner of chiral-odd DiFF
- simple product

$$\boxed{h_1(x) \, H_1^{\sphericalangle}(z, M_h)}$$

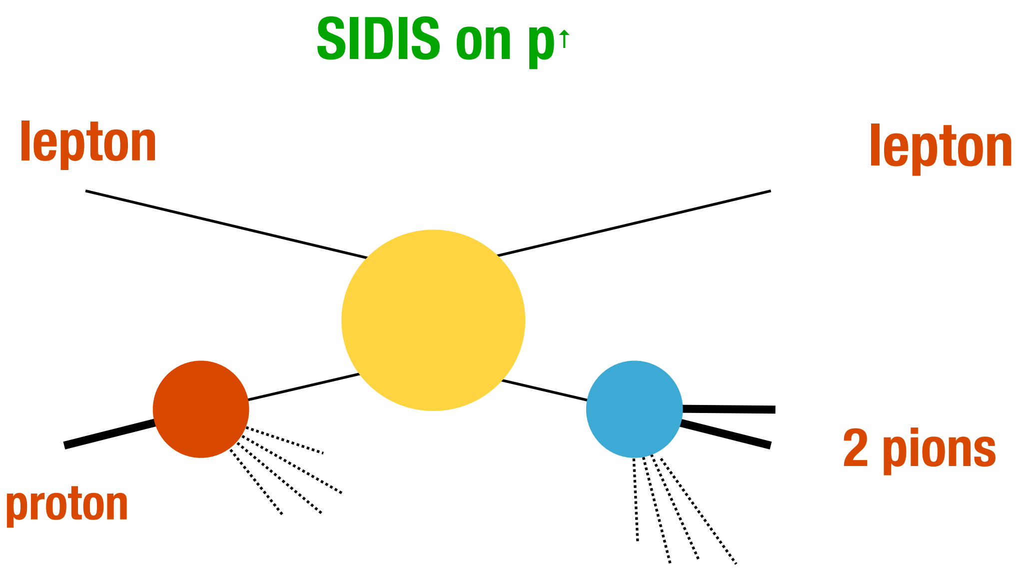
- QCD evolution: DGLAP evolution
- known

[Bacchetta, Radici, Ceccopieri]

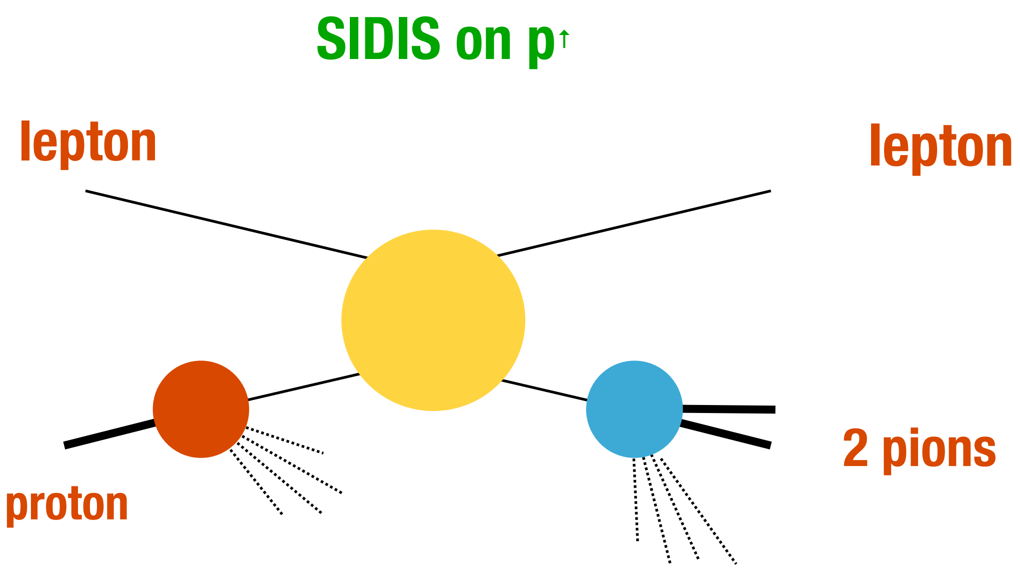
- no need for input Functional Form of the transversity
- direct extraction point by point

Frameworks for DiFFs

Frameworks for DiFFs

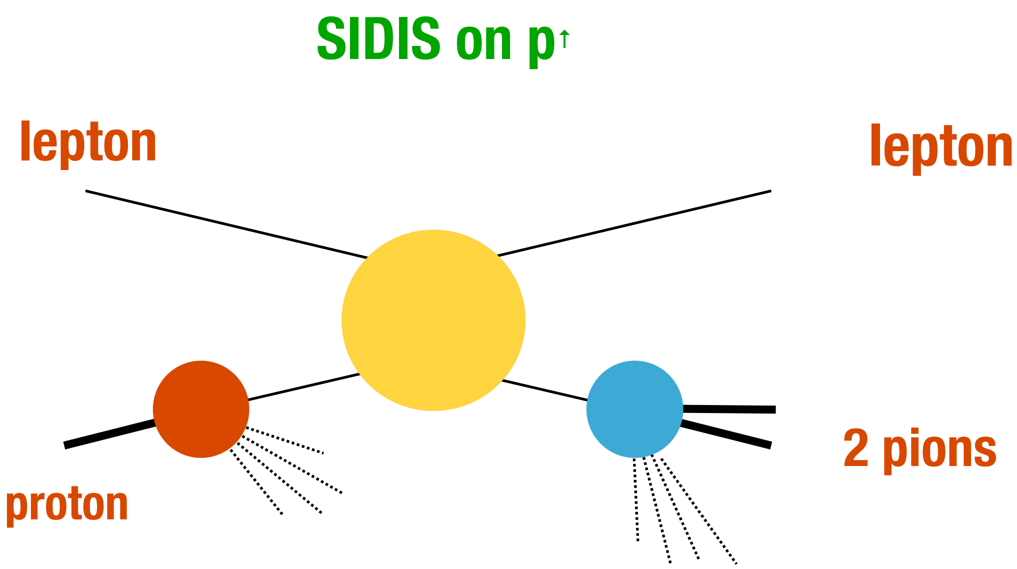


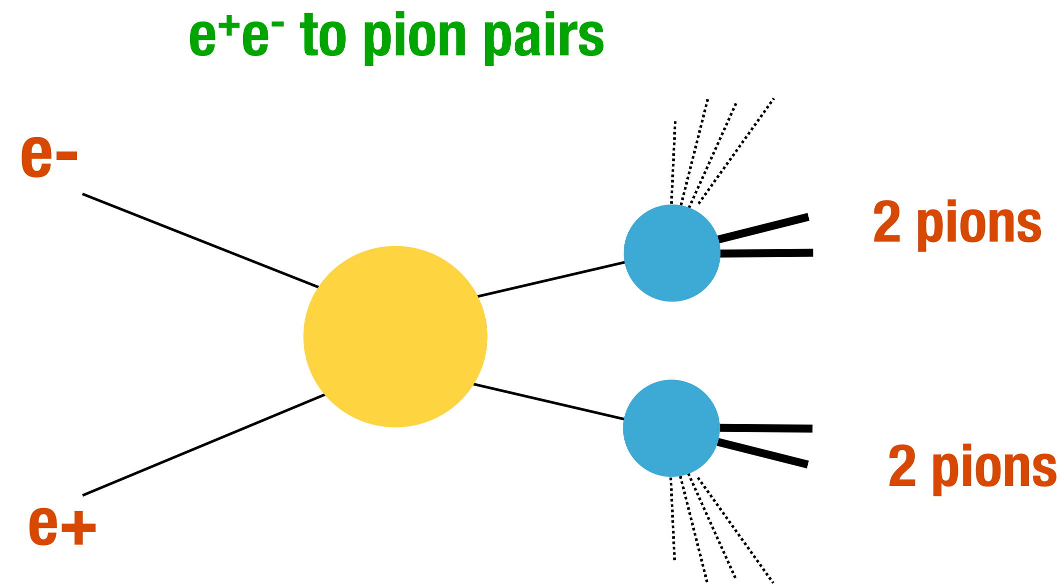
Frameworks for DiFFs



Talks by
N. Makke
C. Braun
S. Gliske

Frameworks for DiFFs





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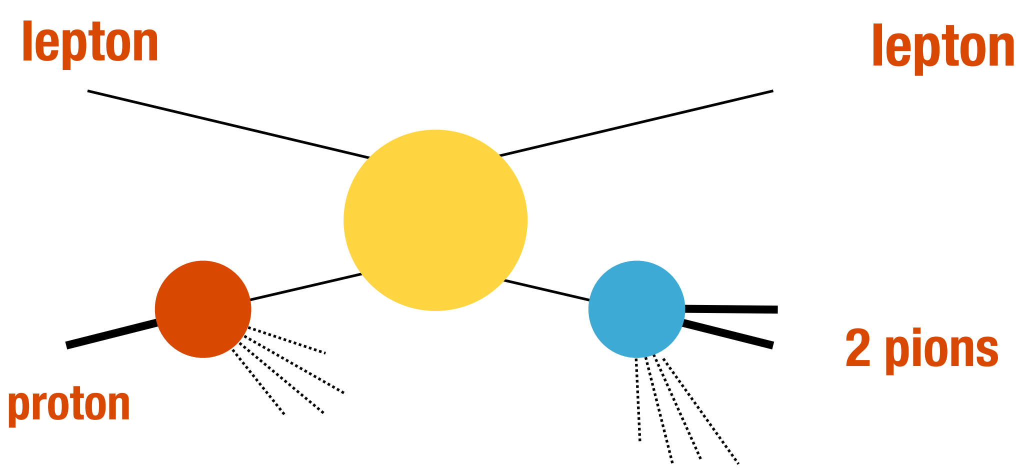
Frameworks for DiFFs

SIDIS on p↑



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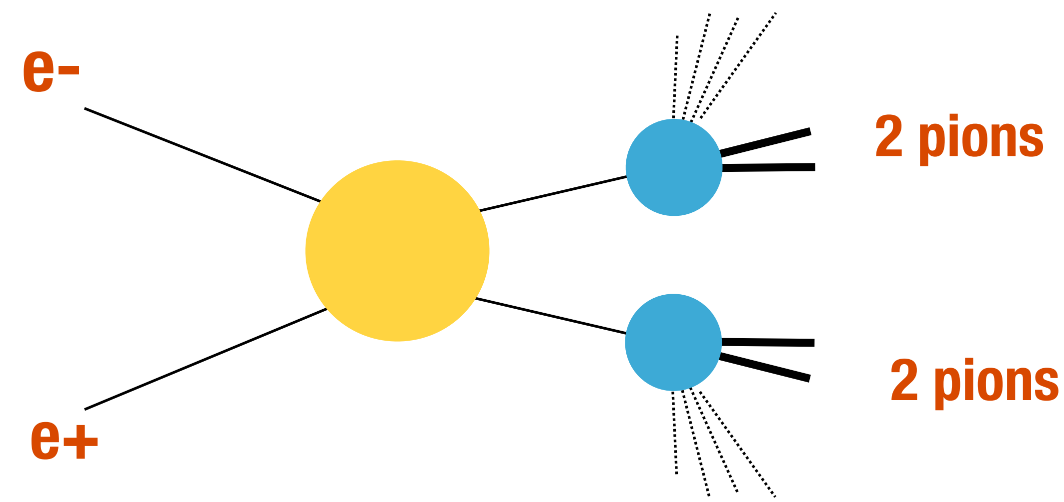
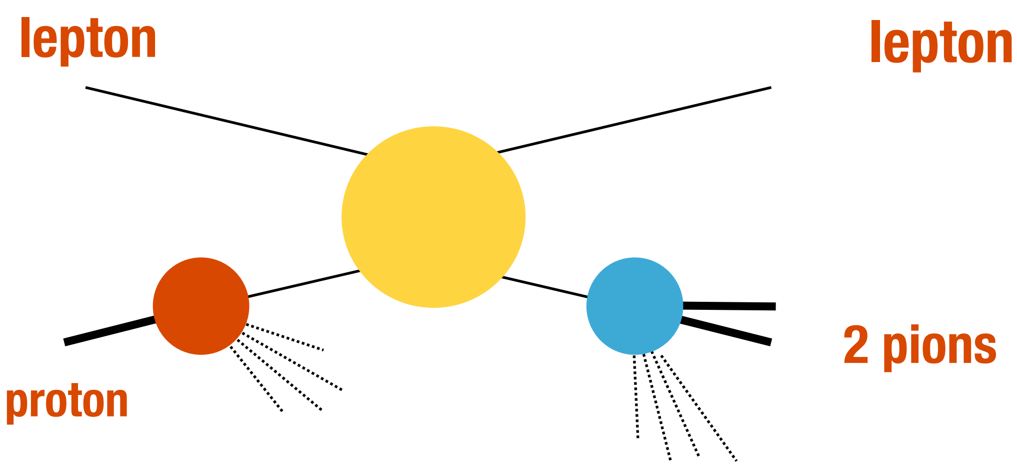
e^+e^- to pion pairs



Talk by
I. Garzia

Frameworks for DiFFs

SIDIS on p↑



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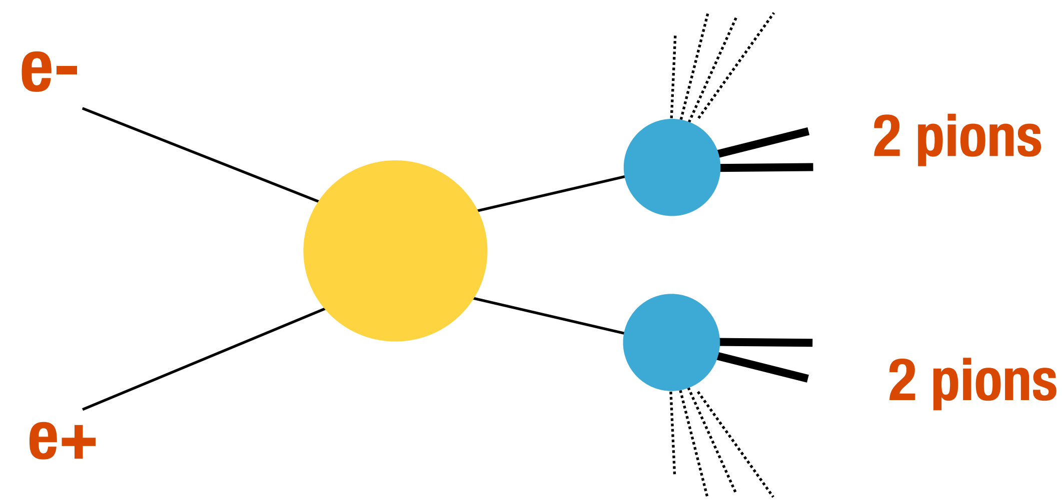

Talk by
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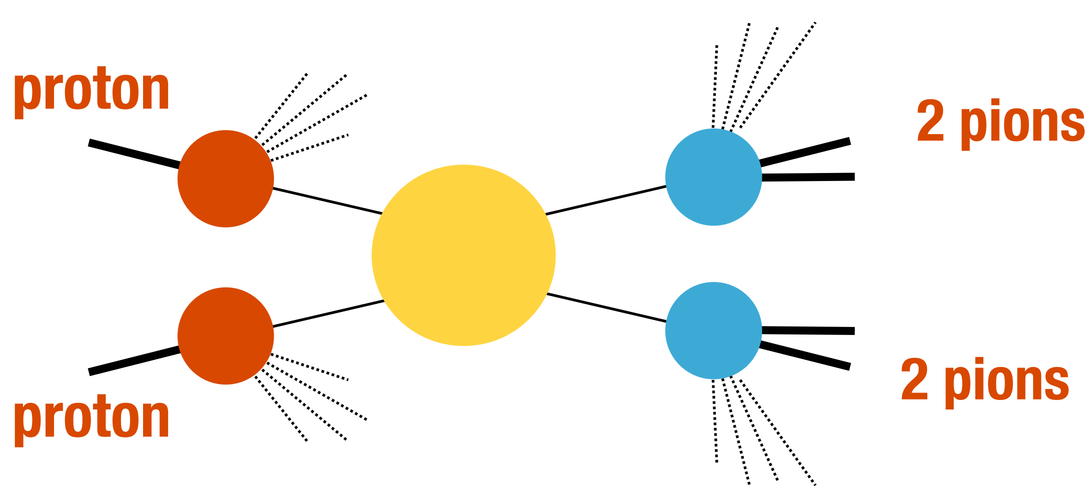
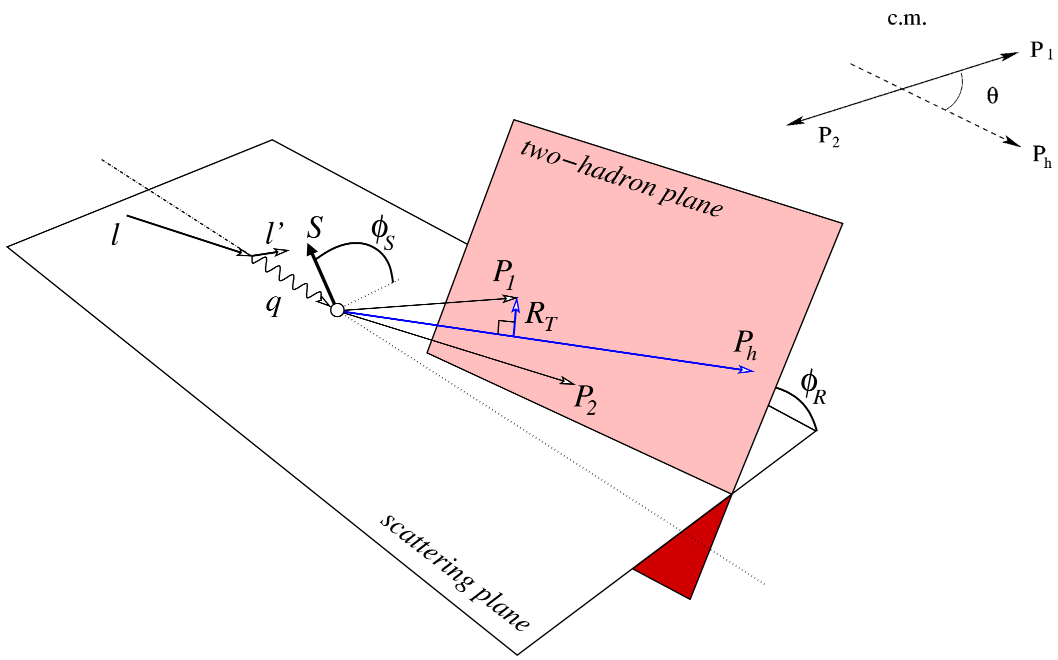
pp↑ to pion pairs



SIDIS production of pion pairs

@ COMPASS & HERMES



Chiral-odd DiFF:

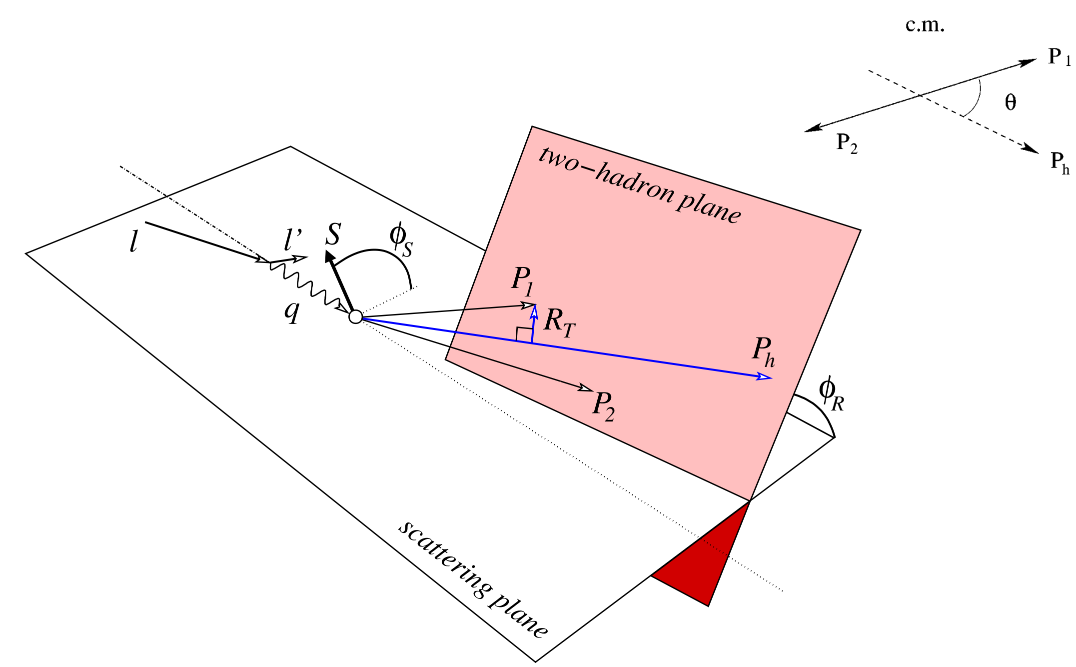
Distribution of hadrons inside the jet
is related to the

Direction of the transverse polarization of the fragmenting quarks

$$A_{\mathrm{DIS}}(x,z,M_h^2,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, \frac{|\bar{R}|}{M_h}\, H_{1,sp}^{q\to\pi^+\pi^-}(z,M_h^2,Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2) \quad D_1^{q\to\pi^+\pi^-}(z,M_h^2,Q^2)}$$

SIDIS production of pion pairs

@ COMPASS & HERMES



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$$A_{\mathrm{DIS}}(x,z,M_h^2,Q^2) = -C_y \frac{\sum_q e_q^2\, \boxed{h_1^q(x,Q^2)}}{\sum_q e_q^2\, f_1^q(x,Q^2)} \boxed{\frac{\frac{|\bar{R}|}{M_h} H_{1,sp}^{q\to\pi^+\pi^-}(z,M_h^2,Q^2)}{D_1^{q\to\pi^+\pi^-}(z,M_h^2,Q^2)}}$$

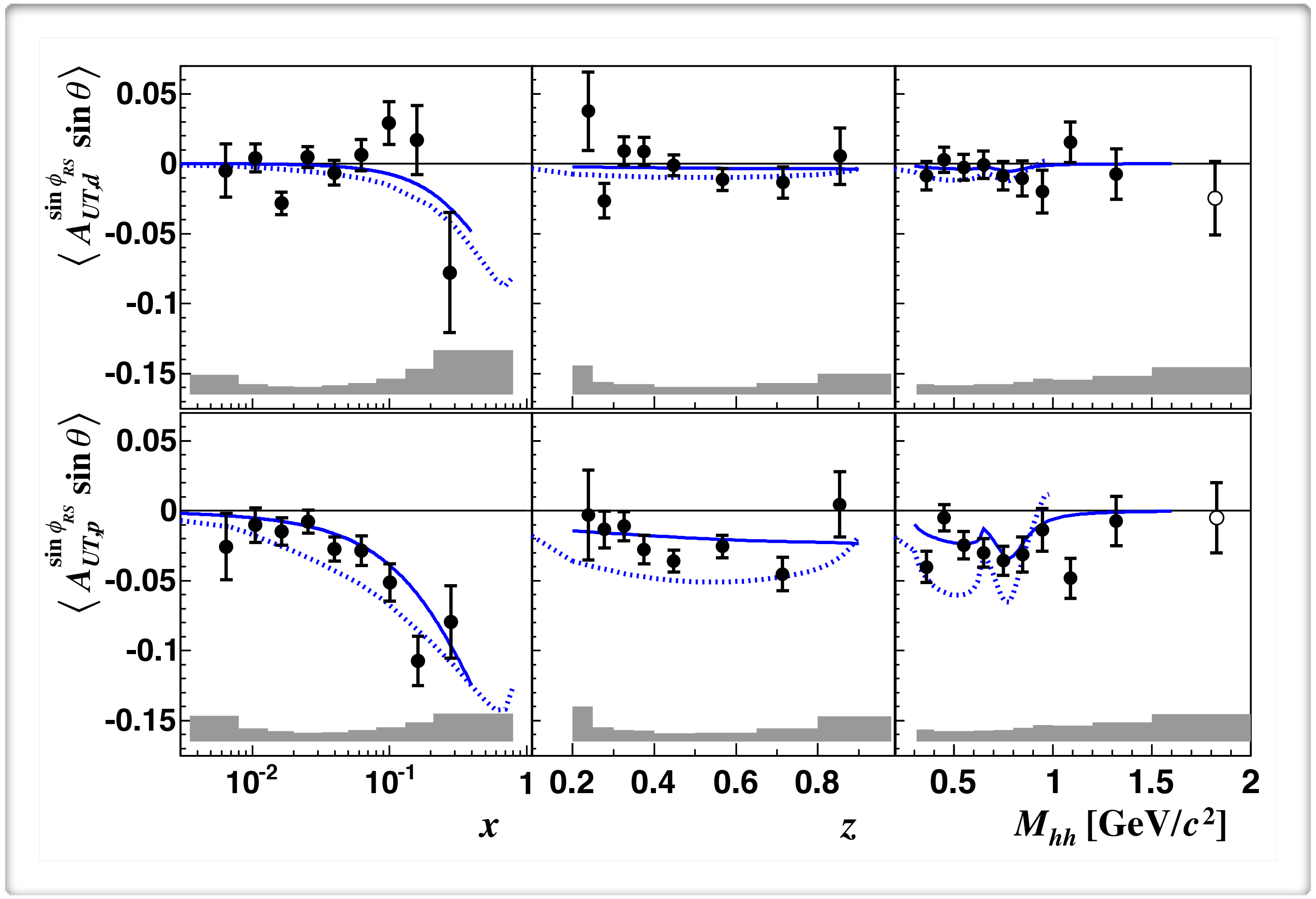
Knowledge on DiFFs leads to $h_1(x, Q^2)$

SIDIS production of pion pairs

@ COMPASS & HERMES

2002-4 Deuteron Data

2007 Proton Data



SIDIS production of pion pairs

@ COMPASS & HERMES

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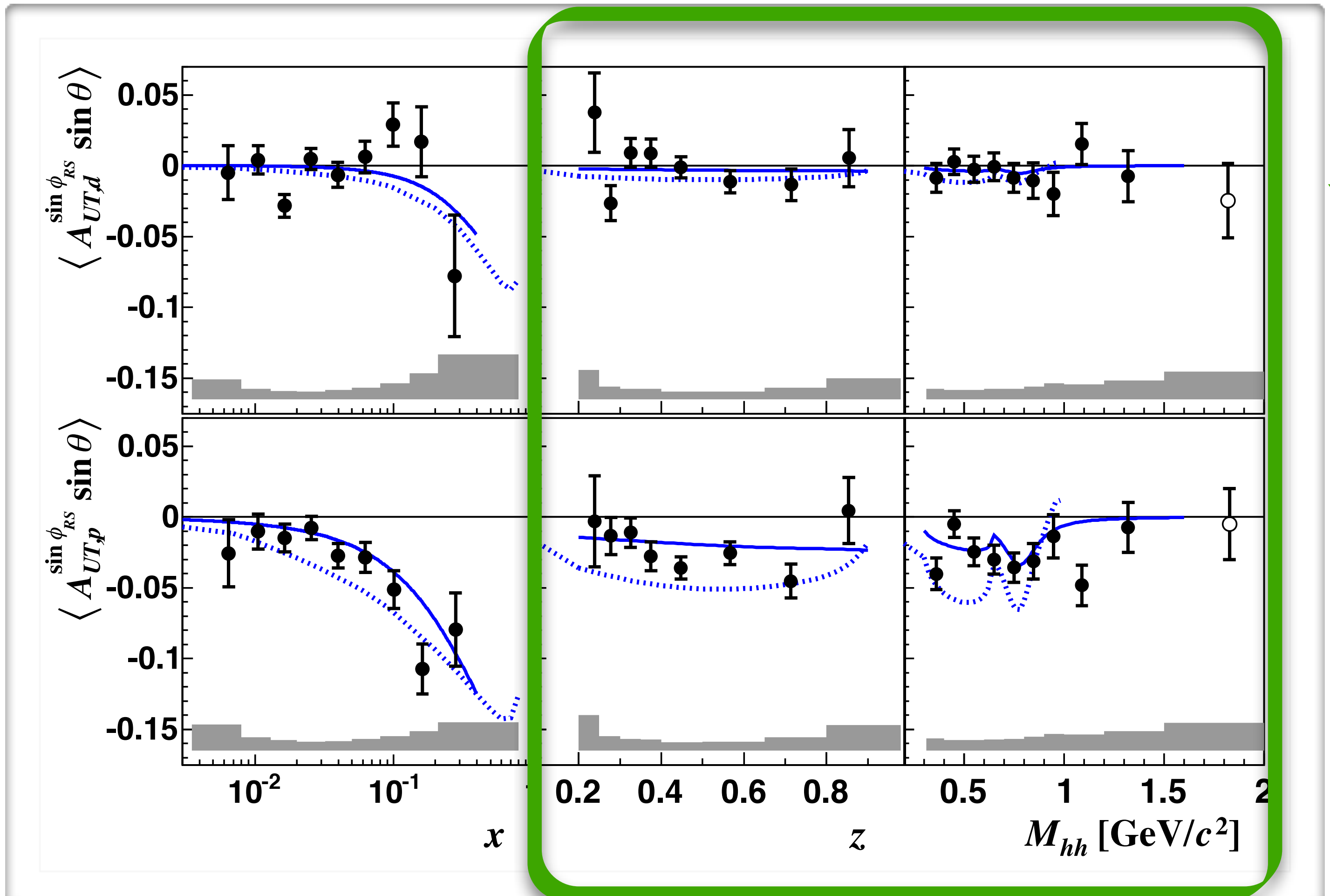

(z, M_h)-dpdence determined by DiFF from Belle

[A.C., Bacchetta, Radici, Bianconi, Phys.Rev. D85]

SIDIS production of pion pairs

@ COMPASS & HERMES

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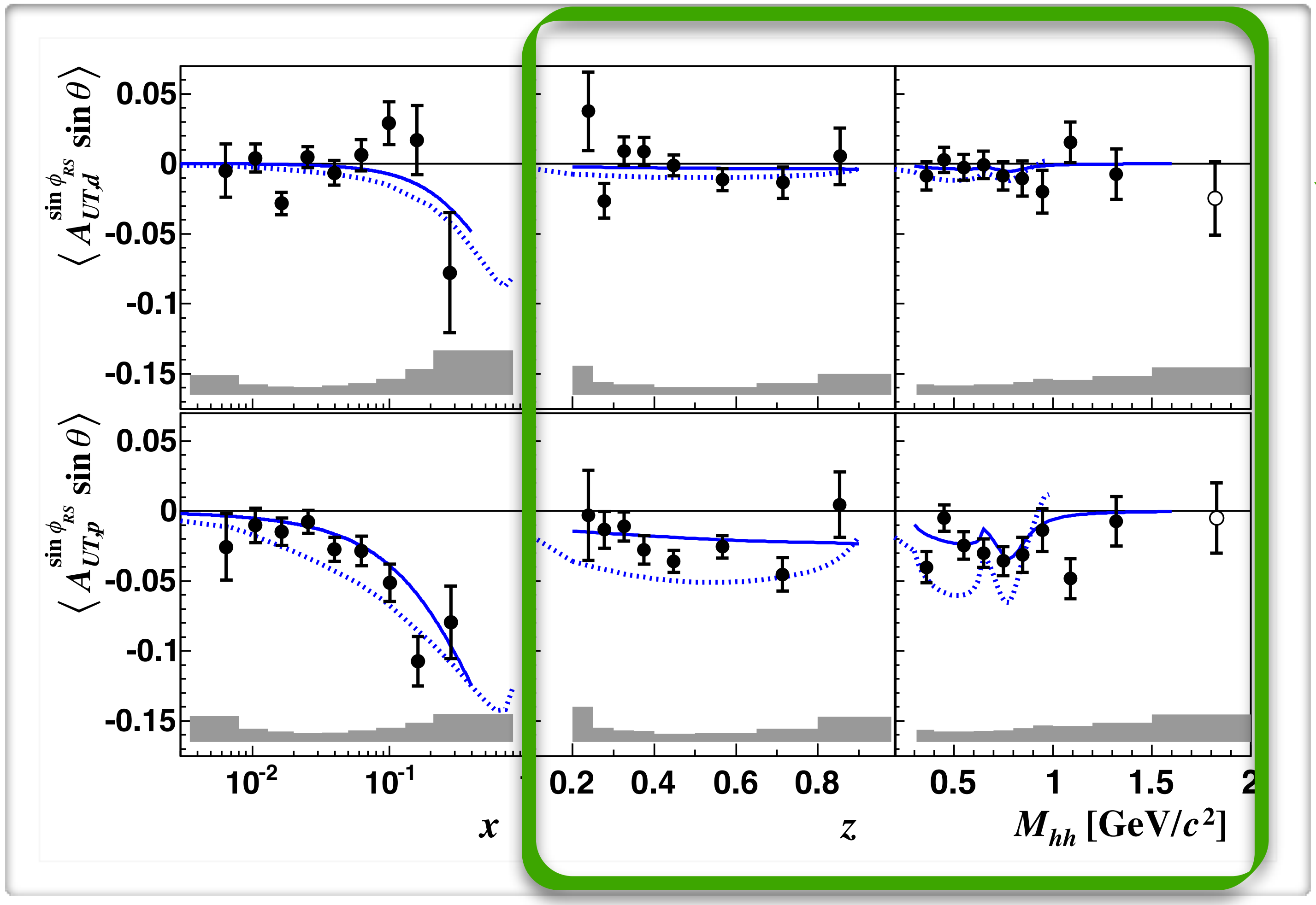


(z, M_h)-dpdence determined by DiFF from Belle

[A.C., Bacchetta, Radici, Bianconi, Phys.Rev. D85]

COMPASS range: $0.2<z<1$ & $0.29<M_h<1.29$ GeV

$$n_q(Q^2) = \int dz dM_h \, D_1^q(z, M_h; Q^2)$$

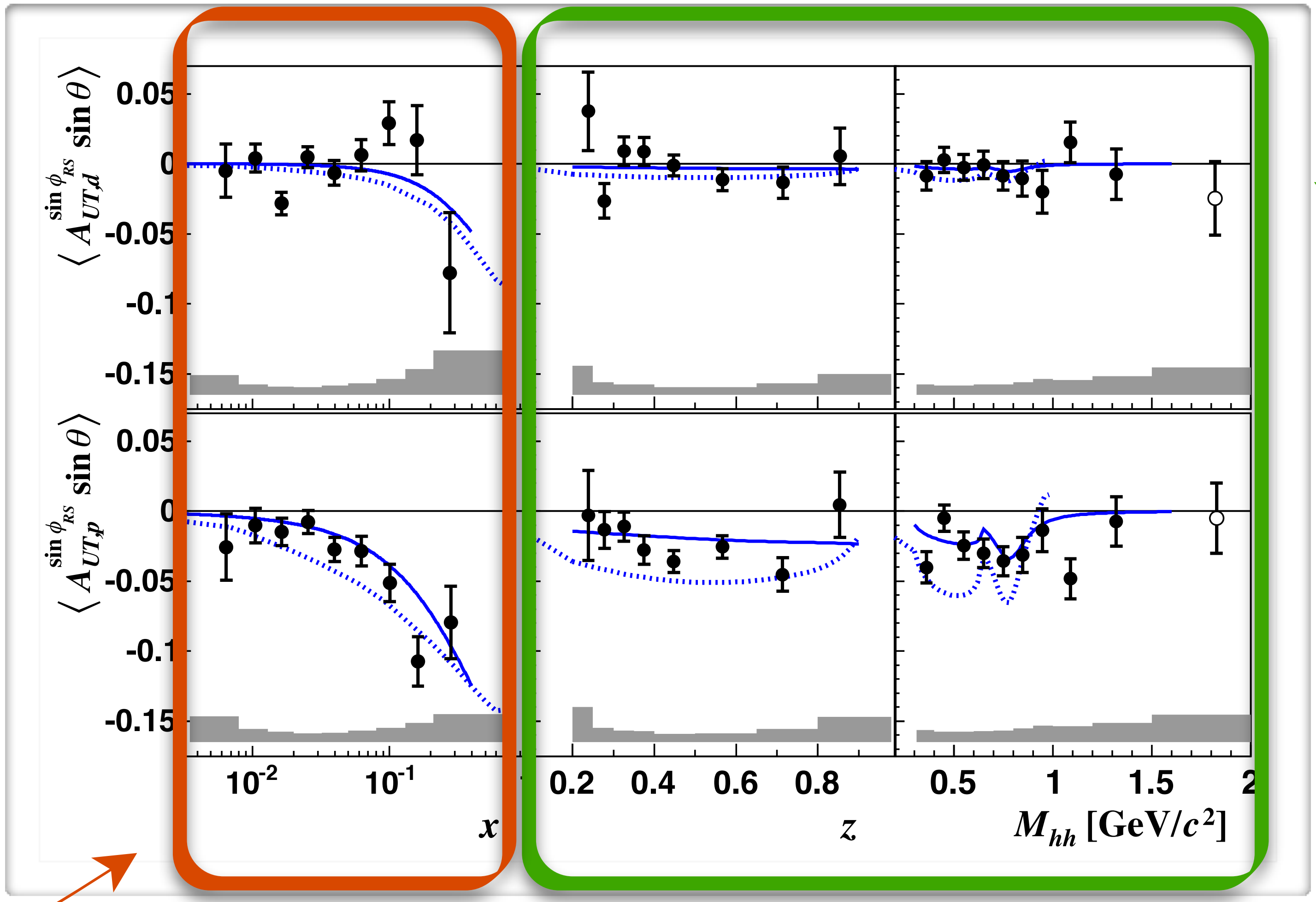
$$n_q^{\uparrow}(Q^2) = \int dz dM_h \, \frac{|\mathbf{R}|}{M_h} \, H_{1,sp}^{\sphericalangle q}(z, M_h; Q^2)$$

SIDIS production of pion pairs

@ COMPASS & HERMES

2002-4 Deuteron Data

2007 Proton Data



(z, M_h)-dpdence determined by DiFF from Belle

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x-dependence only from Transversity

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Transversity from $A_{UT}^{\sin(\Phi_R+\Phi_S)\sin\theta}$

$$A_{\rm DIS}(x,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\, n_q(Q^2)}$$

Using symmetries for DiFFs:

$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -\overline{H}_1^{\sphericalangle,u} = \overline{H}_1^{\sphericalangle,d}$$

$$D_1^u = D_1^d = \overline{D}_1^u = \overline{D}_1^d\,;\quad D_1^s = \overline{D}_1^s\,,\quad D_1^c = \overline{D}_1^c$$

Proton

$$x h_1^{u_v}(x,Q^2) - \tfrac{1}{4}\, x h_1^{d_v}(x,Q^2) \propto -A_{\rm DIS}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

Deuteron

$$x h_1^{u_v}(x,Q^2) + x h_1^{d_v}(x,Q^2) \propto -\frac{5}{3} A_{\rm DIS}(x,Q^2)\frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)}\, x\left(f_1^{u+\bar{u}} + f_1^{d+\bar{d}} + \frac{2}{5} f_1^{s+\bar{s}}\right)$$

and combinations of both ...

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Deuteron

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and combinations of both ...

We take results for our analysis
from pion pair production in e^+e^- annihilation at Belle

Transversity from e p$^{\uparrow}$→e' ($\pi^+\pi^-$) X @ HERMES

$$xh_1^{u_v}(x,Q^2) - \frac{1}{4}\, xh_1^{d_v}(x,Q^2) = -C_y^{-1}\, A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

with 1-to-100 GeV^2 evolution correction:
small corrections

HERMES range: -0.259^{-1} (± 25% theo. err.) from fit

integrated in mean values

Transversity from e p$^{\uparrow}\to$e' ($\pi^+\pi^-$) X @ HERMES

$$\boxed{x h_1^{u_v}(x,Q^2) - \frac{1}{4}\, x h_1^{d_v}(x,Q^2) = -C_y^{-1}\, A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)}$$

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Transversity from e p$^{\uparrow}\to$e' ($\pi^+\pi^-$) X @ COMPASS 2007

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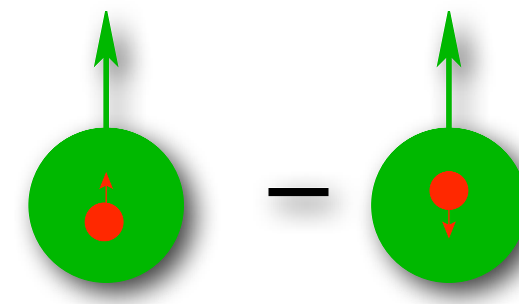
with 1-to-100 GeV^2 evolution correction:
negligible corrections

COMPASS range: -0.208^{-1} **(± 19% theo. err.)** **from fit**

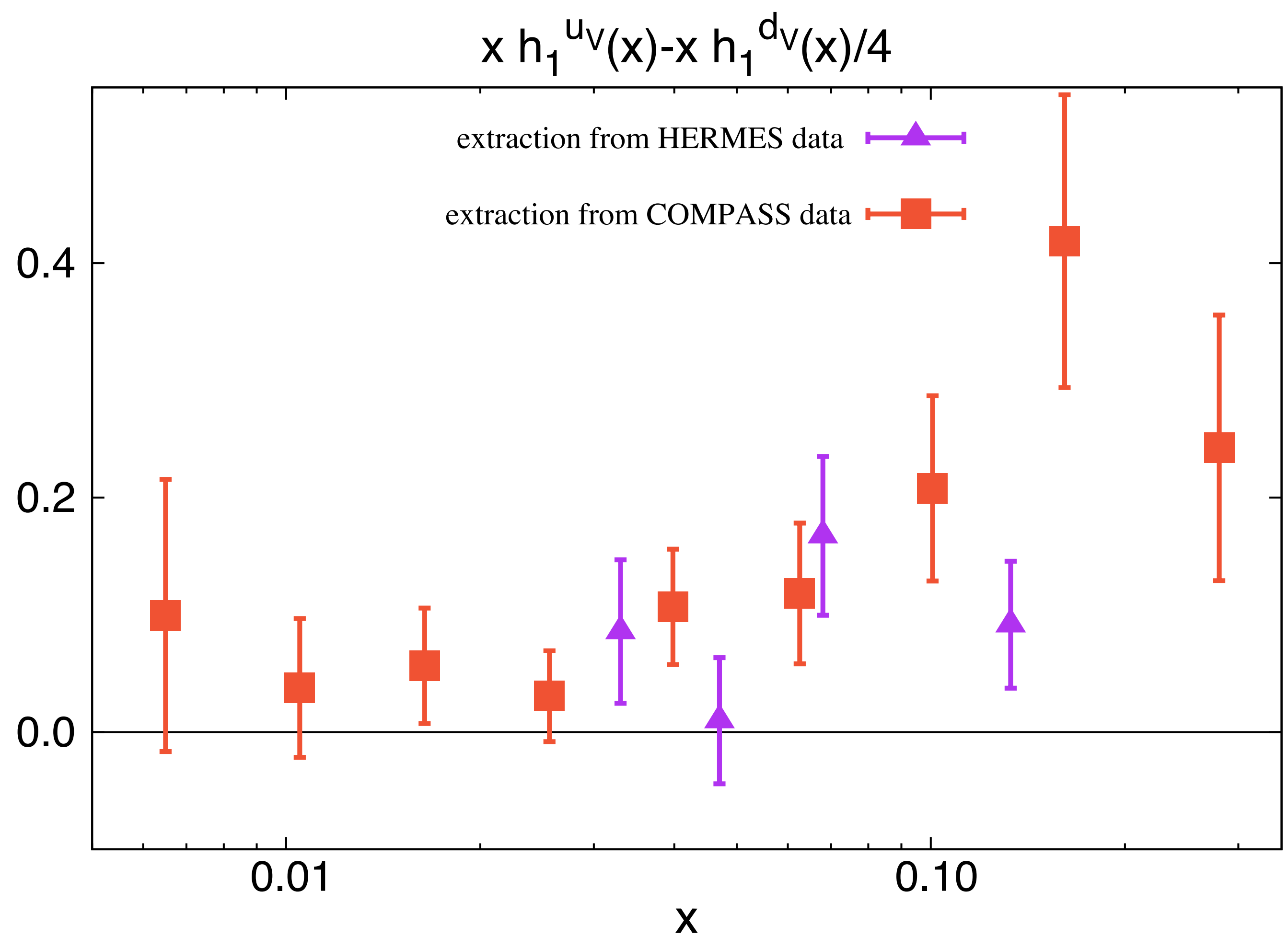
Transversity from Proton data



Transversity from pion pair production SIDIS off transversely polarized target

- from HERMES data
- DiFF analysis

  point by point from fit
- PRL 107



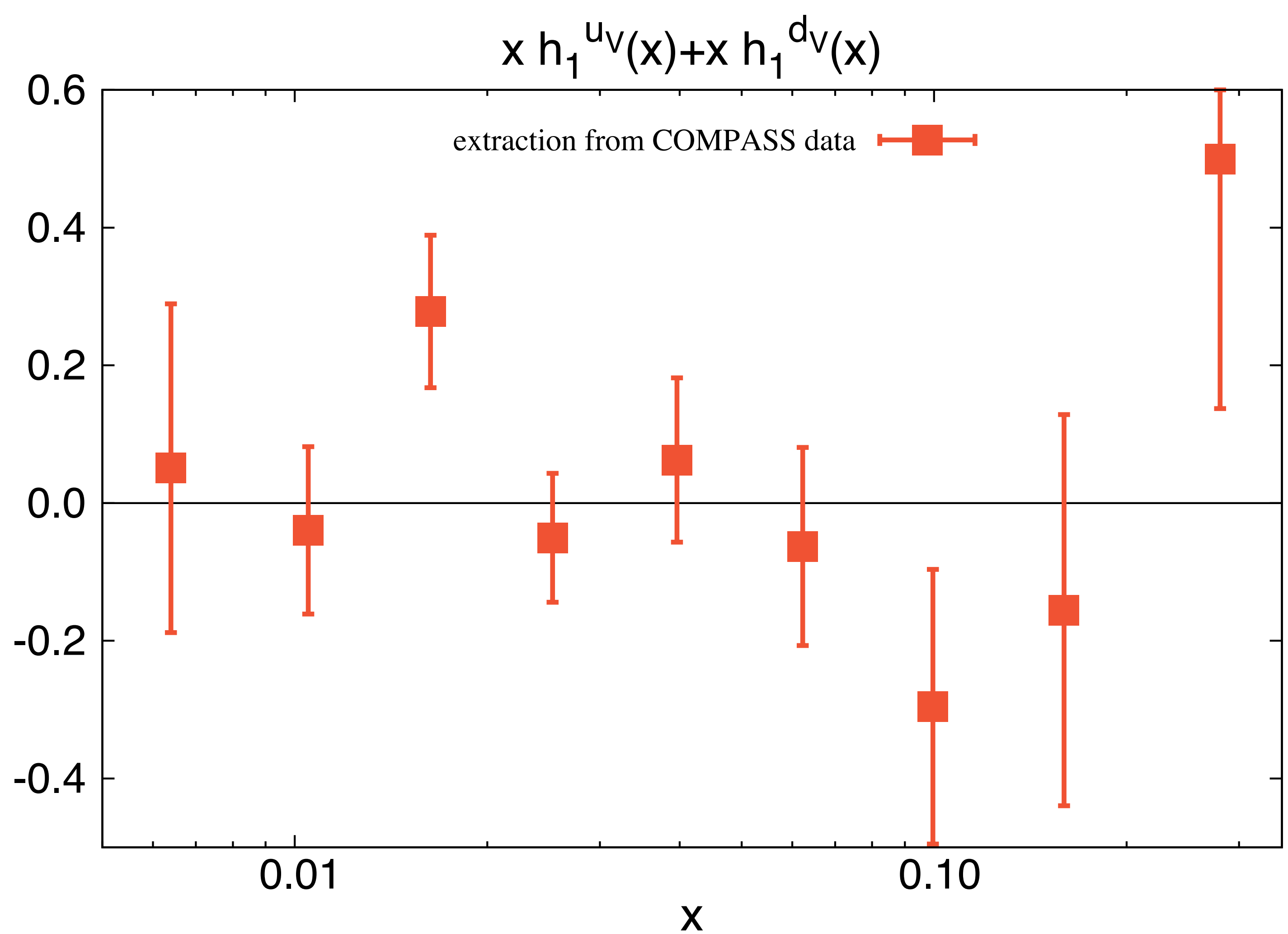
- from COMPASS data
- DiFF analysis

  point by point from fit
- JHEP 1303

$f_1(x)$ from MSTW08

Transversity from Deuteron data

COMPASS 2002-2004



- from COMPASS data
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- JHEP 1303

$f_1(x)$ from MSTW08

Fitting the Valence Transversities

Fitting the Valence Transversities

Constraints from first principles

- ✦ **Soffer bound**

$$2|h_1^q(x,Q^2)| \leq |f_1^q(x,Q^2) + g_1^q(x,Q^2)| \equiv 2\,\mathrm{SB}^q(x,Q^2)$$

- ✦ **$h_1(x=1)=0$** ; the parton model predicts $h_1(x=0)=0$ but too restrictive in QCD

Fitting the Valence Transversities

Constraints from first principles

- ✦ Soffer bound

$$2|h_1^q(x,Q^2)| \leq |f_1^q(x,Q^2) + g_1^q(x,Q^2)| \equiv 2\,\mathrm{SB}^q(x,Q^2)$$

- ✦ $h_1(x=1)=0$; the parton model predicts $h_1(x=0)=0$ but too restrictive in QCD

QCD evolution with HOPPET code

- ✦ of the Soffer bound: LO evolution of $f_1(x)$ from MSTW08 & $g_1(x)$ from DSS

- ✦ of the DiFF & h_1: LO as in previous papers

Fitting the Valence Transversities

Constraints from first principles

- ✦ Soffer bound $$2|h_1^q(x,Q^2)| \leq |f_1^q(x,Q^2) + g_1^q(x,Q^2)| \equiv 2\,\mathrm{SB}^q(x,Q^2)$$
- ✦ $h_1(x=1)=0$; the parton model predicts $h_1(x=0)=0$ but too restrictive in QCD

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- ✦ of the DiFF & h_1: LO as in previous papers

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Fitting the Valence Transversities

Constraints from first principles

✦ Soffer bound

$$2|h_1^q(x,Q^2)| \leq |f_1^q(x,Q^2) + g_1^q(x,Q^2)| \equiv 2\,\mathrm{SB}^q(x,Q^2)$$

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with FF defined [-1,1]

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The Functional Form

@ Q_0^2

$$x\, h_1^{q_V}(x) = \tanh\left(x^{1/2}\,(A_q + B_q\,x + C_q\,x^2 + D_q\,x^3)\right)\;\left(x\,\mathrm{SB}^q(x) + x\,\mathrm{SB}^{\bar{q}}(x)\right)$$

1st order polynomial

$$A_q + B_q\,x$$

2nd order polynomial

$$A_q + B_q\,x + C_q\,x^2$$

3rd order polynomial

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judicious choice for integrability of the transversities

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Rigid version

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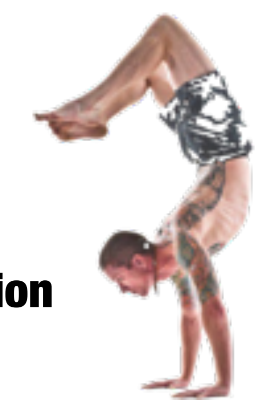

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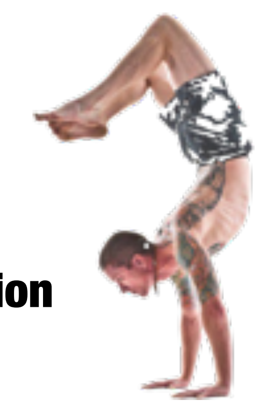

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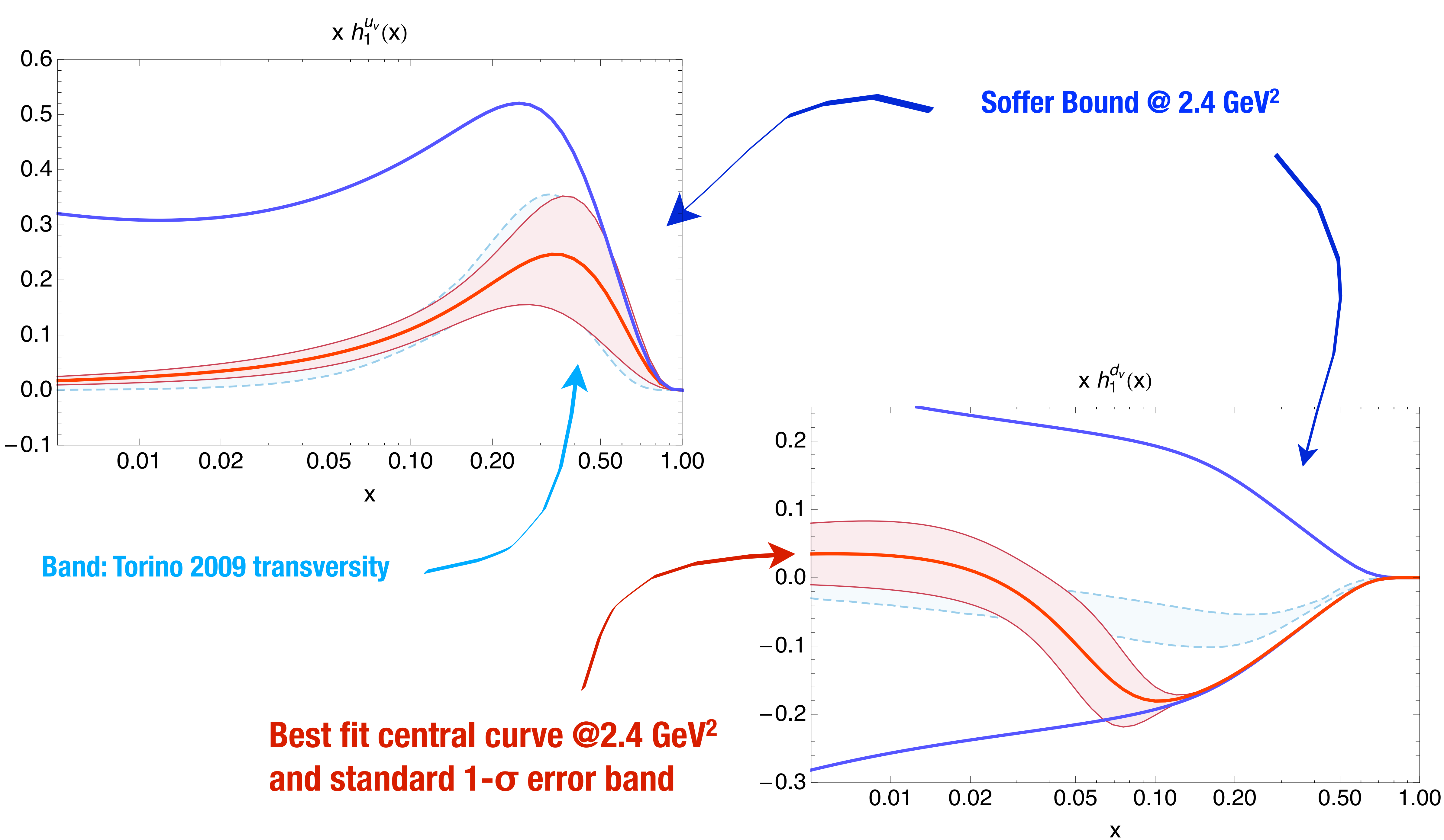
Extra-flexible version

Our Rigid Functional Form *1st order polynomial*



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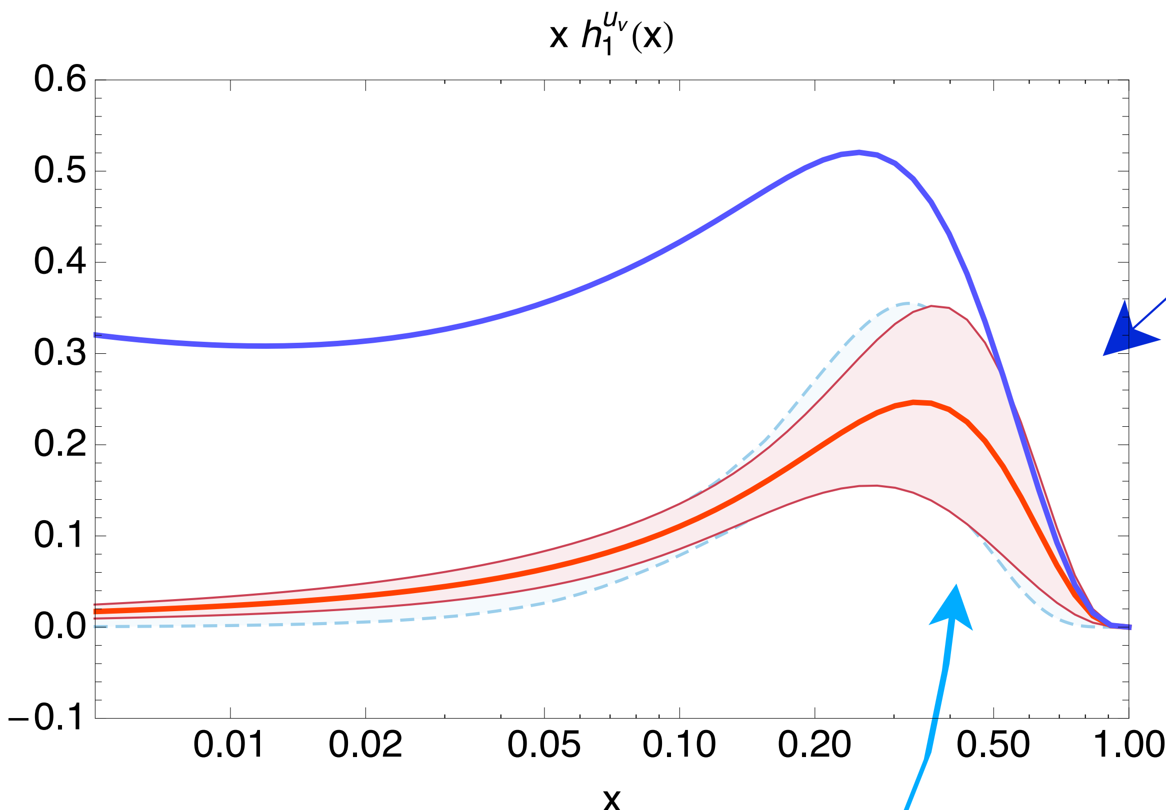
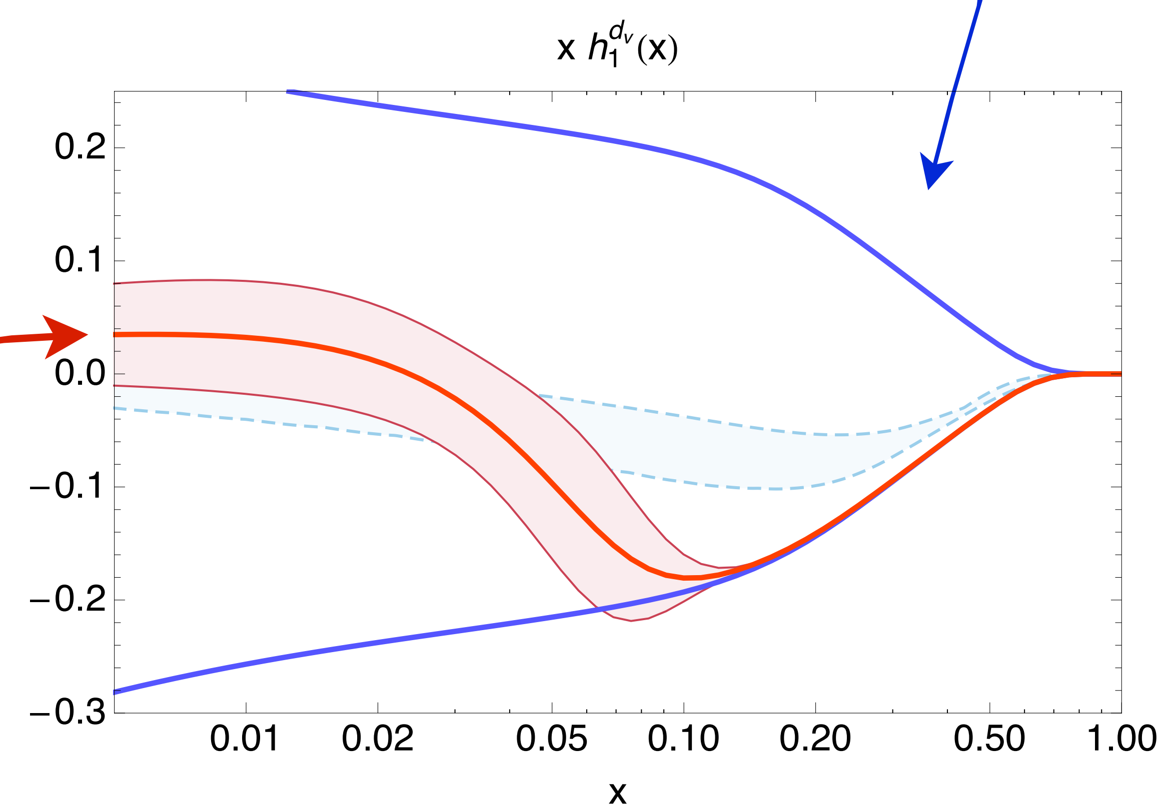


Soffer Bound @ 2.4 GeV^2



Band: Torino 2009 transversity

Best fit central curve @2.4 GeV^2
and standard 1-σ error band

Rigid version

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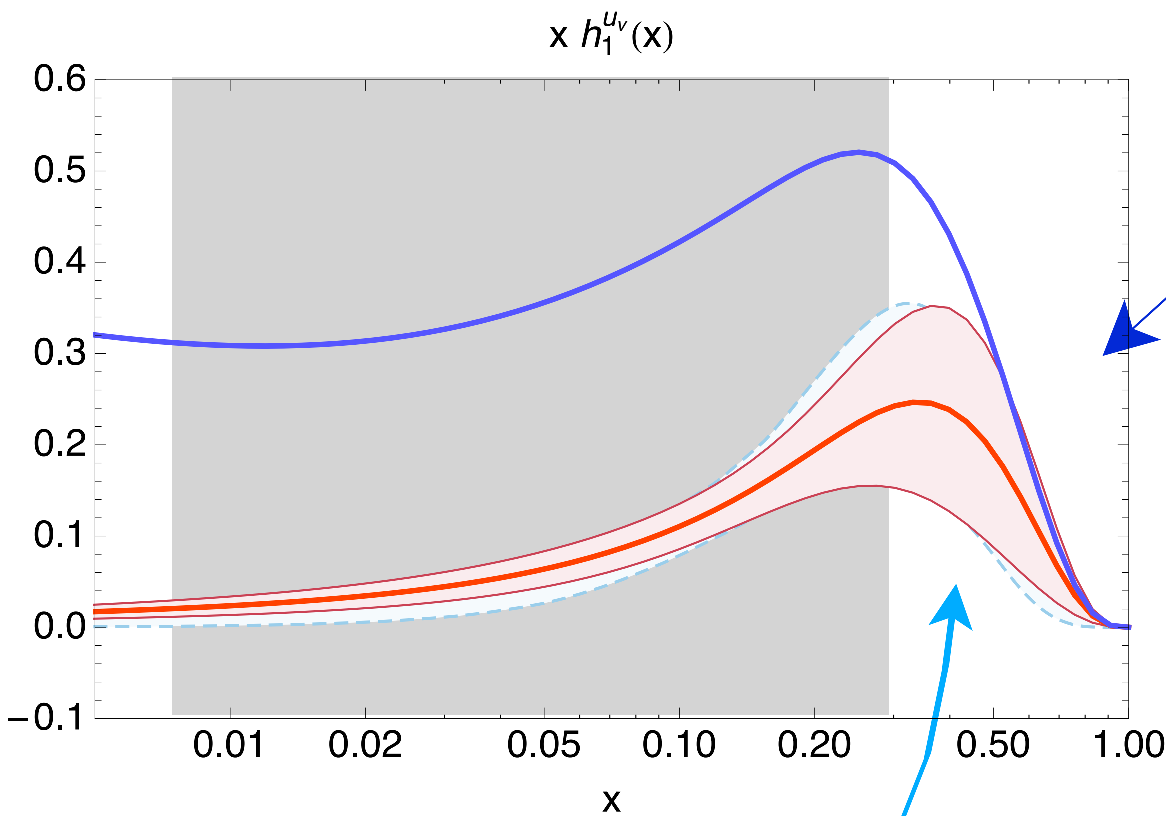

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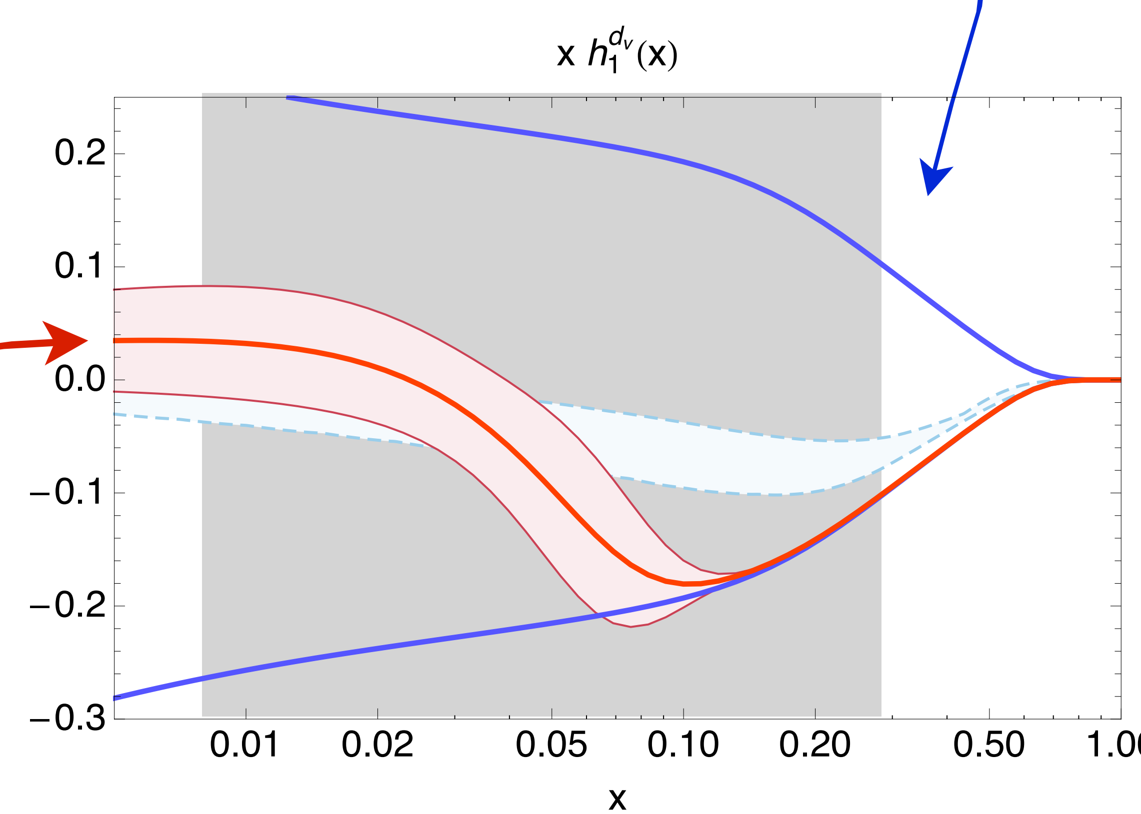

Best fit central curve @2.4 GeV^2
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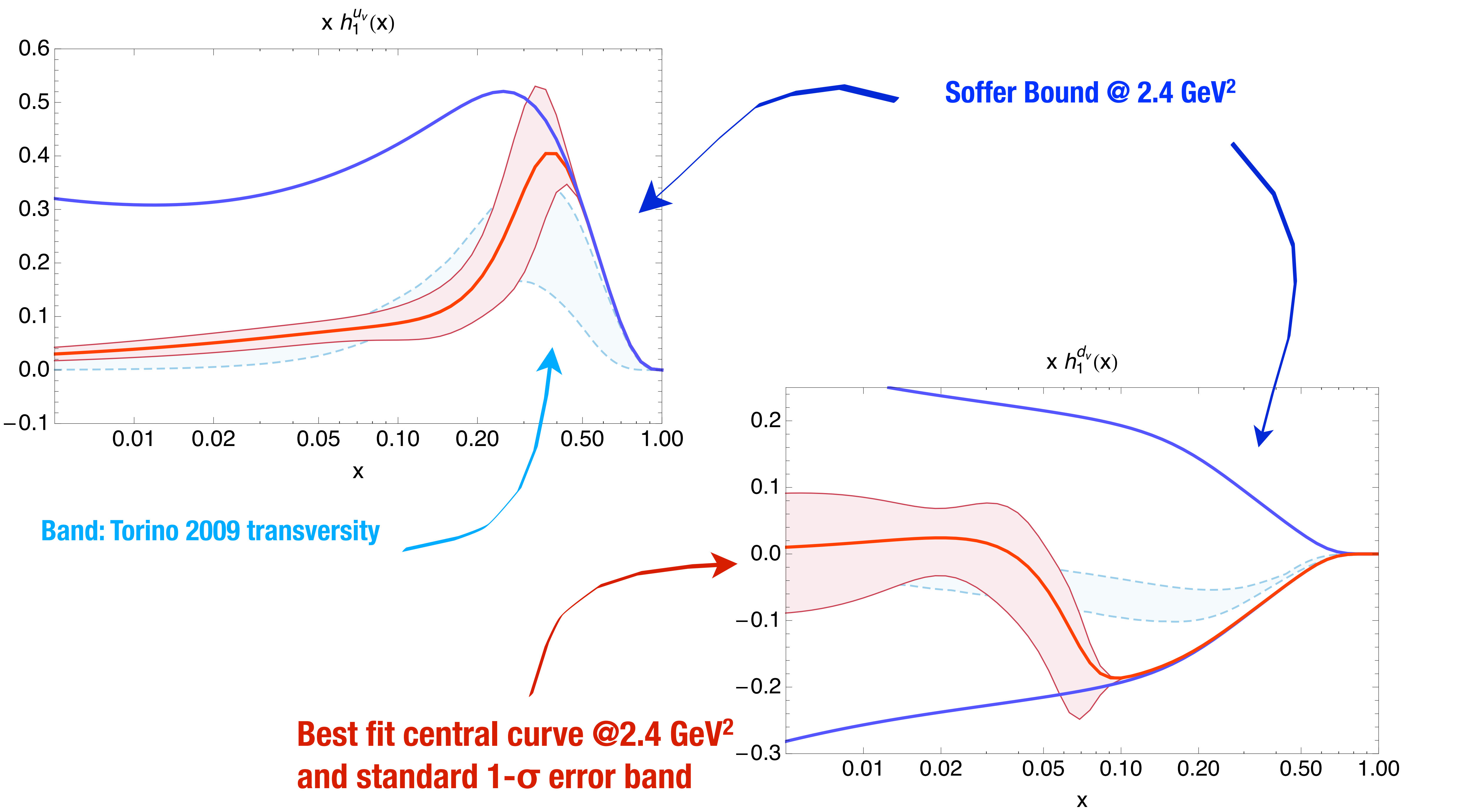
Rigid version

Our Flexible Functional Form *2nd order polynomial*



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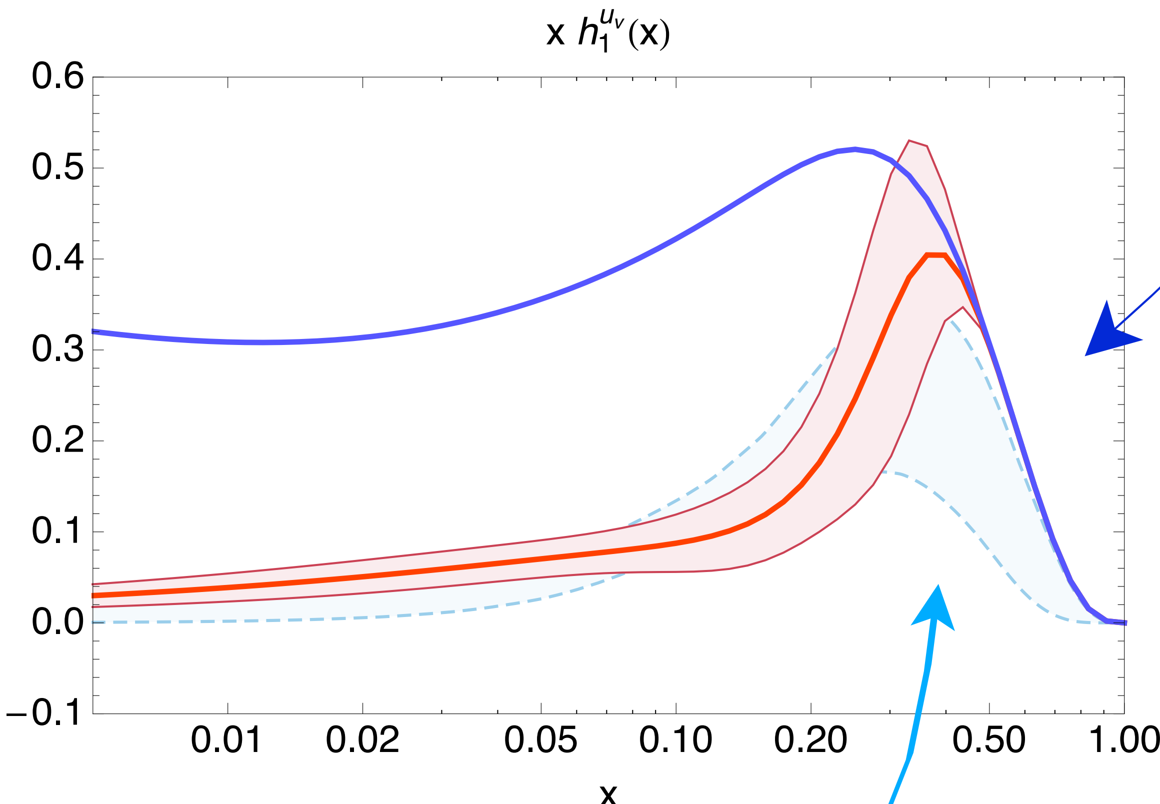

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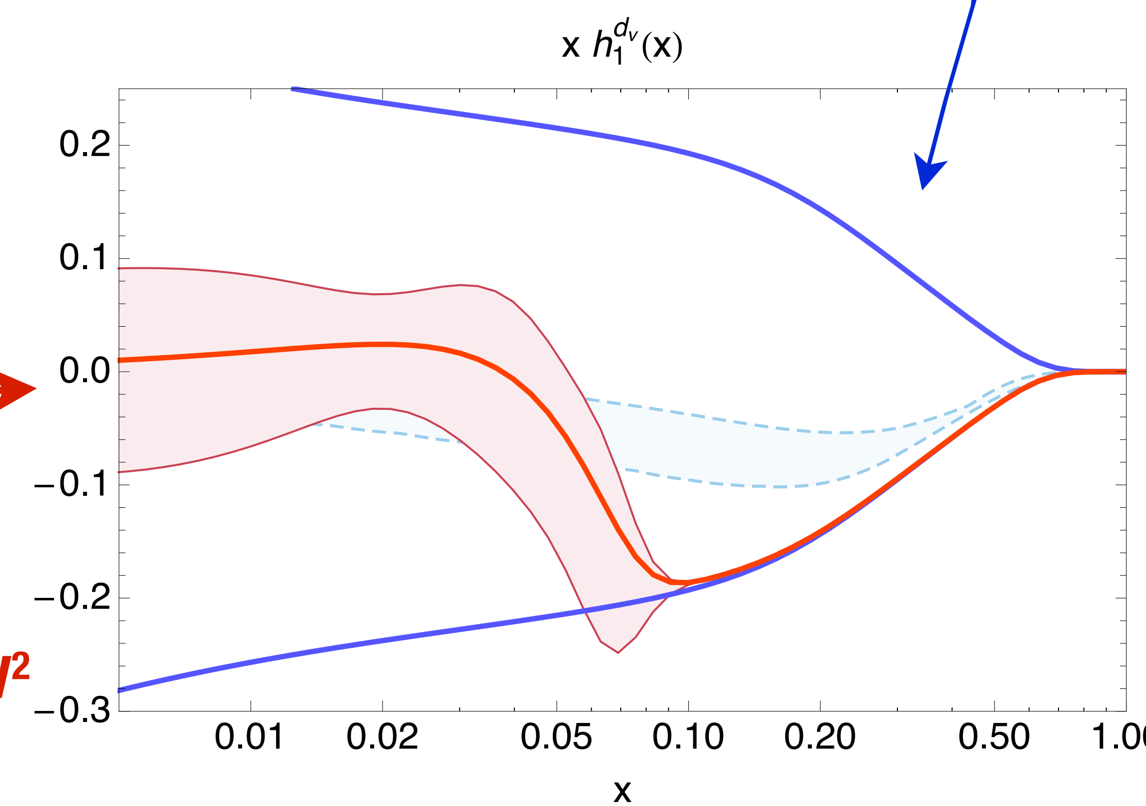

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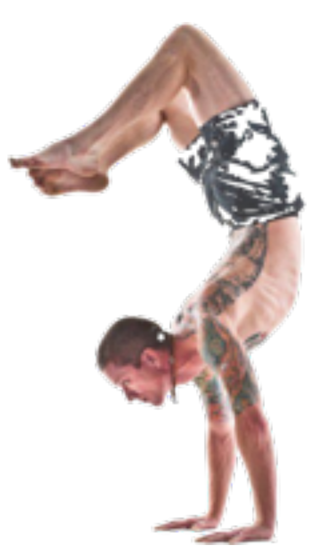

Flexible version

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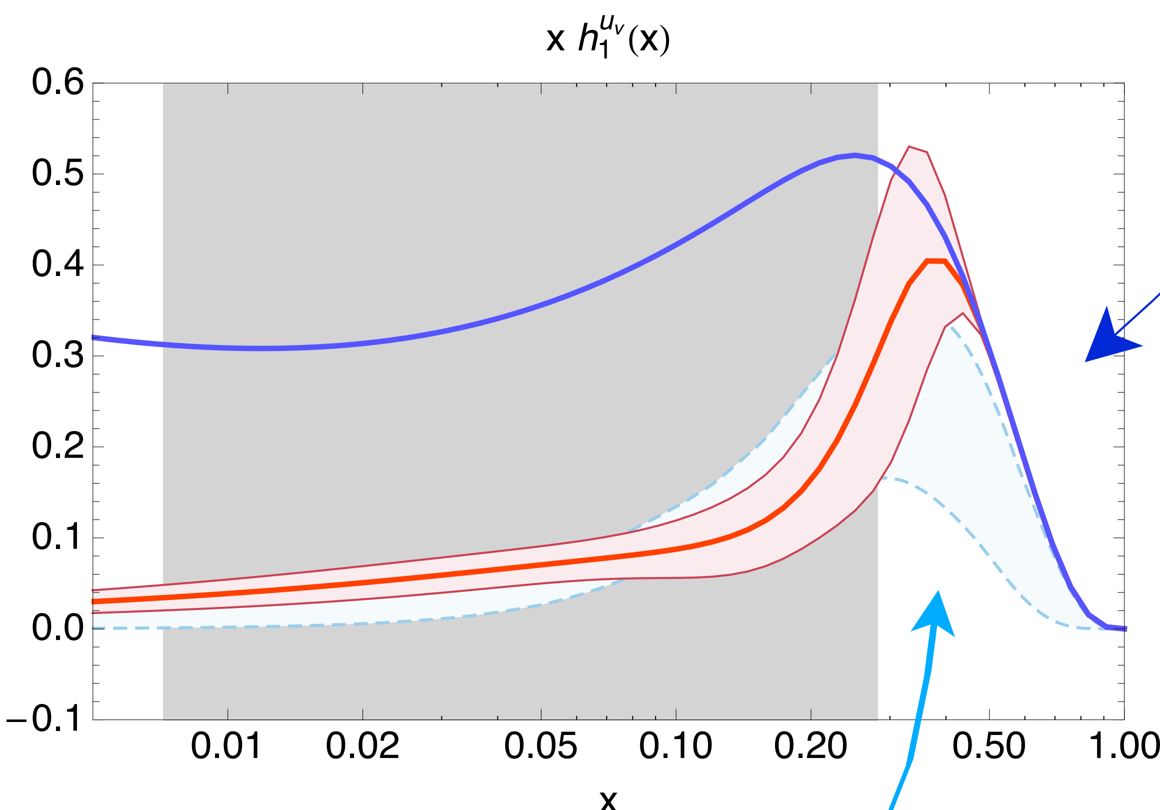

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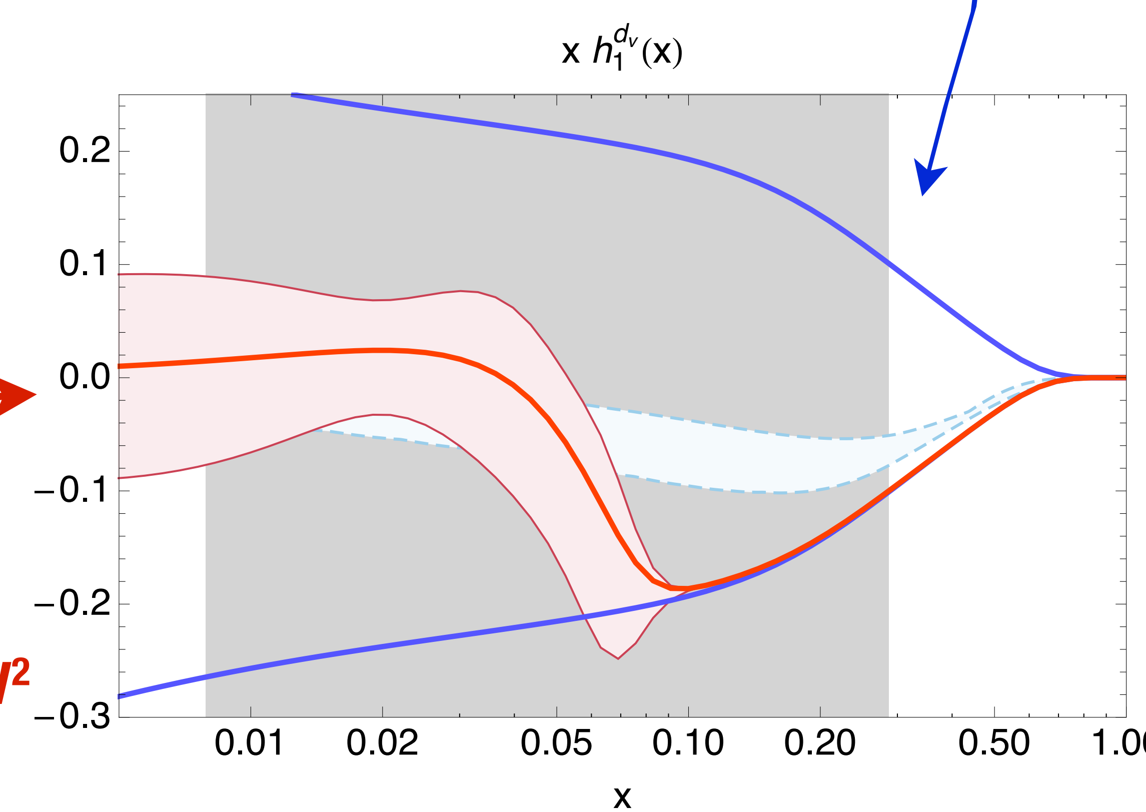

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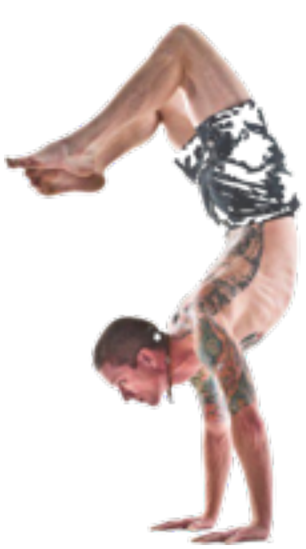

Flexible version

The Error Analysis: *the Monte Carlo approach*

Too small errors w.r.t. ABSENCE of data

- ✦ the error is smaller where there are NO data → low and large-x !!!
- ✦ standard error propagation dictated by error on parameters

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✦ redo the fit *n* times

✦ keep the 1σ distributed resulting "transversities", at each data point

✦ the error band is now made by 68% of the *n* replica point by point

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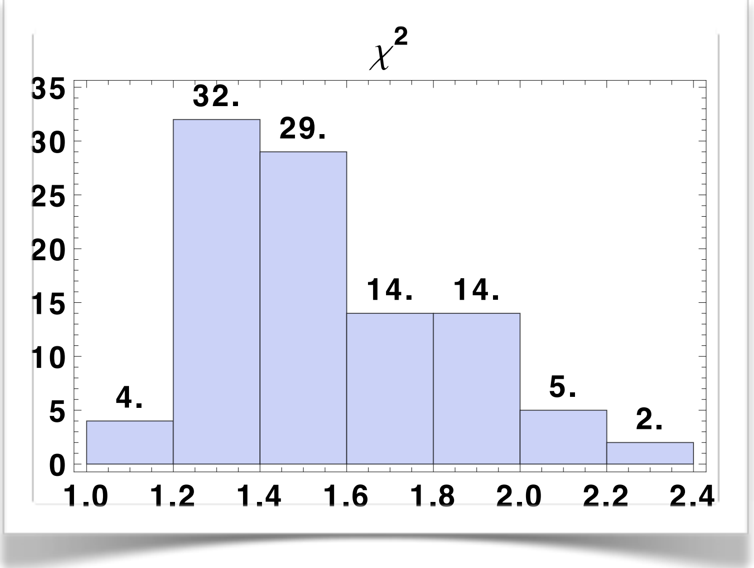
Distribution of the χ^2 for

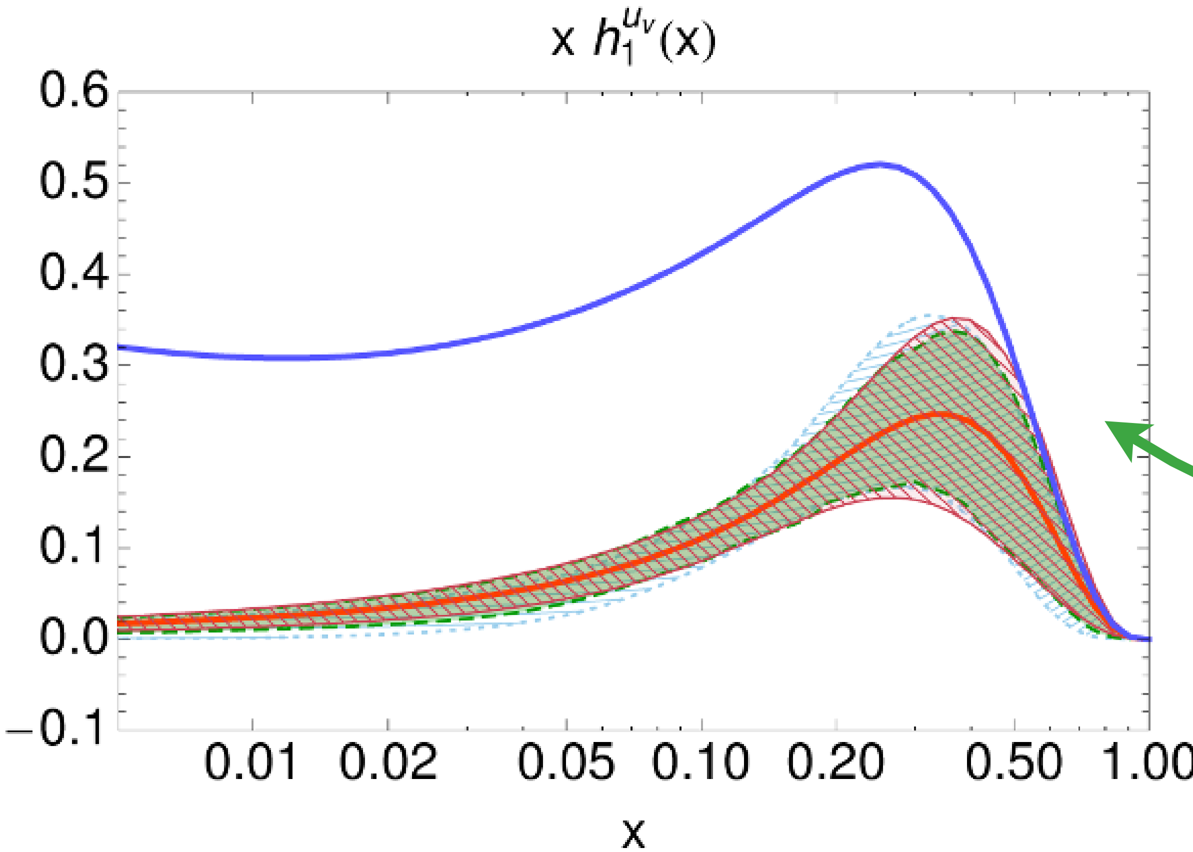
➡ *n=100 replica*

➡ *our flexible functional form*

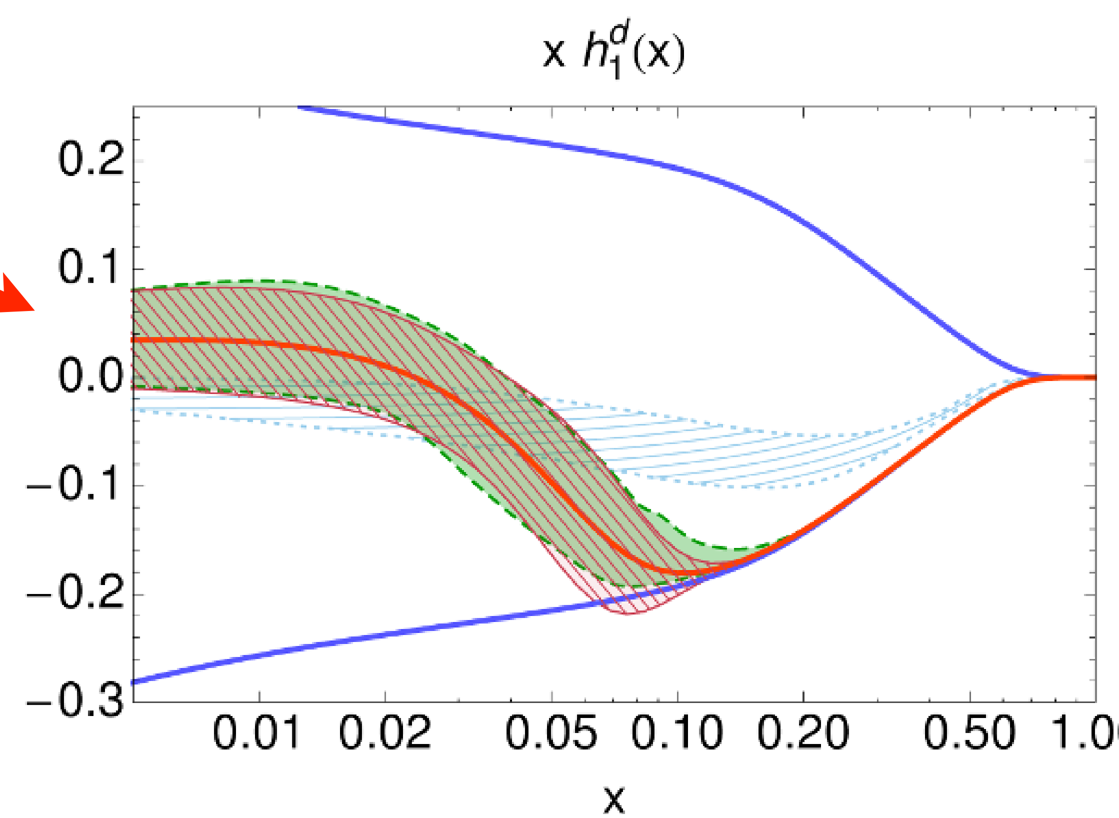


The Error Analysis: *the Monte Carlo approach*

1st order polynomial



1σ error band from replicas @2.4 GeV²



Best fit central curve @2.4 GeV² and standard 1σ error band

The Error Analysis: *the Monte Carlo approach*

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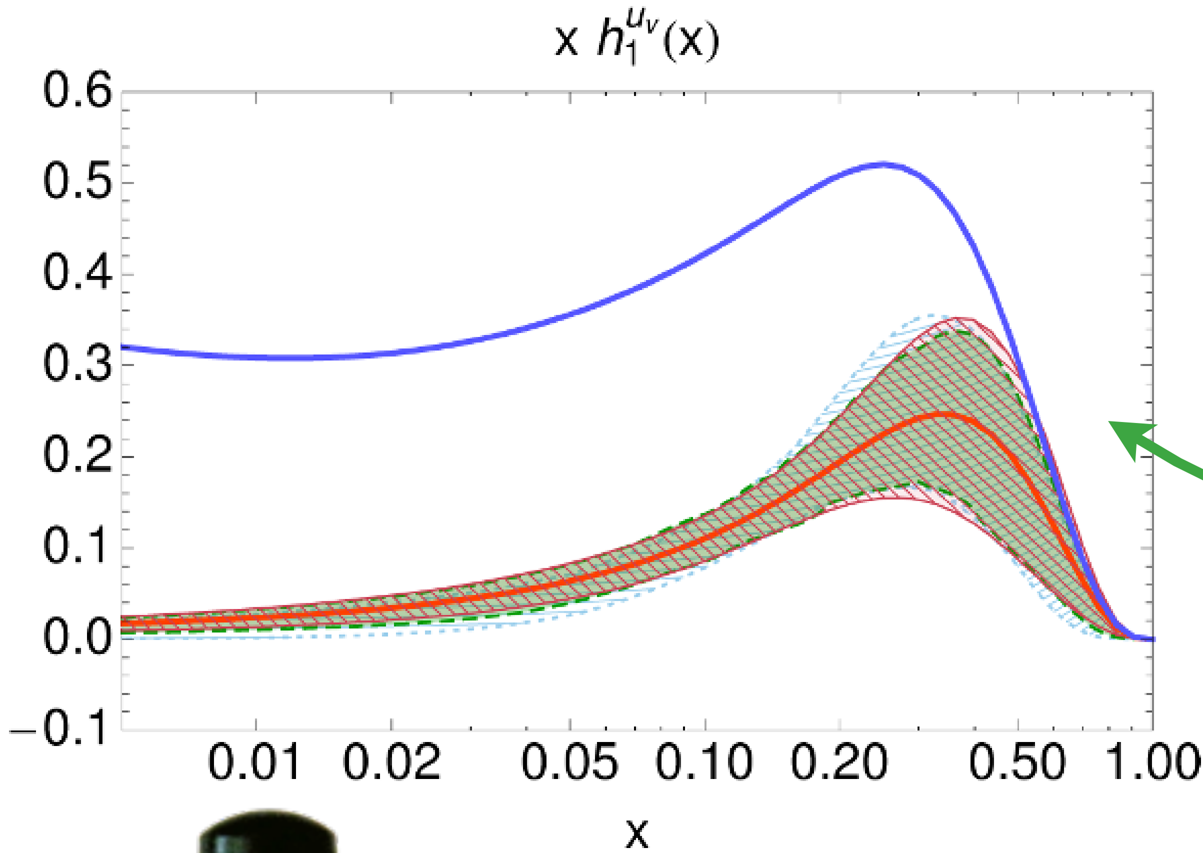

1σ error band from replicas @2.4 GeV^2

Best fit central curve @2.4 GeV^2
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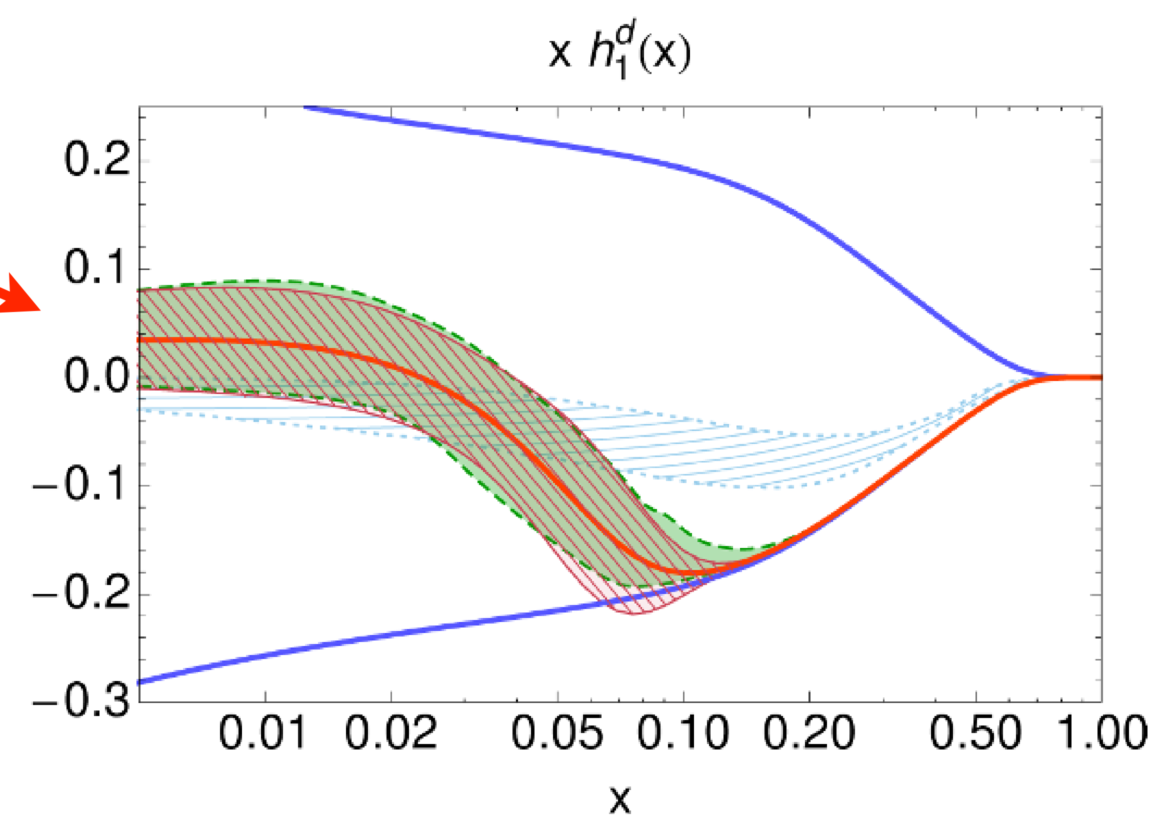
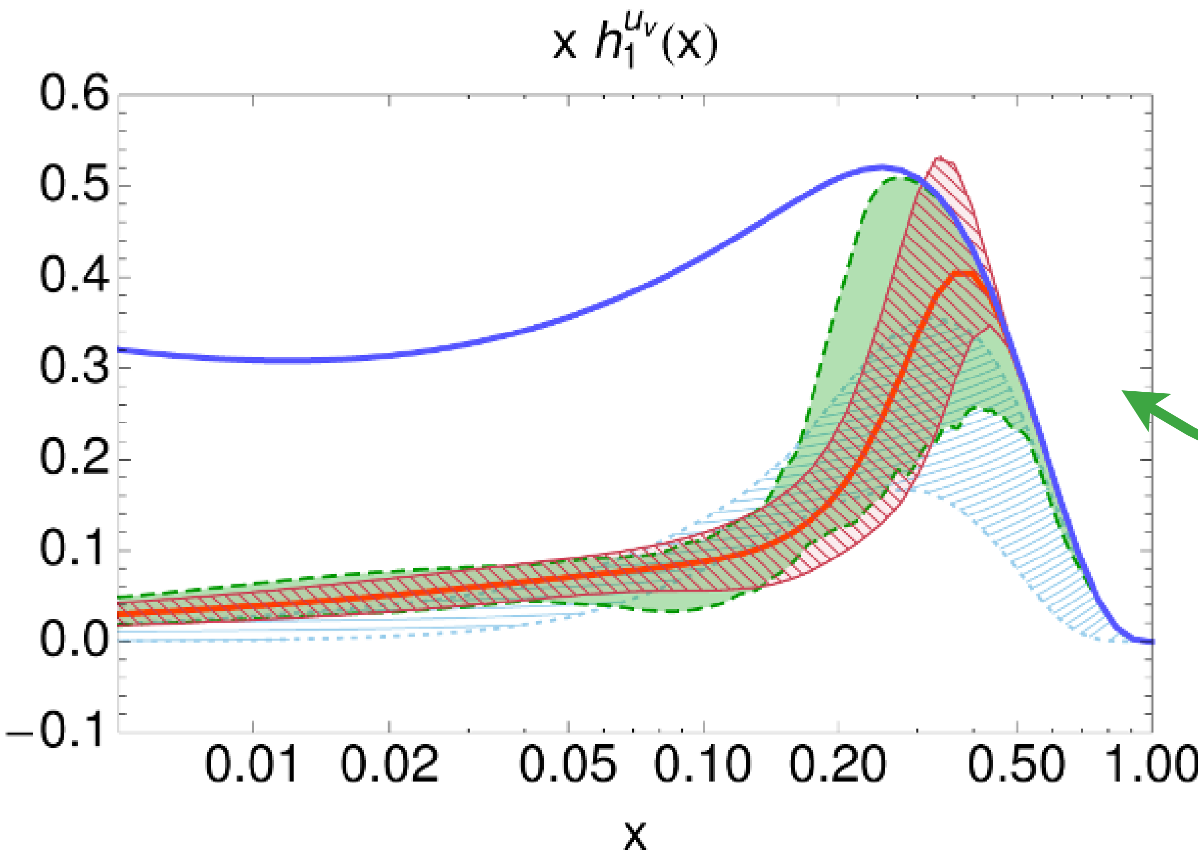


Rigid version

The Error Analysis: *the Monte Carlo approach*

2nd order polynomial



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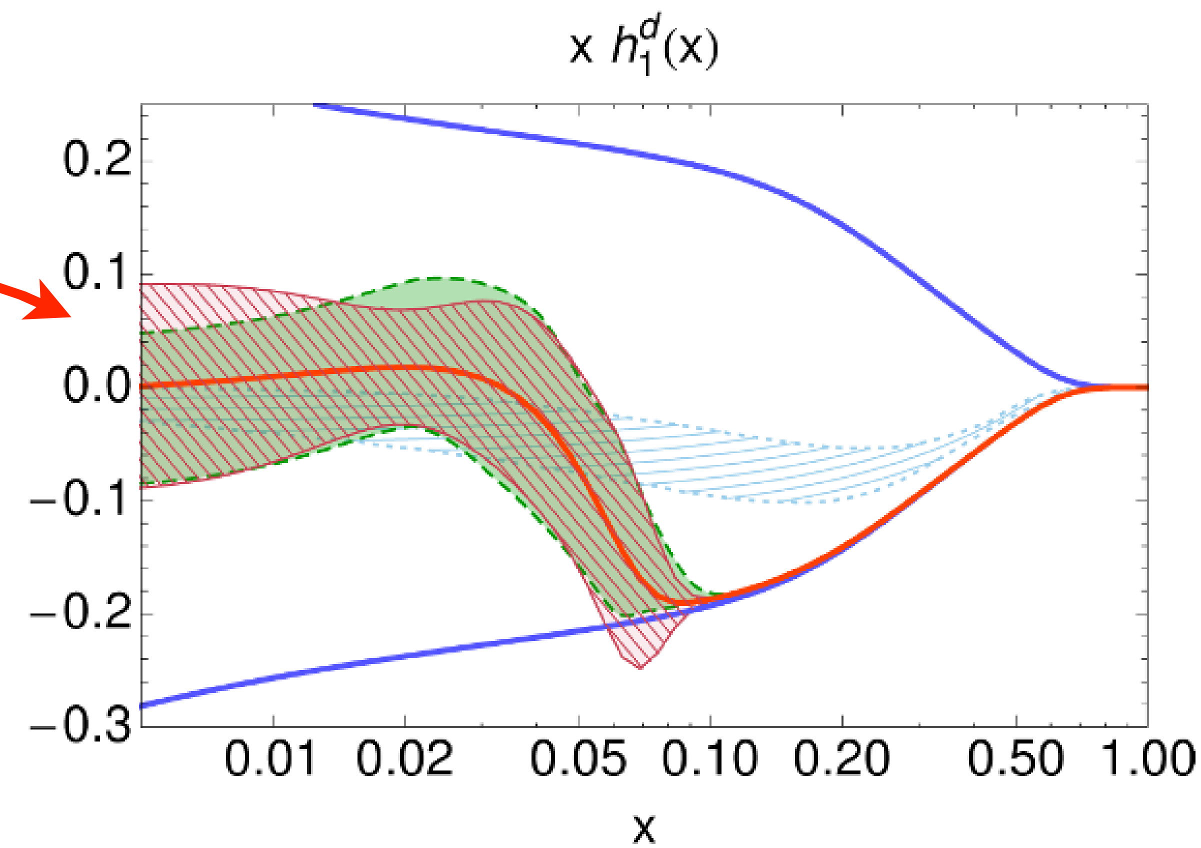

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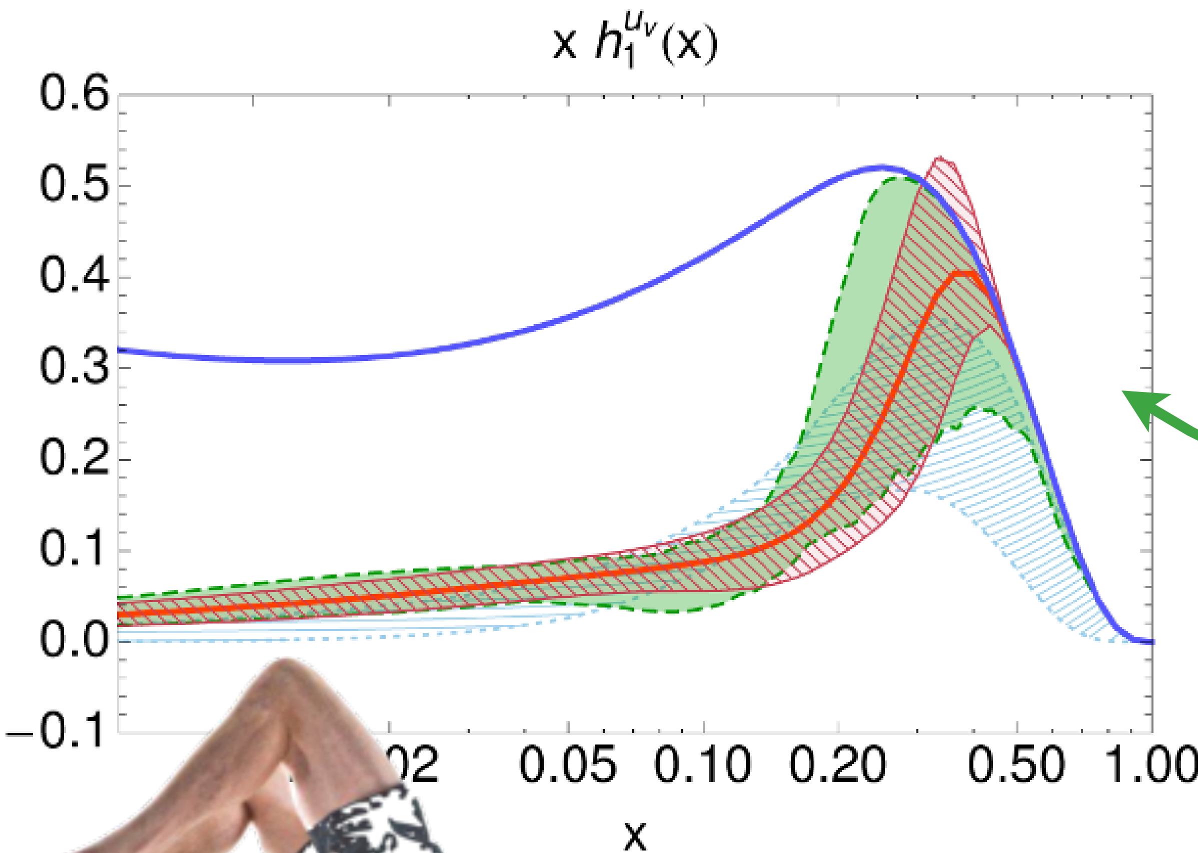

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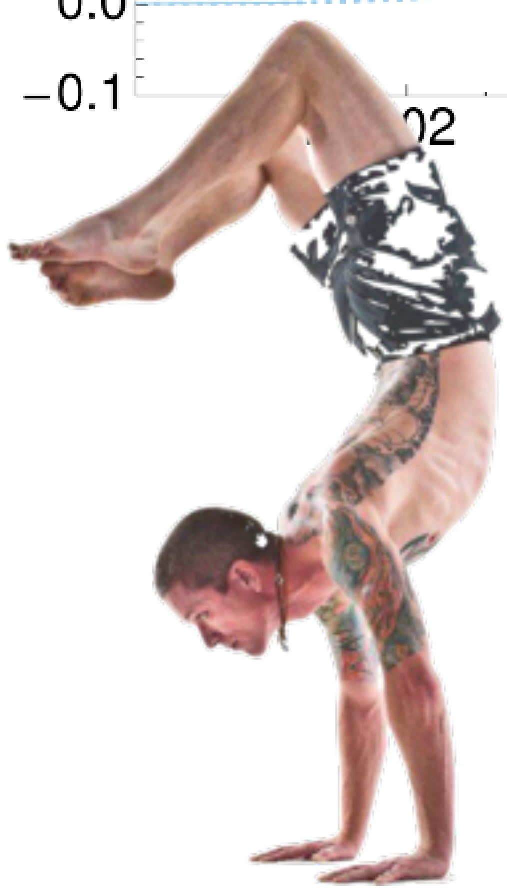
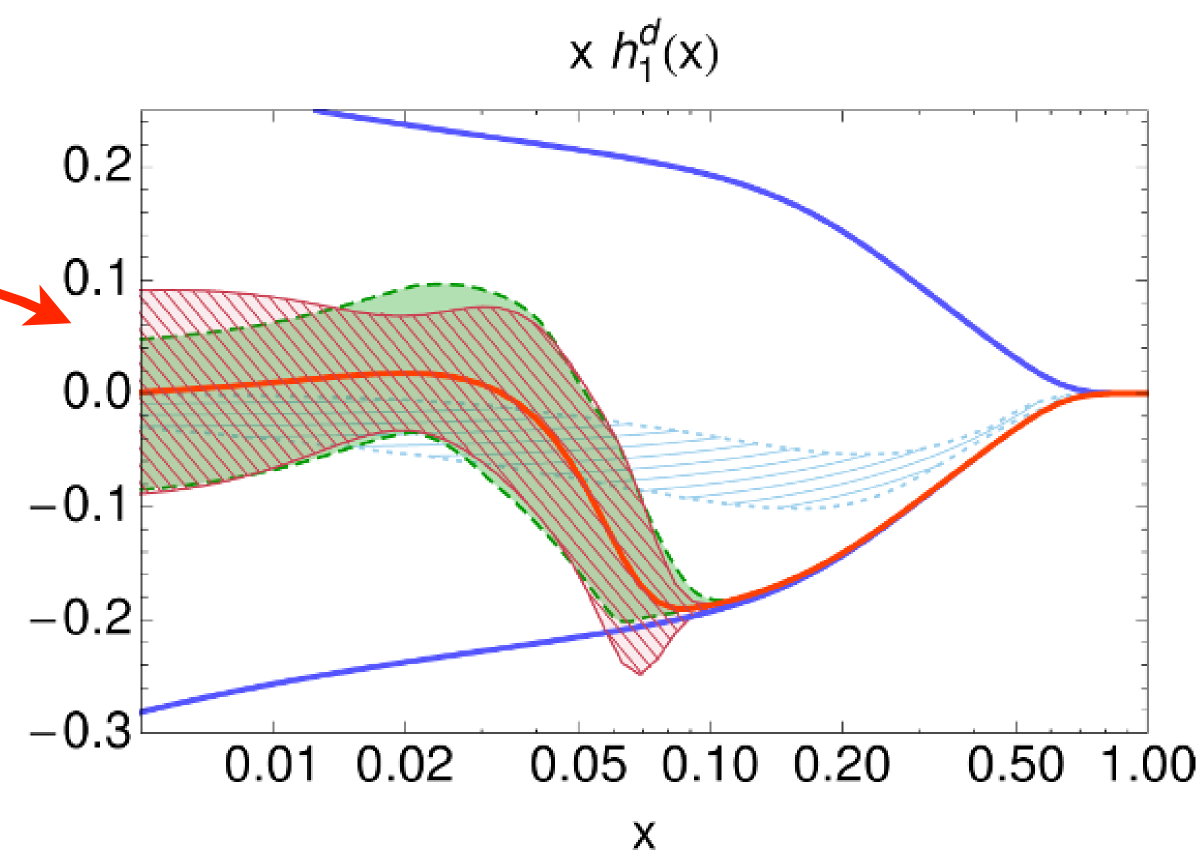


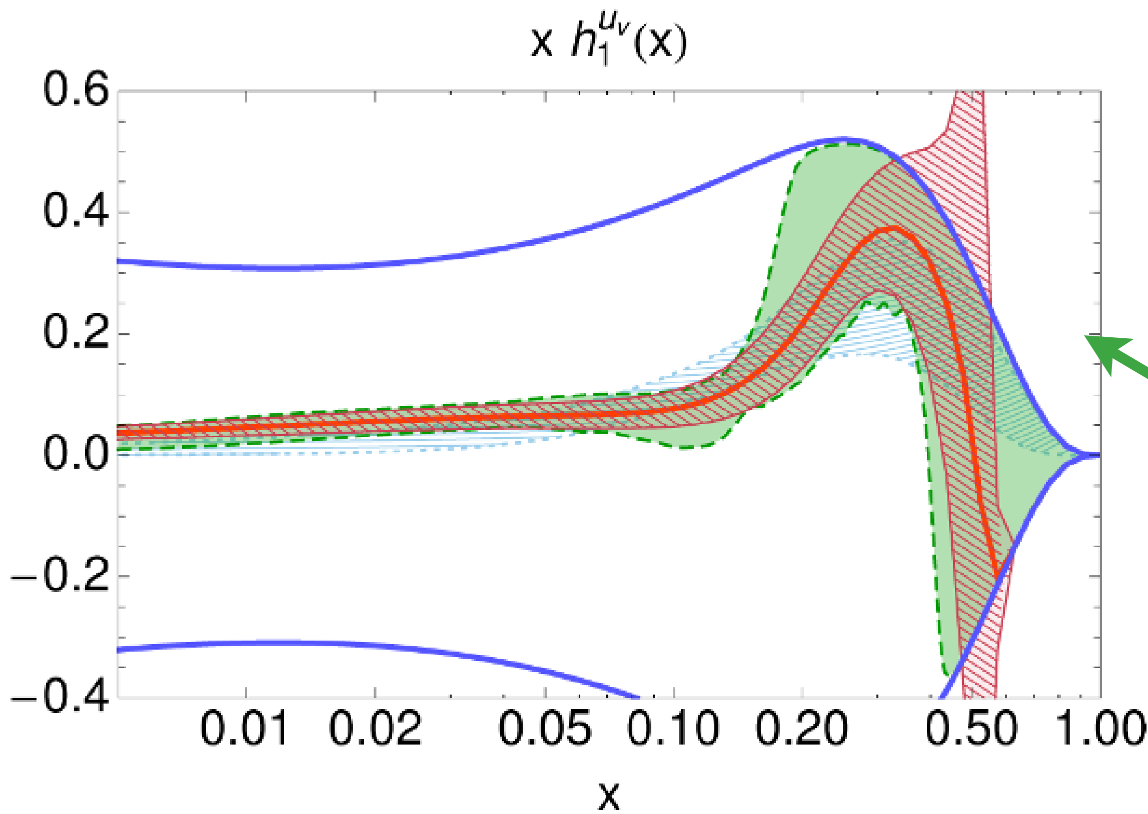
Best fit central curve @2.4 GeV² and standard 1σ error band

Flexible version



The Error Analysis: *the Monte Carlo approach*

3rd order polynomial



1σ error band from replicas @2.4 GeV²

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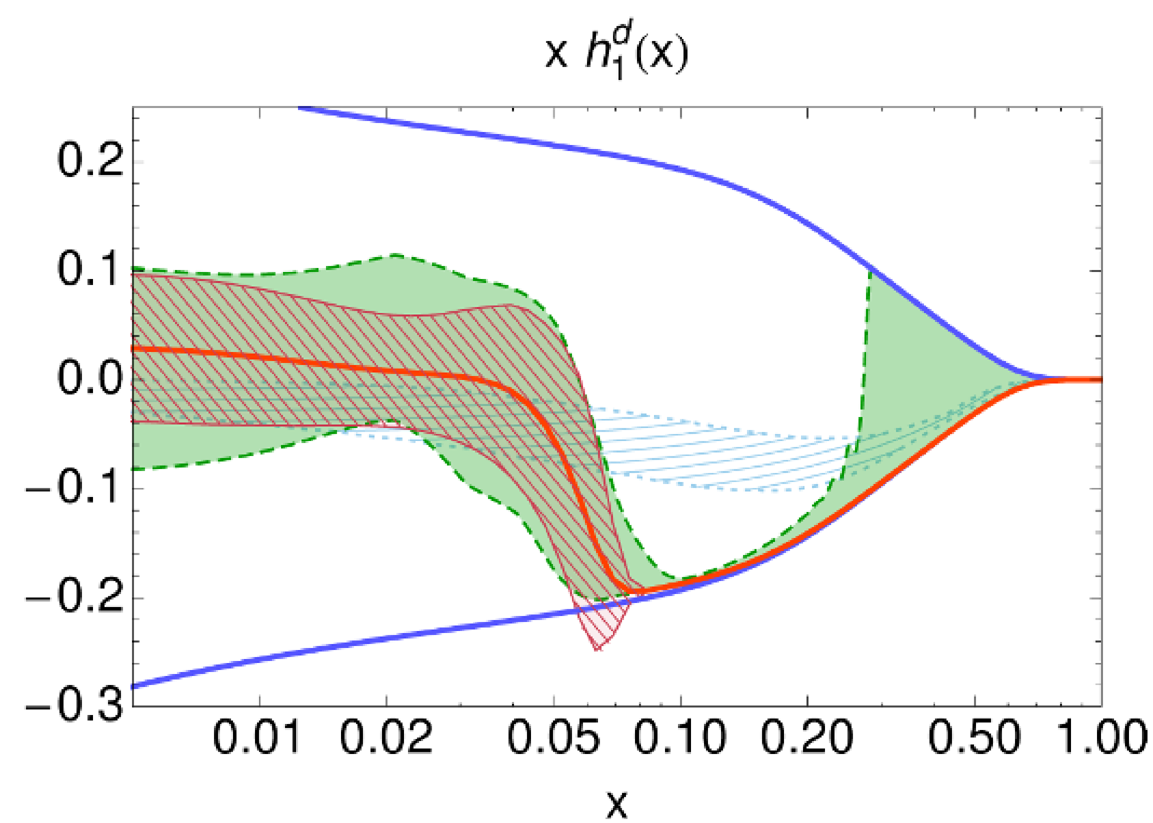

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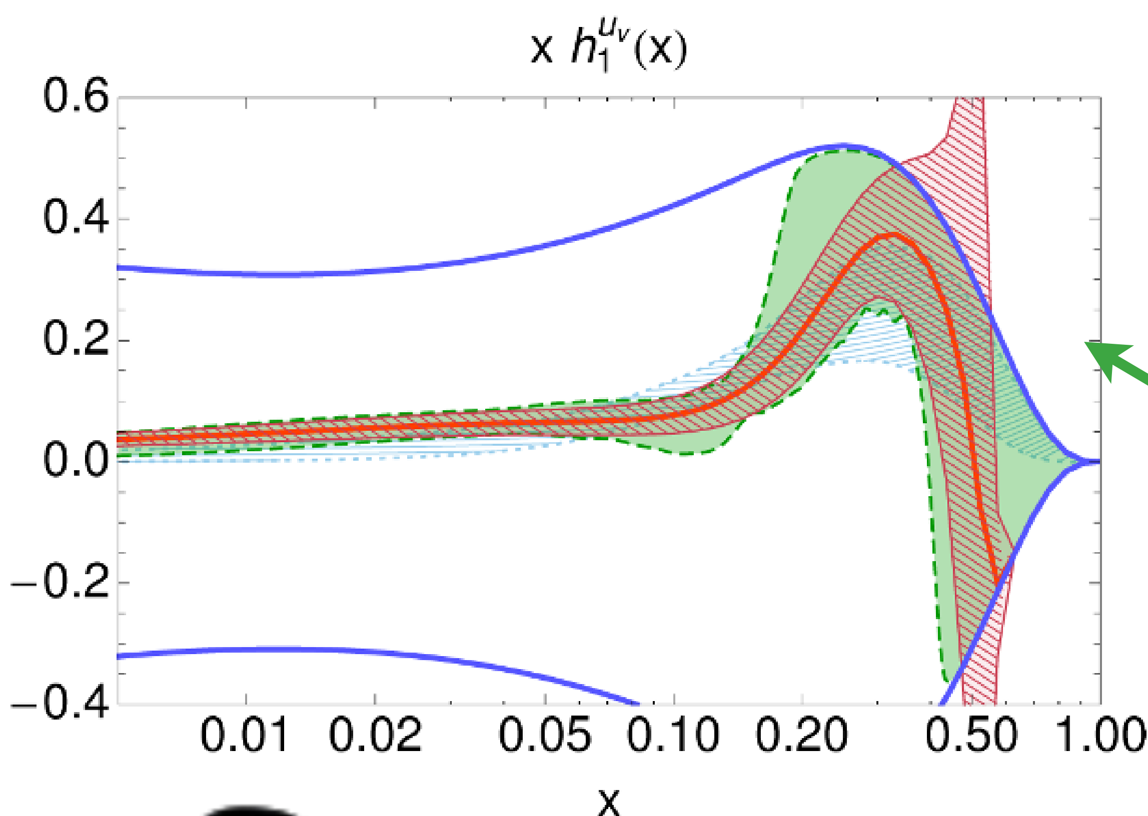

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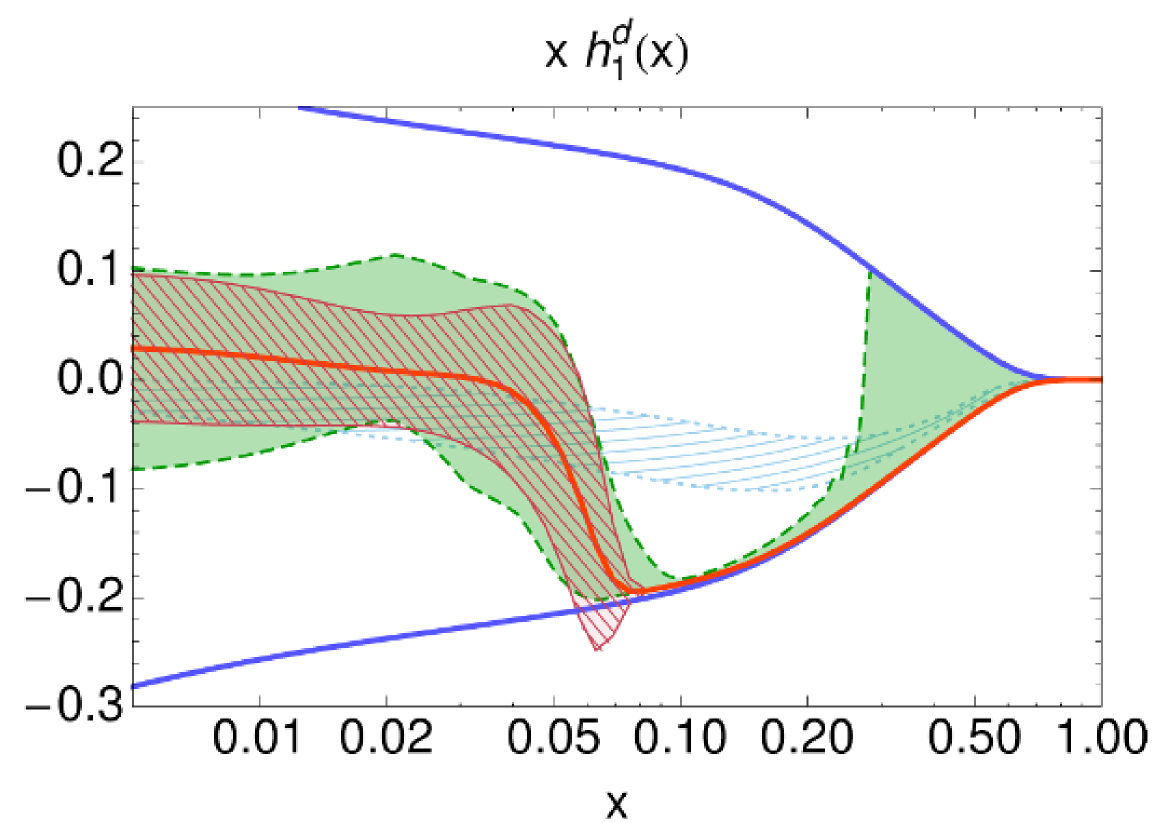
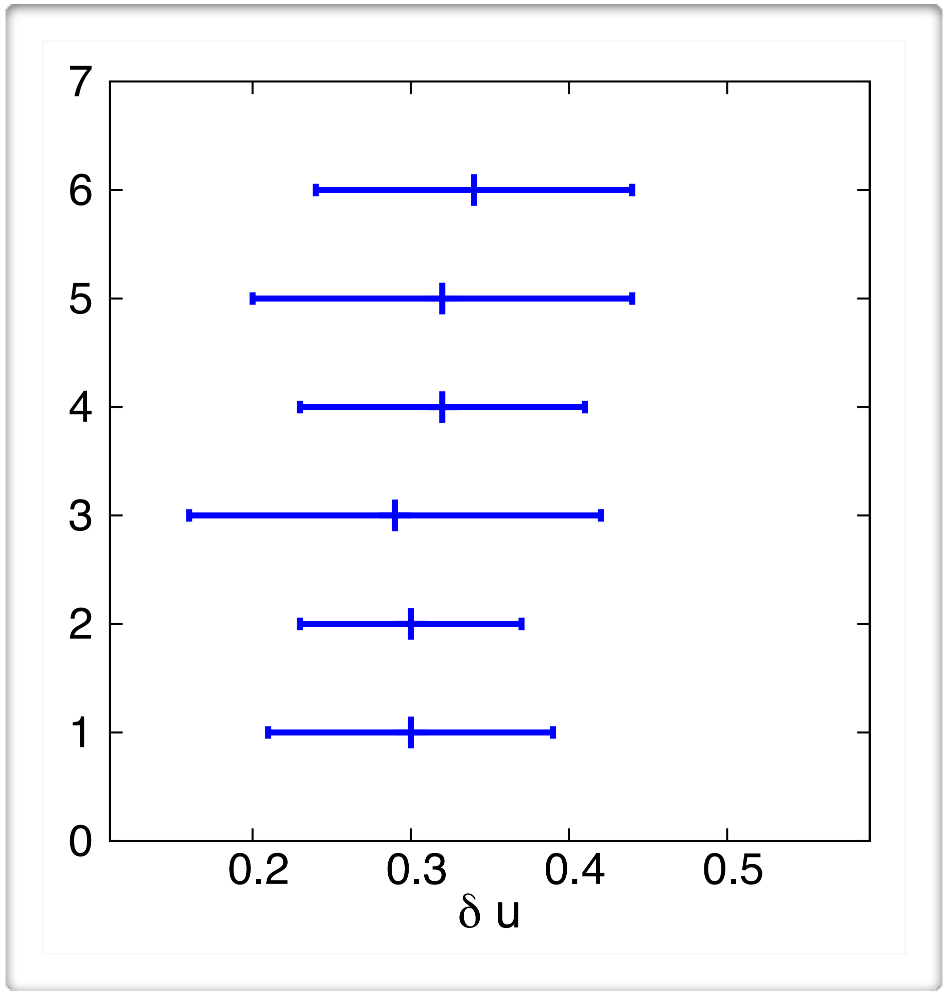
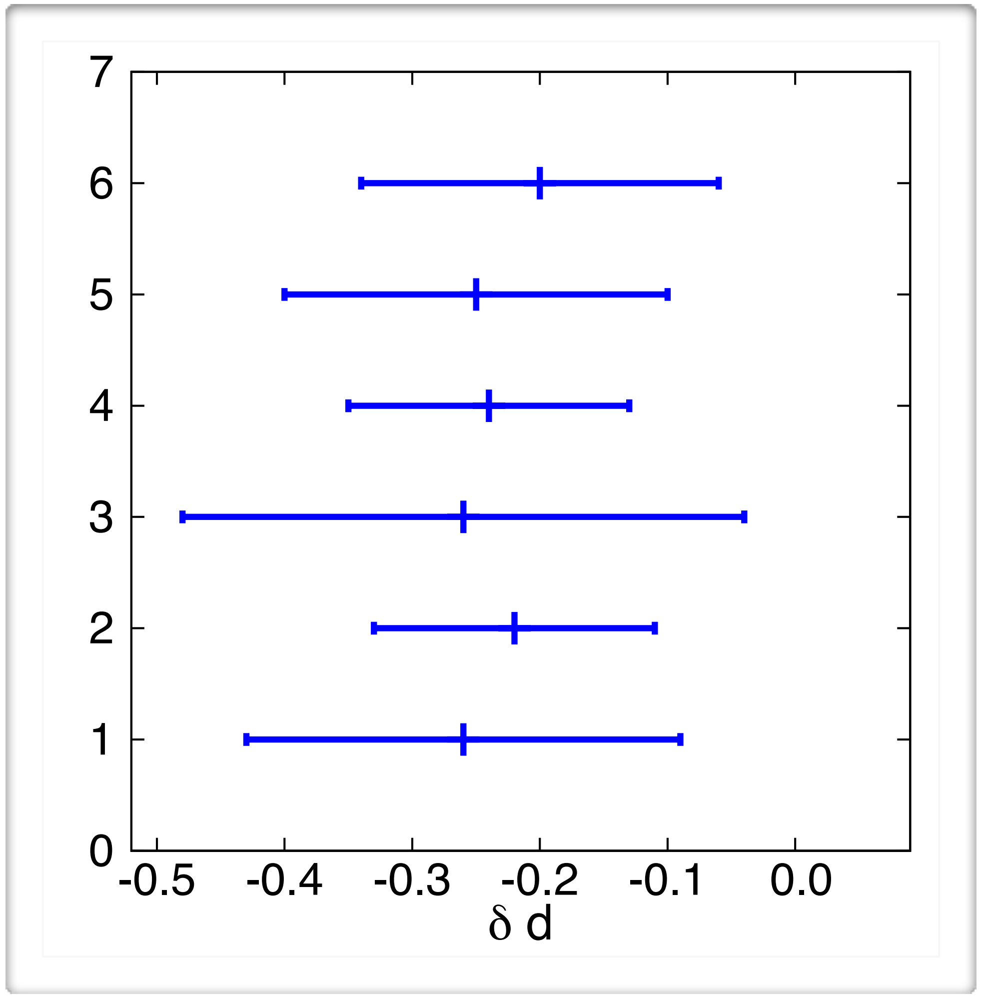
Extra-flexible version



Tensor Charge *where we have data*



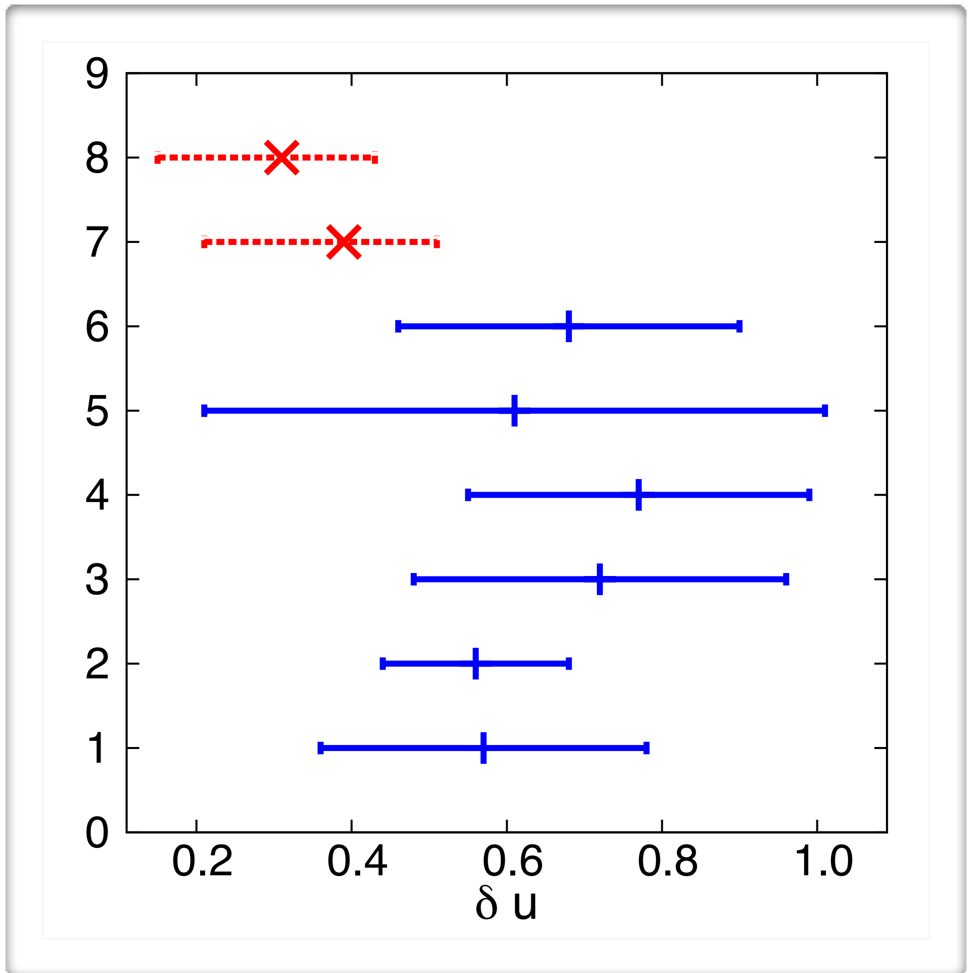
6. MC extra flexible
5. standard extra flexible
4. MC flexible
3. standard flexible
2. MC rigid
1. standard rigid



$$\delta q = \int_{6.4\times 10^{-3}}^{0.28} dx\, h_1^{q_v}(x)$$

Tensor Charge

full range 10^{-10}- 1



8. fit of A_0

7. fit of A_{12}

6. MC extra flexible

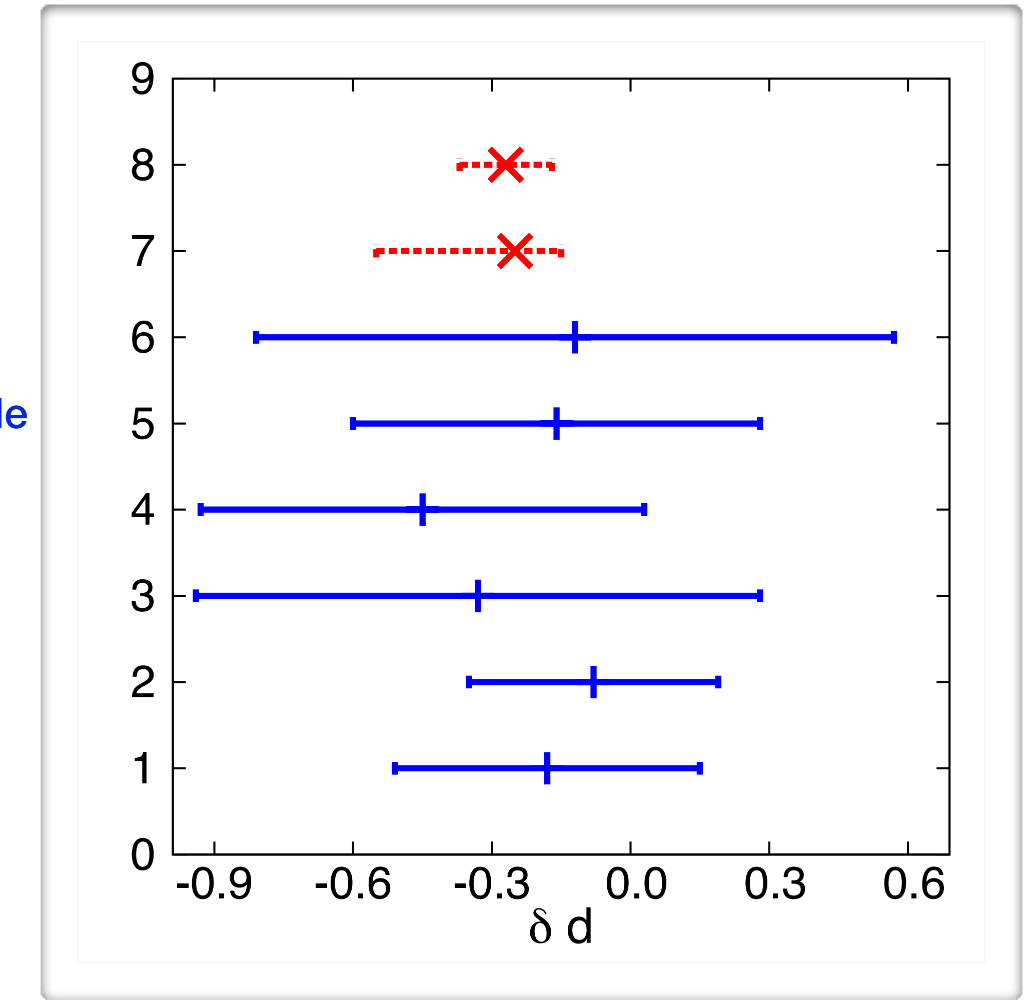
5. standard extra flexible

4. MC flexible

3. standard flexible

2. MC rigid

1. standard rigid



$$\delta q = \int_{\sim 0}^{1} dx \, h_1^{q_v}(x)$$

Conclusion

Extraction of valence transversities from collinear framework

- Transversity via DiFF
  - Flavor decomposition thanks to the available proton and deuteron data
  - Fits for h_1^u & h_1^d [Bacchetta, A.C., Radici, JHEP 1303 (2013) 119]
  - *Functional Form* crucial to standard fitting procedure
    - ➡ Highly unconstrained outside data range
    - ➡ *Important!* e.g., for tensor charge
    - ➡ We NEED more data at higher x-values → JLab@12GeV
  - Monte Carlo-like error analysis
    - ➡ Compatible with standard analysis
    - ➡ Bigger errorbands

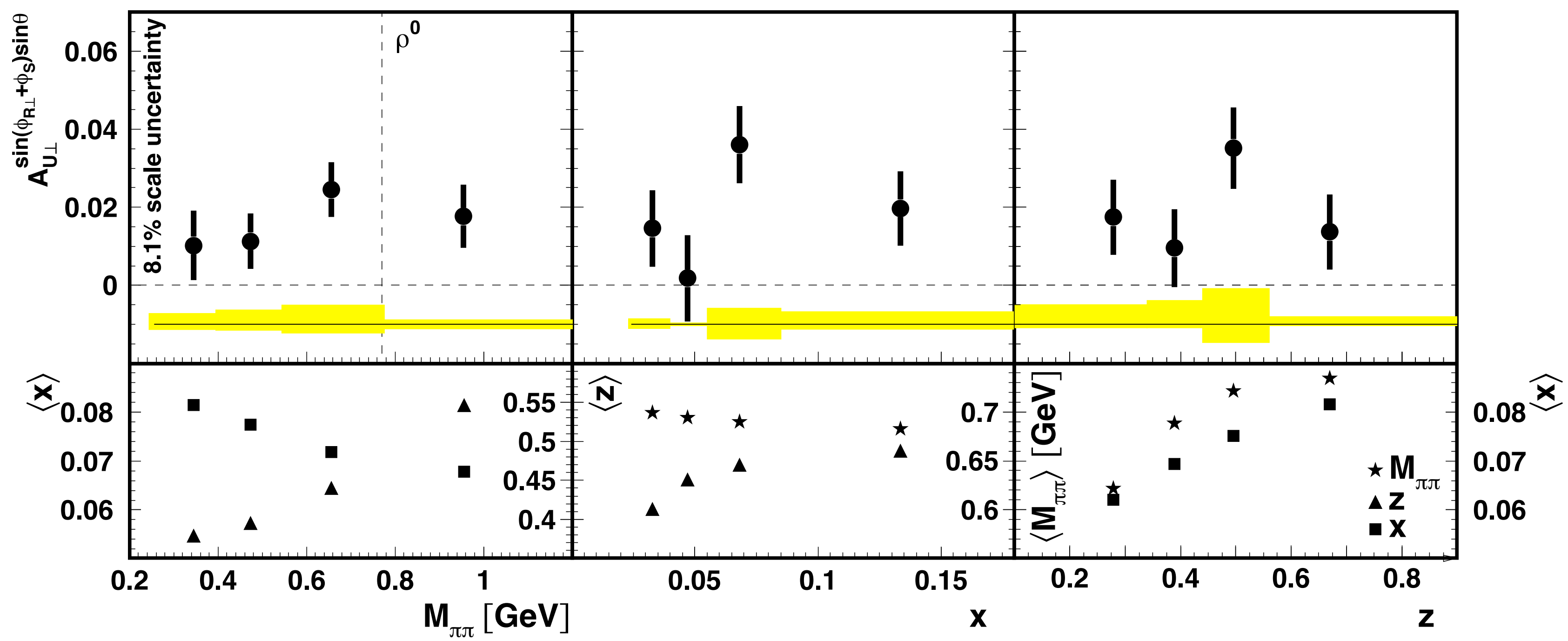
Outlook

- **Dihadron Fragmentation Functions**
  - **Fits** in (z, M_h, Q^2) with more accurate Q^2 evolution [Bacchetta, Bianconi, Courtoy, Radici]
  - **Data for Unpolarized DiFF**
- **Transversity via DiFF**
  - **Flavor decomposition** we need Kaon data from Belle as well
  - **Fits for h_1^u & h_1^d** we need data for x>0.3 !

Back-up slides

$A_{UT}^{\sin(\Phi_R+\Phi_S)\sin\theta}$ @ HERMES

[JHEP 06, 017 (2008)]



✦ integrated over 0.5<Mh<1GeV

✦ integrated over 0.2<z<1

$$n_q(Q^2) \quad = \int dz\, dM_h^2\, D_1^{q\to\pi^+\pi^-}(z, M_h^2, Q^2)$$

$$\boxed{A_{\mathrm{DIS}}(x,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\, n_q(Q^2)}}$$

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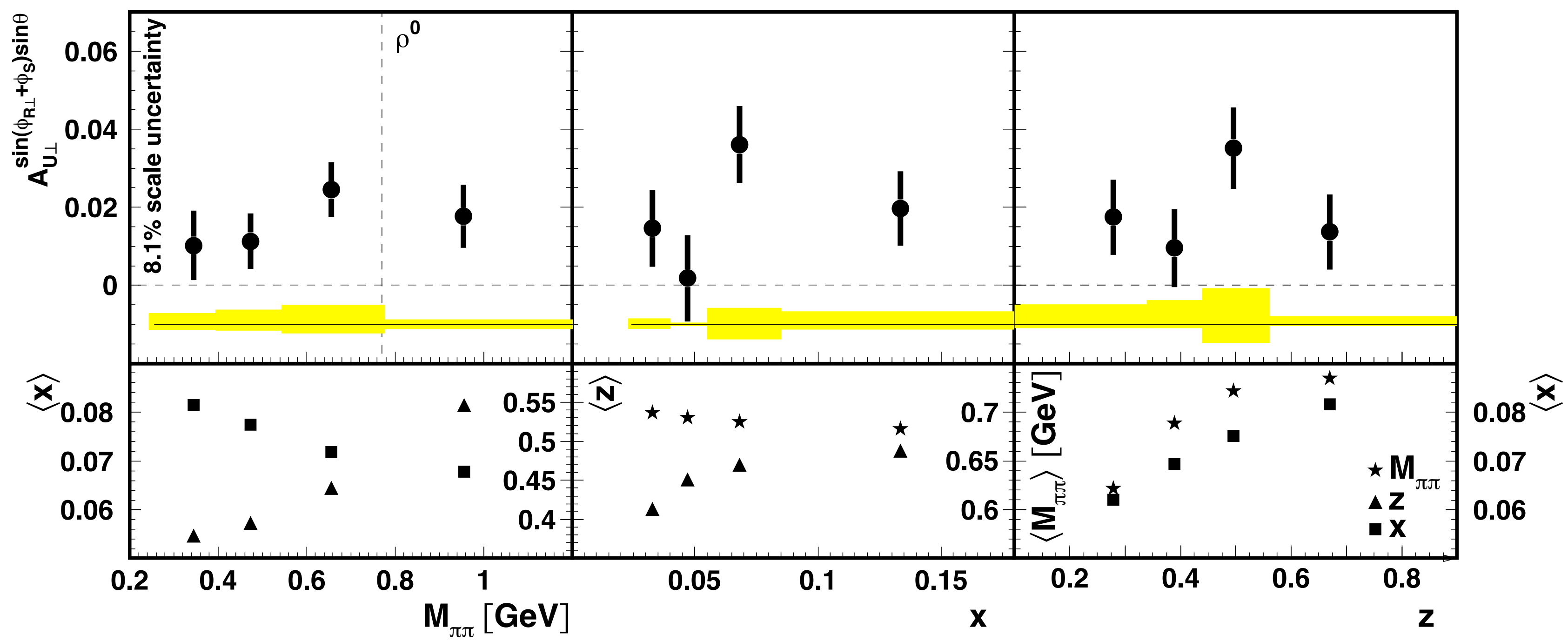

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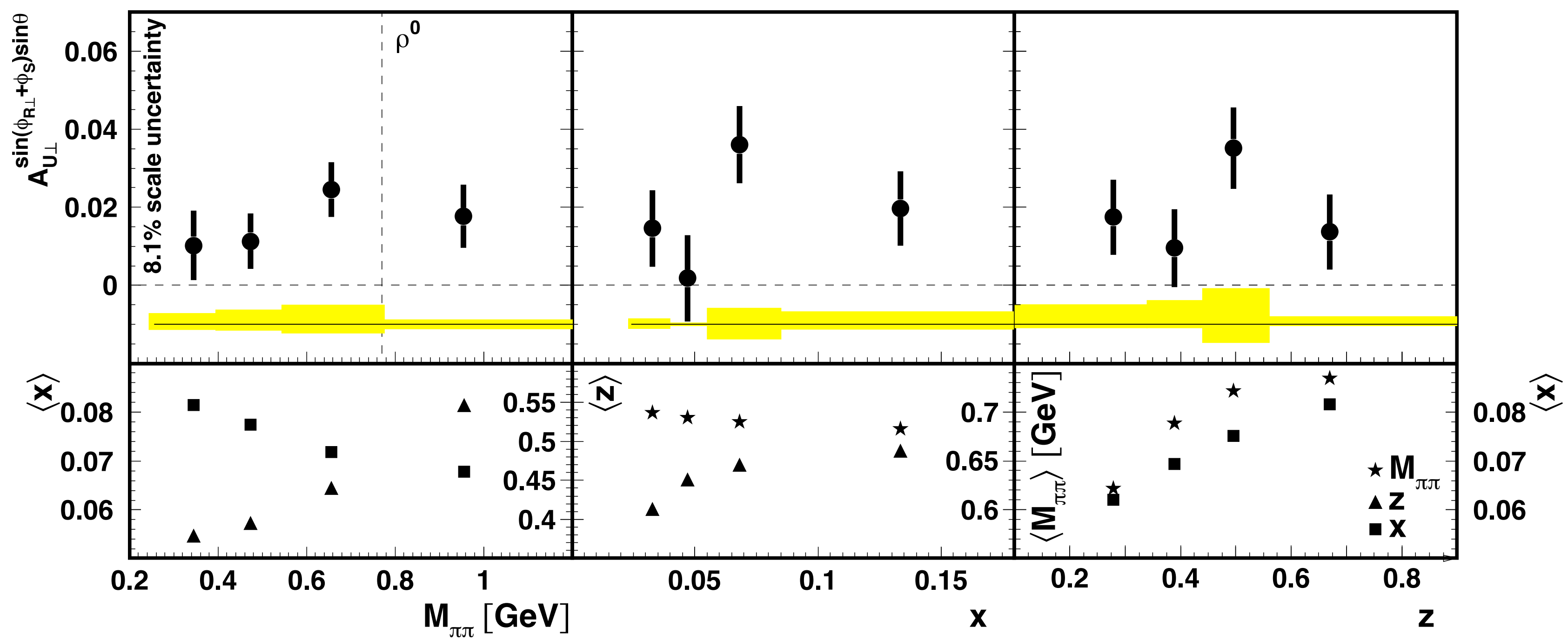

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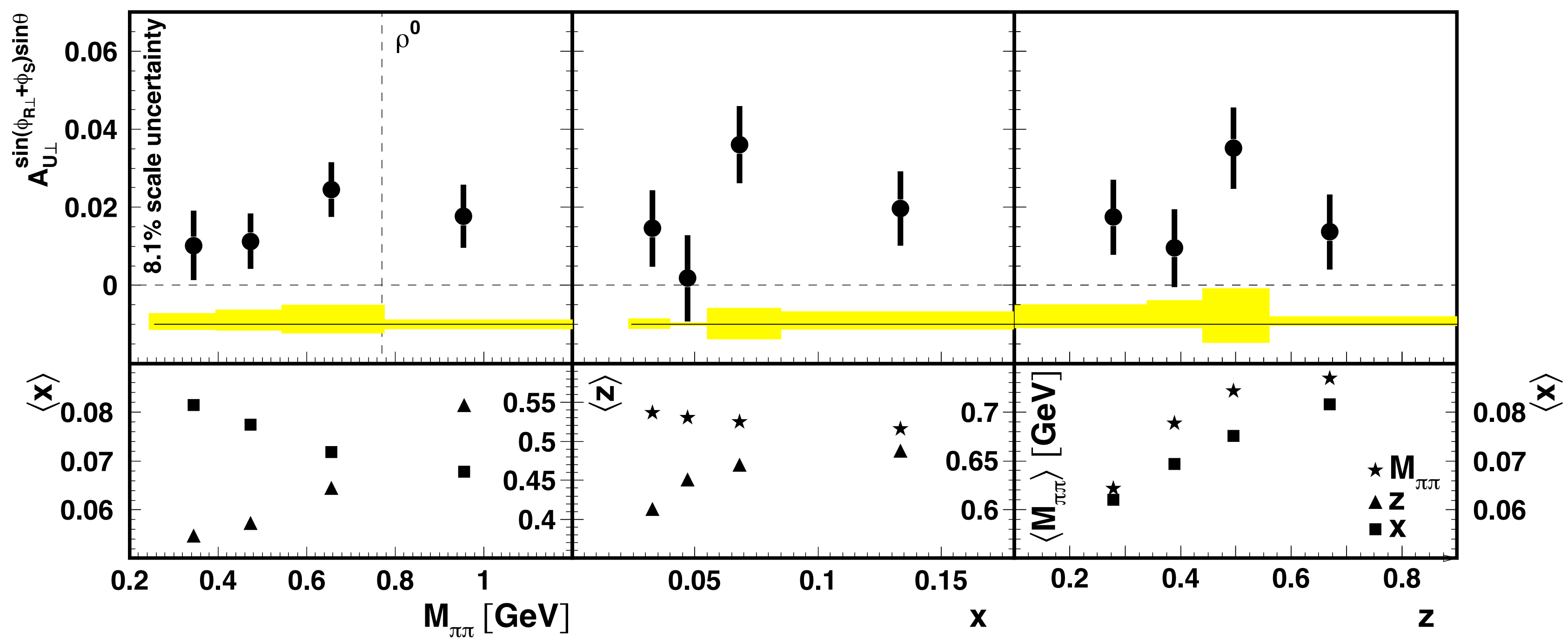
[JHEP 06, 017 (2008)]

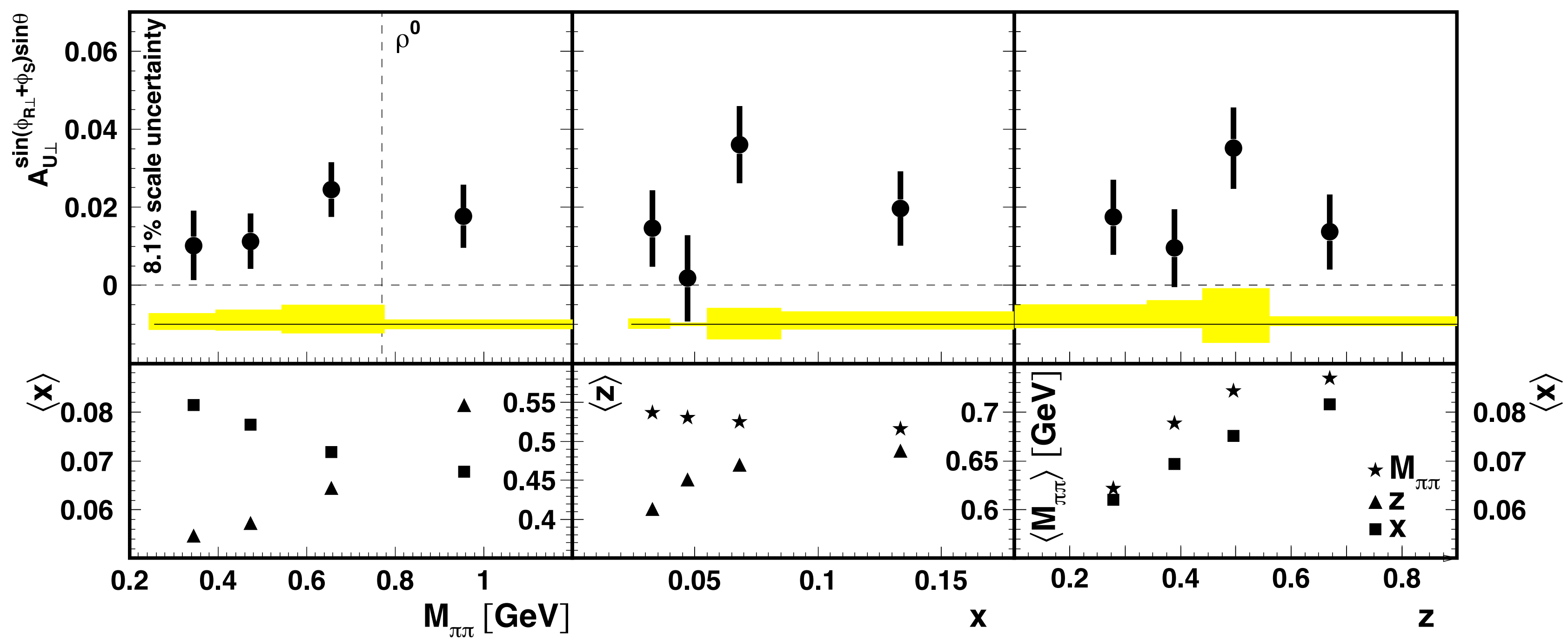


$$A_{\mathrm{DIS}}(x,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\, n_q(Q^2)}$$

$$xh_1^{u_v}(x,Q^2) - \tfrac{1}{4}\, xh_1^{d_v}(x,Q^2) = -C_y^{-1}\, A_{\mathrm{DIS}}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

$A_{UT}^{\sin(\Phi_R+\Phi_S)\sin\theta}$ @ HERMES

[JHEP 06, 017 (2008)]

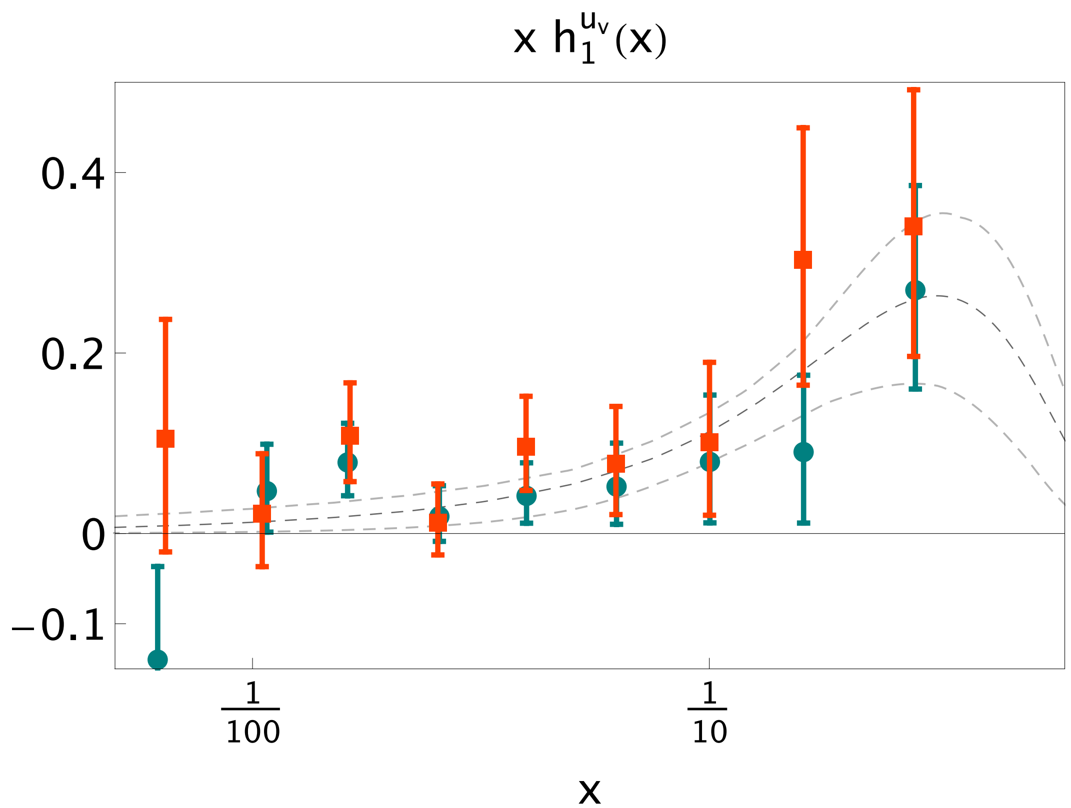


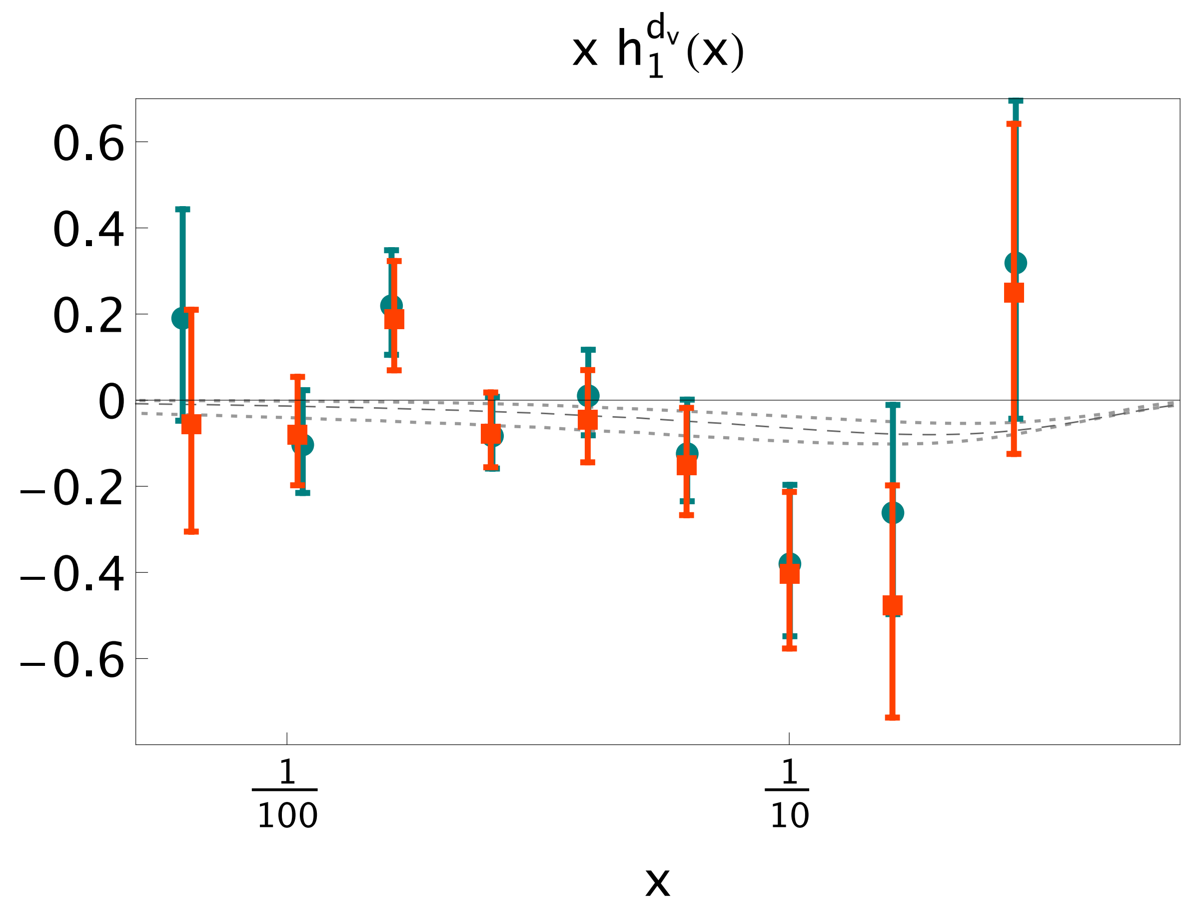
$$A_{\rm DIS}(x,Q^2) = -C_y \frac{\sum_q e_q^2\, h_1^q(x,Q^2)\, n_q^{\uparrow}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\, n_q(Q^2)}$$

$$x h_1^{u_v}(x,Q^2) - \tfrac{1}{4}\, x h_1^{d_v}(x,Q^2) = -C_y^{-1}\, A_{\rm DIS}(x,Q^2)\, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2}\, x f_1^{q+\bar{q}}(x,Q^2)$$

Off the record: COMPASS data on Proton 2010

2nd order polynomial





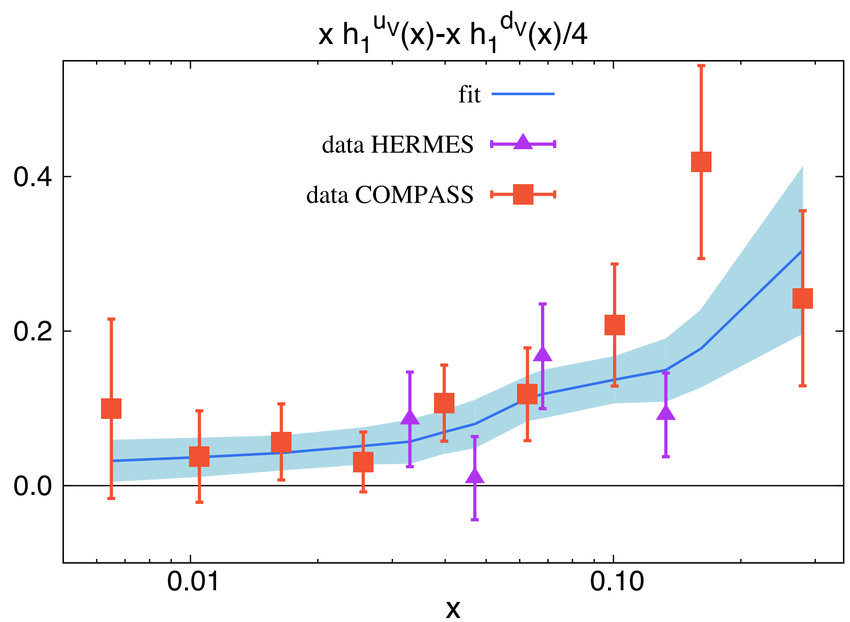
COMPASS 2004 (P) & 2007 (D)

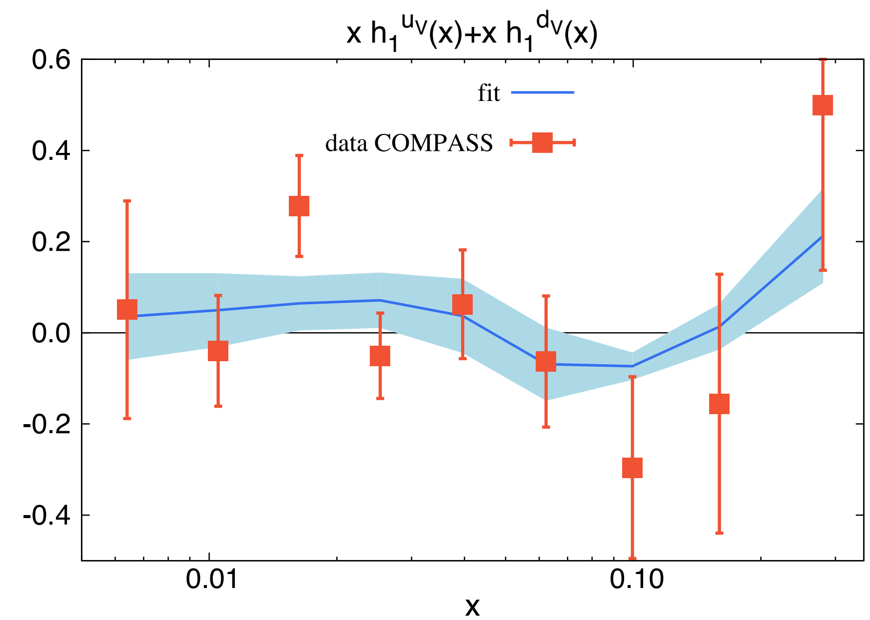
COMPASS 2010 (P) & 2007 (D)

Comparison with extraction

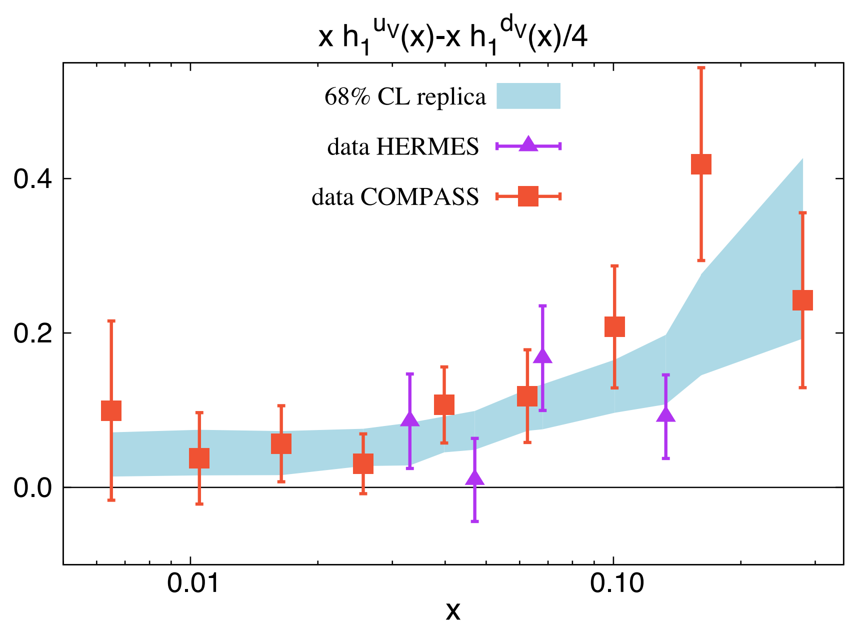
PROTON

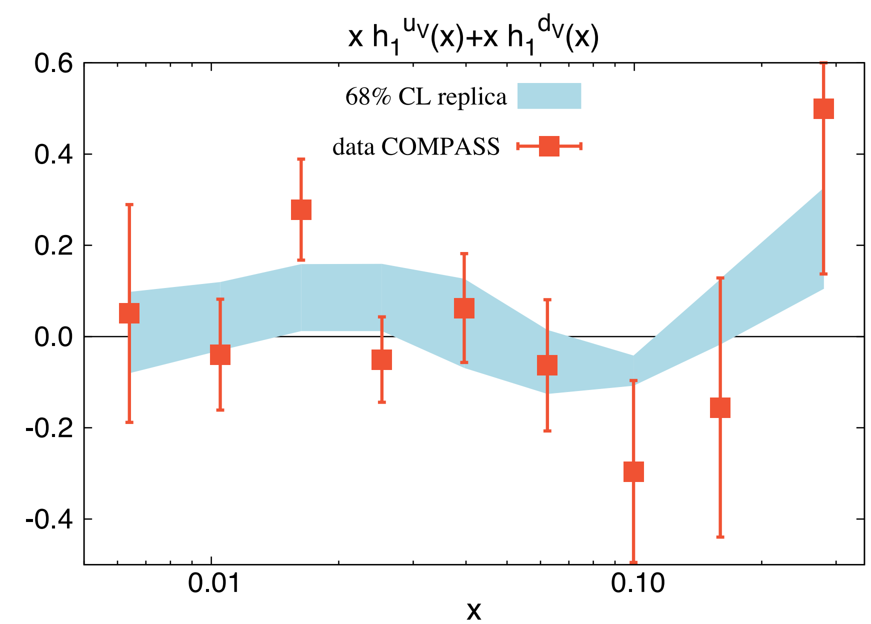
DEUTERON





flexible functional form

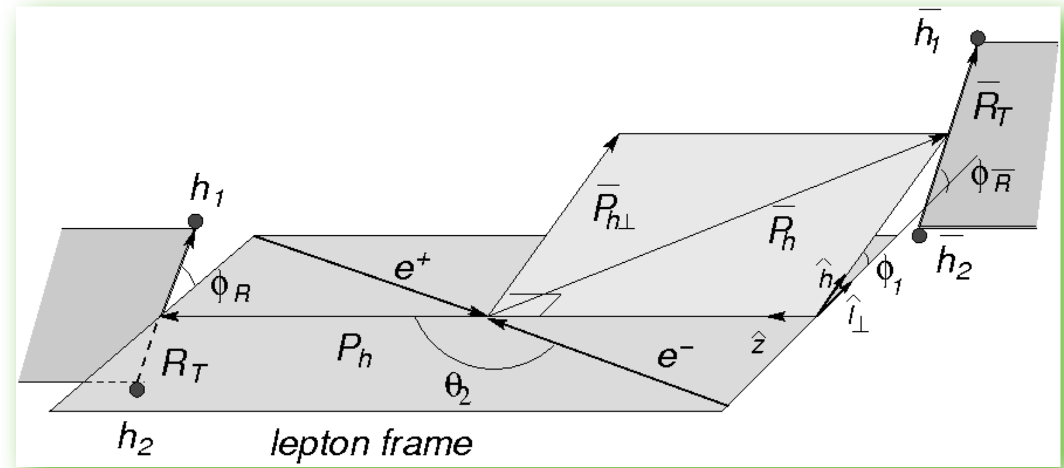




replica with flexible ff

Semi-Inclusive production of pion pair in e⁺e⁻ annihilation @Belle

[Belle, Phys.Rev.Lett.107.072004]

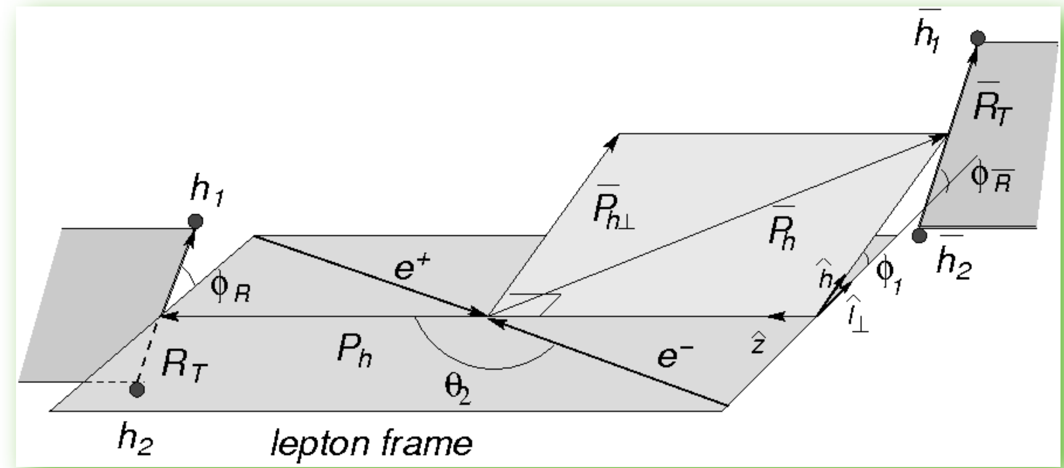


- 2 hemispheres
- azimuthal modulation between the 2 hemispheres

$$A_{e+e-}(z, M_h^2, \bar{z}, \bar{M}_h^2) \propto -f(\theta_2)\, g(\theta)\, g(\bar{\theta}) \, \frac{\sum_q e_q^2 \, H_1^{\sphericalangle\, q}(z, M_h^2)\; H_1^{\sphericalangle\, q}(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2)\, D_1^q(\bar{z}, \bar{M}_h^2)}$$

Semi-Inclusive production of pion pair in e^+e^- annihilation @Belle

[Belle, Phys.Rev.Lett.107.072004]



- ✦ 2 hemispheres
- ✦ azimuthal modulation between the 2 hemispheres

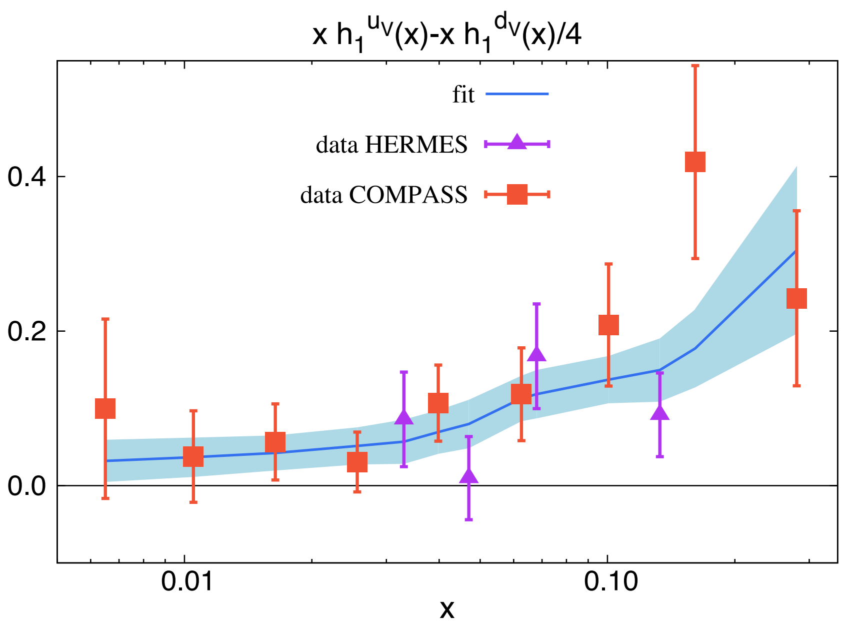
$$A_{e+e-}(z, M_h^2, \bar{z}, \bar{M}_h^2) \propto -f(\theta_2)\ g(\theta)\ g(\bar{\theta})\ \frac{\sum_q e_q^2\, H_1^{\sphericalangle\, q}(z, M_h^2)\ H_1^{\sphericalangle\, q}(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2)\, D_1^q(\bar{z}, \bar{M}_h^2)}$$

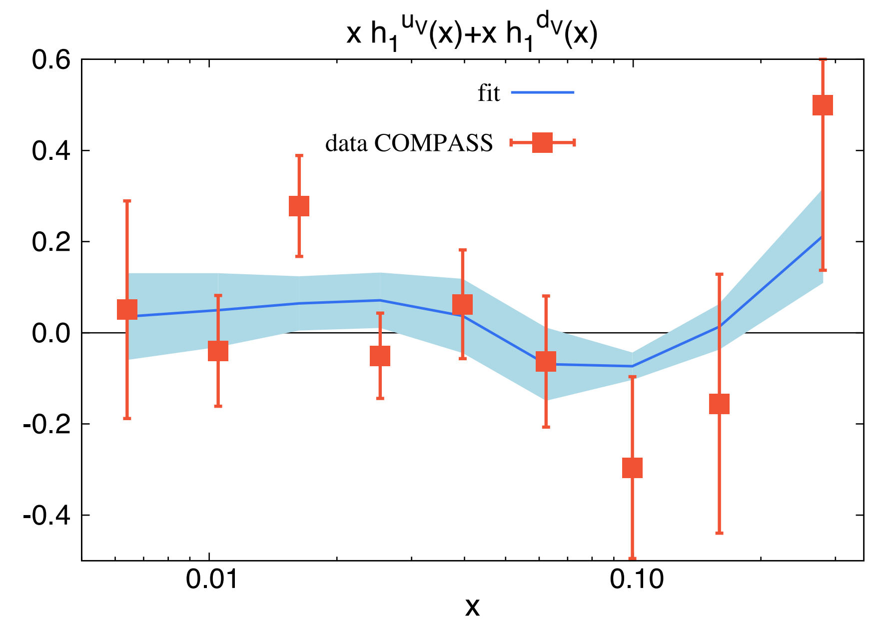
Two ways of analyzing the DiFFs

- ✦ **1st analysis:** direct analysis from experimental data [Bacchetta, A.C., Radici, PRL 107 (2011)]
- ✦ **2nd analysis:** analysis from fit of the data [A.C., Bacchetta, Radici, Bianconi, Phys.Rev. D85]

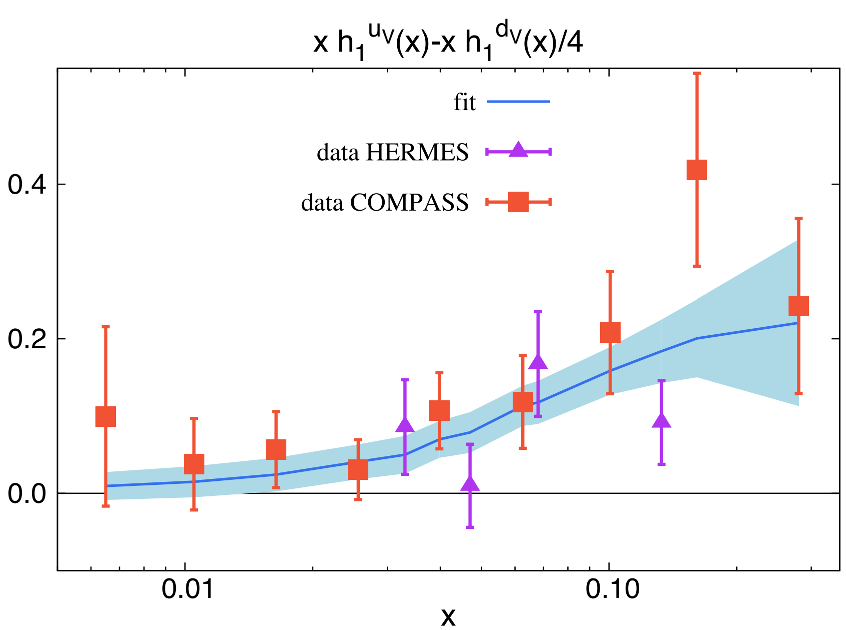
Comparison with extraction

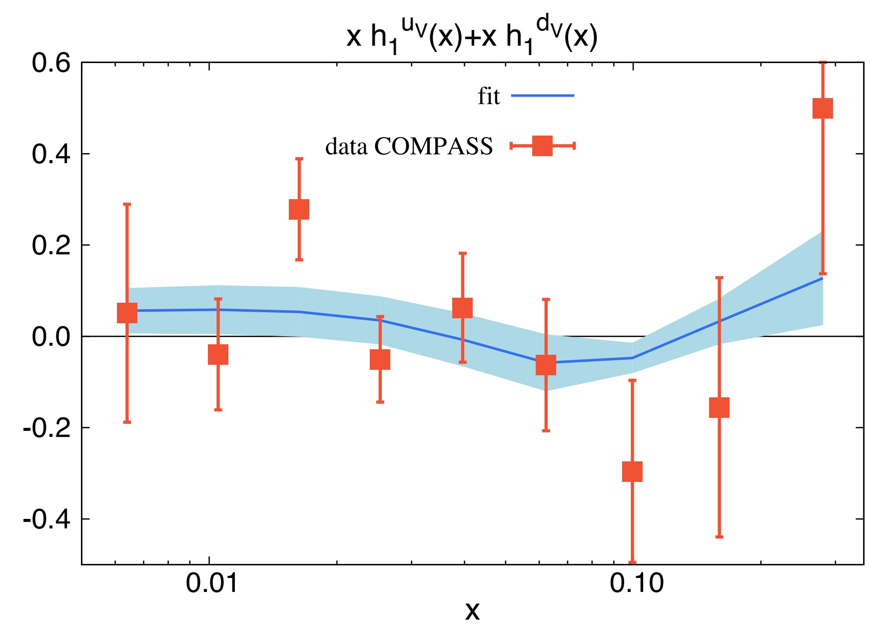
PROTON **DEUTERON**





flexible functional form





rigid functional form

Monte Carlo Approach:

some illustrations

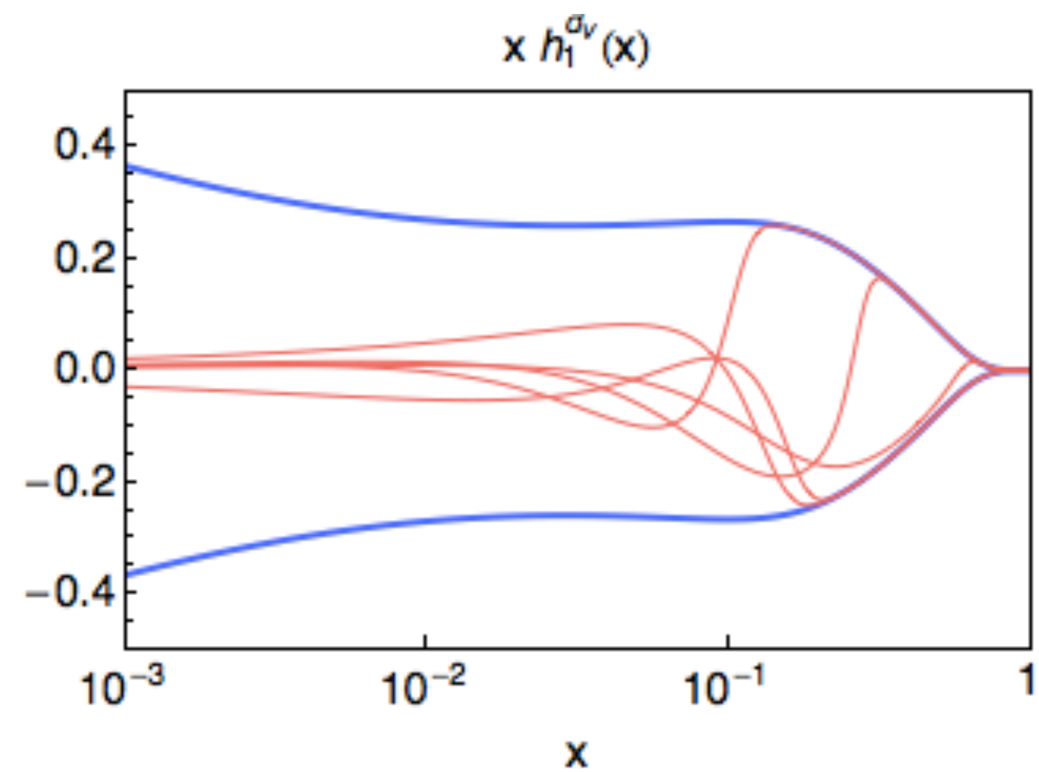
Can we find “unforeseen” replica?

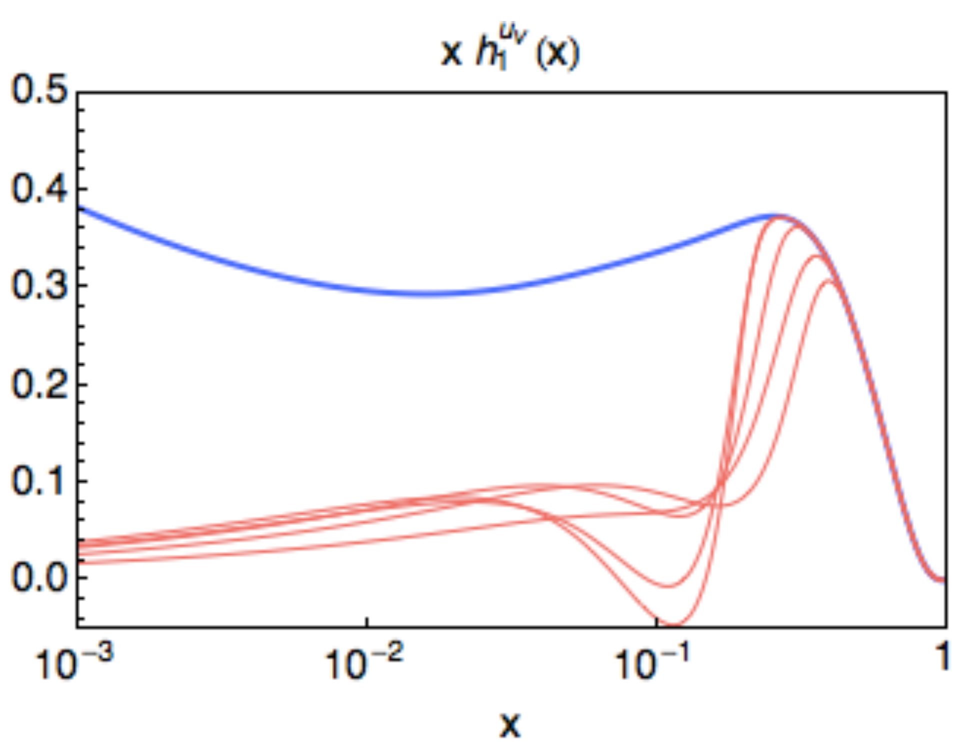
Monte Carlo Approach:

some illustrations

Can we find "unforeseen" replica?

Yes, here at 1GeV^2





χ^2/dof

1.56557
1.42199
1.79911
2.07397
1.75523