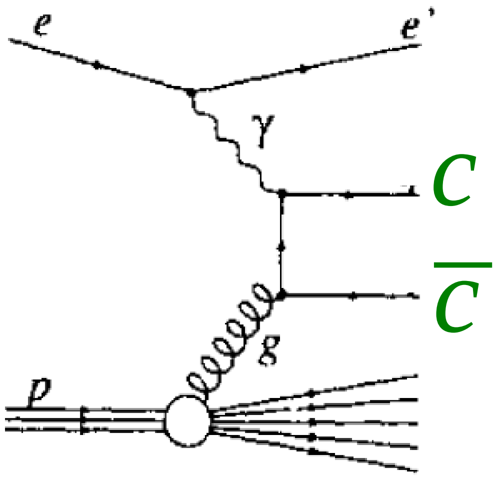
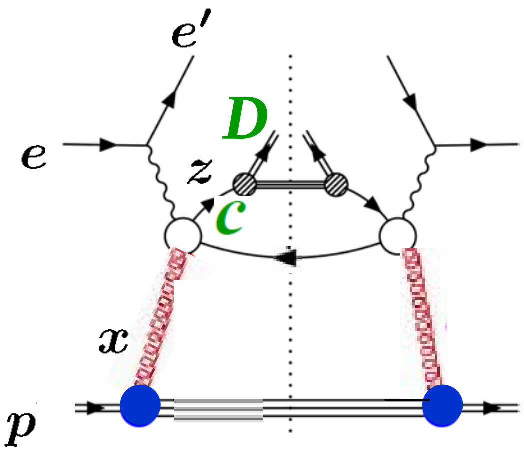
Gluon correlations in the transversely polarized nucleon at twist three

Kazuhiro Tanaka (Juntendo U)

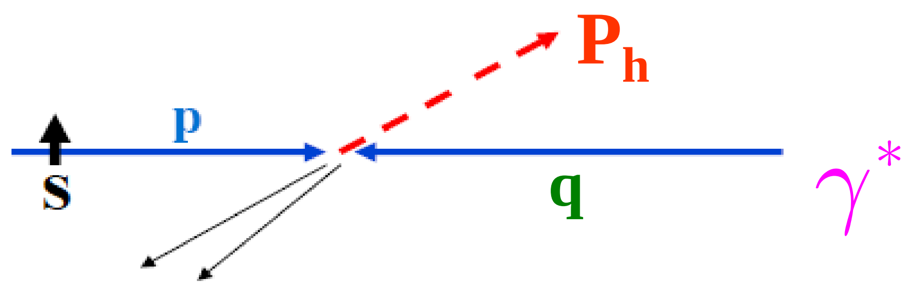
PRD85 ('12) 114026
JHEP1302 ('13) 003

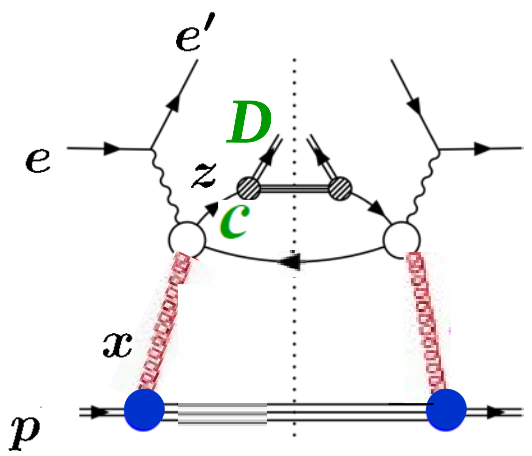
SIDIS $ep \to eDX$



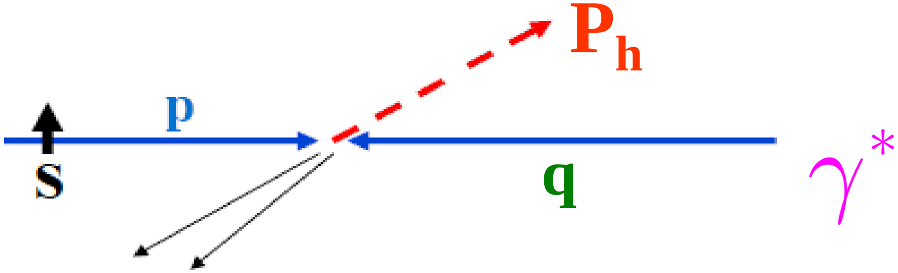


SSA in SIDIS $ep^{\uparrow} \to eDX$



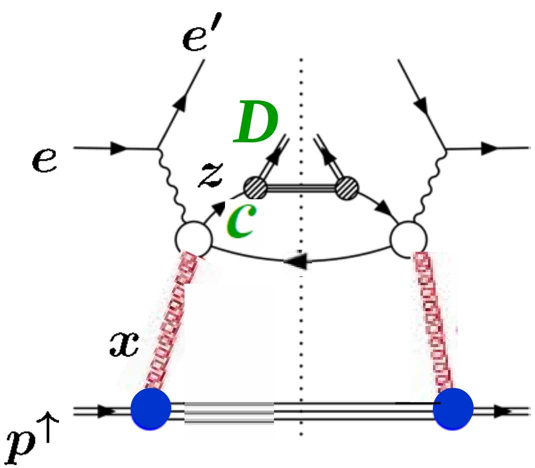


SSA in SIDIS $ep^{\uparrow} \to eDX$



$$d\sigma \sim \vec{S}_{\perp} \cdot \left(\vec{p} \times \vec{P_h}\right)$$

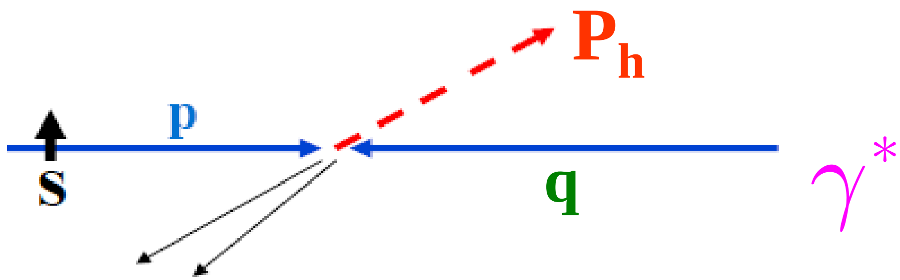
$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



$S_{\perp}$

1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born

SSA in **SIDIS** $ep^{\uparrow} \to eDX$



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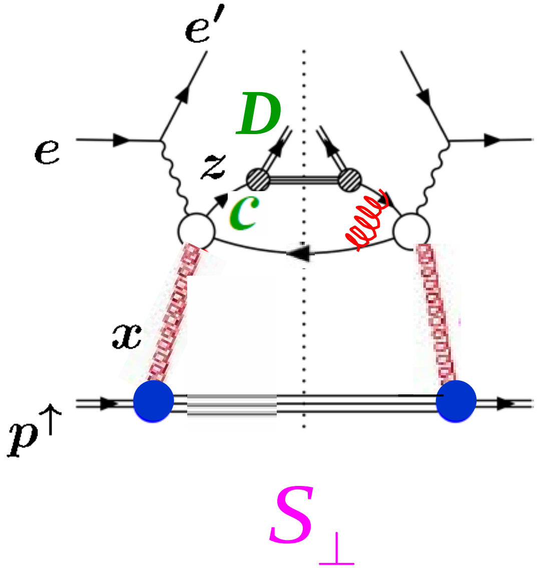

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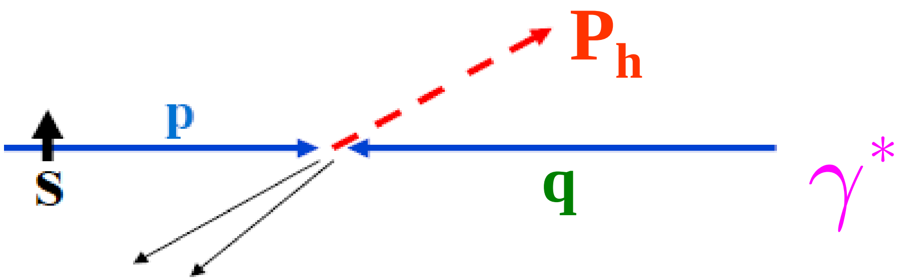
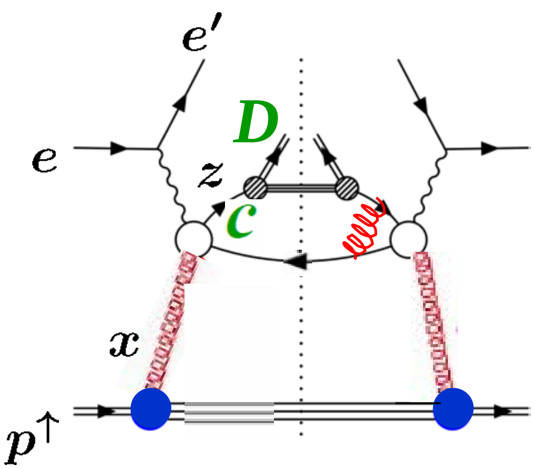


$$d\sigma \sim \vec{S}_{\perp} \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

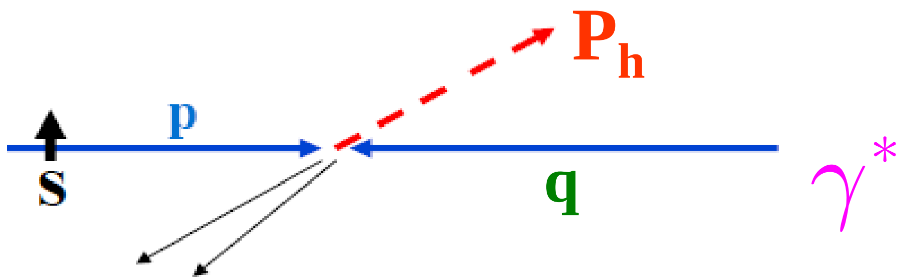
at twist-2

1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon ○
2. helicity flip by one unit ✗
3. interaction phase: beyond Born △



$S_{\perp}$

SSA in **SIDIS** $ep^{\uparrow} \to eDX$

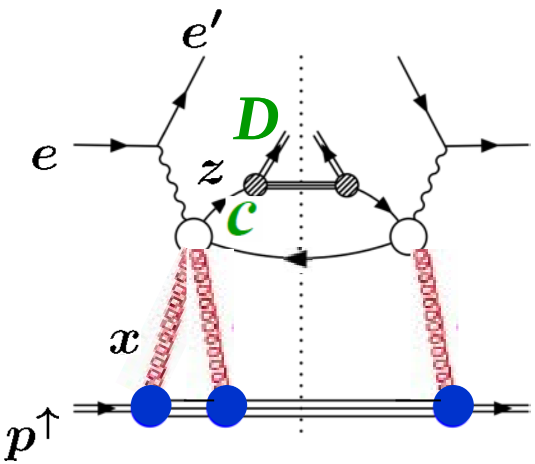


$$d\sigma \sim \vec{S}_{\perp} \cdot \left(\vec{p} \times \vec{P}_h\right)$$

$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

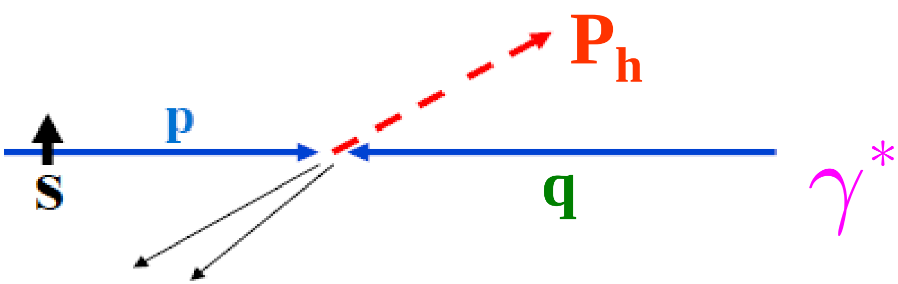
at twist-3

1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon ○
2. helicity flip by one unit ○
3. interaction phase: beyond Born ○



$S_{\perp}$

SSA in **SIDIS** $ep^{\uparrow} \to eDX$

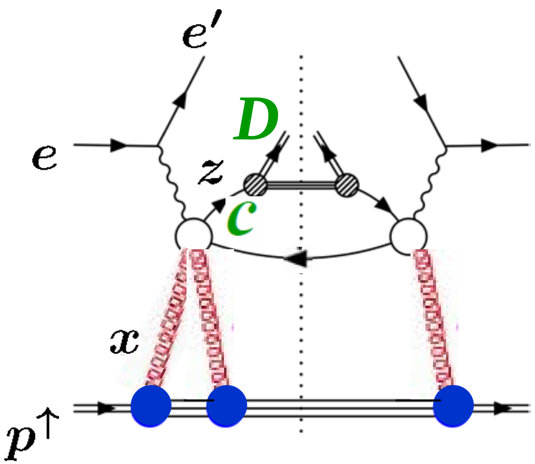


$$d\sigma \sim \vec{S}_{\perp} \cdot \left(\vec{p} \times \vec{P}_h \right)$$

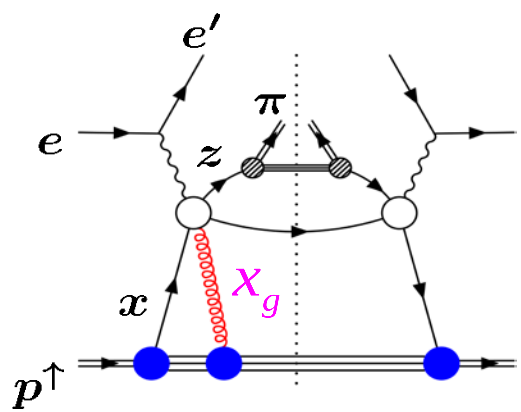
$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

at twist-3

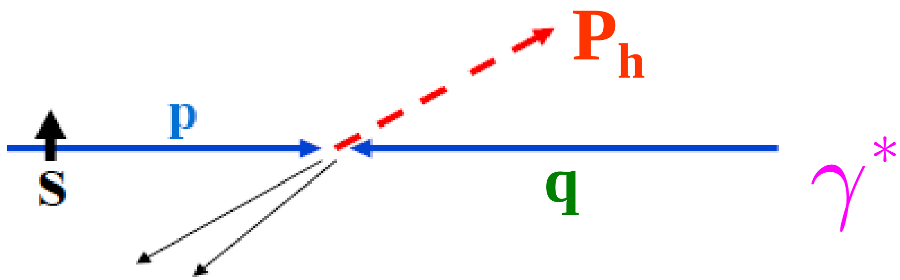
1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon ○
2. helicity flip by one unit ○
3. interaction phase: beyond Born ○



$S_{\perp}$



SSA in SIDIS $ep^{\uparrow} \to eDX$



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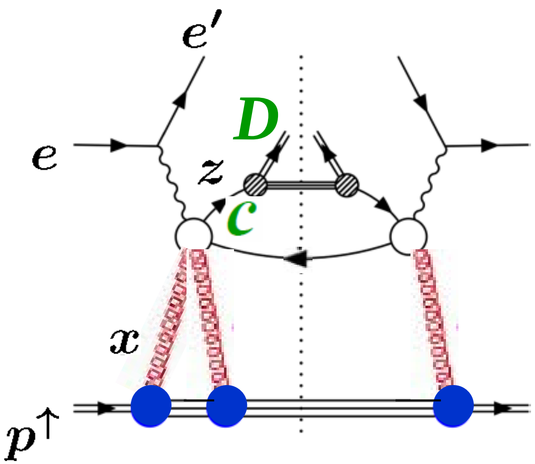
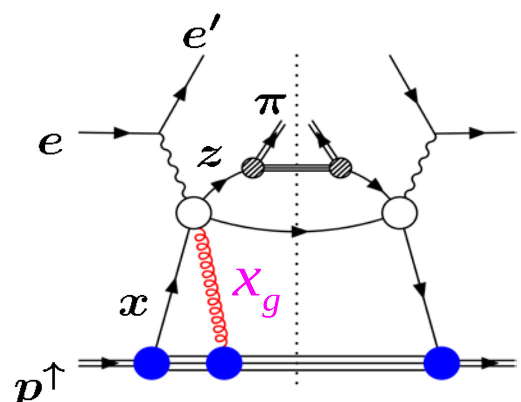
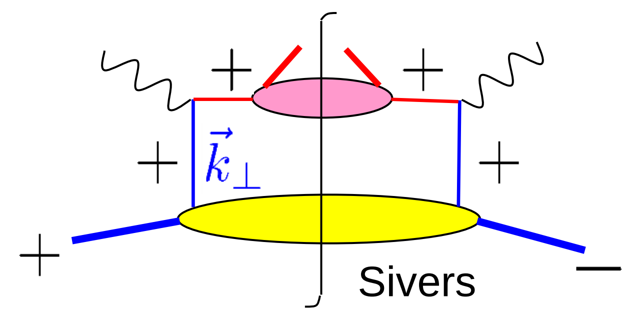


$S_{\perp}$

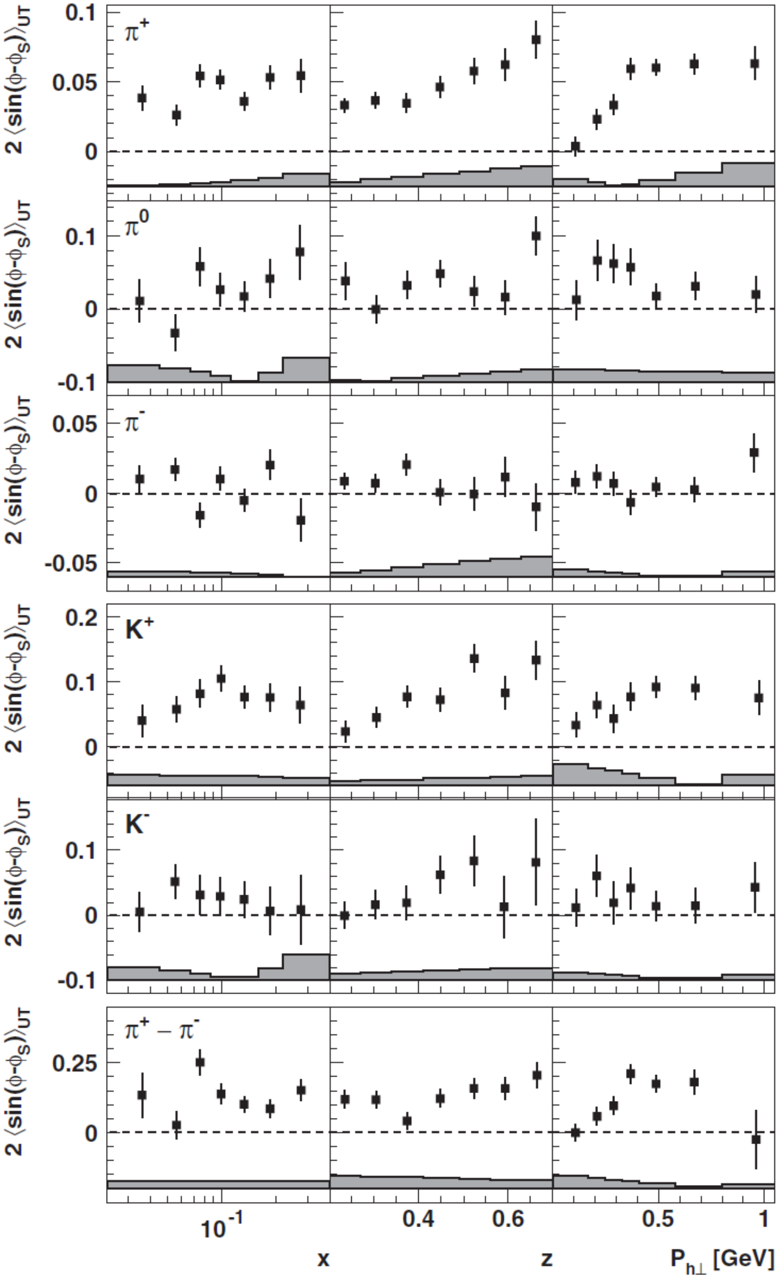


$P_{h\perp} \gg \Lambda_{QCD}$



$P_{h\perp} \sim \Lambda_{QCD}$

Sivers asymmetry



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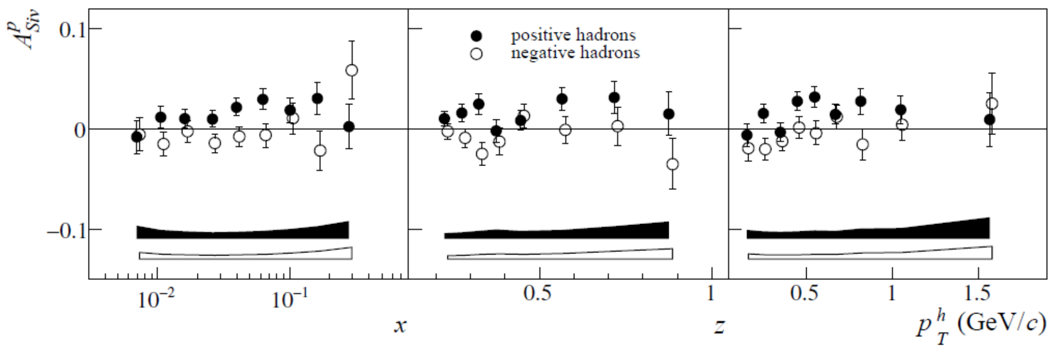

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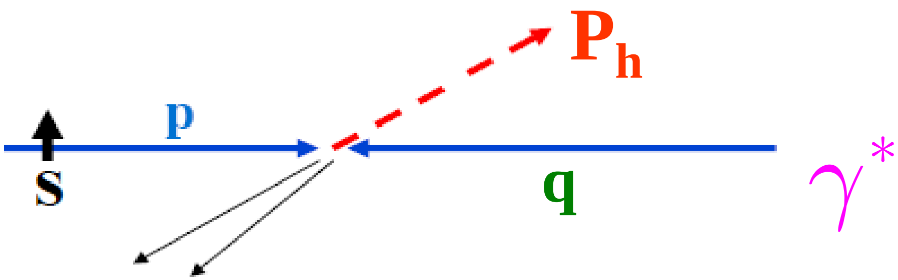
SSA in **SIDIS** $ep^{\uparrow} \rightarrow eDX$



$$d\sigma \sim \vec{S}_{\perp} \cdot \left(\vec{p} \times \vec{P}_h\right)$$

$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

at twist-3

1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon ○
2. helicity flip by one unit ○
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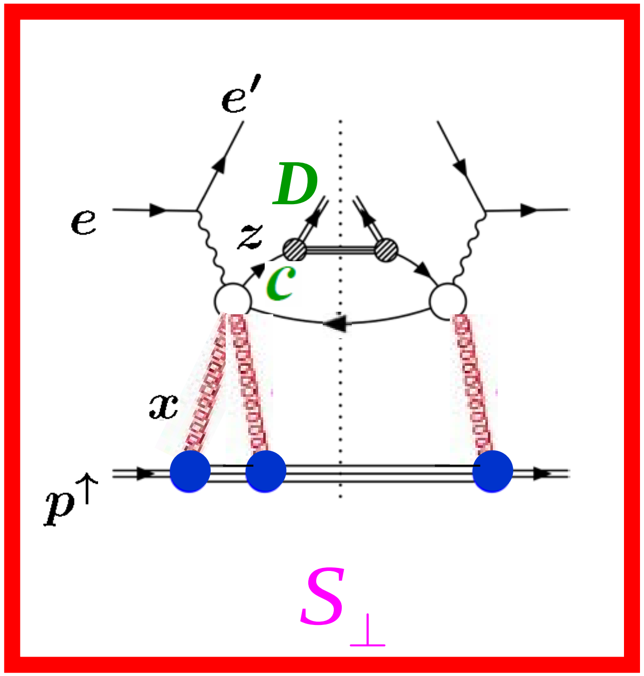
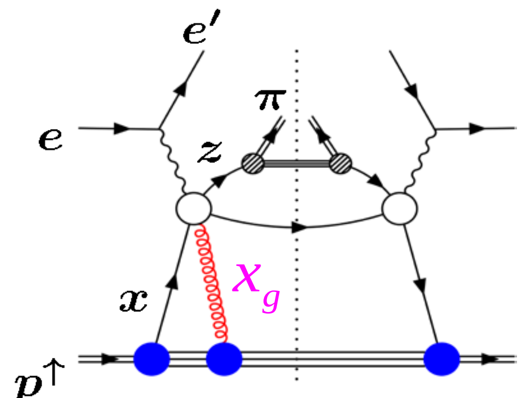
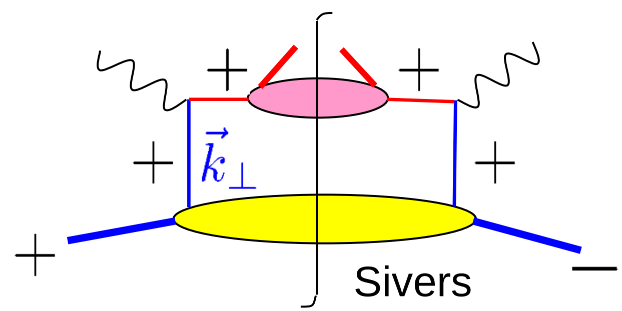


$m_D \neq 0 \quad m_c \neq 0$

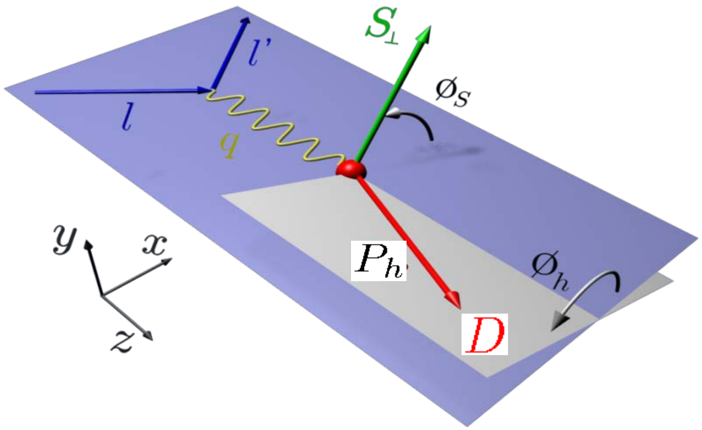


$P_{h\perp} \gg \Lambda_{QCD}$



$P_{h\perp} \sim \Lambda_{QCD}$

$\star$ Kinematics for $e(\ell) + p(p, S_\perp) \to e(\ell') + D(P_h) + X$



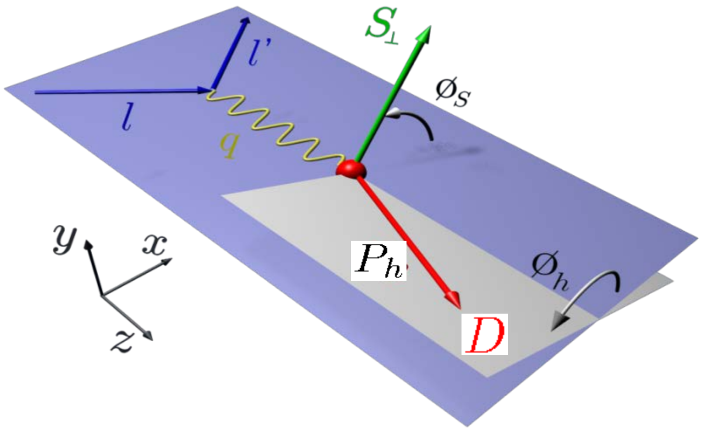
$$S_{ep} = (\ell + p)^2$$
$$q = \ell - \ell'$$
$$x_{bj} = \frac{Q^2}{2p \cdot q}$$
$$z_f = \frac{p \cdot P_h}{p \cdot q}$$

$P_{h\perp}$: $\perp$-mom. of final D

ϕ_h: azimuth. angle of hadron plane

ϕ_S: azimuth. angle of $\vec{S}_\perp$

⋆ Kinematics for $e(\ell) + p(p, S_\perp) \to e(\ell') + D(P_h) + X$



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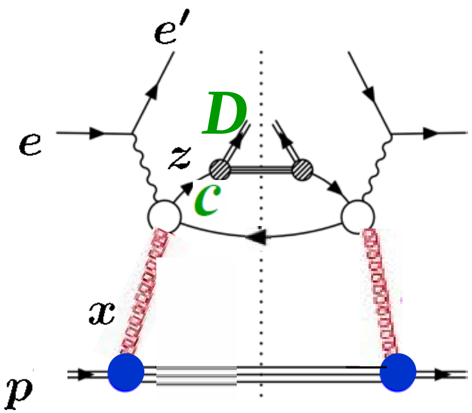


ϕ_h: azimuth. angle of hadron plane

ϕ_S: azimuth. angle of $\vec{S}_\perp$

$$\frac{d^5\sigma^{\rm unpol}_{\rm twist\text{-}2}}{dx_{bj}dQ^2dz_f dP^2_{h\perp}d\phi_h} = \frac{\alpha^2_{em}\alpha_s e^2_c}{8\pi x^2_{bj}z^2_f S^2_{ep}Q^2}\frac{1}{4}\sum_{k=1}^{4}\mathcal{A}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P^2_{h\perp}}{z^2_f Q^2} - \left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right) + \frac{m^2_c}{\hat{z}^2Q^2}\right)$$
$$\times\sum_{a=c,\bar{c}} D_a(z)g(x)\,\hat{\sigma}^U_k$$
$$= \boldsymbol{\sigma^U_1 + \sigma^U_2\cos(\phi_h) + \sigma^U_3\cos(2\phi_h)}$$

twist-3 SSA for $ep^{\uparrow} \to eDX$

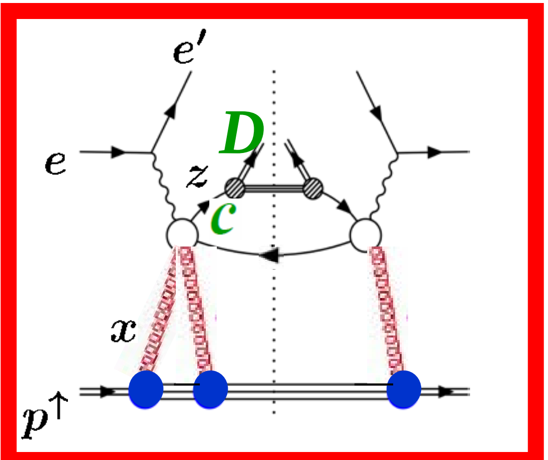
Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

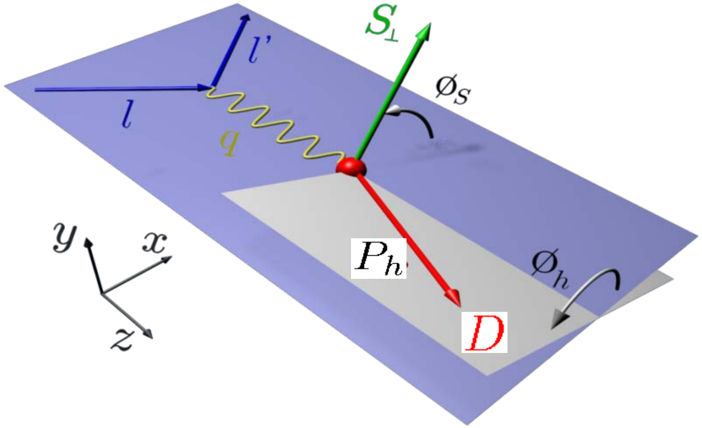
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$





twist-3 SSA for $ep^{\uparrow} \to eDX$

Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

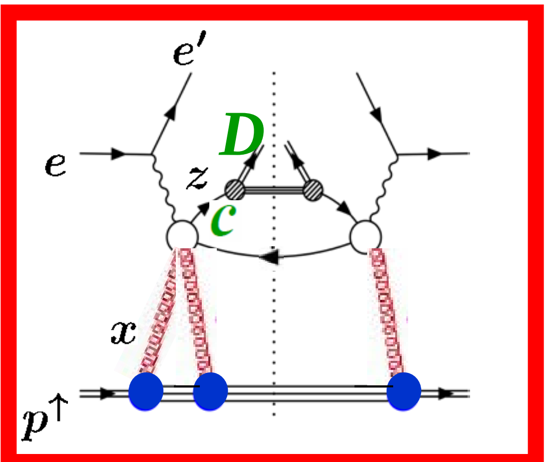
$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$

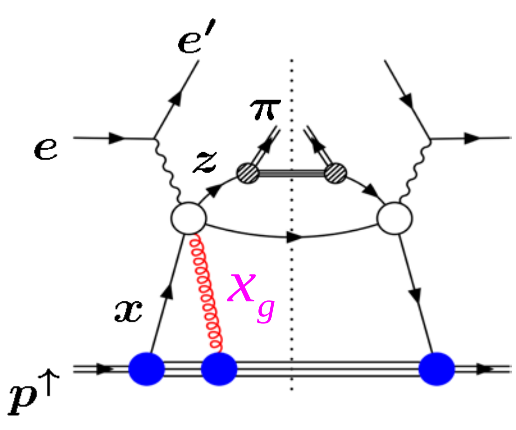
- Receives contribution from 4 functions $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$, and all of them contribute both as derivative and nonderivative terms.

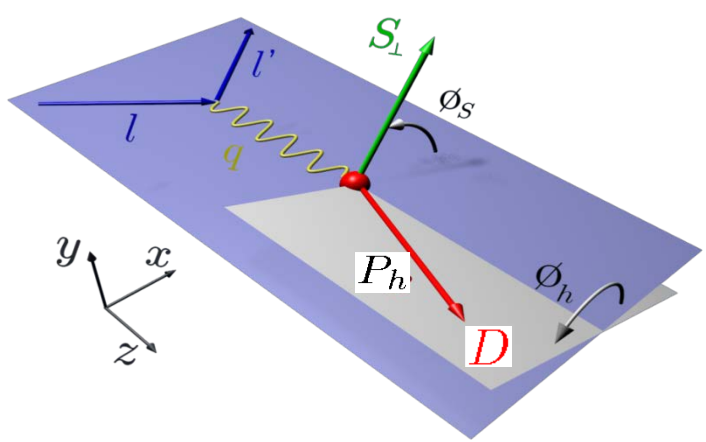
$\Delta\hat{\sigma}_k^1 \neq \Delta\hat{\sigma}_k^2$
$\Delta\hat{\sigma}_k^3 \neq \Delta\hat{\sigma}_k^4$

-5 structure functions with different azimuthal dependence

similar as $ep^{\uparrow} \to e\pi X$ from quark-gluon correlation







twist-3 SSA for $ep^{\uparrow} \to eDX$

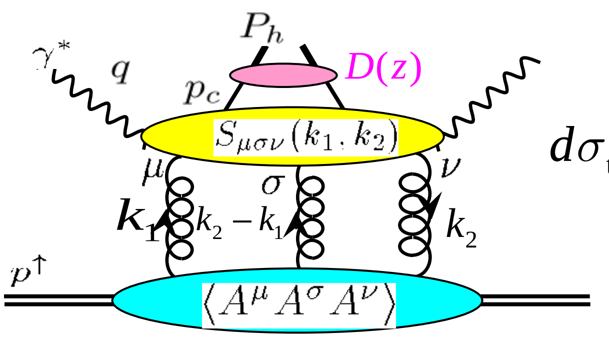
Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$



$$d\sigma_{\text{twist-3}}\sim\int dx_1dx_2dz\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i=x_ip}\otimes\begin{Bmatrix}N_{\nu\mu\sigma}(x_1,x_2)\\O_{\nu\mu\sigma}(x_1,x_2)\end{Bmatrix}\otimes D(z)$$

$$n^\mu=\left(0,n^-,\mathbf{0}_\perp\right)\qquad p\cdot n=p^+n^-=1$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x_1,x_2)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x_2,x_2-x_1)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x_1,x_1-x_2)\right]$$

$$O^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x_1,x_2)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x_2,x_2-x_1)+g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x_1,x_1-x_2)\right]$$

Permutation symmetry

twist-3 SSA for $ep^\uparrow \to eDX$

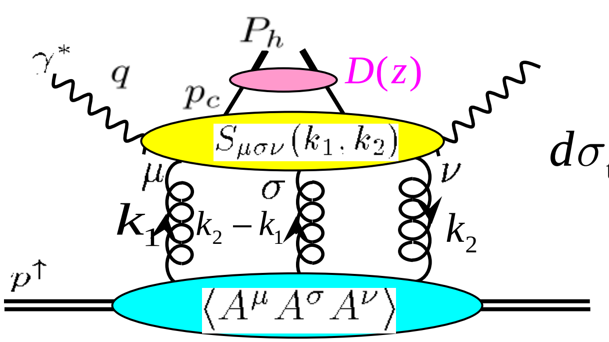
Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$



$$d\sigma_{\text{twist-3}}\sim\int dx_1dx_2dz\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i=x_ip}\otimes\left\{\begin{matrix}N_{\nu\mu\sigma}(x_1,x_2)\\ O_{\nu\mu\sigma}(x_1,x_2)\end{matrix}\right\}\otimes D(z)$$

$$n^\mu=\left(0,n^-,\mathbf{0}_\perp\right)\qquad p\cdot n=p^+n^-=1$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\left\langle p\,S_\perp\right|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)\left|p\,S_\perp\right\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\left\langle p\,S_\perp\right|F^{\mu n}(0)D_\perp^\sigma(\zeta n)F^{\nu n}(\lambda n)\left|p\,S_\perp\right\rangle\qquad D_\perp^\sigma=\partial_\perp^\sigma-igA_\perp^\sigma$$

twist-3 SSA for $ep^\uparrow \to eDX$

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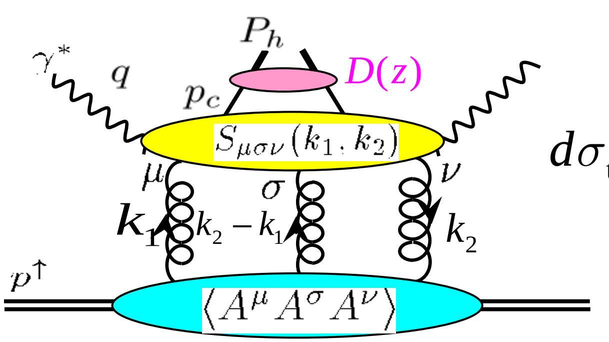
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$



$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i=x_i p} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1,x_2) \\ O_{\nu\mu\sigma}(x_1,x_2) \end{Bmatrix} \otimes D(z)$$

$$n^\mu=\left(0,n^-,\mathbf{0}_\perp\right) \qquad p\cdot n = p^+n^- = 1$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\, S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\, S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x_1,x_2)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x_2,x_2-x_1)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x_1,x_1-x_2)\right]$$

$$O^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x_1,x_2)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x_2,x_2-x_1)+g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x_1,x_1-x_2)\right]$$

Permutation symmetry

twist-3 SSA for $ep^{\uparrow} \to eDX$

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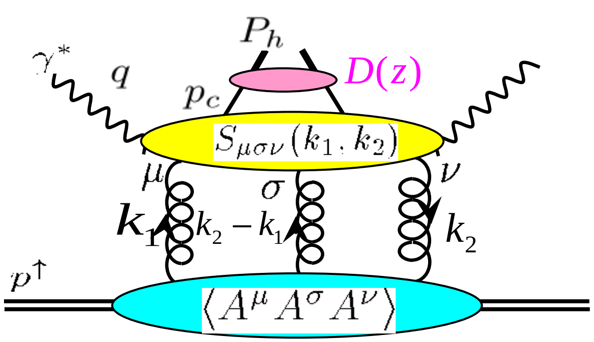
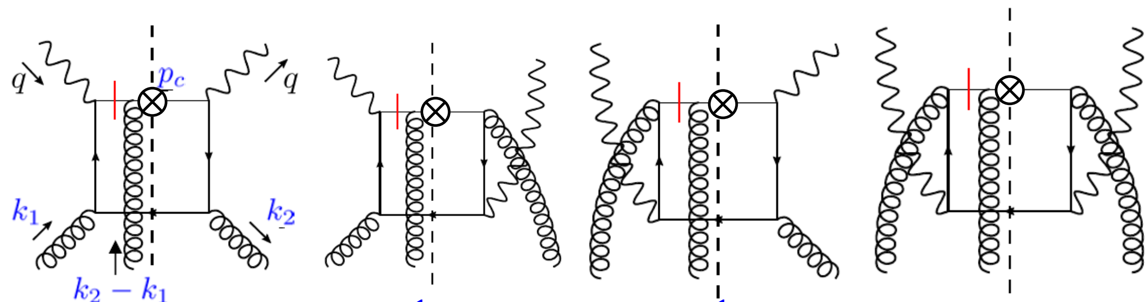
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$





$$\frac{1}{k^2-m_c^2+i\varepsilon}=\mathrm{P}\frac{1}{k^2-m_c^2}-i\pi\delta(k^2-m_c^2)$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x_1,x_2)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x_2,x_2-x_1)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x_1,x_1-x_2)\right]$$

$$O^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x_1,x_2)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x_2,x_2-x_1)+g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x_1,x_1-x_2)\right]$$

Permutation symmetry

twist-3 SSA for $ep^{\uparrow} \to eDX$

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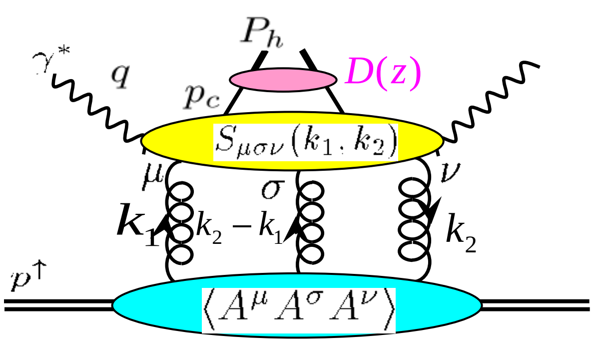
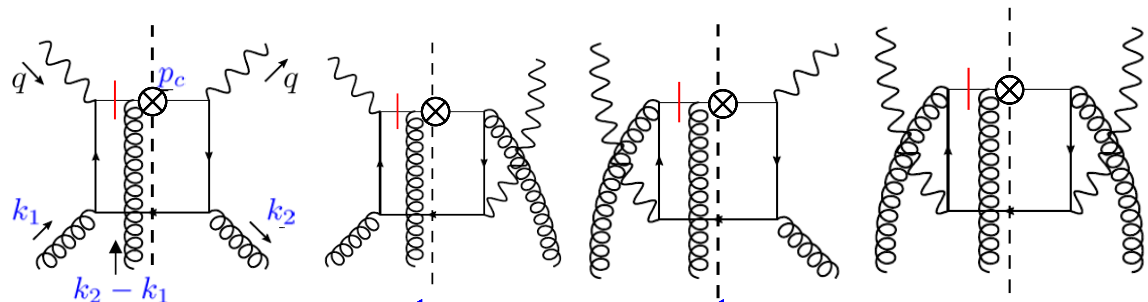
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_fdP_{h\perp}^2d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$





$$\frac{1}{k^2-m_c^2+i\varepsilon}=\mathrm{P}\frac{1}{k^2-m_c^2}-i\pi\delta(k^2-m_c^2)$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x,x)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x,x)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x,0)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x,0)\right]$$

$$O^{\nu\mu\sigma}(x,x)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x,x)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x,0)+g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x,0)\right]$$

Permutation symmetry

twist-3 SSA for $ep^{\uparrow} \to eDX$

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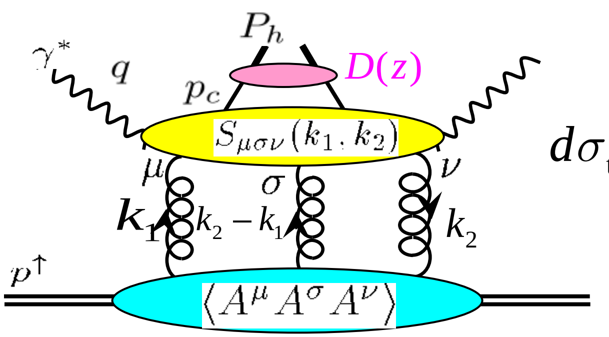
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$



$$d\sigma_{\text{twist-3}}\sim\int dx_1dx_2dz\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i=x_ip}\otimes\left\{\begin{matrix}N_{\nu\mu\sigma}(x_1,x_2)\\O_{\nu\mu\sigma}(x_1,x_2)\end{matrix}\right\}\otimes D(z)$$

$$n^\mu=\left(0,n^-,\mathbf{0}_\perp\right)\qquad p\cdot n=p^+n^-=1$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x,x)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x,x)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x,0)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x,0)\right]$$

$$O^{\nu\mu\sigma}(x,x)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x,x)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x,0)+g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x,0)\right]$$

Permutation symmetry

twist-3 SSA for $ep^\uparrow \to eDX$

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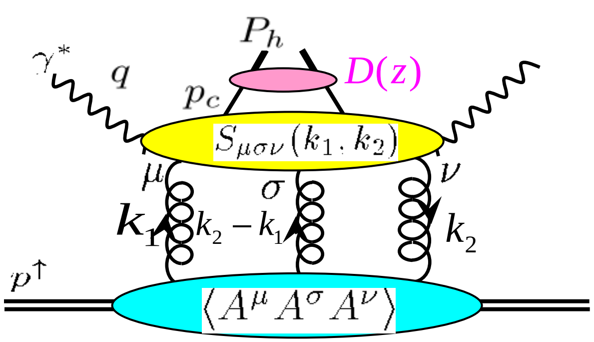
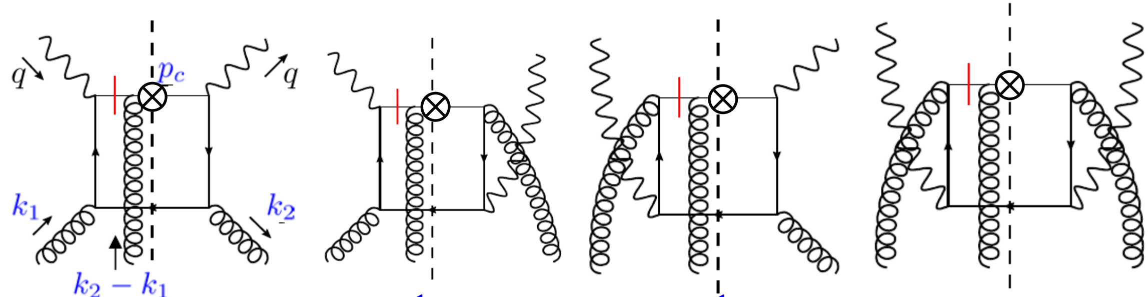
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$





$$\frac{1}{k^2-m_c^2+i\varepsilon}=\mathrm{P}\frac{1}{k^2-m_c^2}-i\pi\delta(k^2-m_c^2)$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x,x)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x,x)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x,0)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x,0)\right]$$

$$O^{\nu\mu\sigma}(x,x)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x,x)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x,0)+g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x,0)\right]$$

Permutation symmetry

Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005

Koike, K.T., Yoshida, PRD83('11) 114014

$$\hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}$$

$$q_T \equiv \frac{P_{h\perp}}{z_f}$$

$$\Delta\hat{\sigma}_1^1 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-2\hat{x}+2\hat{x}^2+12\hat{x}\,\hat{z}(1-\hat{x})(1-\hat{z})) + 2m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2)) - 4m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_2^1 = \frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}\{Q^2\hat{z}(1-\hat{x})(1-\hat{z}) - m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_3^1 = \frac{16\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\{Q^2\hat{z}(1-\hat{x})(1-\hat{z}) - m_c^2\hat{x}\}\{Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_4^1 = \frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^2\hat{z}(1-\hat{x})(1-\hat{z}) - m_c^2\hat{x}\}\{Q^2\hat{z}(1-\hat{z}) + m_c^2\}, \qquad \Delta\hat{\sigma}_8^1 = \Delta\hat{\sigma}_9^1 = 0,$$

$$\Delta\hat{\sigma}_1^2 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-4\hat{x}+4\hat{x}^2+24\hat{x}\,\hat{z}(1-\hat{x})(1-\hat{z})) + 4m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2)) - 8m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_2^2 = 2\Delta\hat{\sigma}_2^1,$$

$$\Delta\hat{\sigma}_3^2 = 2\Delta\hat{\sigma}_3^1,$$

$$\Delta\hat{\sigma}_4^2 = -\frac{16q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x})^2,$$

$$\Delta\hat{\sigma}_8^2 = \frac{16\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1-\hat{x})(1-\hat{z}) - m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^2 = -\frac{16q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_1^3 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2) - 2m_c^2),$$

$$\Delta\hat{\sigma}_2^3 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^3 = -\frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1-8\hat{x}+8\hat{x}^2) - 8m_c^2Q^2\hat{x}\,\hat{z}(1-2\hat{x})(1-\hat{z}) + 8m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_4^3 = -\frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-\hat{z}) + m_c^2)(Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_8^3 = -\frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1-2\hat{x})(1-\hat{z}) - 2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^3 = -\frac{32q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-\hat{z}) + m_c^2),$$

$$\Delta\hat{\sigma}_1^4 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-4\hat{x})(1-\hat{z}) - 4m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2) - 2m_c^2),$$

$$\Delta\hat{\sigma}_2^4 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-4\hat{x})(1-\hat{z}) - 4m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^4 = \frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1+12\hat{x}-16\hat{x}^2) + 4m_c^2Q^2\hat{x}\,\hat{z}(3-8\hat{x})(1-\hat{z}) - 16m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_4^4 = -\frac{32q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^4\hat{z}^2(1-\hat{z})^2(1+\hat{x}-4\hat{x}^2) + m_c^2Q^2\hat{x}\,\hat{z}(1-8\hat{x})(1-\hat{z}) - 4m_c^4\hat{x}^2),$$

$$\Delta\hat{\sigma}_8^4 = \frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1+2\hat{x})(1-\hat{z}) + 2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^4 = -\frac{32q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-\hat{z})(1+\hat{x}) + m_c^2\hat{x}),$$

$$\left(m_c \to 0 \text{ for } ep^{\uparrow} \to e\pi X \text{ case}\right)$$

Estimates of twist-3 SSA for $ep^\uparrow \to eD^0X$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

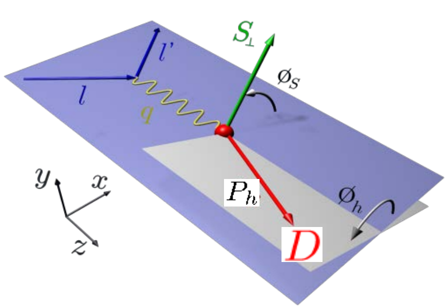
$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$= \sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$

$$A_N:\ \frac{\mathcal{F}_1}{\sigma_1^U},\quad \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U}$$

$$\left(\frac{d^5\sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2dx_{bj}dz_f dP_{h\perp}^2 d\phi_h}=\sigma_1^U+\sigma_2^U\cos\phi_h+\sigma_3^U\cos 2\phi_h\right)$$

$$\langle 1\rangle,\langle\cos\phi\rangle,\langle\cos 2\phi\rangle,\ \langle\sin\phi\rangle,\langle\sin 2\phi\rangle$$



Estimates of twist-3 SSA for $ep^{\uparrow} \to eD^0 X$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$

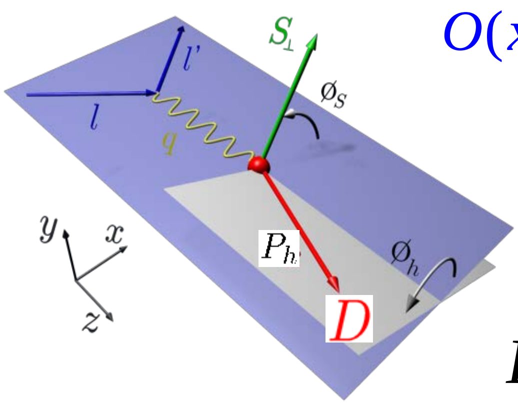
$$A_N:\frac{\mathcal{F}_1}{\sigma_1^U},\ \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U}$$

$$\left(\frac{d^5\sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2dx_{bj}dz_f dP_{h\perp}^2 d\phi_h}=\sigma_1^U+\sigma_2^U\cos\phi_h+\sigma_3^U\cos 2\phi_h\right)$$

guided by $pp^{\uparrow}\to DX$ at RHIC

Koike, Yoshida, PRD84 ('11) 014026

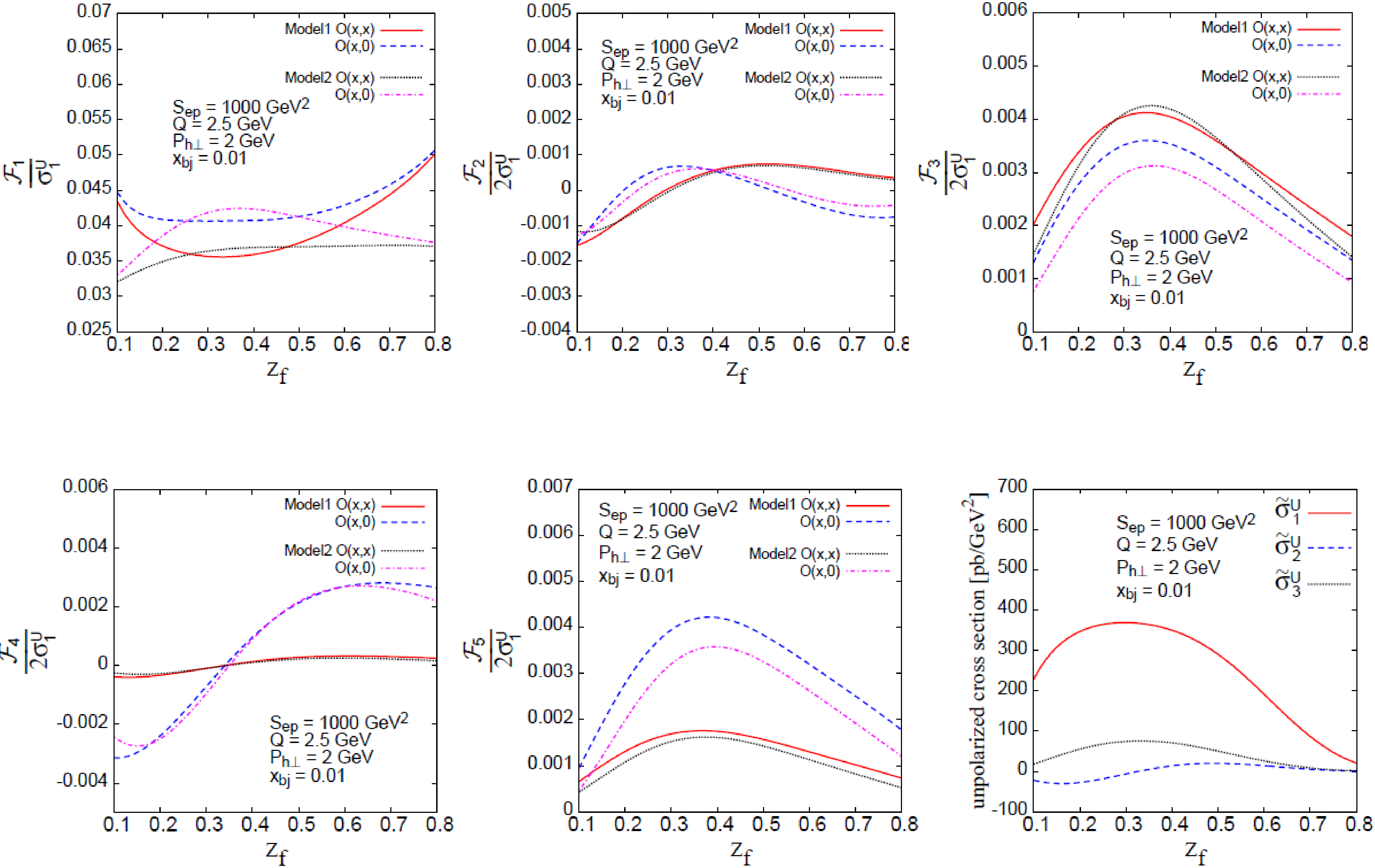
$$O(x,x)=O(x,0)=\begin{cases}0.004xg(x) & \textbf{Model1}\\ 0.001\sqrt{x}\,g(x) & \textbf{Model2}\end{cases}$$



$g(x,\mu)$: **CTEQ6L** $\quad \mu=\sqrt{Q^2+m_c^2+P_{h\perp}^2}$

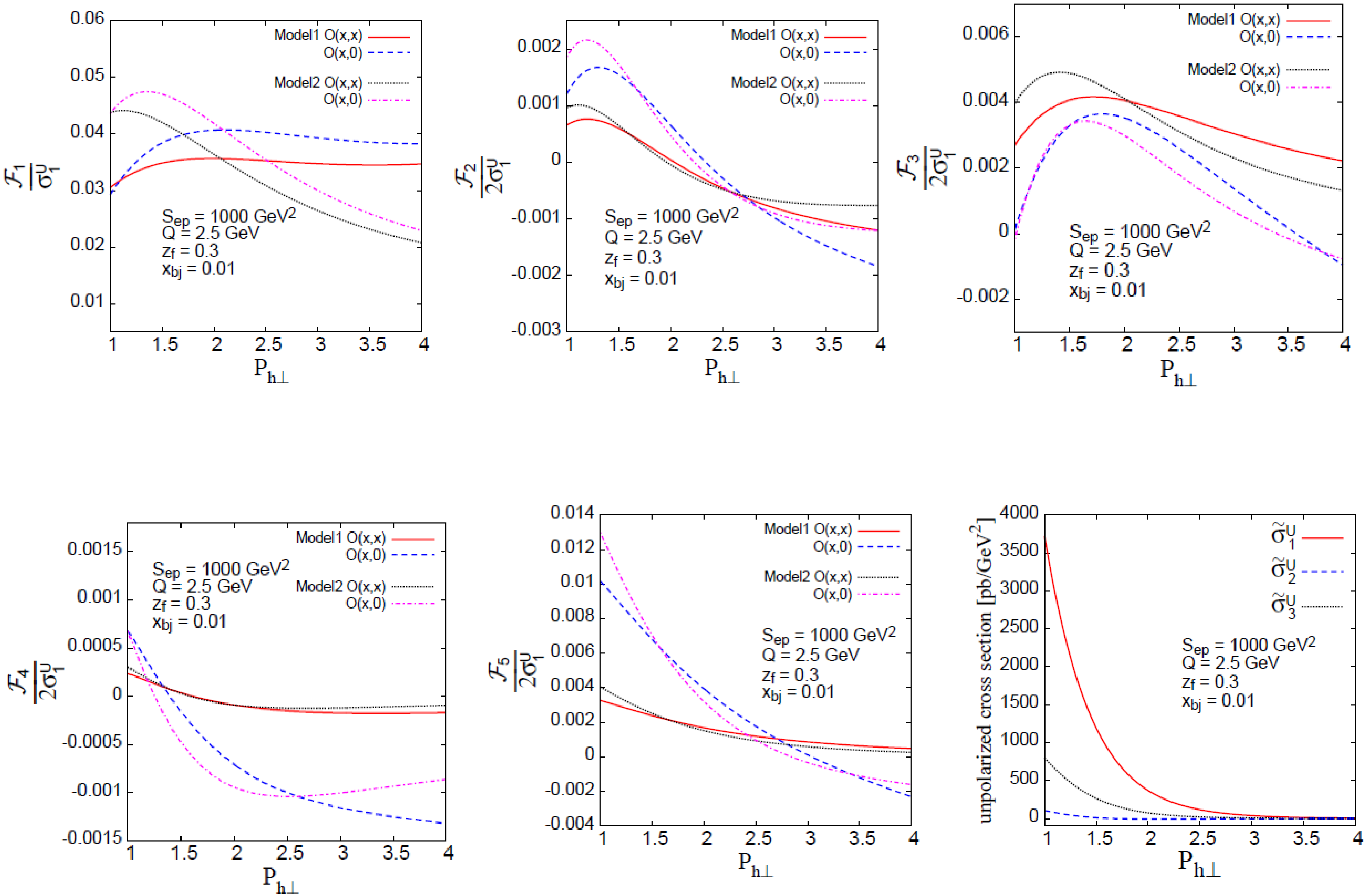
$D_c(z)$: **Kneesch-Kniehl-Kramer-Schienbein08**

Beppu, Koike, K.T., Yoshida, PRD85 ('12) 114026



$$\frac{d^5\sigma^{\rm unpol}_{\rm twist\text{-}2}}{dx_{bj}\,dy\,dz_f\,dP_{h\perp}^2\,d\phi_h} = \tilde{\sigma}_1^U + \tilde{\sigma}_2^U \cos\phi_h + \tilde{\sigma}_3^U \cos 2\phi_h$$

Beppu, Koike, K.T., Yoshida, PRD85 ('12) 114026



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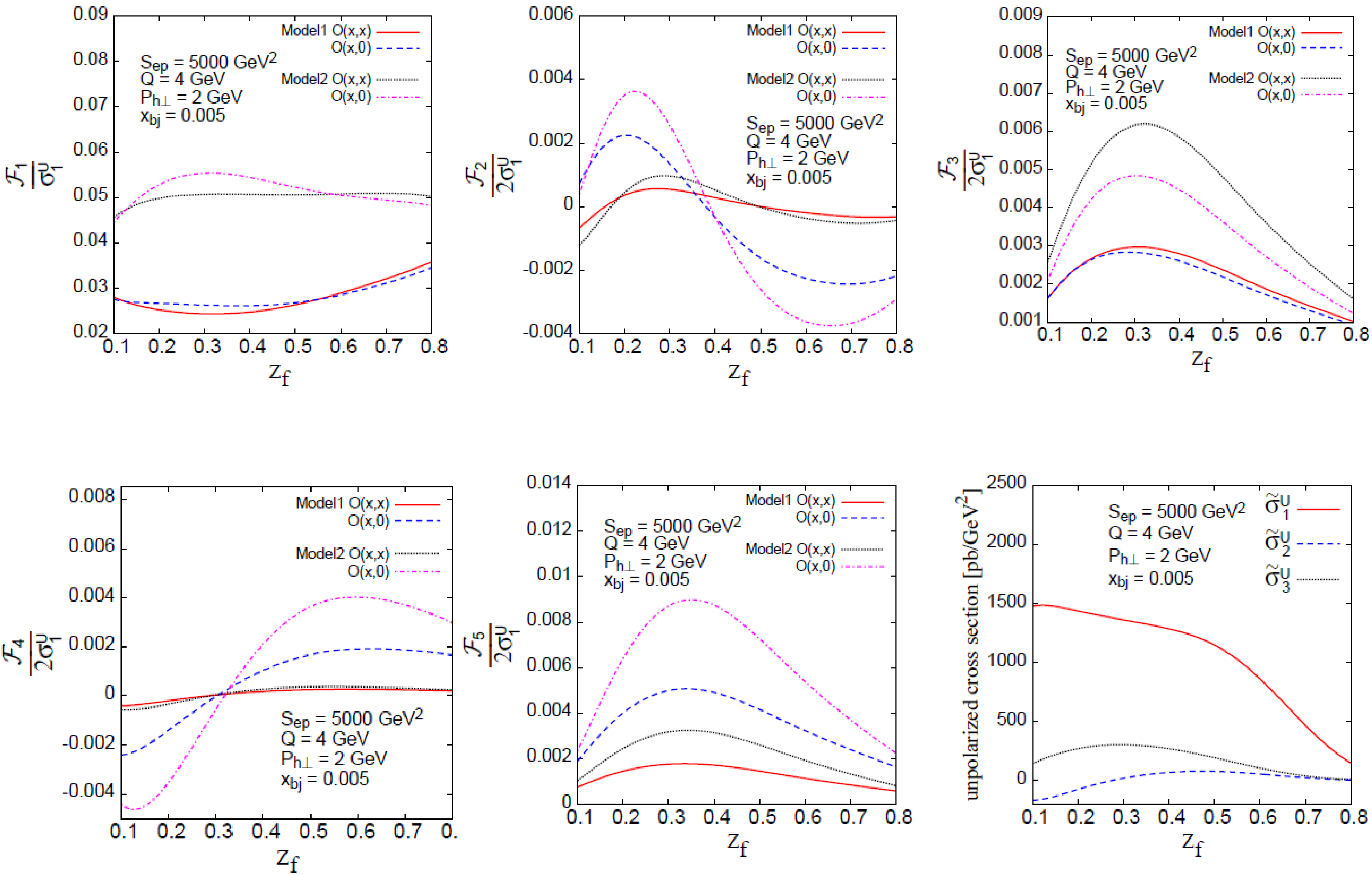
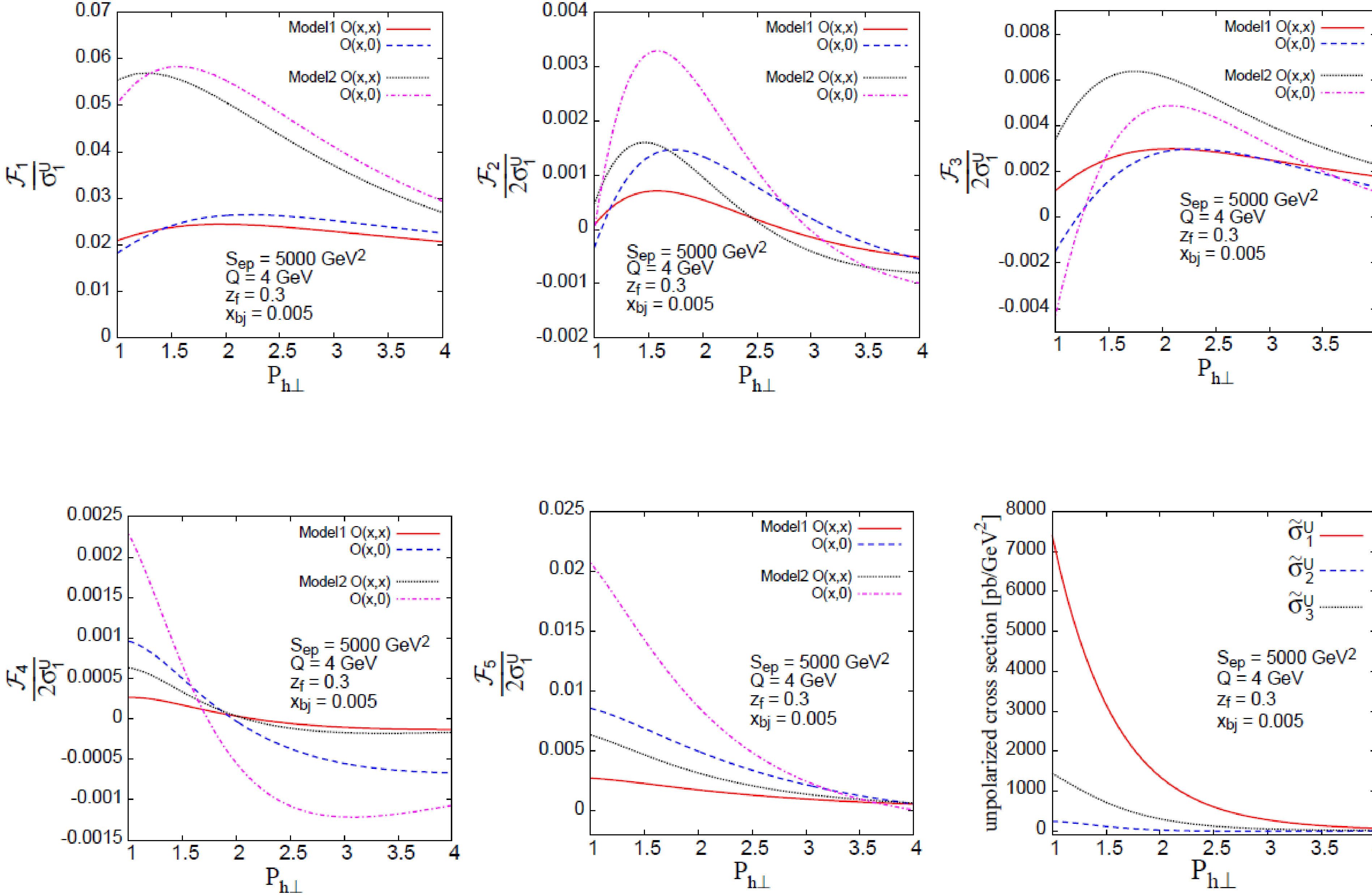


Beppu, Koike, K.T., Yoshida, PRD85 ('12) 114026



twist-3 SSA for $ep^{\uparrow} \to eDX$

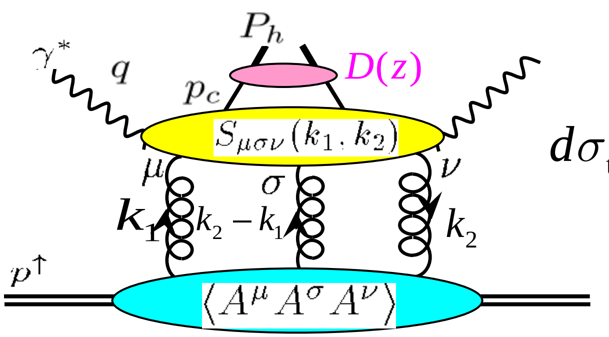
Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$



$$d\sigma_{\text{twist-3}}\sim\int dx_1dx_2dz\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i=x_ip}\otimes\begin{Bmatrix}N_{\nu\mu\sigma}(x_1,x_2)\\O_{\nu\mu\sigma}(x_1,x_2)\end{Bmatrix}\otimes D(z)$$

$$n^\mu=\left(0,n^-,\mathbf{0}_\perp\right)\qquad p\cdot n=p^+n^-=1$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$i\int\frac{d\lambda}{2\pi}\frac{d\zeta}{2\pi}e^{i\lambda x_1+i\mu(x_2-x_1)}\langle p\,S_\perp|F^{\mu n}(0)D_\perp^\sigma(\zeta n)F^{\nu n}(\lambda n)|p\,S_\perp\rangle\qquad D_\perp^\sigma=\partial_\perp^\sigma-igA_\perp^\sigma$$

$$i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\, S_\perp\right|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)\left|p\, S_\perp\right\rangle$$

$$=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$-i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\, S_\perp\right|F^{\mu n}(0)D_\perp^{\sigma}(\zeta n)F^{\nu n}(\lambda n)\left|p\, S_\perp\right\rangle$$

$$=M^{\nu\mu\sigma}(x_1,x_2)$$

$$i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\langle p\,S_\perp|F_a^{\mu n}(0)gF_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)|p\,S_\perp\rangle$$

$$=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$-i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\langle p\,S_\perp|F^{\mu n}(0)D_\perp^{\sigma}(\zeta n)F^{\nu n}(\lambda n)|p\,S_\perp\rangle$$

$$=M^{\nu\mu\sigma}(x_1,x_2)$$

$$i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\,S_\perp\left|F_a^{\mu n}(0)\,gF_c^{\sigma n}(\zeta n)\,F_b^{\nu n}(\lambda n)\right|p\,S_\perp\right\rangle$$

$$=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x_1,x_2)-g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x_2,x_2-x_1)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x_1,x_1-x_2)\right]$$

$$-i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\,S_\perp\left|F^{\mu n}(0)\,D_\perp^{\sigma}(\zeta n)\,F^{\nu n}(\lambda n)\right|p\,S_\perp\right\rangle$$

$$=M^{\nu\mu\sigma}(x_1,x_2)$$

$$M^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}M_1(x_1,x_2)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}M_2(x_1,x_2)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}M_2(x_2,x_1)\right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\ S\rangle = -\frac{x}{2}\Big[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu p n} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S \,|\, \overline{\psi}(0)\gamma^{\sigma}\gamma_5 \psi(\lambda n) \,|\, p\ S\rangle = 2M_N \Big[\Delta g(x) (S\cdot n)\, p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \cdots \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\ S \rangle = -\frac{x}{2}\left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} \left(S\cdot n\right) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S \,|\, \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) \,|\, p\ S \rangle = 2M_N \left[\Delta g(x) \left(S\cdot n\right) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \cdots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S_{\perp} \,|\, F^{\mu +}(0) F^{-+}(\lambda n) \,|\, p\ S_{\perp} \rangle = x \mathcal{G}_T(x) i\epsilon^{\mu -+\alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^{+} A^{-} = -\frac{1}{\partial_{-}}\left(D_{\perp j} F^{j+} + g\bar{\psi} t^{a} \gamma^{+} \psi t^{a} \right) \qquad D_{\nu} F^{\mu\nu} = g\bar{\psi} t^{a} \gamma^{\mu} \psi t^{a}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\ S \rangle = -\frac{x}{2} \left[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | \bar\psi(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | p\ S \rangle = 2M_N \left[\Delta g(x) (S\cdot n) p^\sigma + g_T(x) S_\perp^\sigma + \cdots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S_\perp | F^{\mu +}(0) F^{-+}(\lambda n) | p\ S_\perp \rangle = x \mathcal{G}_T(x) i\epsilon^{\mu -+\alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-} \left(D_{\perp j} F^{j+} + g\bar\psi t^a \gamma^+ \psi t^a \right) \qquad D_\nu F^{\mu\nu} = g\bar\psi t^a \gamma^\mu \psi t^a$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\ S_\perp \middle| F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) \middle| p\ S_\perp \right\rangle = M^{\nu\mu\sigma}(x_1, x_2)$$

$$M^{\nu\mu\sigma}(x_1,x_2) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} M_1(x_1,x_2) + g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} M_2(x_1,x_2) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} M_2(x_2,x_1) \right]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\ S_\perp \middle| \bar\psi(0) \gamma^\mu g F^{\alpha n}(\mu n) \psi(\lambda n) \middle| p\ S_\perp \right\rangle = M_N p^\mu \epsilon^{\alpha pn S_\perp} G_F(x_1, x_2)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\ S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | \bar{\psi}(0)\gamma^{\sigma}\gamma_5 \psi(\lambda n) | p\ S \rangle = 2M_N \left[\Delta g(x)(S\cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \cdots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S_{\perp} | F^{\mu +}(0) F^{-+}(\lambda n) | p\ S_{\perp} \rangle = x\mathcal{G}_T(x) i\epsilon^{\mu -+\alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^{+} A^{-} = -\frac{1}{\partial_{-}} \left(D_{\perp j} F^{j+} + g\bar{\psi} t^a \gamma^{+} \psi t^a \right) \qquad D_{\nu} F^{\mu\nu} = g\bar{\psi} t^a \gamma^{\mu} \psi t^a$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\ S_{\perp} \middle| F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) \middle| p\ S_{\perp} \right\rangle = M^{\nu\mu\sigma}(x_1, x_2)$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} M_1(x_1, x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} M_2(x_1, x_2) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} M_2(x_2, x_1) \right]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\ S_{\perp} \middle| \bar{\psi}(0)\gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) \middle| p\ S_{\perp} \right\rangle = M_N p^{\mu} \epsilon^{\alpha pn S_{\perp}} G_F(x_1, x_2)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x') \right)$$

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\, S\rangle = -\frac{x}{2}\Big[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu p n} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S | \bar\psi(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | p\, S\rangle = 2M_N \Big[\Delta g(x) (S\cdot n) p^\sigma + g_T(x) S_\perp^\sigma + \cdots \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S_\perp | F^{\mu +}(0) F^{-+}(\lambda n) | p\, S_\perp\rangle = x \mathcal{G}_T(x) i\epsilon^{\mu -+\alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-}\left(D_{\perp j} F^{j+} + g\bar\psi t^a \gamma^+ \psi t^a \right) \qquad D_\nu F^{\mu\nu} = g\bar\psi t^a \gamma^\mu \psi t^a$$

$$-i\int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p\, S_\perp \middle| F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) \middle| p\, S_\perp \right\rangle = M^{\nu\mu\sigma}(x_1, x_2)$$

$$M^{\nu\mu\sigma}(x_1,x_2) = 2iM_N \Big[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n\alpha} M_1(x_1,x_2) + g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n\alpha} M_2(x_1,x_2) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n\alpha} M_2(x_2,x_1) \Big]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p\, S_\perp \middle| \bar\psi(0) \gamma^\mu g F^{\alpha n}(\mu n) \psi(\lambda n) \middle| p\, S_\perp \right\rangle = M_N p^\mu \epsilon^{\alpha p n S_\perp} G_F(x_1, x_2)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \big(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x, x') + G_F(x'-x, x') \big)$$

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\, S \rangle = -\frac{x}{2} \Big[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle p\, S\,|\,F^{\mu n}(0)F^{\nu n}(\lambda n)\,|\,p\, S\rangle = -\frac{x}{2}\Big[G(x)g_\perp^{\mu\nu} + \Delta G(x)\, i\epsilon^{\mu\nu pn}(S\cdot n)M_N + 2\mathcal{G}_T(x)\, i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots\Big]$$

$$z^\nu \frac{\partial}{\partial z^\nu} F^{\mu z}(0)\tilde F_\mu{}^\rho(z) - z^\nu \frac{\partial}{\partial z_\rho} F^{\mu z}(0)\tilde F_{\mu\nu}(z) \qquad z^2 \to 0$$

$$= i\int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz)\tilde F_\mu{}^z(z) - i\int_0^1 dt F^{\mu z}(0) g F_\mu{}^z(tz)\tilde F^{\rho z}(z) + \int_0^1 du \Big(i\int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz)\tilde F_\mu{}^z(uz)$$

$$- i\int_0^1 dt F^{\mu z}(0) g u F_\mu{}^z(tuz)\tilde F^{\rho z}(uz) - i\int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz)\tilde F_\mu{}^z(uz)$$

$$- F^{\mu z}(0)\epsilon^\rho{}_{\mu z\sigma} D_\lambda F^{\lambda\sigma}(uz) + F^{\mu z}(0)\overleftarrow{D}_\mu \tilde F^{\rho z}(uz) + F^{\rho z}(0) D^\mu \tilde F_\mu{}^z(uz)\Big) - F^{\mu z}(0)\epsilon^\rho{}_{\mu z\sigma} D_\lambda F^{\lambda\sigma}(z) + F^{\mu z}(0)\overleftarrow{D}_\mu \tilde F^{\rho z}(z)$$

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\, S\rangle = -\frac{x}{2}\Big[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$z^{\nu} \frac{\partial}{\partial z^{\nu}} F^{\mu z}(0) \tilde{F}_{\mu}{}^{\rho}(z) - z^{\nu} \frac{\partial}{\partial z_{\rho}} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \qquad z^2 \to 0$$

$$= i\int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_{\mu}{}^{z}(z) - i\int_0^1 dt F^{\mu z}(0) g F_{\mu}{}^{z}(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \Big(i\int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_{\mu}{}^{z}(uz)$$

$$- i\int_0^1 dt F^{\mu z}(0) g u F_{\mu}{}^{z}(tuz) \tilde{F}^{\rho z}(uz) - i\int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_{\mu}{}^{z}(uz)$$

$$- F^{\mu z}(0) \epsilon^{\rho}{}_{\mu z\sigma} D_{\lambda} F^{\lambda\sigma}(uz) + F^{\mu z}(0) \overleftarrow{D}_{\mu} \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^{\mu} \tilde{F}_{\mu}{}^{z}(uz) \Big) - F^{\mu z}(0) \epsilon^{\rho}{}_{\mu z\sigma} D_{\lambda} F^{\lambda\sigma}(z) + F^{\mu z}(0) \overleftarrow{D}_{\mu} \tilde{F}^{\rho z}(z)$$

$$i\int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\, S_{\perp} \middle| F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) \middle| p\, S_{\perp} \right\rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \cdots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \Big[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} N(x_1, x_1 - x_2) \Big]$$

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\, S \rangle = -\frac{x}{2}\Big[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$z^\nu \frac{\partial}{\partial z^\nu} F^{\mu z}(0) \tilde{F}_\mu{}^\rho(z) - z^\nu \frac{\partial}{\partial z_\rho} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z)$$

$$z^2 \to 0$$

$$= i\int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_\mu{}^z(z) - i\int_0^1 dt F^{\mu z}(0) g F_\mu{}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \Big(i\int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_\mu{}^z(uz)$$

$$-i\int_0^1 dt F^{\mu z}(0) g u F_\mu{}^z(tuz) \tilde{F}^{\rho z}(uz) - i\int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_\mu{}^z(uz)$$

$$-F^{\mu z}(0) \epsilon^\rho_{\ \mu z\sigma} D_\lambda F^{\lambda\sigma}(uz) + F^{\mu z}(0) \overleftarrow{D}_\mu \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^\mu \tilde{F}_\mu{}^z(uz) \Big) - F^{\mu z}(0) \epsilon^\rho_{\ \mu z\sigma} D_\lambda F^{\lambda\sigma}(z) + F^{\mu z}(0) \overleftarrow{D}_\mu \tilde{F}^{\rho z}(z)$$

$$i\int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\, S_\perp \middle| F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) \middle| p\, S_\perp \right\rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \cdots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \Big[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} N(x_1, x_1 - x_2) \Big]$$

$$D_\nu F^{\mu\nu} = g\bar{\psi} t^a \gamma^\mu \psi t^a \qquad D_\mu \tilde{F}^{\mu\nu} = 0$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\, S_\perp \middle| \bar{\psi}(0) \gamma^\mu g F^{\alpha n}(\mu n) \psi(\lambda n) \middle| p\, S_\perp \right\rangle = M_N p^\mu \epsilon^{\alpha pn S_\perp} G_F(x_1, x_2)$$

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\ S \rangle = -\frac{x}{2}\left[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \right]$$

$$z^\nu \frac{\partial}{\partial z^\nu} F^{\mu z}(0) \tilde{F}_\mu{}^\rho(z) - z^\nu \frac{\partial}{\partial z_\rho} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \qquad z^2 \to 0$$

$$= i\int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_\mu{}^z(z) - i\int_0^1 dt F^{\mu z}(0) g F_\mu{}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \Big(i\int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_\mu{}^z(uz)$$

$$- i\int_0^1 dt F^{\mu z}(0) g u F_\mu{}^z(tuz) \tilde{F}^{\rho z}(uz) - i\int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_\mu{}^z(uz)$$

$$- F^{\mu z}(0) \epsilon^\rho{}_{\mu z\sigma} D_\lambda F^{\lambda\sigma}(uz) + F^{\mu z}(0) \overleftarrow{D}_\mu \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^\mu \tilde{F}_\mu{}^z(uz) \Big) - F^{\mu z}(0) \epsilon^\rho{}_{\mu z\sigma} D_\lambda F^{\lambda\sigma}(z) + F^{\mu z}(0) \overleftarrow{D}_\mu \tilde{F}^{\rho z}(z)$$

$$i\int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\ S_\perp \middle| F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) \middle| p\ S_\perp \right\rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \cdots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} N(x_1, x_1 - x_2) \right]$$

$$D_\nu F^{\mu\nu} = g\bar{\psi} t^a \gamma^\mu \psi t^a \qquad D_\mu \tilde{F}^{\mu\nu} = 0$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \left\langle p\ S_\perp \middle| \bar{\psi}(0) \gamma^\mu g F^{\alpha n}(\mu n) \psi(\lambda n) \middle| p\ S_\perp \right\rangle = M_N p^\mu \epsilon^{\alpha pn S_\perp} G_F(x_1, x_2)$$

$$2x^2 \frac{\partial}{\partial x} \mathcal{G}_T(x) + x\Delta G(x) = \int dx' P \frac{1}{x - x'} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) (2N(x, x - x') - 2N(x', x' - x))$$

$$+ \int dx' P \frac{1}{x - x'} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) (4N(x, x') - 6N(x', x' - x) - 6N(x, x - x'))$$

$$+ \int dx' P \frac{1}{x - x'} P \frac{1}{x} (8N(x', x' - x) + 8N(x, x - x')) + \int dx' \left(P\frac{1}{x} - \frac{\partial}{\partial x} \right) (G_F(x' + x, x') + G_F(x' - x, x'))$$

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\, S \rangle = -\frac{x}{2}\Big[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu p n} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \Big(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x,x') + G_F(x'-x,x') \Big)$$

$$\mathcal{G}_T(x) = \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$- \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \Big[P\frac{1}{x'-x''} \big(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\big)(2N(x',x'-x'') - 2N(x'',x''-x'))$$

$$+ P\frac{1}{x'-x''} \big(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\big)(4N(x',x'') - 6N(x'',x''-x') - 6N(x',x'-x''))$$

$$+ P\frac{1}{x'-x''} P\frac{1}{x'} (8N(x'',x''-x') + 8N(x',x'-x'')) + \left(P\frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x''+x',x'') + G_F(x''-x',x'')) \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\, S \rangle = -\frac{x}{2}\left[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu p n} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \right]$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2}\int dx' \left(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x,x') + G_F(x'-x,x') \right)$$

$$\mathcal{G}_T(x) = \frac{1}{2}\int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$-\int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2}\Big[P\frac{1}{x'-x''}\left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\right)(2N(x',x'-x'') - 2N(x'',x''-x'))$$

$$+ P\frac{1}{x'-x''}\left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\right)(4N(x',x'') - 6N(x'',x''-x') - 6N(x',x'-x''))$$

$$+ P\frac{1}{x'-x''} P\frac{1}{x'}(8N(x'',x''-x') + 8N(x',x'-x'')) + \left(P\frac{1}{x'} - \frac{\partial}{\partial x'} \right)(G_F(x''+x',x'') + G_F(x''-x',x''))\Big]$$

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$$M_1(x_1,x_2) = \mathcal{P}\frac{N(x_1,x_2)}{x_2 - x_1}$$

$$M_2(x_1,x_2) = \mathcal{P}\frac{N(x_2,x_2-x_1)}{x_1-x_2} + \delta(x_1-x_2)x_1^2\Big[\int_{x_1}^{\epsilon(x_1)} dx' \int dx'' \frac{1}{x'^3}\Big\{ P\frac{2x'-x''}{(x'-x'')^2}(N(x',x'-x'')$$

$$- N(x'',x''-x')) + P\frac{1}{x'-x''}(2N(x',x'') - N(x'',x''-x') - N(x',x'-x''))\Big\}$$

$$-\frac{1}{2}\int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^3}(G_F(x''+x',x'') + G_F(x''-x',x'')) - \frac{1}{4}\int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'}\Big]$$

$$\left\langle pS_\perp \right| \bar{\psi}(0) gF^{\mu+}(\zeta n) \psi(\lambda n) \left| pS_\perp \right\rangle = \frac{M_N}{4} \not{p} S_{\perp\alpha} p_\beta \varepsilon^{\alpha\beta\mu+} G_F + i \frac{M_N}{4} \gamma_5 \not{p} S_\perp^\mu \widetilde{G}_F$$

$$D_\perp^\mu(\zeta n) \qquad G_D \qquad \widetilde{G}_D$$

$$G_D(x_1, x_2) = P \frac{1}{x_1 - x_2} G_F(x_1, x_2),$$

$$\widetilde{G}_D(x_1, x_2) = \delta(x_1 - x_2) \widetilde{g}(x_1) + P \frac{1}{x_1 - x_2} \widetilde{G}_F(x_1, x_2)$$

$$\tilde{g}(x) = -x \int_x^{\epsilon(x)} dx_1 \left[\frac{2\Delta q(x_1)}{x_1} + \frac{1}{x_1^2} P \int_{-1}^{1} dx_2 \left\{ \frac{G_F(x_1, x_2)}{x_1 - x_2} + (3x_1 - x_2) \frac{\tilde{G}_F(x_1, x_2)}{(x_1 - x_2)^2} \right\} \right]$$

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$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle p\ S\,|\,F^{\mu n}(0)F^{\nu n}(\lambda n)\,|\,p\ S\rangle=-\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu}+\Delta G(x)i\epsilon^{\mu\nu pn}(S\cdot n)M_N+2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}M_N+\cdots\Big]$$

$$\mathcal{G}_T(x)=-\frac{1}{2x^2}\int dx'\big(8M_2(x,x')-4M_2(x',x)-4M_1(x',x)+G_F(x'+x,x')+G_F(x'-x,x')\big)$$

$$\mathcal{G}_T(x)=\frac{1}{2}\int_x^{\epsilon(x)}dx'\frac{\Delta G(x')}{x'}$$

$$-\int_x^{\epsilon(x)}dx'\int dx''\frac{1}{2x'^2}\Big[P\frac{1}{x'-x''}\Big(\frac{\partial}{\partial x'}-\frac{\partial}{\partial x''}\Big)(2N(x',x'-x'')-2N(x'',x''-x'))$$

$$+P\frac{1}{x'-x''}\Big(\frac{\partial}{\partial x'}+\frac{\partial}{\partial x''}\Big)(4N(x',x'')-6N(x'',x''-x')-6N(x',x'-x''))$$

$$+P\frac{1}{x'-x''}P\frac{1}{x'}(8N(x'',x''-x')+8N(x',x'-x''))+\left(P\frac{1}{x'}-\frac{\partial}{\partial x'}\right)(G_F(x''+x',x'')+G_F(x''-x',x''))\Big]$$

$$\epsilon(x)=\frac{x}{|x|}$$

$$M_1(x_1,x_2)=\mathcal{P}\frac{N(x_1,x_2)}{x_2-x_1}$$

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$$M_2(x_1,x_2)=\mathcal{P}\frac{N(x_2,x_2-x_1)}{x_1-x_2}+\delta(x_1-x_2)x_1^2\Big[\int_{x_1}^{\epsilon(x_1)}dx'\int dx''\frac{1}{x'^3}\Big\{P\frac{2x'-x''}{(x'-x'')^2}(N(x',x'-x'')$$

$$-N(x'',x''-x'))+P\frac{1}{x'-x''}(2N(x',x'')-N(x'',x''-x')-N(x',x'-x''))\Big\}$$

$$-\frac{1}{2}\int_x^{\epsilon(x)}dx'\int dx''\frac{1}{2x'^3}(G_F(x''+x',x'')+G_F(x''-x',x''))-\frac{1}{4}\int_x^{\epsilon(x)}dx'\frac{\Delta G(x')}{x'}\Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\ S \rangle = -\frac{x}{2}\Big[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\mathcal{G}_T(x) = \frac{1}{2}\int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'}$$

$$-\int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2}\Big[P\frac{1}{x'-x''}\Big(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\Big)\big(2N(x', x'-x'') - 2N(x'', x''-x')\big)$$

$$+ P\frac{1}{x'-x''}\Big(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\Big)\big(4N(x', x'') - 6N(x'', x''-x') - 6N(x', x'-x'')\big)$$

$$+ P\frac{1}{x'-x''} P\frac{1}{x'}\big(8N(x'', x''-x') + 8N(x', x'-x'')\big) + \left(P\frac{1}{x'} - \frac{\partial}{\partial x'}\right)\big(G_F(x''+x', x'') + G_F(x''-x', x'')\big)\Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S \,|\, \bar\psi(0)\gamma^\sigma \gamma_5 \psi(\lambda n) \,|\, p\ S \rangle = 2M_N \Big[\Delta g(x) (S\cdot n)\, p^\sigma + g_T(x) S_\perp^\sigma + \cdots \Big]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1}\left[\Delta q(x_1) + \frac{1}{2} P\int_{-1}^{1} dx_2 \frac{1}{x_1 - x_2}\left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}\right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}\right) \tilde{G}_F(x_1, x_2) \right\}\right]$$

WW

genuine twist-3

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p\ S \rangle = -\frac{x}{2}\Big[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\mathcal{G}_T(x) = \frac{1}{2}\int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'}$$

WW

$$- \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2}\Big[P\frac{1}{x'-x''}\Big(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\Big)(2N(x', x'-x'') - 2N(x'', x''-x'))$$

genuine twist-3

$$+ P\frac{1}{x'-x''}\Big(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\Big)(4N(x', x'') - 6N(x'', x''-x') - 6N(x', x'-x''))$$

$$+ P\frac{1}{x'-x''} P\frac{1}{x'}(8N(x'', x''-x') + 8N(x', x'-x'')) + \left(P\frac{1}{x'} - \frac{\partial}{\partial x'} \right)(G_F(x''+x', x'') + G_F(x''-x', x'')) \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\ S | \bar{\psi}(0) \gamma^{\sigma}\gamma_5 \psi(\lambda n) | p\ S \rangle = 2M_N \Big[\Delta g(x) (S\cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \cdots \Big]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1}\Bigg[\Delta q(x_1) + \frac{1}{2} P\int_{-1}^{1} dx_2 \frac{1}{x_1 - x_2}\left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}\right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}\right) \tilde{G}_F(x_1, x_2) \right\} \Bigg]$$

WW **genuine twist-3**

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S \,|\, F^{\mu n}(0) F^{\nu n}(\lambda n) \,|\, p\, S \rangle = -\frac{x}{2} \Big[G(x) g_\perp^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu p n} (S\cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\mathcal{G}_T(x) = \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'}$$

WW

$$- \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \Big[P \frac{1}{x'-x''} \big(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\big) \big(2N(x', x'-x'') - 2N(x'', x''-x')\big)$$

genuine twist-3

$$+ P \frac{1}{x'-x''} \big(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\big) \big(4N(x', x'') - 6N(x'', x''-x') - 6N(x', x'-x'')\big)$$

$$+ P \frac{1}{x'-x''} P \frac{1}{x'} \big(8N(x'', x''-x') + 8N(x', x'-x'')\big) + \left(P\frac{1}{x'} - \frac{\partial}{\partial x'} \right) \big(G_F(x''+x', x'') + G_F(x''-x', x'')\big) \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\, S \,|\, \bar\psi(0) \gamma^\sigma \gamma_5 \psi(\lambda n) \,|\, p\, S \rangle = 2M_N \Big[\Delta g(x) (S\cdot n) p^\sigma + g_T(x) S_\perp^\sigma + \cdots \Big]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1} \left[\Delta q(x_1) + \frac{1}{2} P \int_{-1}^{1} dx_2 \frac{1}{x_1 - x_2} \left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \tilde{G}_F(x_1, x_2) \right\} \right]$$

WW   **genuine twist-3**

$$\int dx\, g_T(x) = \int dx\, \Delta q(x) \equiv \Delta q \qquad \int_{-1}^{1} dx\, \mathcal{G}_T(x) = \int_0^1 dx\, \Delta G(x) = a_0 \equiv \Delta G$$

$$\langle p\, S_\perp | -\!\!\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\langle p\, S_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p\, S_\perp | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p\, S_\perp \rangle$$

$$\mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^{\alpha} - F^{\mu\alpha} A^{\beta}$$

$$A^{\alpha}(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

$$\langle p\, S_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

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$$\frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

transverse spin sum rule

$$\langle p\, S_\perp \,|\, -\!\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) \,|\, p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

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transverse spin sum rule

$$L \overset{?}{=} L_q + L_g$$

$$\langle p\, S_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

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transverse spin sum rule

$$L \stackrel{?}{=} L_q + L_g$$

$$J_q + J_g = \frac{1}{2}$$

$$W^\mu = -\frac{1}{2}\epsilon^\mu{}_{\nu\rho\sigma} p^\nu \int d^3x \mathcal{M}^{+\rho\sigma} = W^\mu_q + W^\mu_g$$

$$\frac{\langle p\, S_\perp | W^j_{q,g} | p\, S_\perp \rangle}{2p^+ (2\pi)^3 \delta^3(0)} \equiv J_{q,g} S^j_\perp$$

$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g})$$

Ji, Xiong, Yuan, PLB717 ('12) 214

$$\bar{p}^\mu = \frac{p^\mu + p'^\mu}{2} \qquad \Delta^\mu = p'^\mu - p^\mu$$

$$\mathcal{M}^{\lambda\mu\nu}_{q,g} = x^\mu T^{\lambda\nu}_{q,g} - x^\nu T^{\lambda\mu}_{q,g}$$

$$\langle p'\, S' | T^{\mu\nu}_{q,g} | p\, S \rangle = \bar{u}(p',S')\left[A_{q,g}\gamma^{(\mu}\bar{p}^{\nu)} + B_{q,g}\frac{\bar{p}^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g} M g^{\mu\nu}\right] u(p,S)$$

$$\langle p\, S_\perp | -\!\!\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p\, S_\perp | \mathcal{M}^{+\alpha\beta}_{g\text{-spin}} | p\, S_\perp \rangle \qquad \mathcal{M}^{\mu\alpha\beta}_{g\text{-spin}} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

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$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g})$$

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$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g}) + \frac{p^3}{2(p^0 + M_N)} \bar{C}_{q,g}$$

Hatta, K.T., Yoshida, JHEP1302 ('13) 003

$$\bar{p}^\mu = \frac{p^\mu + p'^\mu}{2} \qquad \Delta^\mu = p'^\mu - p^\mu$$

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$$\langle p\, S_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p\, S_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p\, S_\perp | \mathcal{M}^{+\alpha\beta}_{g\text{-spin}} | p\, S_\perp \rangle$$

$$\mathcal{M}^{\mu\alpha\beta}_{g\text{-spin}} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

$$A^\alpha(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

$$\frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

transverse spin sum rule

$$L \overset{?}{=} L_q + L_g$$

$$J_q + J_g = \frac{1}{2}$$

$$W^\mu = -\frac{1}{2}\epsilon^\mu{}_{\nu\rho\sigma} p^\nu \int d^3x \mathcal{M}^{+\rho\sigma} = W^\mu_q + W^\mu_g$$

$$\frac{\langle p\, S_\perp | W^j_{q,g} | p\, S_\perp \rangle}{2p^+(2\pi)^3 \delta^3(0)} \equiv J_{q,g} S^j_\perp$$

$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g})$$

Ji, Xiong, Yuan, PLB717 ('12) 214

$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g}) + \frac{p^3}{2(p^0 + M_N)} \bar{C}_{q,g}$$

Hatta, K.T., Yoshida, JHEP1302 ('13) 003

$$\bar{p}^\mu = \frac{p^\mu + p'^\mu}{2} \qquad \Delta^\mu = p'^\mu - p^\mu$$

$$\mathcal{M}^{\lambda\mu\nu}_{q,g} = x^\mu T^{\lambda\nu}_{q,g} - x^\nu T^{\lambda\mu}_{q,g}$$

$$\langle p'\, S' | T^{\mu\nu}_{q,g} | p\, S \rangle = \bar{u}(p', S')\Big[A_{q,g}\gamma^{(\mu}\bar{p}^{\nu)} + B_{q,g}\frac{\bar{p}^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g} M g^{\mu\nu}\Big] u(p, S)$$

$$L = L_q + L_g \qquad \left(\frac{1}{2} = J_q + J_g\right)$$

frame dependent!

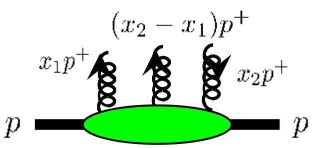
Summary Twist-three gluonic correlations in $|p\,S_\perp\rangle$

single- and double-spin asymm.

SSA in SIDIS $ep^\uparrow \to eDX$ at $P_{h\perp} \gg \Lambda_{\rm QCD}$

Twist-3 mechanism from three-gluon correlation inside the nucleon

photon-gluon fusion



Factorization formula for twist-3 SSA

convoluted with F-type correl. fns. $\left\{ \begin{array}{l} N(x,x),\ O(x,x),\ \dfrac{dN(x,x)}{dx},\ \dfrac{dO(x,x)}{dx} \\ N(x,0),\ O(x,0),\ \dfrac{dN(x,0)}{dx},\ \dfrac{dO(x,0)}{dx} \end{array} \right.$

5 different azimuthal dependences

Numerical estimates of A_N : $\langle 1 \rangle$, $\langle \sin 2\phi \rangle$ ~ % level

good chance to access multi-gluon effects

Exact relation between $\langle p\,S_\perp | F^{+\perp}F^{+\perp}F^{+\perp} | p\,S_\perp\rangle$ & $\langle p\,S_\perp | F^{+\perp}D^{\perp}F^{+\perp} | p\,S_\perp\rangle$

$$\mathcal{G}_T(x) = \frac{1}{2}\int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} + [\text{genuine tw.3}] \qquad g_T(x) = \int_x^{\epsilon(x)} \frac{dx'}{x'} \Delta q(x') + [\text{genuine tw.3}]$$

$$\int_{-1}^{1} dx\,\mathcal{G}_T(x) = \int_0^1 dx\,\Delta G(x) = a_0 \equiv \Delta G, \quad \int dx\, g_T(x) = \int dx\,\Delta q(x) \equiv \Delta q$$ ➡ transverse spin SR