# Predictions of Diffractive, Elastic, Total, and Total-Inelastic pp Cross Sections vs LHC Measurements

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# CONTENTS

- The total pp cross section at LHC is predicted in a special fully unitarized parton model which does not employ eikonalization and does not depend on knowledge of the $\rho$-value.
- The following diffractive cross sections are described in this model based on a LO QCD approach:
  - SD – SD1/SD2, single dissociation (one/the other proton dissociates).
  - DD - double dissociation (both protons dissociate).
  - CD – central dissociation (neither proton dissociates, but there is central production of particle).
- This approach allows a unique determination of the Regge triple Pomeron coupling (*PPP*).
- Details can be found in ICHEP 2012, 6 July 2012, arXiv:1205.1446 (talk by Robert Ciesielski and KG).

# DIFFRACTION IN QCD

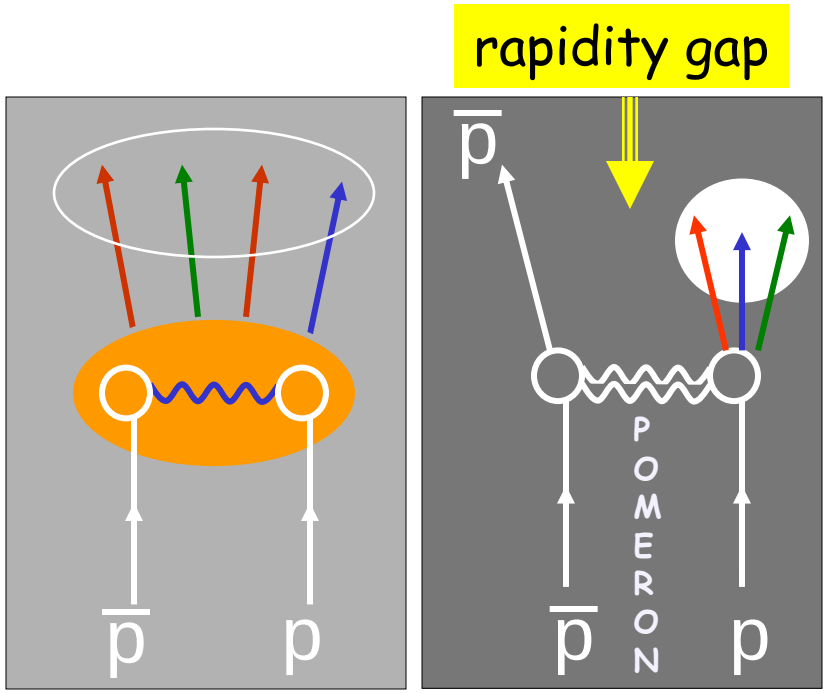
## Non-diffractive events

- color-exchange ➔ η-gaps exponentially suppressed

## Diffractive events

- Colorless vacuum exchange

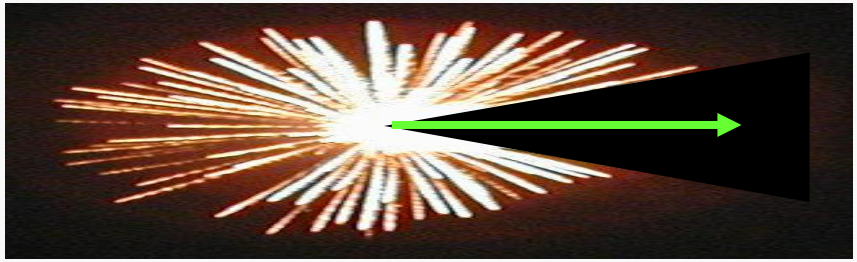
➔ η-gaps not exp'ly suppressed

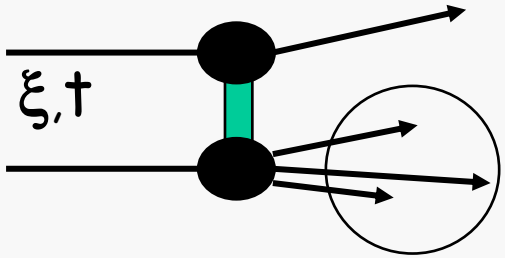


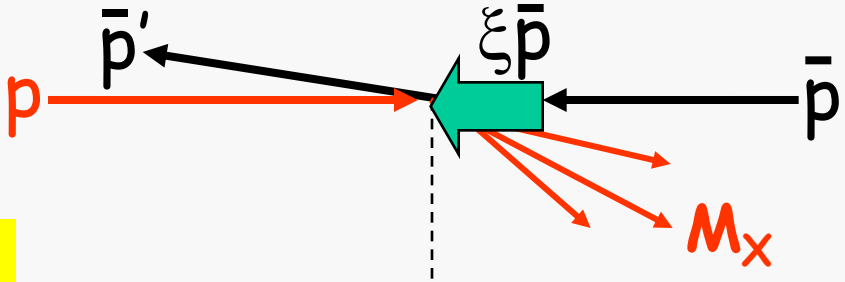

Goal: probe the QCD nature of the diffractive exchange

# DEFINITIONS

## SINGLE DIFFRACTION

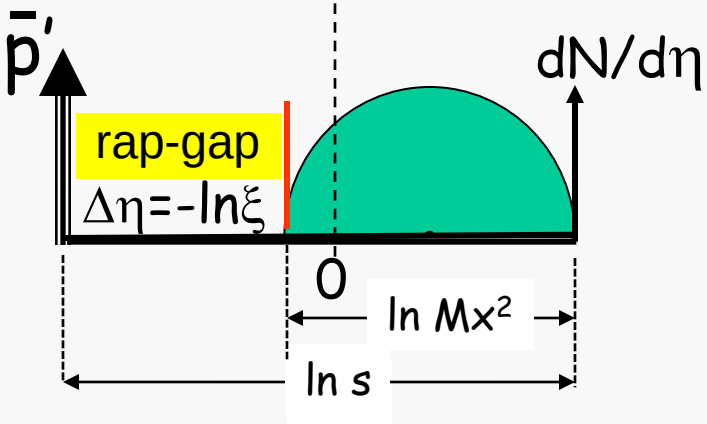





$$1 - x_L \equiv \xi = \frac{M_X^2}{s}$$

Forward momentum loss

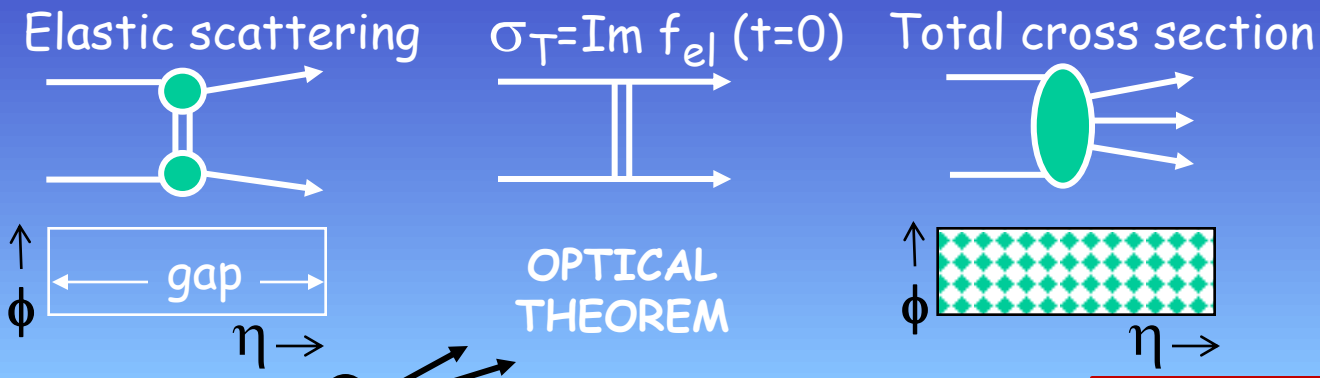


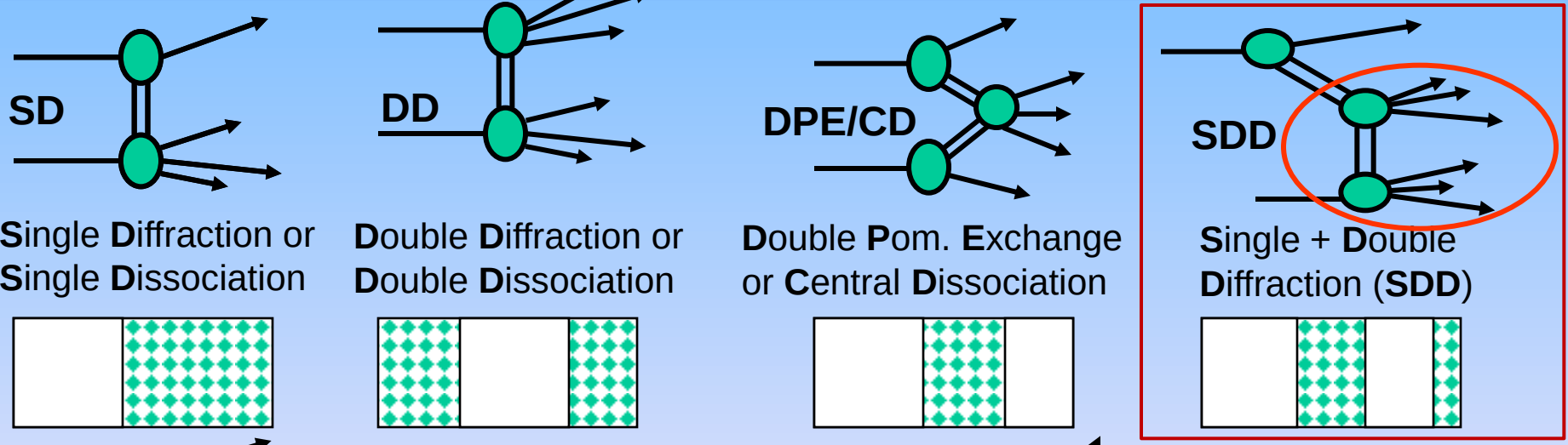

$$\xi^{CAL} = \frac{\sum_{i=1}^{all} E_T^{i\text{-tower}}\, e^{-\eta_i}}{\sqrt{s}}$$

**since no radiation →**
no price paid for increasing diffractive-gap width

$$\left(\frac{d\sigma}{d\Delta\eta}\right)_{t=0} \approx \text{constant} \Rightarrow \frac{d\sigma}{d\xi} \propto \frac{1}{\xi} \Rightarrow \frac{d\sigma}{dM^2} \propto \frac{1}{M^2}$$

# DIFFRACTION AT CDF



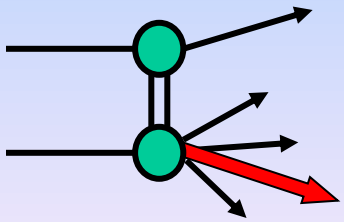

**S**ingle **D**iffraction or **S**ingle **D**issociation

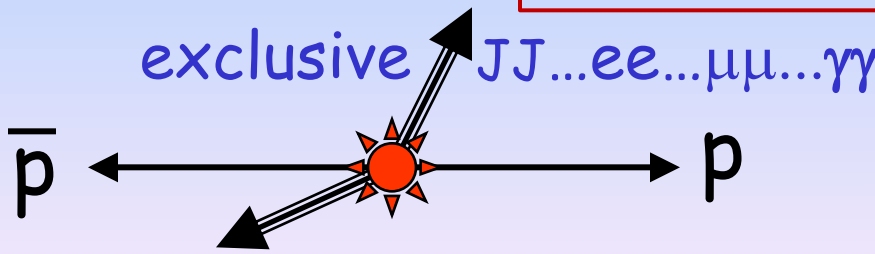
**D**ouble **D**iffraction or **D**ouble **D**issociation

**D**ouble **P**om. **E**xchange or **C**entral **D**issociation

**S**ingle + **D**ouble **D**iffraction (**SDD**)

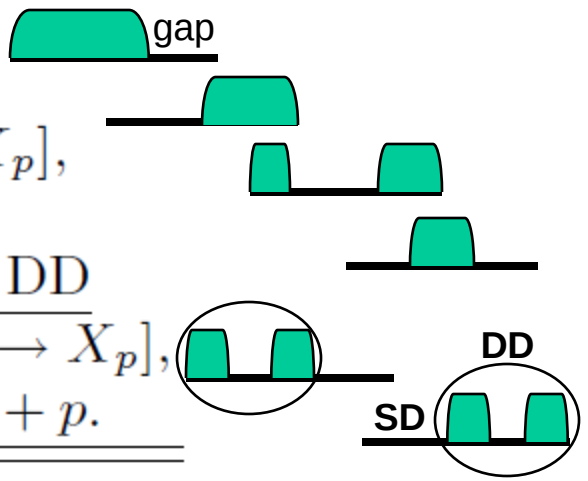




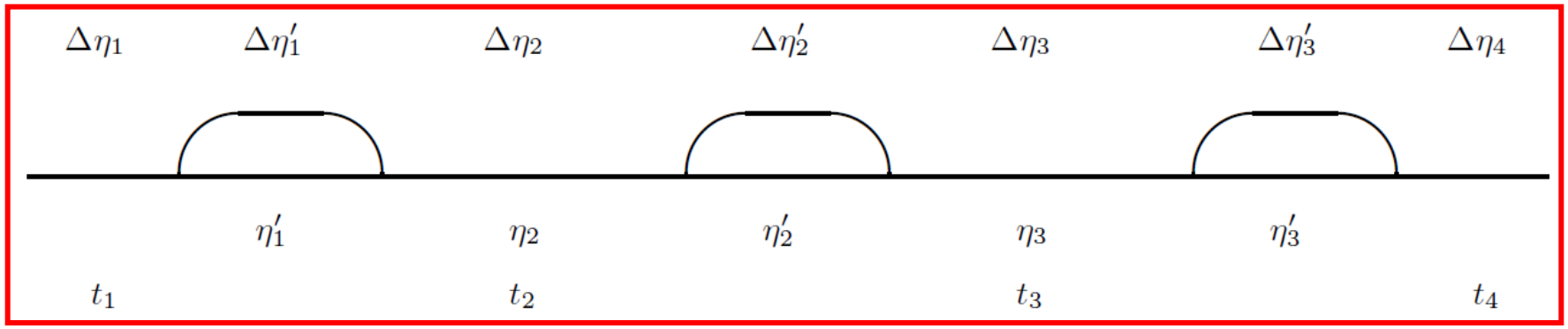



# Basic and combined diffractive processes

| acronym | basic diffractive processes |
|---|---|
| $\mathbf{SD}_{\bar{p}}$ | $\bar{p}p \to \bar{p} + \text{gap} + [p \to X_p]$, |
| $\mathbf{SD}_{p}$ | $\bar{p}p \to [\bar{p} \to X_{\bar{p}}] + \text{gap} + p$, |
| **DD** | $\bar{p}p \to [\bar{p} \to X_{\bar{p}}] + \text{gap} + [p \to X_p]$, |
| **DPE** | $\bar{p}p \to \bar{p} + \text{gap} + X_c + \text{gap} + p$, |
| | 2-gap combinations of SD and DD |
| $\mathbf{SDD}_{\bar{p}}$ | $\bar{p}p \to \bar{p} + \text{gap} + X_c + \text{gap} + [p \to X_p]$, |
| $\mathbf{SDD}_{p}$ | $\bar{p}p \to [\bar{p} \to X_{\bar{p}}]\text{gap} + X_c + \text{gap} + p$. |

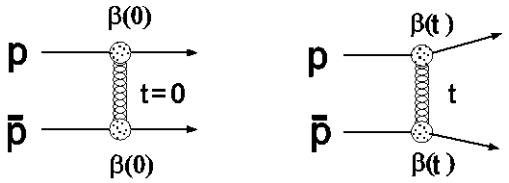



4-gap diffractive process-Snowmass 2001- http://arxiv.org/pdf/hep-ph/0110240

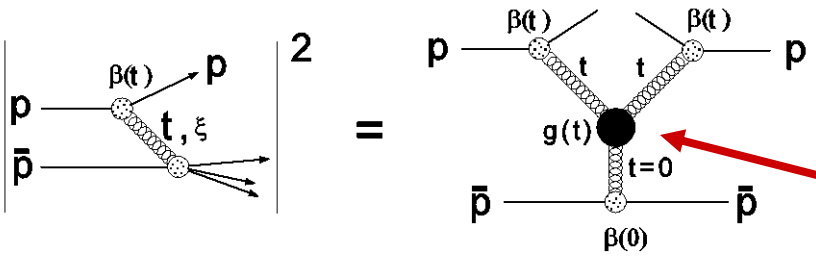


# Regge theory – values of $s_0$ & $g_{PPP}$?

KG-PLB 358, 379 (1995)




SINGLE DIFFRACTION DISSOCIATION




Parameters:

- $s_0$, $s_0'$ and g(t)
- set $s_0' = s_0$ (universal *IP* )
- determine $s_0$ and $g_{PPP}$ – **how?**

$\alpha(t)=\alpha(0)+\alpha' t \quad \alpha(0)=1+\varepsilon$

$$\sigma_T = \beta_1(0)\beta_2(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1} = \sigma_0^{p\bar{p}}\left(\frac{s}{s_0}\right)^{\epsilon} \qquad (1)$$

$$\frac{d\sigma_{el}}{dt} = \frac{\beta_1^2(t)\beta_2^2(t)}{16\pi}\left(\frac{s}{s_0}\right)^{2[\alpha(t)-1]}$$

$$= \frac{\sigma_T^2}{16\pi}\left(\frac{s}{s_0}\right)^{2\alpha' t} F^4(t) \approx \frac{\sigma_T^2}{16\pi} e^{b_{el}(s)t} \qquad (2)$$

$$F^4(t) \approx e^{b_{0,el}t} \quad \Rightarrow \quad b_{el}(s) = b_{0,el} + 2\alpha' \ln\left(\frac{s}{s_0}\right) \qquad (3)$$

$$\frac{d^2\sigma_{sd}}{dtd\xi}$$

$$= \frac{\beta_1^2(t)}{16\pi}\,\xi^{1-2\alpha(t)}\left[\beta_2(0)\,g(t)\left(\frac{s'}{s_0'}\right)^{\alpha(0)-1}\right]$$

$$= f_{\mathcal{P}/p}(\xi,t)\;\sigma_T^{\mathcal{P}\bar{p}}(s',t) \qquad (4)$$

# A complication … ➔ Unitarity!

$$\left(\frac{d\sigma_{el}}{dt}\right)_{t=0} \sim \left(\frac{s}{s_o}\right)^{2\epsilon}, \ \sigma_t \sim \left(\frac{s}{s_o}\right)^{\epsilon}, \ \text{and} \ \sigma_{sd} \sim \left(\frac{s}{s_o}\right)^{2\epsilon}$$

- $\sigma_{sd}$ grows faster than $\sigma_t$ as $s$ increases *
  ➔ unitarity violation at high $s$

(similarly for partial x-sections in impact parameter space)

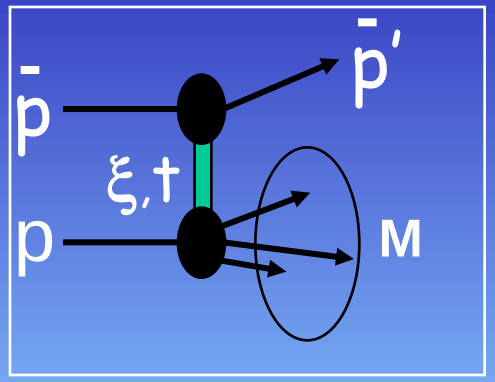
- the unitarity limit is already reached at $\sqrt{s} \sim 2$ TeV !

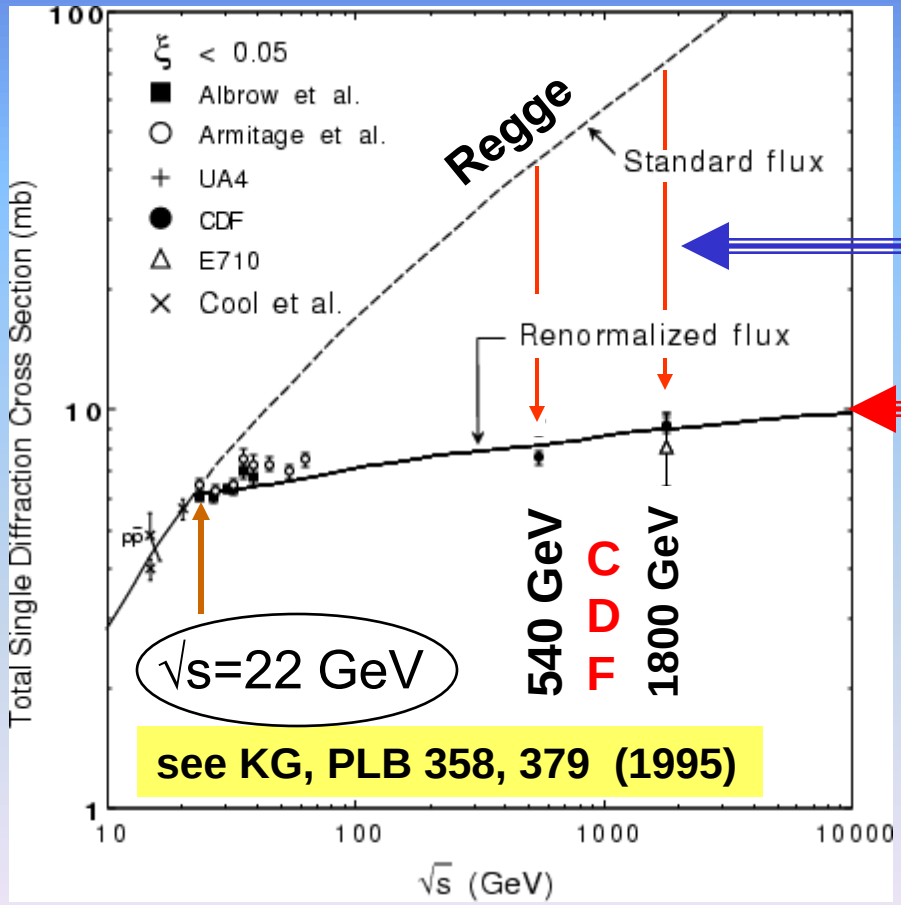
- need unitarization

* similarly for $(d\sigma_{el}/dt)_{t=0}$ w.r.t. $\sigma_t$, but this is handled differently in RENORM

# FACTORIZATION BREAKING IN SOFT DIFFRACTION

➔diffractive x-section suppressed relative to Regge prediction as √s increases



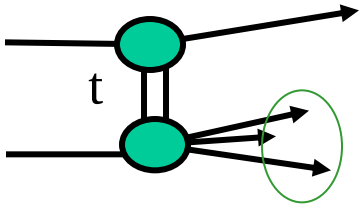

Factor of ~8 (~5) suppression at √s = 1800 (540) GeV

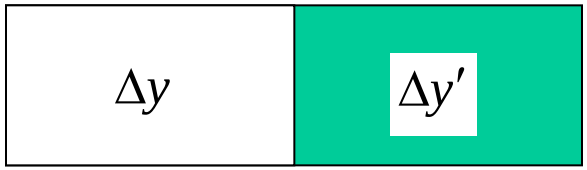
**RENORMALIZATION**

Interpret flux as gap formation probability that saturates when it reaches unity

# Single diffraction renormalized - 1

KG ➔ CORFU-2001: http://arxiv.org/abs/hep-ph/0203141




2 independent variables: $t, \Delta y$

color factor

$$\kappa = \frac{g_{IP-IP-IP}(t)}{\beta_{IP-p-p}(0)} \approx 0.17$$

$$\frac{d^2\sigma}{dt\, d\Delta y} = C \cdot F_p^2(t) \cdot \left\{ e^{(\varepsilon + \alpha' t)\Delta y} \right\}^2 \cdot \kappa \cdot \left\{ \sigma_o\, e^{\varepsilon\, \Delta y'} \right\}$$

gap probability

subenergy x-section

Gap probability ➔ (re)normalize to unity

# Single diffraction renormalized - 2

color factor

$$\kappa = \frac{g_{IP-IP-IP}(t)}{\beta_{IP-p-p}(0)} \approx 0.17$$

Experimentally:

$$\kappa = \frac{g_{IP-IP-IP}}{\beta_{IP-p}} = 0.17 \pm 0.02, \qquad \varepsilon = 0.104$$

**KG&JM, PRD 59 (114017) 1999**

QCD: $$\kappa = \mathrm{f}_g \times \frac{1}{N_c^2 - 1} + \mathrm{f}_q \times \frac{1}{N_c} \xrightarrow{Q^2 = 1} \approx 0.75 \times \frac{1}{8} + 0.25 \times \frac{1}{3} = 0.18$$

# Single diffraction renormalized - 3

$$\frac{d^2\sigma_{sd}(s,M^2,t)}{dM^2dt} = \left[\frac{\sigma_\circ}{16\pi}\sigma_\circ^{I\!Pp}\right] \frac{s^{2\epsilon}}{N(s,s_o)} \frac{e^{bt}}{(M^2)^{1+\epsilon}}$$

$$b = b_0 + 2\alpha' \ln\frac{s}{M^2} \qquad s_o^{\rm CMG} = (3.7 \pm 1.5)\ {\rm GeV}^2$$

$$N(s,s_o) \equiv \int_{\xi_{\rm min}}^{\xi_{\rm max}} d\xi \int_{t=0}^{-\infty} dt\, f_{I\!P/p}(\xi,t) \overset{s\to\infty}{\to} \sim s_o^\epsilon \frac{s^{2\epsilon}}{\ln s}$$

$$\frac{d^2\sigma_{sd}(s,M^2,t)}{dM^2dt} \overset{s\to\infty}{\to} \sim \ln s\, \frac{e^{bt}}{(M^2)^{1+\epsilon}}$$

$$\sigma_{sd} \xrightarrow{s\to\infty} \sim \frac{\ln s}{b \to \ln s} \Rightarrow const$$

set to unity
➔ determines $s_o$

# $M^2$ distribution: data

➔ $d\sigma/dM^2|_{t=-0.05}$ ~ independent of s over 6 orders of magnitude!

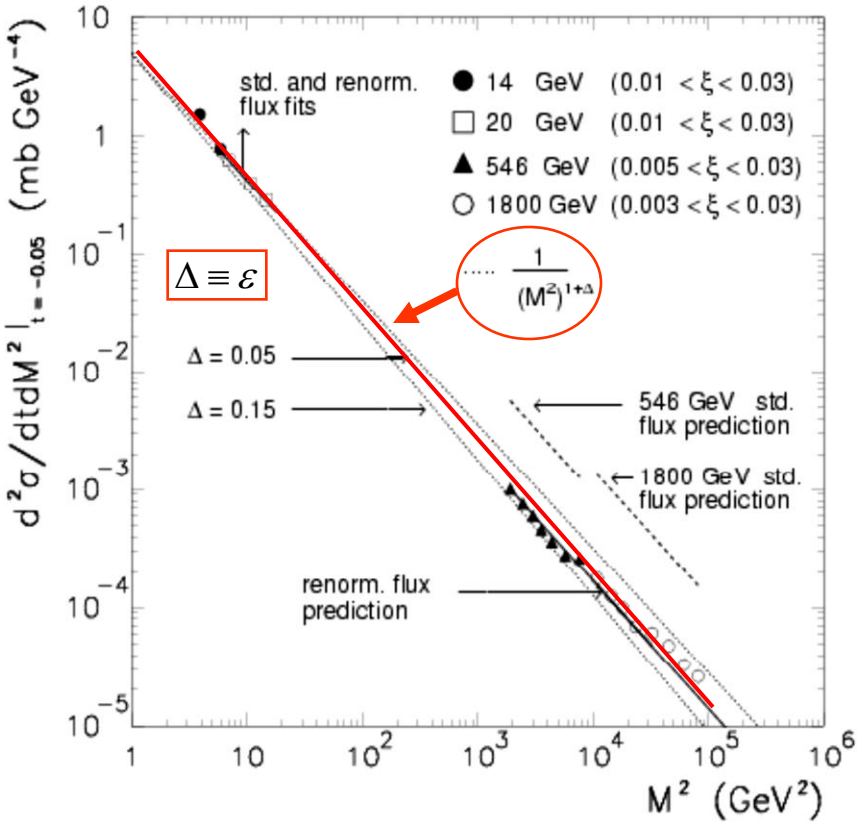
Regge → data

$$\frac{d\sigma}{dM^2} \propto \frac{s^{2\varepsilon} \to 1}{(M^2)^{1+\varepsilon}}$$

Independent of s over 6 orders of magnitude in $M^2$

➔ $M^2$ scaling

**KG&JM, PRD 59 (1999) 114017**



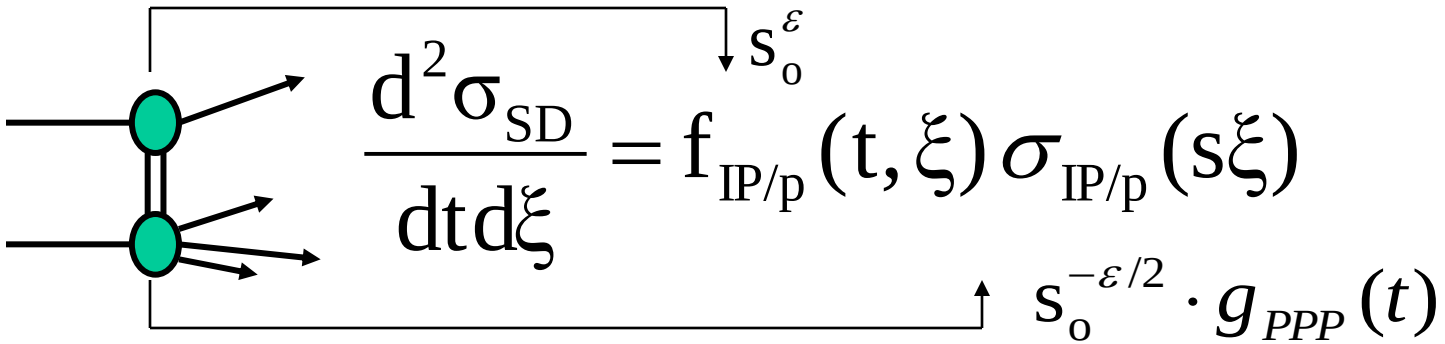

➔ factorization breaks down to ensure $M^2$ scaling

# Scale $s_0$ and *PPP* coupling

Pomeron flux: interpret as gap probability

➔set to unity: determines $g_{PPP}$ and $s_0$

KG, PLB 358 (1995) 379



$$\frac{d^2\sigma_{SD}}{dtd\xi} = f_{IP/p}(t,\xi)\,\sigma_{IP/p}(s\xi)$$

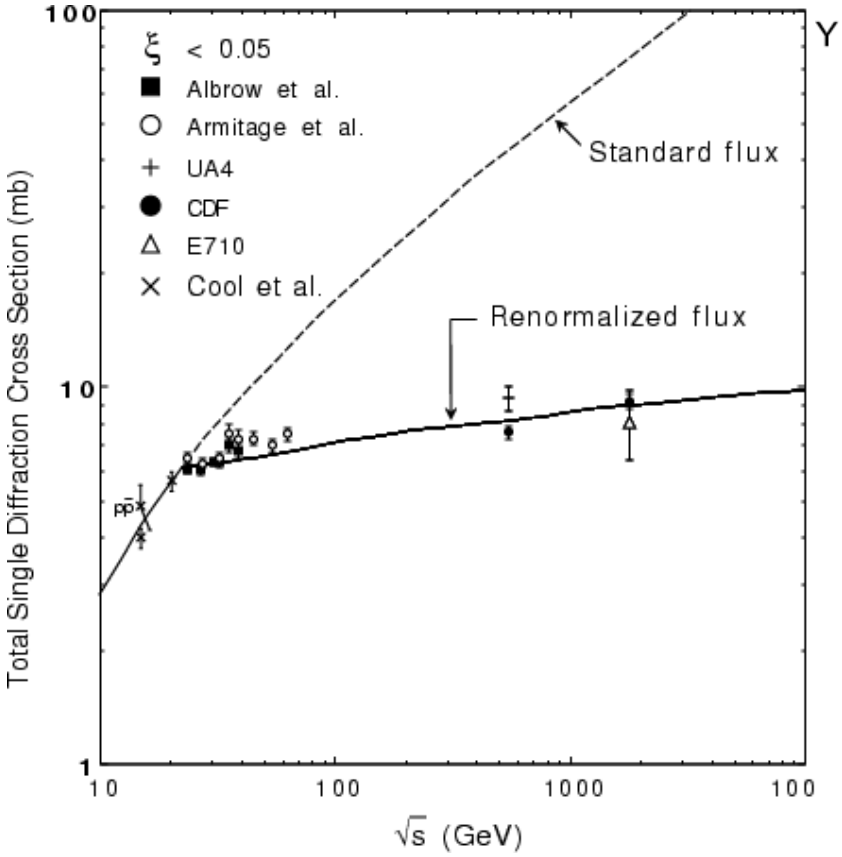
$s_o^{\varepsilon}$

$s_o^{-\varepsilon/2}\cdot g_{PPP}(t)$

Pomeron-proton x-section

- Two free parameters: $s_o$ and $g_{PPP}$
- Obtain product $g_{PPP}\bullet s_o^{\varepsilon/2}$ from $\sigma_{SD}$
- Renormalized Pomeron flux determines $s_o$
- Get unique solution for $g_{PPP}$

# Saturation at low $Q^2$ and small-x



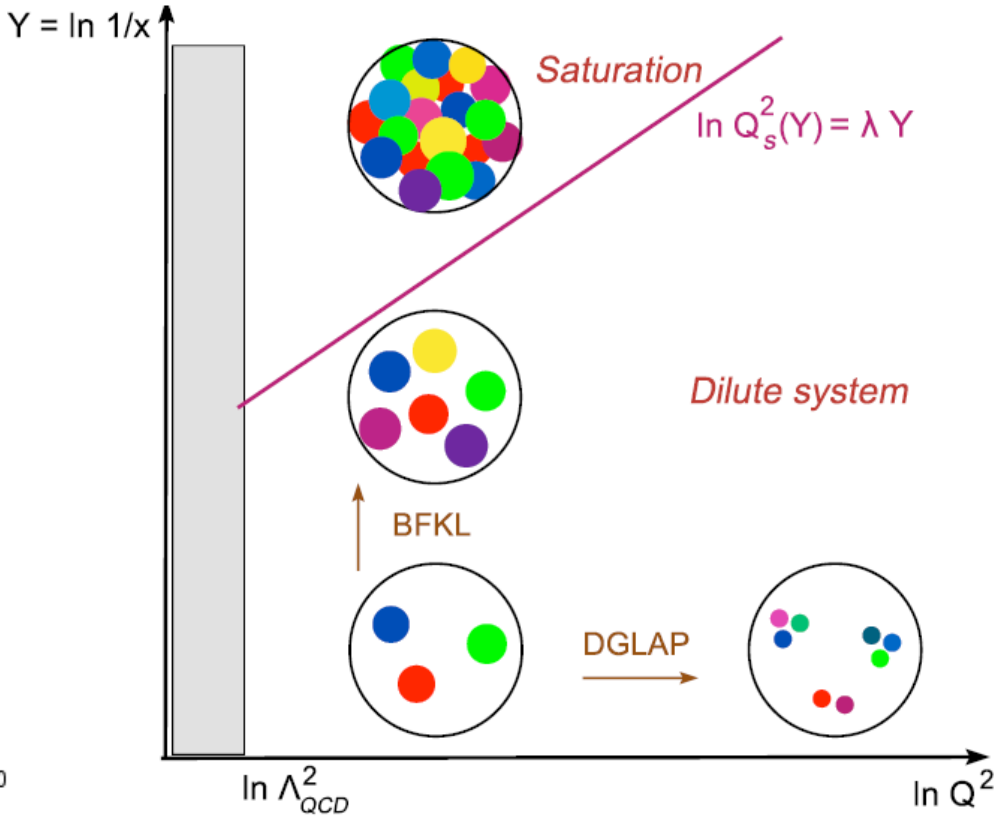
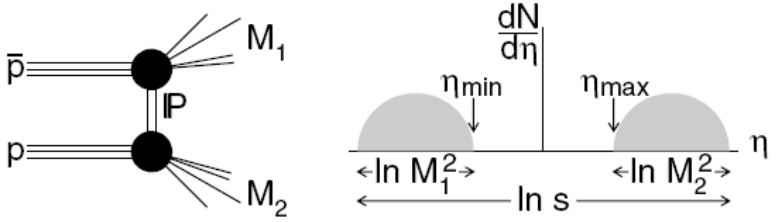



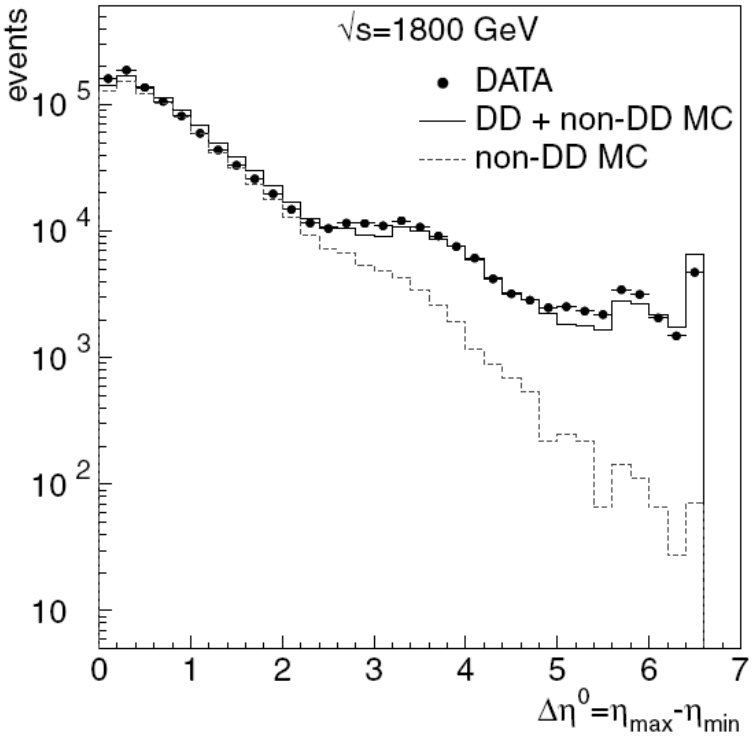

figure from a talk by Edmond Iancu

# DD at CDF



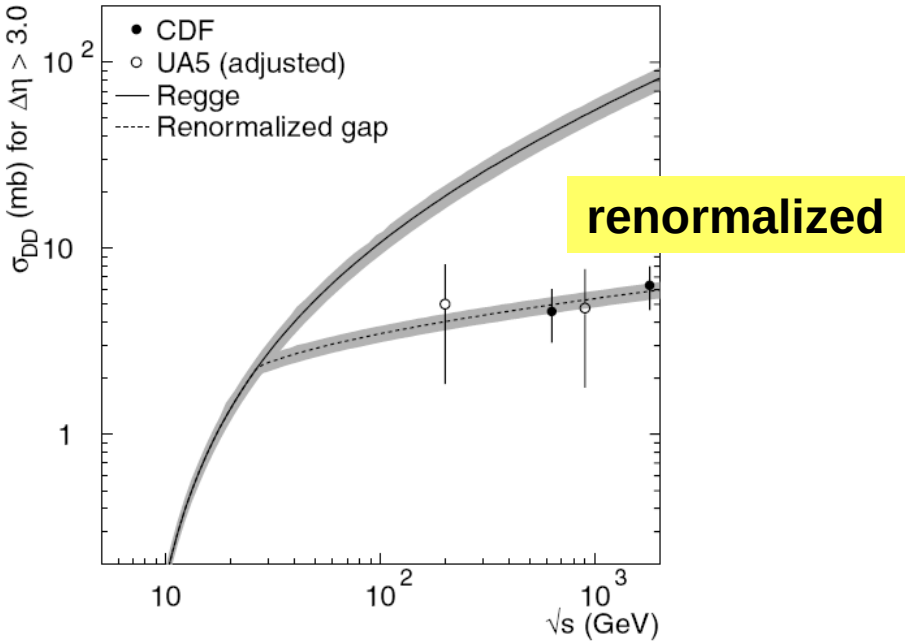

$$\frac{d^3\sigma_{DD}}{dt dM_1^2 dM_2^2} = \frac{d^2\sigma_{SD}}{dt dM_1^2}\frac{d^2\sigma_{SD}}{dt dM_2^2} \Big/ \frac{d\sigma_{el}}{dt}$$

$$= \frac{[\kappa\beta_1(0)\beta_2(0)]^2}{16\pi}\frac{s^{2\epsilon}e^{b_{DD}t}}{(M_1^2M_2^2)^{1+2\epsilon}}$$



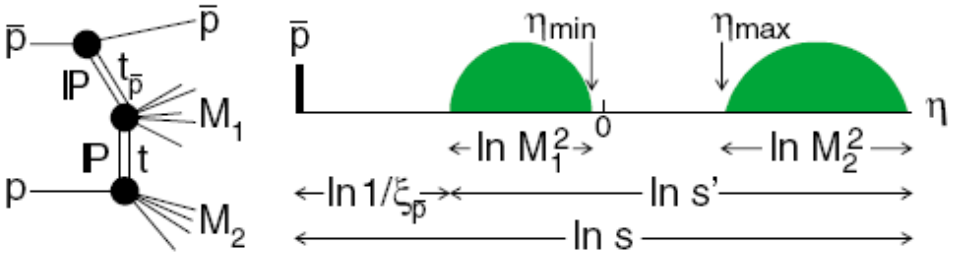

$$\frac{d^3\sigma_{DD}}{dt d\Delta\eta d\eta_c} = \left[\frac{\kappa\beta^2(0)}{16\pi}e^{2[\alpha(t)-1]\Delta\eta}\right]\left[\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right]$$

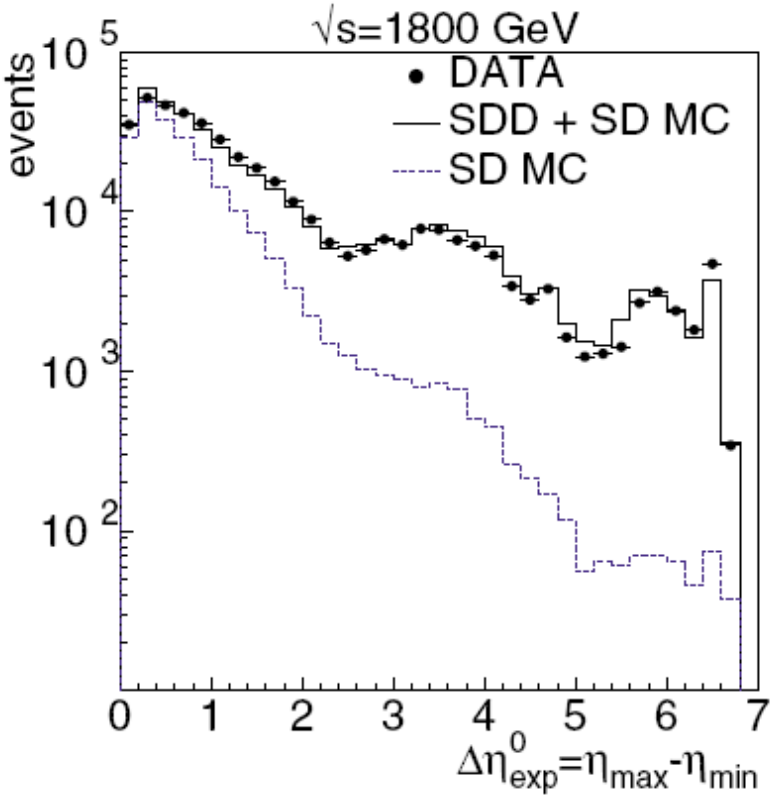
gap probability  x-section




**renormalized**

# SDD at CDF

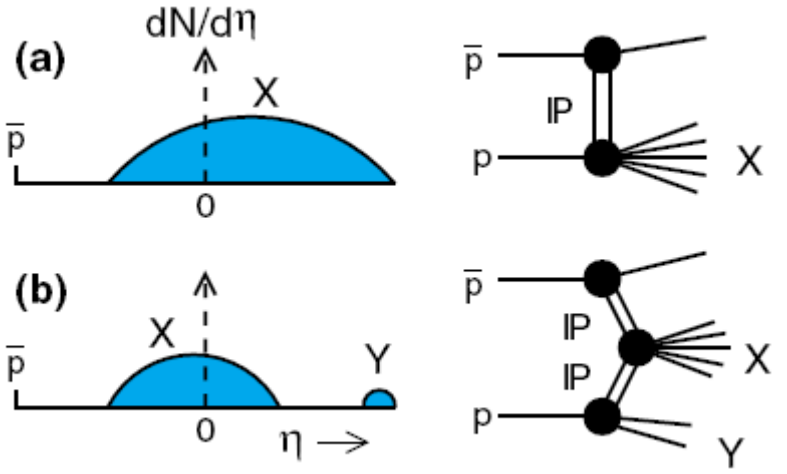



- Excellent agreement between data and MBR (MinBiasRockefeller) MC

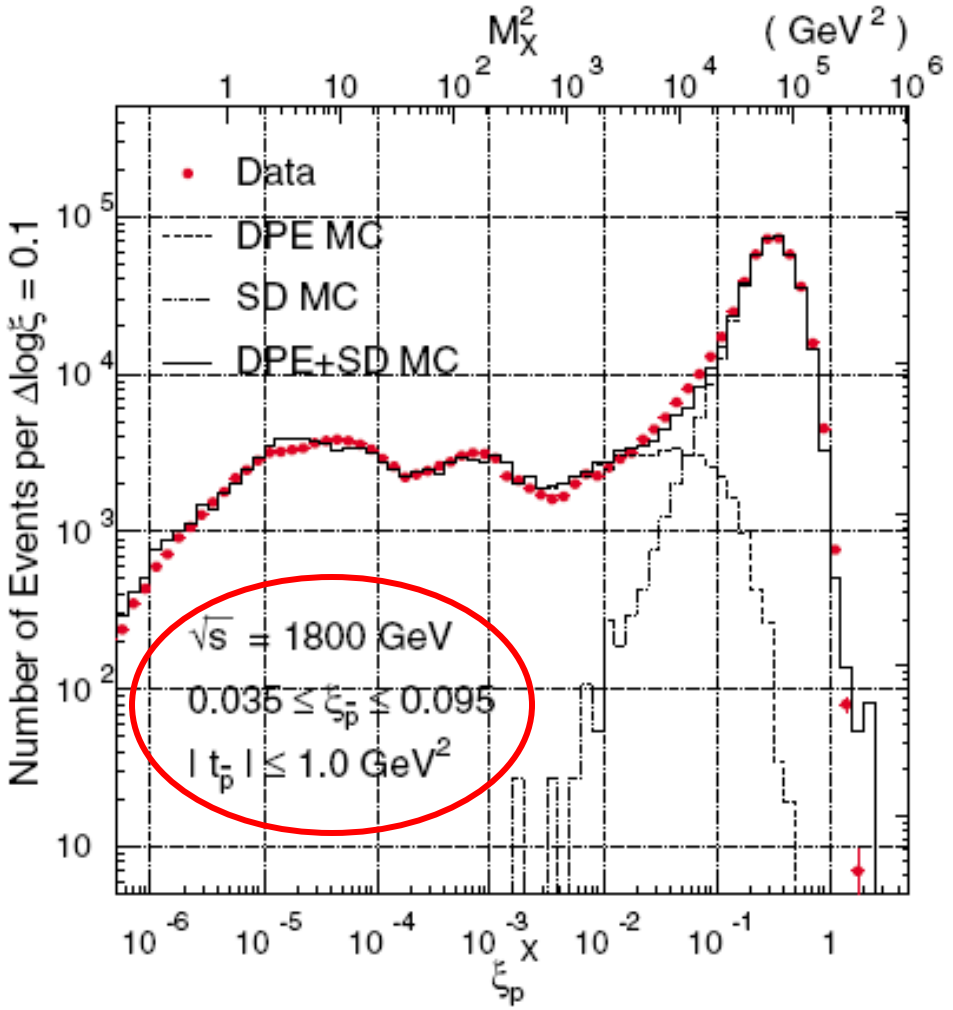
$$\frac{d^5\sigma}{dt_{\bar{p}}dtd\xi_{\bar{p}}d\Delta\eta d\eta_c} = \left[\frac{\beta(t)}{4\sqrt{\pi}}\, e^{[\alpha(t_{\bar{p}})-1]\ln(1/\xi)}\right]^2 \times \kappa\left\{\kappa\left[\frac{\beta(0)}{4\sqrt{\pi}}\, e^{[\alpha(t)-1]\Delta\eta}\right]^2 \kappa\left[\beta^2(0)\left(\frac{s''}{s_0}\right)^{\epsilon}\right]\right\}$$

# CD/DPE at CDF




- Excellent agreement between data and MBR

➔ low and high masses are correctly implemented



# Difractive x-sections

$$\frac{d^2\sigma_{SD}}{dtd\Delta y} = \frac{1}{N_{\rm gap}(s)}\left[\frac{\beta^2(t)}{16\pi}e^{2[\alpha(t)-1]\Delta y}\right]\cdot\left\{\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right\},$$

$$\frac{d^3\sigma_{DD}}{dtd\Delta ydy_0} = \frac{1}{N_{\rm gap}(s)}\left[\frac{\kappa\beta^2(0)}{16\pi}e^{2[\alpha(t)-1]\Delta y}\right]\cdot\left\{\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right\},$$

$$\frac{d^4\sigma_{DPE}}{dt_1dt_2d\Delta ydy_c} = \frac{1}{N_{\rm gap}(s)}\left[\Pi_i\left[\frac{\beta^2(t_i)}{16\pi}e^{2[\alpha(t_i)-1]\Delta y_i}\right]\right]\cdot\kappa\left\{\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right\}$$

$$\beta^2(t) = \beta^2(0)F^2(t)$$

$$F^2(t) = \left[\frac{4m_p^2-2.8t}{4m_p^2-t}\left(\frac{1}{1-\frac{t}{0.71}}\right)^2\right]^2 \approx a_1e^{b_1t}+a_2e^{b_2t}$$

$\alpha_1$=0.9, $\alpha_2$=0.1, $b_1$=4.6 GeV$^{-2}$, $b_2$=0.6 GeV$^{-2}$, $s'=s\,e^{-\Delta y}$, $\kappa$=0.17, $\kappa\beta^2(0)=\sigma_0$, $s_0$=1 GeV$^2$, $\sigma_0$=2.82 mb or 7.25 GeV$^{-2}$

# Total, elastic, and inelastic x-sections

$$\sigma_{\rm ND} = (\sigma_{\rm tot} - \sigma_{\rm el}) - (2\sigma_{\rm SD} + \sigma_{\rm DD} + \sigma_{\rm CD})$$

CMG

R. J. M. Covolan, K. Goulianos, J. Montanha, Phys. Lett. B **389**, 176 (1996)

$$\sigma_{\rm tot}^{p^{\pm}p} = \begin{cases} 16.79 s^{0.104} + 60.81 s^{-0.32} \mp 31.68 s^{-0.54} & \text{for } \sqrt{s} < 1.8 \\ \sigma_{\rm tot}^{\rm CDF} + \frac{\pi}{s_0}\left[\left(\ln\frac{s}{s_F}\right)^2 - \left(\ln\frac{s^{\rm CDF}}{s_F}\right)^2\right] & \text{for } \sqrt{s} \geq 1.8 \end{cases}$$

KG Moriond 2011, arXiv:1105.1916

$\sqrt{s^{\rm CDF}} = 1.8$ TeV, $\sigma_{\rm tot}^{\rm CDF} = 80.03 \pm 2.24$ mb

$\sqrt{s_F} = 22$ GeV $\quad s_0 = 3.7 \pm 1.5$ GeV$^2$

$\sigma_{el}^{p\pm p} = \sigma_{tot} \times (\sigma_{el}/\sigma_{tot})$, with $\sigma_{el}/\sigma_{tot}$ from CMG
small extrapol. from 1.8 to 7 and up to 50 TeV )

# Diffractive and Total pp Cross Sections at LHC

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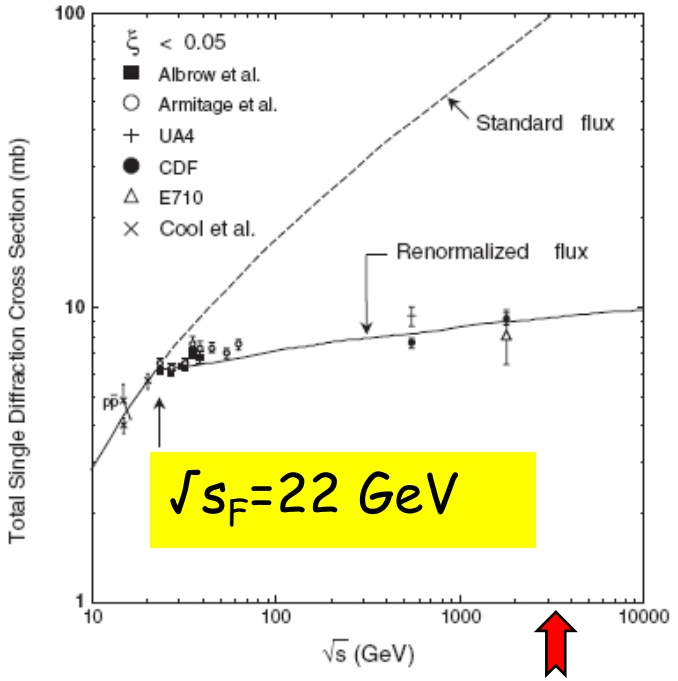
EDS '09




$\sqrt{s_F}$=22 GeV

- Use the Froissart formula as a *saturated* cross section:

$$\sigma_t(s > s_F) = \sigma_t(s_F) + \frac{\pi}{m^2} \cdot \ln^2 \frac{s}{s_F}$$

- This formula should be valid above the *knee* in $\sigma_{sd}$ vs. $\sqrt{s}$ at $\sqrt{s_F} = 22$ GeV (Fig. 1) and therefore valid at $\sqrt{s} = 1800$ GeV.

- Use $m^2 = s_o$ in the Froissart formula multiplied by $1/0.389$ to convert it to $\text{mb}^{-1}$.

- Note that contributions from Reggeon exchanges at $\sqrt{s} = 1800$ GeV are negligible, as can be verified from the global fit of Ref. [7].

- Obtain the total cross section at the LHC:

$$\sigma_t^{\text{LHC}} = \sigma_t^{\text{CDF}} + \frac{\pi}{s_o} \cdot \left( \ln^2 \frac{s^{\text{LHC}}}{s_F} - \ln^2 \frac{s^{\text{CDF}}}{s_F} \right)$$

98 ± 8 mb at 7 TeV
109 ±12 mb at 14 TeV

Main error from $s_0$

# Reduce the uncertainty in $s_0$

## Saturation glueball?

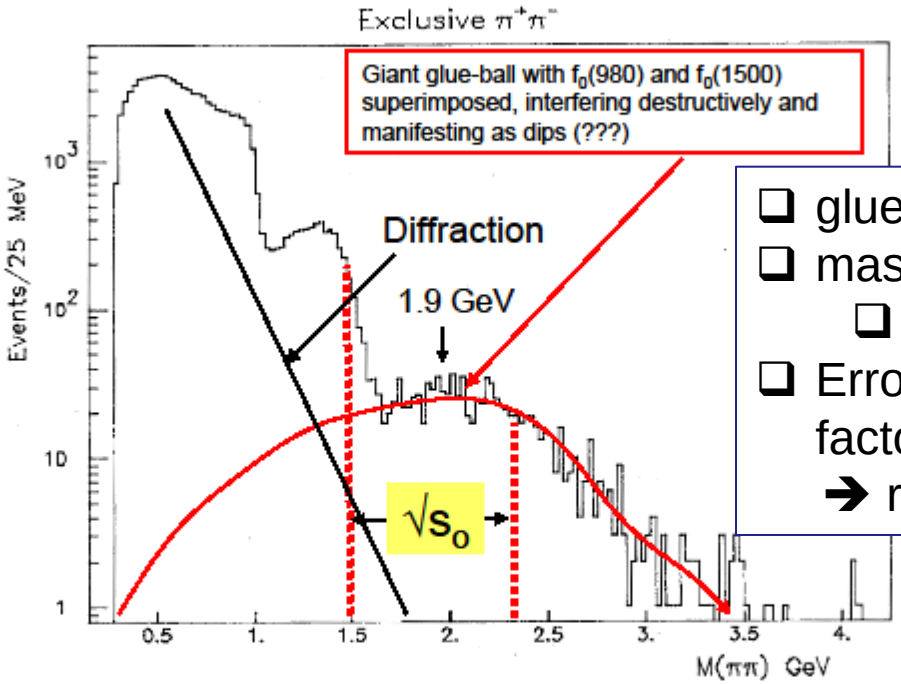
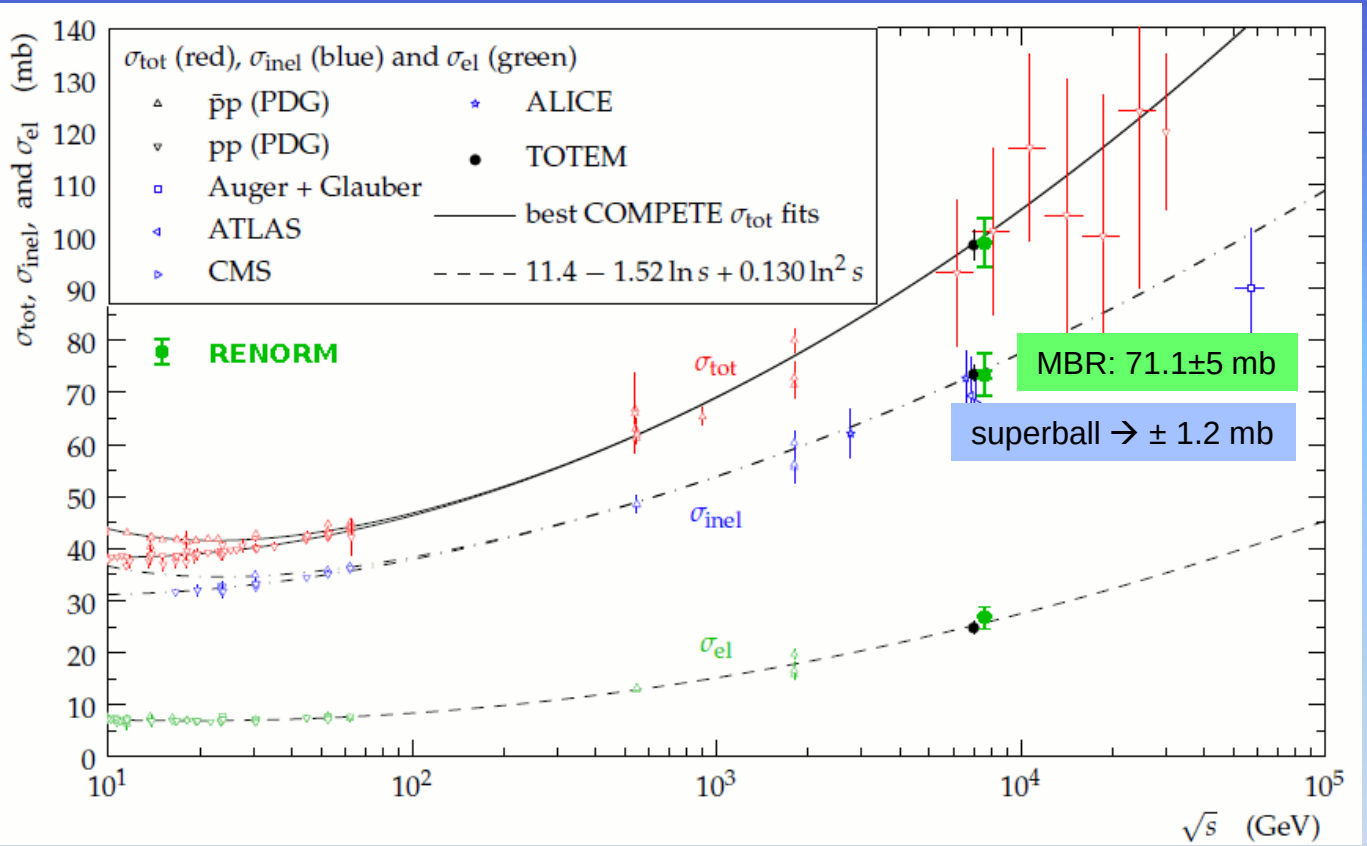



Figure 8: $M_{\pi^-\pi^-}$ spectrum in *DIPE* at the ISR (Axial Field Spectrometer, R807 [97, 98]). Figure from Ref. [98]. **See M.G.Albrow, T.D. Goughlin, J.R. Forshaw, hep-ph>arXiv:1006.1289**

MIAMI 2010, Nov14 -19 Diffraction, saturation, and pp cross sections at the LHC and beyond K. Goulianos 2

- glue-ball-like object ➔ "superball"
- mass →1.9 GeV ➔ $m_s^2$= 3.7 GeV
  - agrees with RENORM $s_o$=3.7
- Error in $s_0$ can be reduced by factor ~4 from a fit to these data!
  ➔ reduces error in $\sigma_t$.

# TOTEM vs PYTHIA8-MBR




**$\sigma_{inrl}^{7\ TeV}$= 72.9 ±1.5 mb**

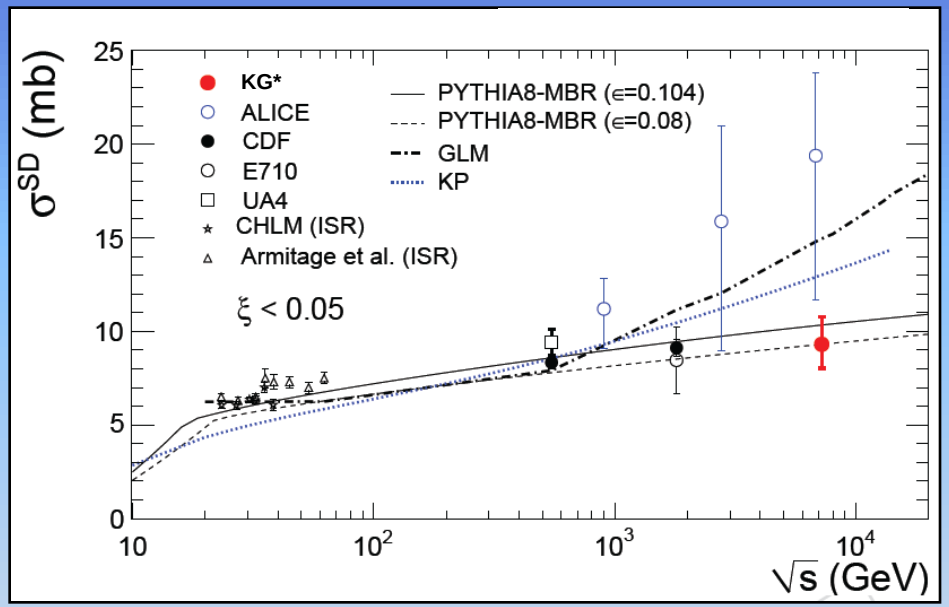
**$\sigma_{inrl}^{8\ TeV}$= 74.7 ±1.7 mb**

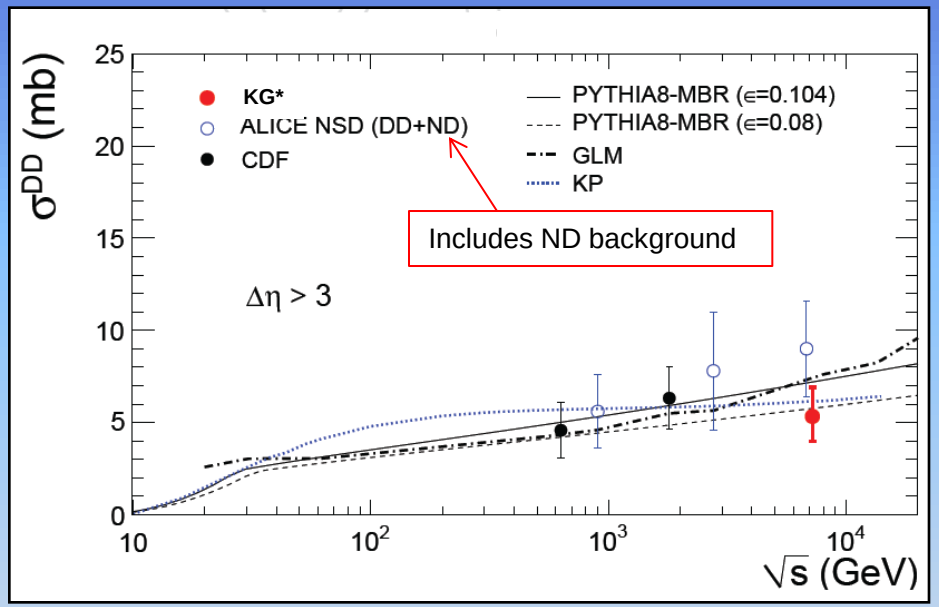
***TOTEM, G. Latino talk at MPI@LHC, CERN 2012***

RENORM: 71.1±1.2 mb

RENORM: 72.3±1.2 mb

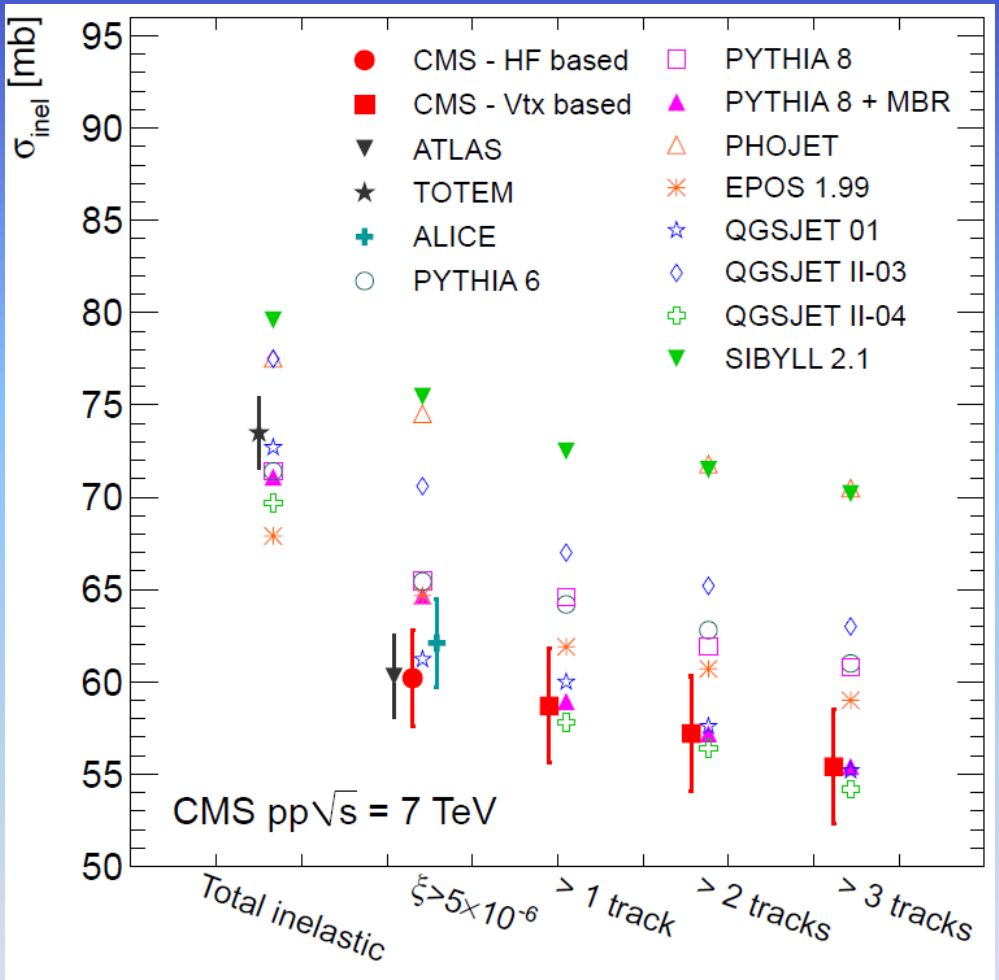
# CMS SD and DD x-sections vs ALICE: measurements and theory models




- KG*: after extrapolation into low ξ from measured CMS data using the MBR model: find details on data in Robert Ciesielski's talk on Wed. at 15:30.

# Total-Inelastic Cross Sections vs model predictions



# Monte Carlo Strategy for the LHC …

## MONTE CARLO STRATEGY

- $\sigma_{tot}$ → from SUPERBALL model
- optical theorem → Im $f_{el}(t=0)$
- dispersion relations → Re $f_{el}(t=0)$
- $\sigma_{el}$ ← using global fit
- $\sigma_{inel} = \sigma_{tot} - \sigma_{el}$
- differential $\sigma_{SD}$ → from RENORM
- use *nesting* of final states for

*pp* collisions at the *P-p* sub-energy $\sqrt{s'}$

$\sigma_T$

↕ optical theorem

Im $f_{el}(t=0)$

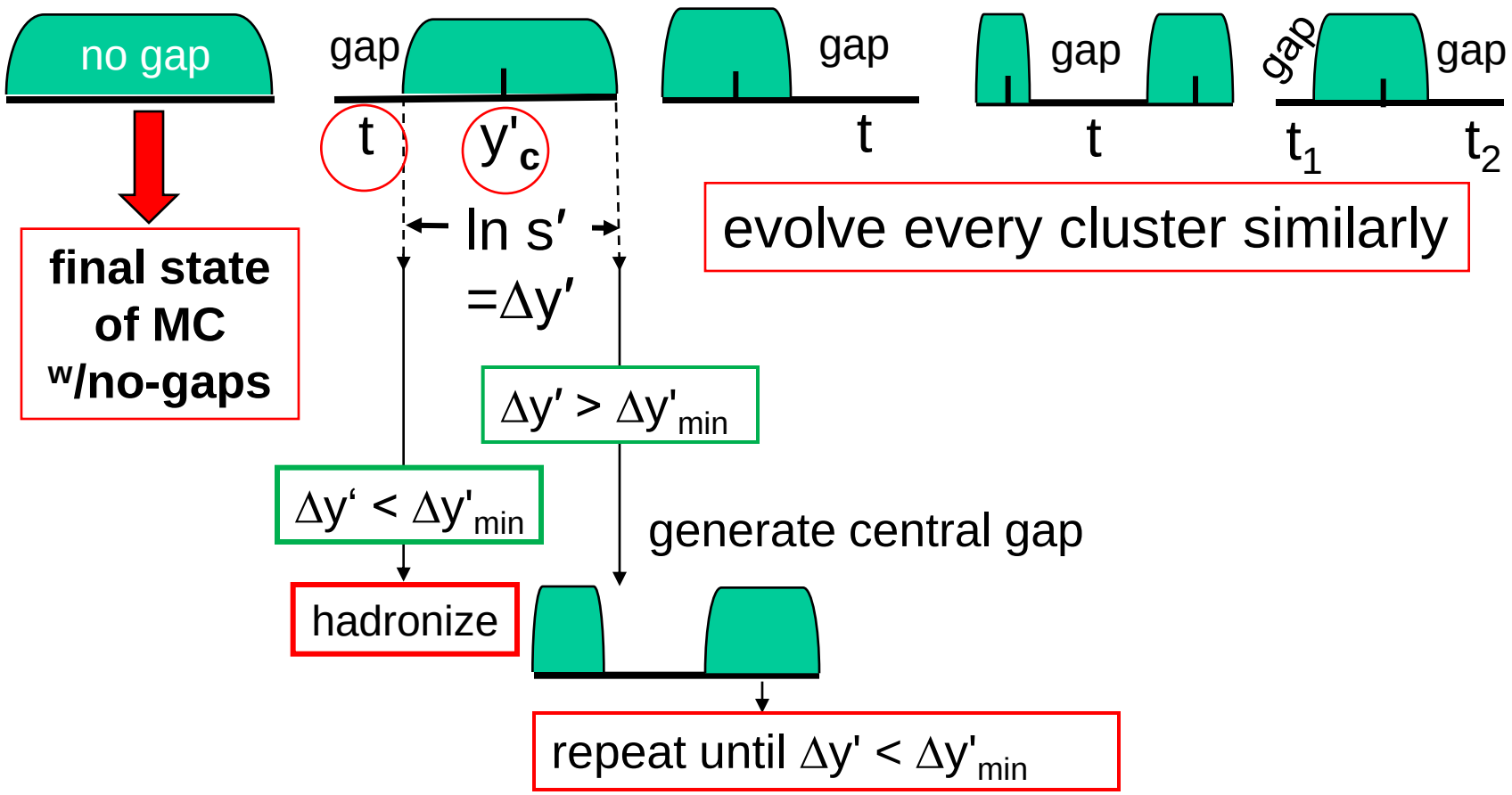
↕ dispersion relations

Re $f_{el}(t=0)$

*Strategy similar to that of MBR used in CDF based on multiplicities from:*
*K. Goulianos, Phys. Lett. B 193 (1987) 151* pp
"A new statistical description of hardonic and $e^+e^-$ multiplicity distributios "

# Monte Carlo algorithm - nesting

Profile of a *pp* inelastic collision



# SUMMARY

- Introduction
- Diffractive cross sections:
  - basic: SD1,SD2, DD, CD (DPE)
  - combined: multigap x-sections

  } **derived from ND and QCD color factors**

  - ND ➔ no diffractive gaps:
    - this is the only final state to be tuned
- Total, elastic, and total inelastic cross sections
- Monte Carlo strategy for the LHC – “nesting”

*Thank you for your attention*