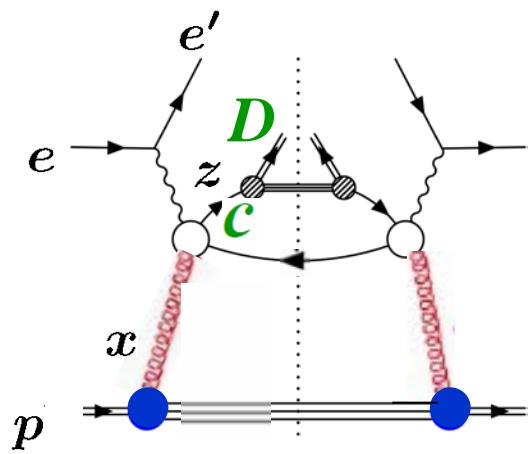
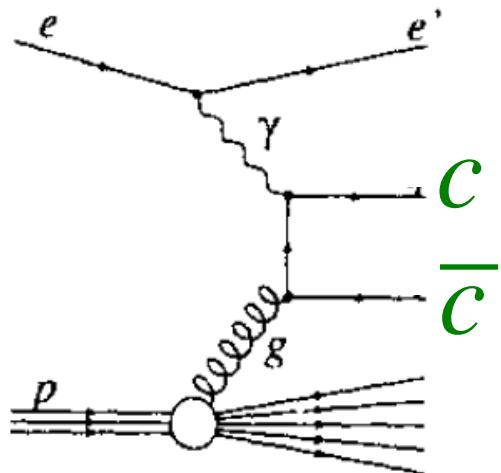


Gluon correlations in the transversely polarized nucleon at twist three

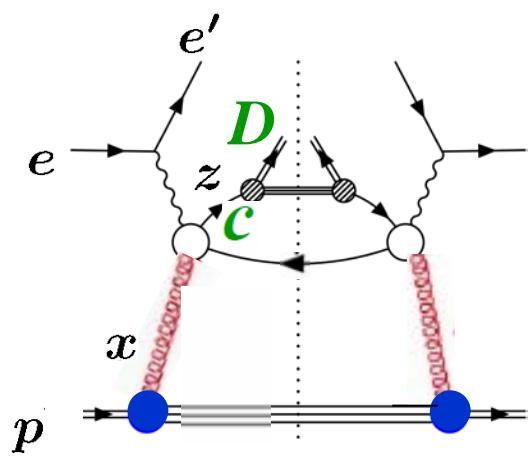
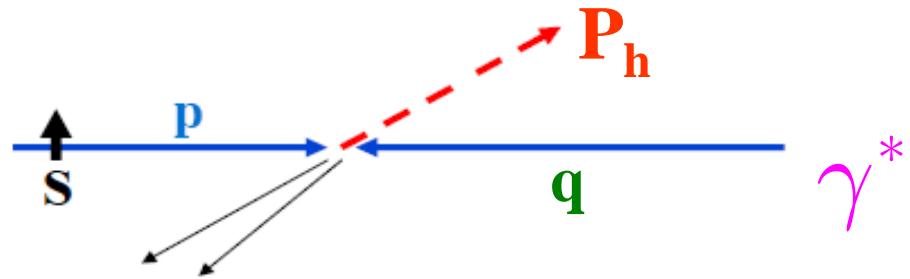
Kazuhiro Tanaka (Juntendo U)

PRD85 ('12) 114026
JHEP1302 ('13) 003

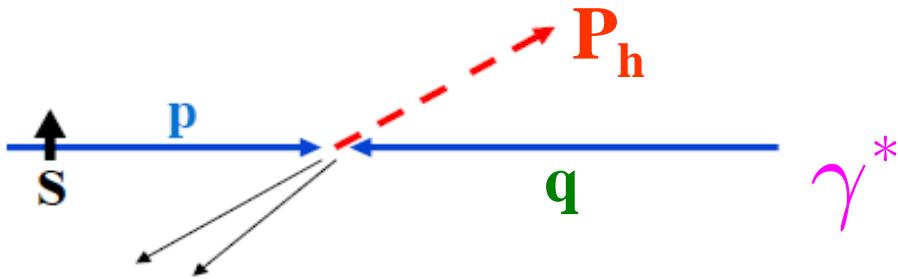
SIDIS $\text{ep} \rightarrow \text{eDX}$



SSA in **SIDIS** $e p^\uparrow \rightarrow e D X$



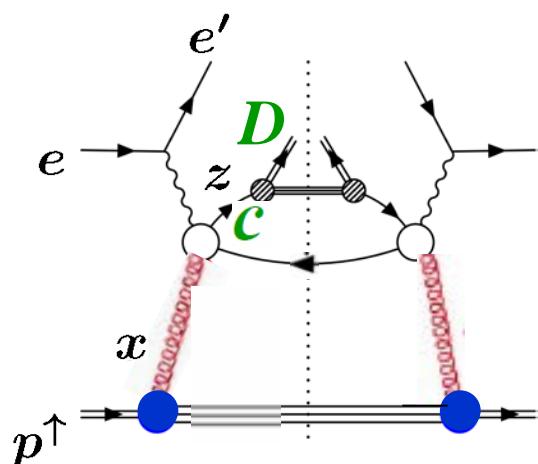
SSA in **SIDIS** $\text{ep}^\uparrow \rightarrow \text{eDX}$



$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

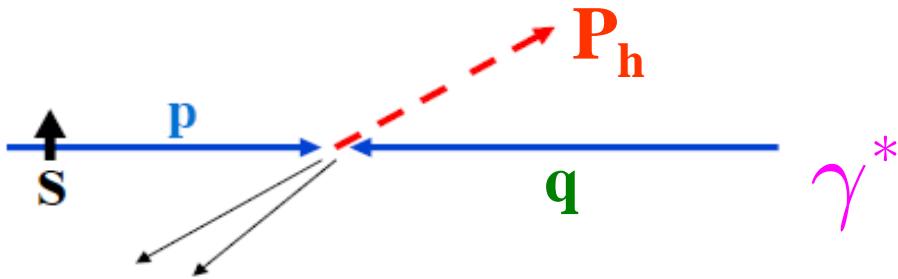
$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



$$S_\perp$$

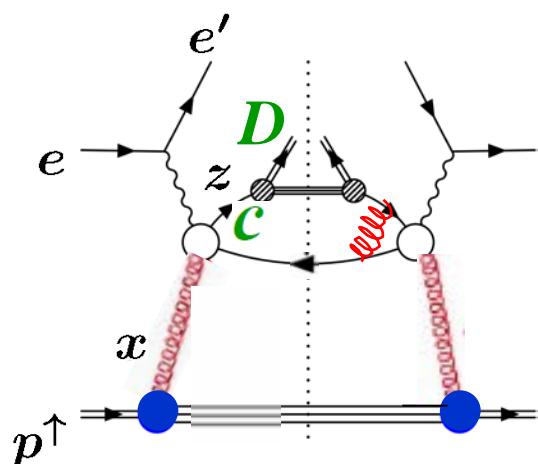
SSA in **SIDIS** $\text{ep}^\uparrow \rightarrow \text{eDX}$



$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

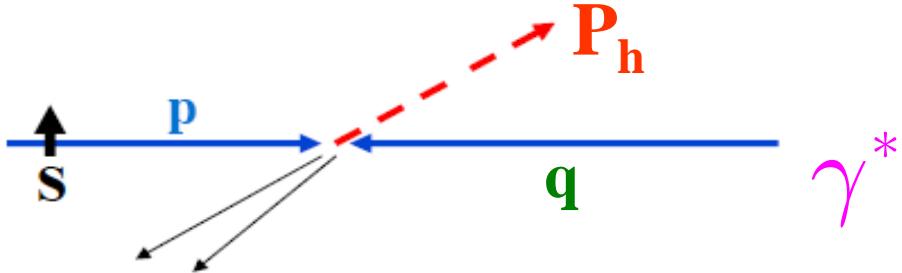
$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



S_\perp

SSA in **SIDIS** $\text{ep}^\uparrow \rightarrow \text{eDX}$

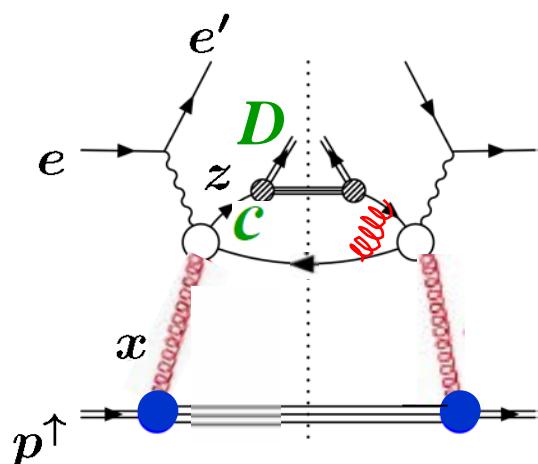


$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

at twist-2

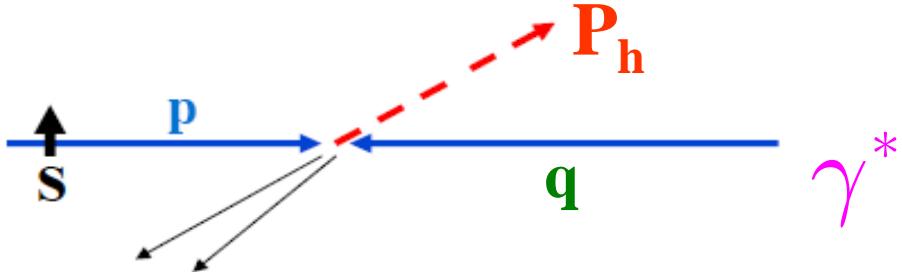
1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



$$S_\perp$$



SSA in **SIDIS** $e p^\uparrow \rightarrow e D X$

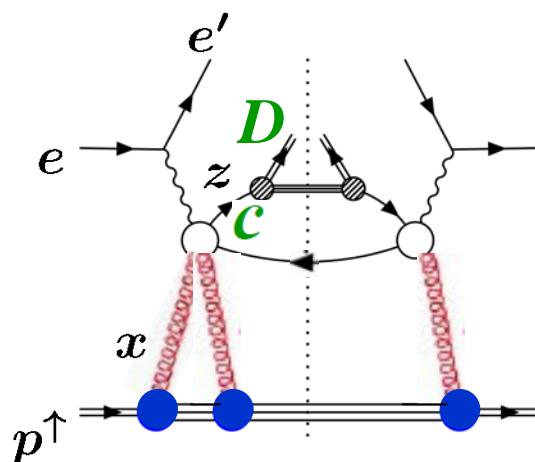


$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

at twist-3

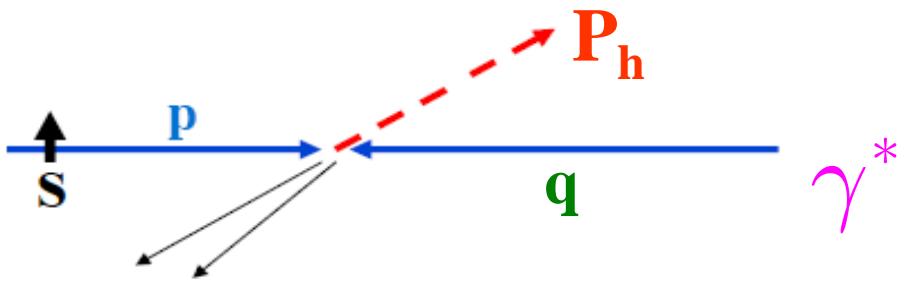
1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



$$S_\perp$$



SSA in **SIDIS** $\text{ep}^\uparrow \rightarrow \text{eDX}$

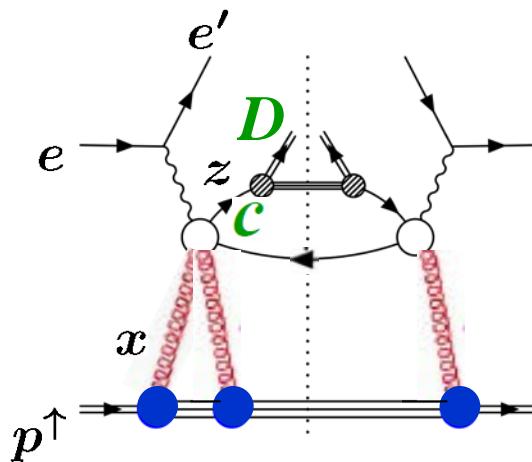


$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

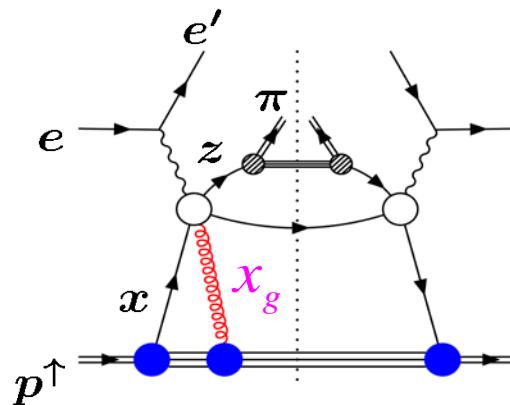
$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

at twist-3

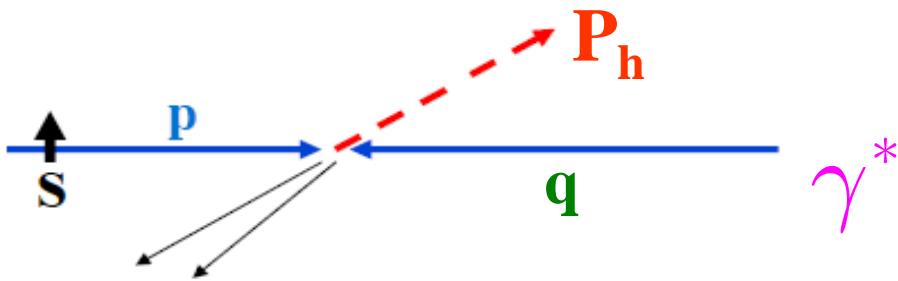
1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



S_\perp



SSA in **SIDIS** $e p^\uparrow \rightarrow e D X$

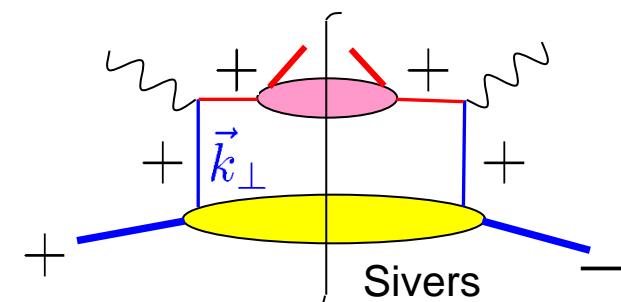
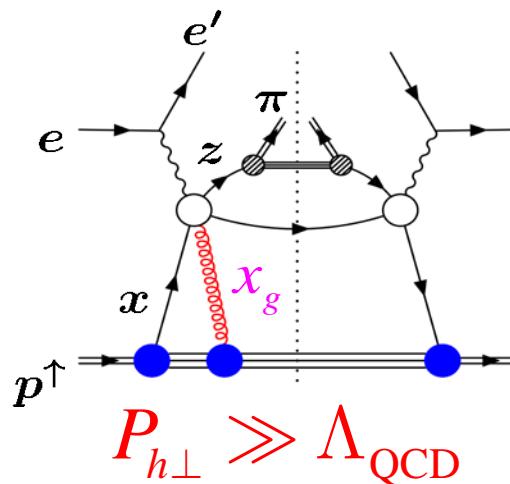
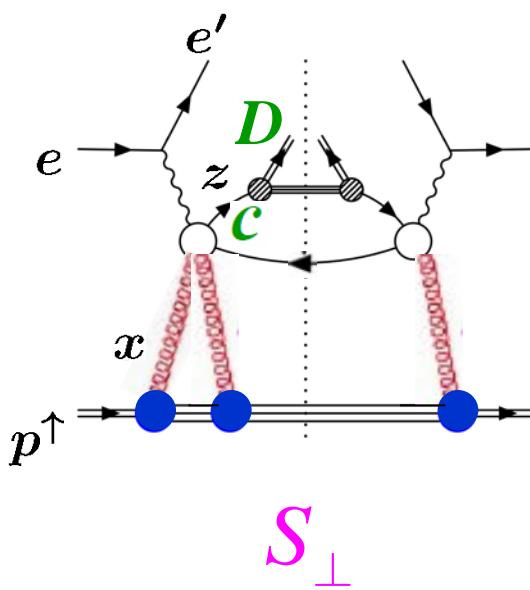


$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

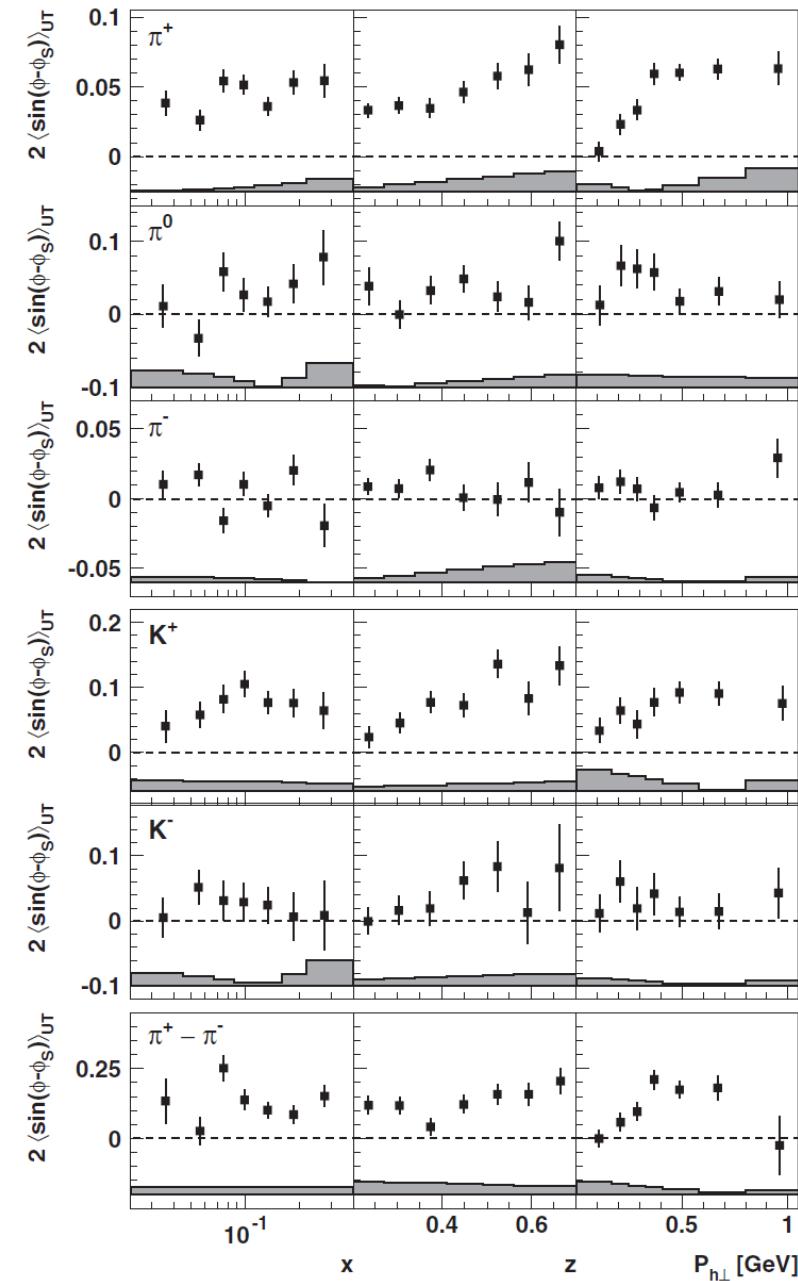
at twist-3

1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



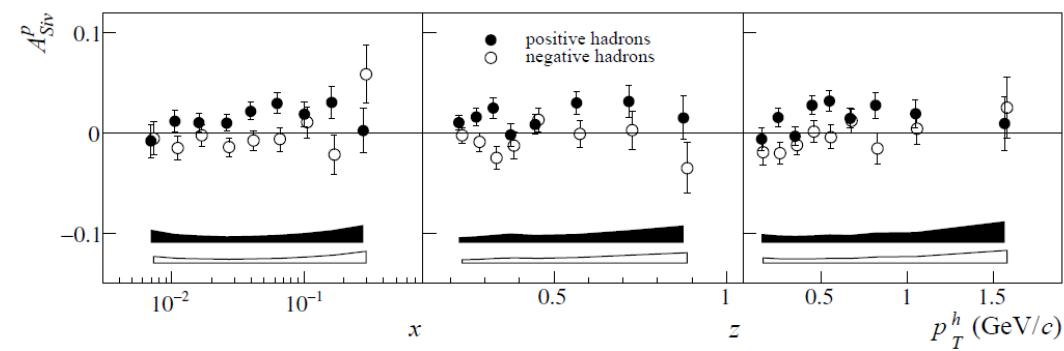
$$P_{h\perp} \gg \Lambda_{\text{QCD}}$$

$$P_{h\perp} \sim \Lambda_{\text{QCD}}$$



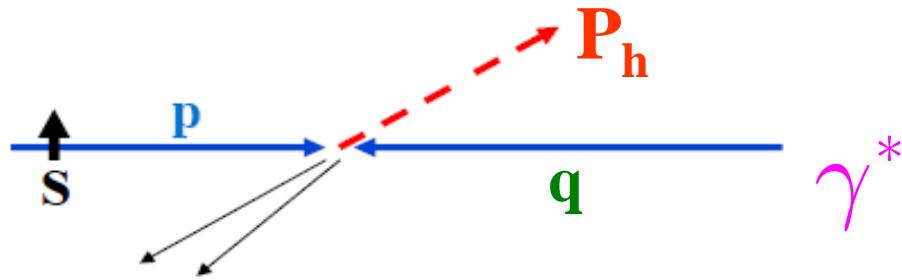
HERMES, PRL103 ('09) 152002

Sivers asymmetry



COMPASS, PLB692 ('10) 240

SSA in **SIDIS** $e p^\uparrow \rightarrow e D X$

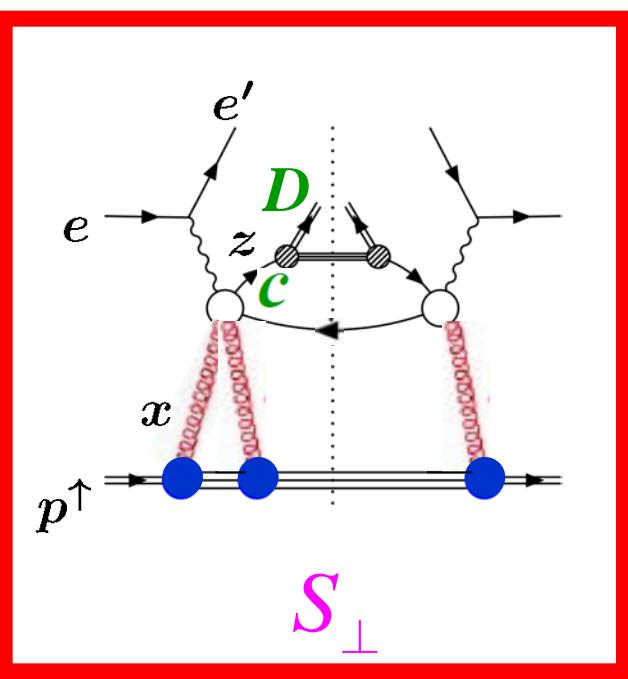


$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

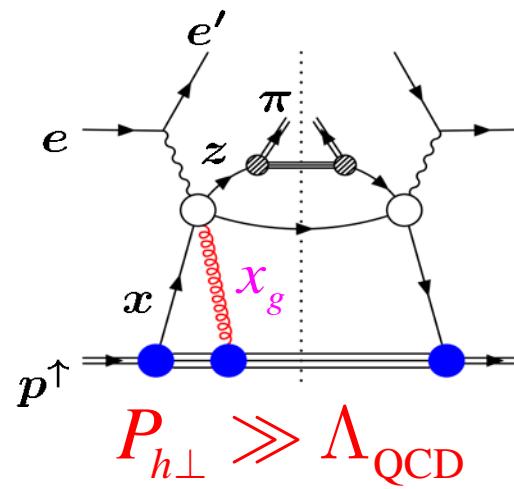
$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

at twist-3

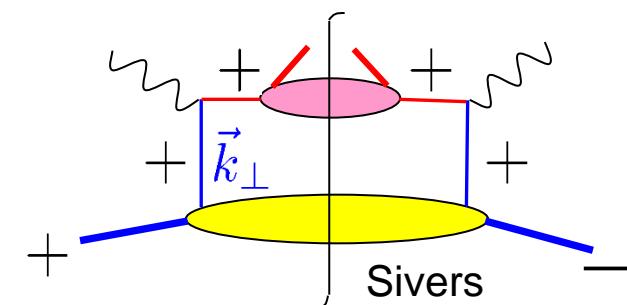
1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



$$m_D \neq 0 \quad m_c \neq 0$$

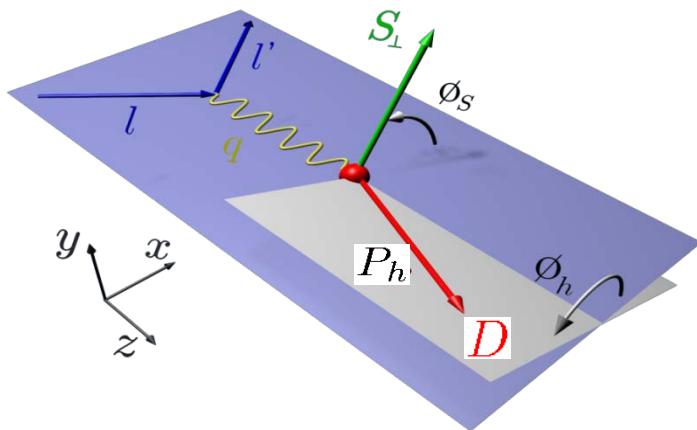


$$P_{h\perp} \gg \Lambda_{\text{QCD}}$$



$$P_{h\perp} \sim \Lambda_{\text{QCD}}$$

★ Kinematics for $e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + D(P_h) + X$



$$S_{ep} = (\ell + p)^2$$

$$q = \ell - \ell'$$

$$x_{bj} = \frac{Q^2}{2p \cdot q}$$

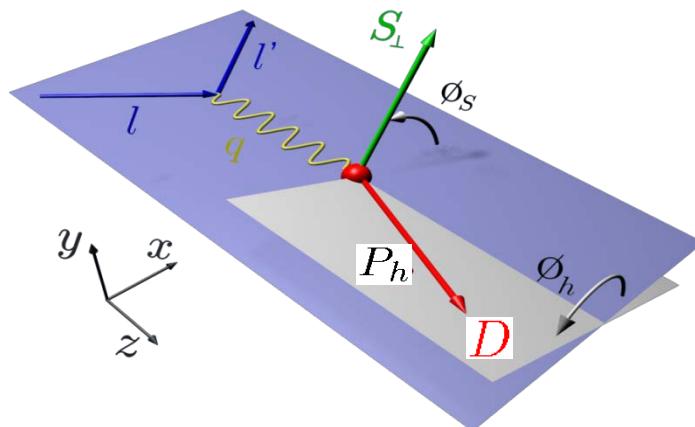
$$z_f = \frac{p \cdot P_h}{p \cdot q}$$

$P_{h\perp}$: \perp -mom. of final D

ϕ_h : azimuth. angle of hadron plane

ϕ_S : azimuth. angle of \vec{S}_\perp

★ Kinematics for $e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + D(P_h) + X$



$$S_{ep} = (\ell + p)^2$$

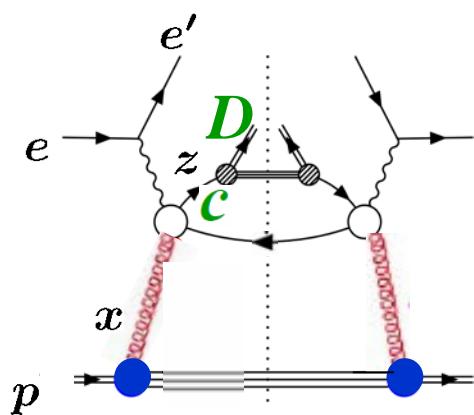
$$q = \ell - \ell'$$

$$x_{bj} = \frac{Q^2}{2p \cdot q}$$

$$z_f = \frac{p \cdot P_h}{p \cdot q}$$

$P_{h\perp}$: \perp -mom. of final D

ϕ_h : azimuth. angle of hadron plane
 ϕ_S : azimuth. angle of \vec{S}_\perp

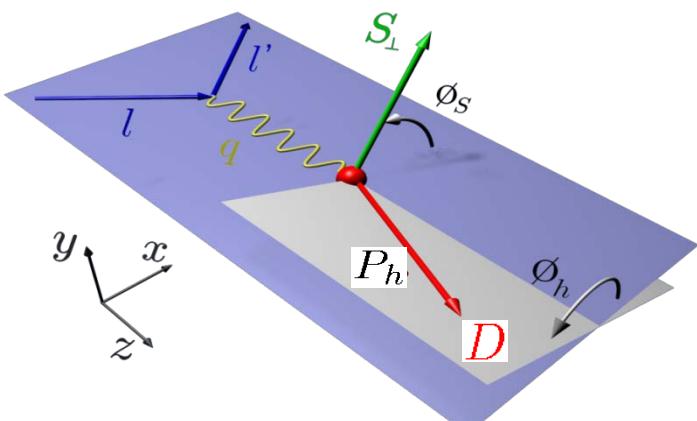
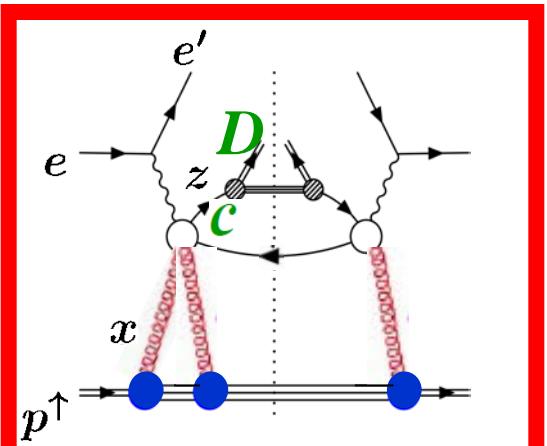


$$\begin{aligned} \frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \frac{1}{4} \sum_{k=1}^4 \mathcal{A}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\quad \times \sum_{a=c, \bar{c}} D_a(z) \mathbf{g}(x) \hat{\sigma}_k^U \\ &= \sigma_1^U + \sigma_2^U \cos(\phi_h) + \sigma_3^U \cos(2\phi_h) \end{aligned}$$

twist-3 SSA for $ep^\uparrow \rightarrow eDX$

Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

$$\begin{aligned}
& \frac{d^5 \sigma_{\text{twist-3}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\
& \times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\
& \quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\
& = \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]
\end{aligned}$$



twist-3 SSA for $ep^\uparrow \rightarrow eDX$

Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

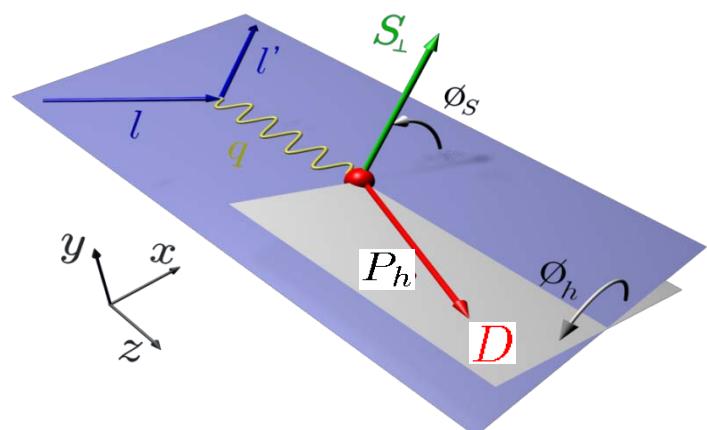
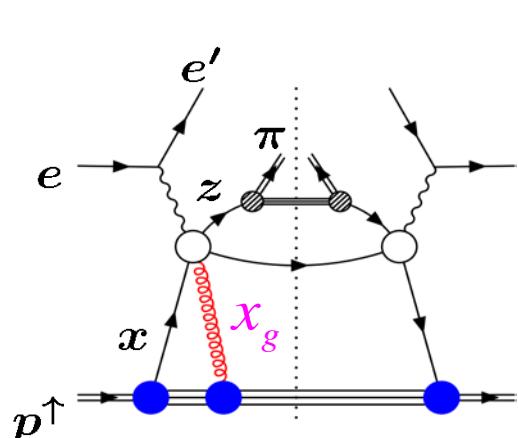
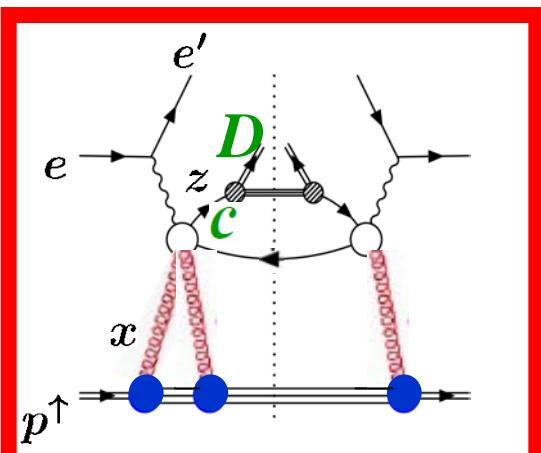
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right)$$

$$\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right.$$

$$\left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right]$$

$$= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]$$

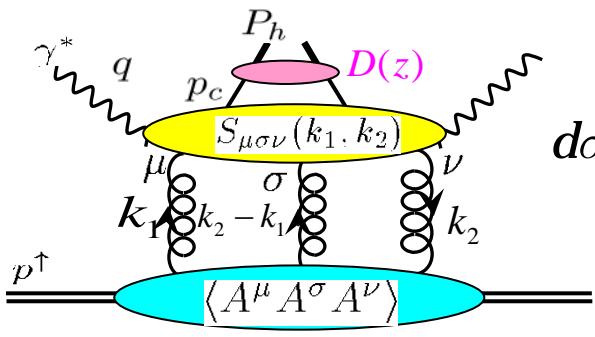
- Receives contribution from 4 functions $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$, and all of them contribute both as derivative and nonderivative terms. $\Delta\hat{\sigma}_k^1 \neq \Delta\hat{\sigma}_k^2$
 $\Delta\hat{\sigma}_k^3 \neq \Delta\hat{\sigma}_k^4$
- 5 structure functions with different azimuthal dependence
similar as $ep^\uparrow \rightarrow e\pi X$ from quark-gluon correlation



twist-3 SSA for $ep^\uparrow \rightarrow eDX$

Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

$$\begin{aligned} \frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\ &= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h] \end{aligned}$$



$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \mathbf{S}_{\mu\beta\nu}(\mathbf{k}_1, \mathbf{k}_2) \mathbf{p}^\beta}{\partial \mathbf{k}_{2\perp}^\sigma} \Bigg|_{\mathbf{k}_i = \mathbf{x}_i \mathbf{p}} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{Bmatrix} \otimes \mathbf{D}(z)$$

$$n^\mu = (0, n^-, \mathbf{0}_\perp) \quad p \cdot n = p^+ n^- = 1$$

$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \langle p \mathcal{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathcal{S}_\perp \rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

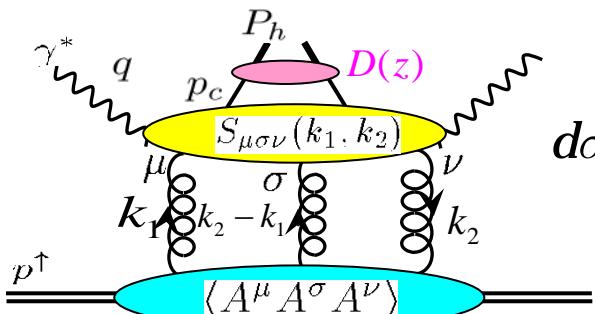
$$\begin{aligned} N^{\nu\mu\sigma}(x_1, x_2) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right] \\ O^{\nu\mu\sigma}(x_1, x_2) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x_1, x_2) + g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x_2, x_2 - x_1) + g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x_1, x_1 - x_2) \right] \end{aligned}$$

Permutation symmetry

twist-3 SSA for $ep^\uparrow \rightarrow eDX$

Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

$$\begin{aligned} \frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\ &= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h] \end{aligned}$$



$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial S_{\mu\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \Bigg|_{k_i=x_i p} \otimes \begin{Bmatrix} N_{v\mu\sigma}(x_1, x_2) \\ O_{v\mu\sigma}(x_1, x_2) \end{Bmatrix} \otimes D(z)$$

$$n^\mu = \left(0, n^-, \mathbf{0}_\perp\right) \quad p \cdot n = p^+ n^- = 1$$

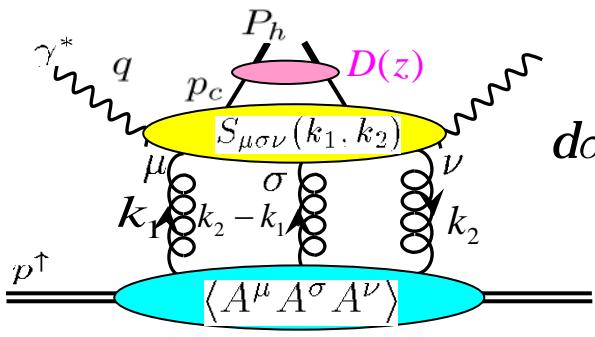
$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \left\langle p \textcolor{violet}{S}_\perp \left| F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) \right| p \textcolor{violet}{S}_\perp \right\rangle = \frac{1}{24} i \textcolor{blue}{f}^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} \textcolor{blue}{d}^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \left\langle p \textcolor{magenta}{S}_\perp \right| F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) \left| p \textcolor{magenta}{S}_\perp \right\rangle \quad D_\perp^\sigma = \partial_\perp^\sigma - ig A_\perp^\sigma$$

twist-3 SSA for $ep^\uparrow \rightarrow eDX$

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$$\begin{aligned} \frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\ &= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h] \end{aligned}$$



$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \mathbf{S}_{\mu\beta\nu}(\mathbf{k}_1, \mathbf{k}_2) \mathbf{p}^\beta}{\partial \mathbf{k}_{2\perp}^\sigma} \Bigg|_{\mathbf{k}_i = \mathbf{x}_i \mathbf{p}} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{Bmatrix} \otimes \mathbf{D}(z)$$

$$n^\mu = (0, n^-, \mathbf{0}_\perp) \quad p \cdot n = p^+ n^- = 1$$

$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \langle p \mathcal{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathcal{S}_\perp \rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

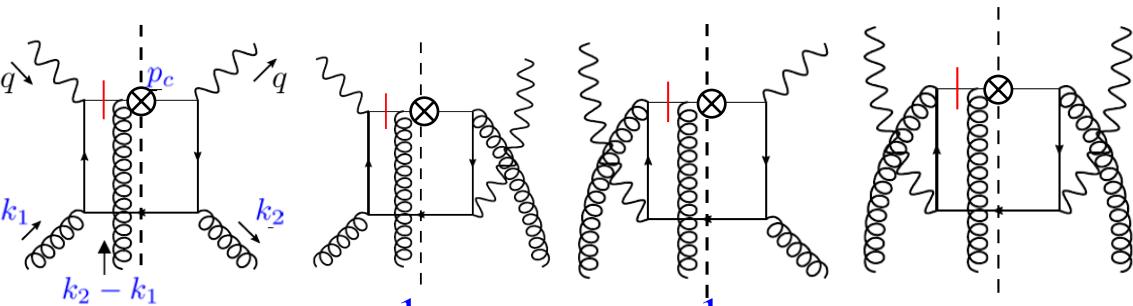
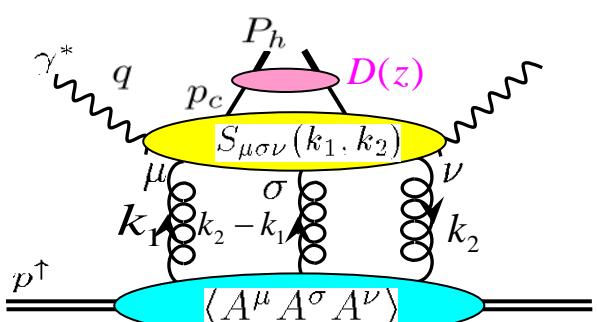
$$\begin{aligned} N^{\nu\mu\sigma}(x_1, x_2) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right] \\ O^{\nu\mu\sigma}(x_1, x_2) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x_1, x_2) + g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x_2, x_2 - x_1) + g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x_1, x_1 - x_2) \right] \end{aligned}$$

Permutation symmetry

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PRD82 ('10) 054005

$$\begin{aligned} \frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\ &= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h] \end{aligned}$$



$$\frac{1}{k^2 - m_c^2 + i\varepsilon} = \text{P} \frac{1}{k^2 - m_c^2} - i\pi\delta(k^2 - m_c^2)$$

$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \langle p \mathcal{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathcal{S}_\perp \rangle = \frac{1}{24} if^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

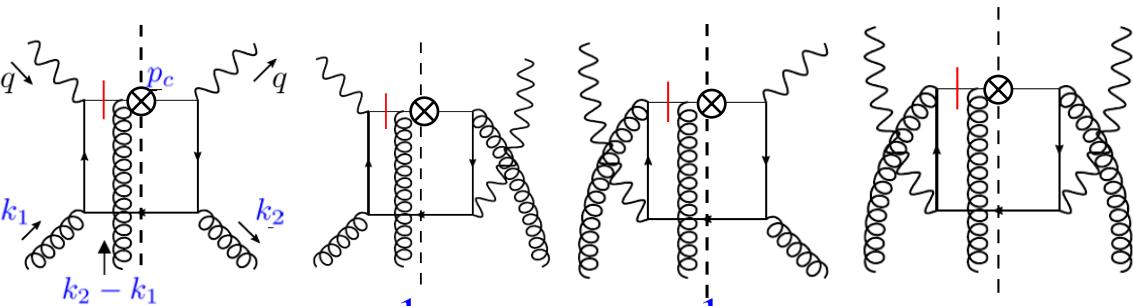
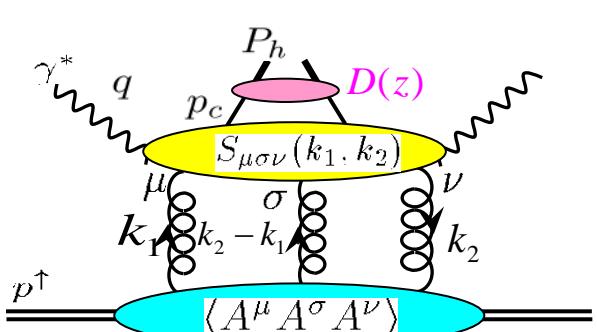
$$\begin{aligned} N^{\nu\mu\sigma}(x_1, x_2) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right] \\ O^{\nu\mu\sigma}(x_1, x_2) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x_1, x_2) + g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x_2, x_2 - x_1) + g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x_1, x_1 - x_2) \right] \end{aligned}$$

Permutation symmetry

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PRD82 ('10) 054005

$$\begin{aligned} \frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\ &= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h] \end{aligned}$$



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$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \langle p \mathcal{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathcal{S}_\perp \rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

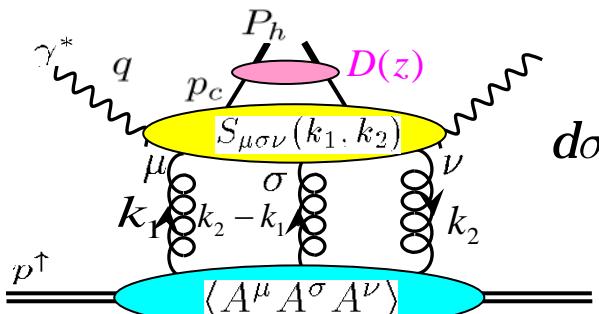
$$\begin{aligned} N^{\nu\mu\sigma}(x, x) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x, x) - g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x, 0) - g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x, 0) \right] \\ O^{\nu\mu\sigma}(x, x) &= 2iM_N \left[g_\perp^{\nu\mu} \mathcal{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x, x) + g_\perp^{\mu\sigma} \mathcal{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x, 0) + g_\perp^{\sigma\nu} \mathcal{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x, 0) \right] \end{aligned}$$

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$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \color{red} S_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \Bigg|_{k_i=x_i p} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{Bmatrix} \otimes D(z)$$

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$$N^{\nu\mu\sigma}(x,x) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x,x) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x,0) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x,0) \right]$$

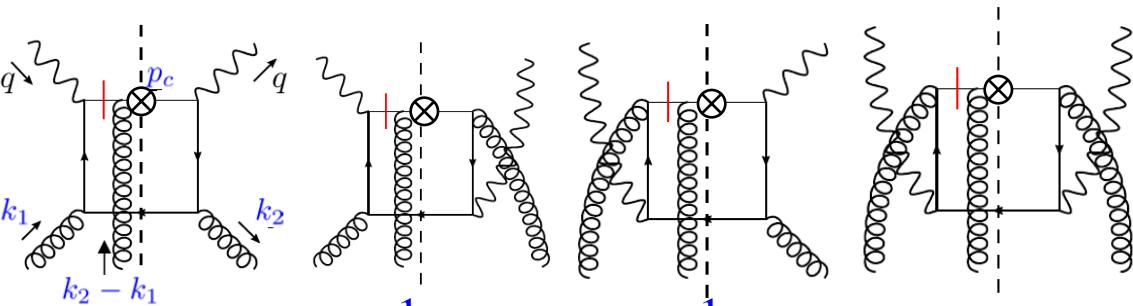
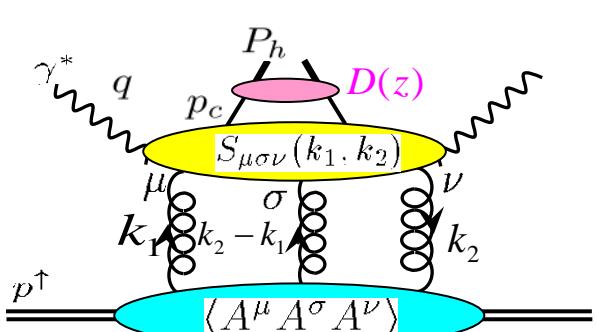
$$O^{\nu\mu\sigma}(x,x) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x,x) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x,0) + g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x,0) \right]$$

Permutation symmetry

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$$\frac{1}{k^2 - m_c^2 + i\varepsilon} = \text{P} \frac{1}{k^2 - m_c^2} - i\pi\delta(k^2 - m_c^2)$$

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Permutation symmetry

$$\begin{aligned}\Delta\hat{\sigma}_1^1 &= \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} \{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-2\hat{x}+2\hat{x}^2+12\hat{x}\hat{z}(1-\hat{x})(1-\hat{z})) \\ &\quad + 2m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-4m_c^4\hat{x}^2\},\end{aligned}$$

$$\Delta\hat{\sigma}_2^1 = \frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}} \{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_3^1 = \frac{16\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3} (1-2\hat{z})\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\}\{Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_4^1 = \frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2} \{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\}\{Q^2\hat{z}(1-\hat{z})+m_c^2\}, \quad \Delta\hat{\sigma}_8^1 = \Delta\hat{\sigma}_9^1 = 0,$$

$$\begin{aligned}\Delta\hat{\sigma}_1^2 &= \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} \{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-4\hat{x}+4\hat{x}^2+24\hat{x}\hat{z}(1-\hat{x})(1-\hat{z})) \\ &\quad + 4m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-8m_c^4\hat{x}^2\},\end{aligned}$$

$$\Delta\hat{\sigma}_2^2 = 2\Delta\hat{\sigma}_2^1,$$

$$\Delta\hat{\sigma}_3^2 = 2\Delta\hat{\sigma}_3^1,$$

$$\Delta\hat{\sigma}_4^2 = -\frac{16q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})^2,$$

$$\Delta\hat{\sigma}_8^2 = \frac{16\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2} (1-2\hat{z})\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_9^2 = -\frac{16q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_1^4 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2),$$

$$\Delta\hat{\sigma}_2^4 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^4 = \frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3} (1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1+12\hat{x}-16\hat{x}^2)+4m_c^2Q^2\hat{x}\hat{z}(3-8\hat{x})(1-\hat{z})-16m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_4^4 = -\frac{32q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^4\hat{z}^2(1-\hat{z})^2(1+\hat{x}-4\hat{x}^2)+m_c^2Q^2\hat{x}\hat{z}(1-8\hat{x})(1-\hat{z})-4m_c^4\hat{x}^2),$$

$$\Delta\hat{\sigma}_8^4 = \frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2} (1-2\hat{z})(Q^2\hat{z}(1+2\hat{x})(1-\hat{z})+2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^4 = -\frac{32q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-\hat{z})(1+\hat{x})+m_c^2\hat{x}),$$

$$\hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}$$

$$q_T \equiv \frac{P_{h\perp}}{z_f}$$

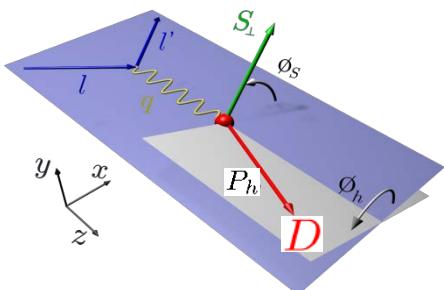
$(m_c \rightarrow 0$ for $ep^\dagger \rightarrow e\pi X$ case)

Estimates of twist-3 SSA for $ep^\uparrow \rightarrow eD^0 X$

$$\begin{aligned}
& \frac{d^5 \sigma_{\text{twist-3}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\
& \times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\
& \quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\
& = \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]
\end{aligned}$$

$$A_N : \frac{\mathcal{F}_1}{\sigma_1^U}, \quad \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U} \quad \left(\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^U + \sigma_2^U \cos \phi_h + \sigma_3^U \cos 2\phi_h \right)$$

$\langle 1 \rangle, \langle \cos \phi \rangle, \langle \cos 2\phi \rangle, \langle \sin \phi \rangle, \langle \sin 2\phi \rangle$



Estimates of twist-3 SSA for $ep^\uparrow \rightarrow eD^0 X$

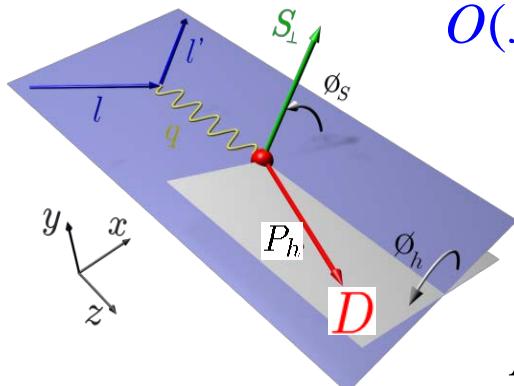
$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep} Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta\left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right)$$

$$\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right.$$

$$\left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right]$$

$$= \sin(\phi_h - \phi_s) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_s) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]$$

$$A_N : \frac{\mathcal{F}_1}{\sigma_1^U}, \quad \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U} \quad \left(\frac{d^5\sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^U + \sigma_2^U \cos \phi_h + \sigma_3^U \cos 2\phi_h \right)$$

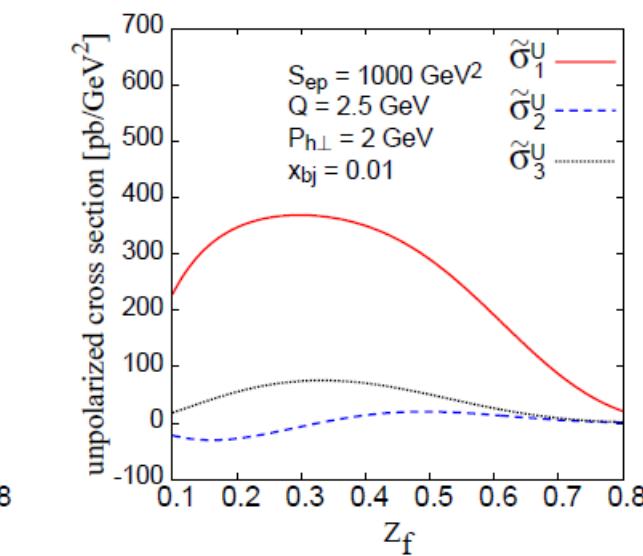
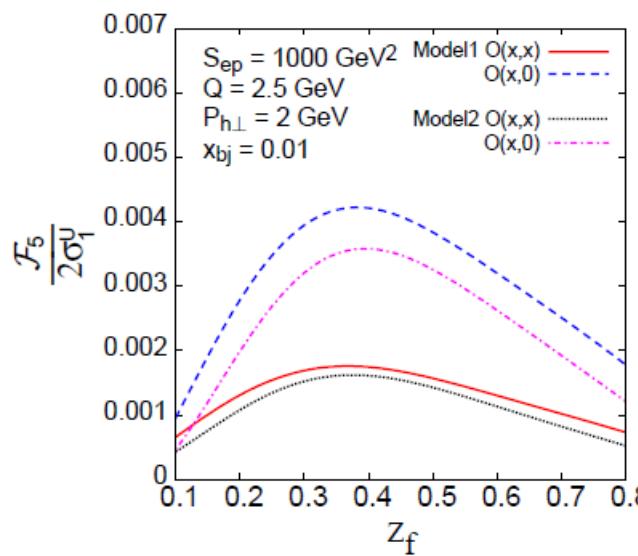
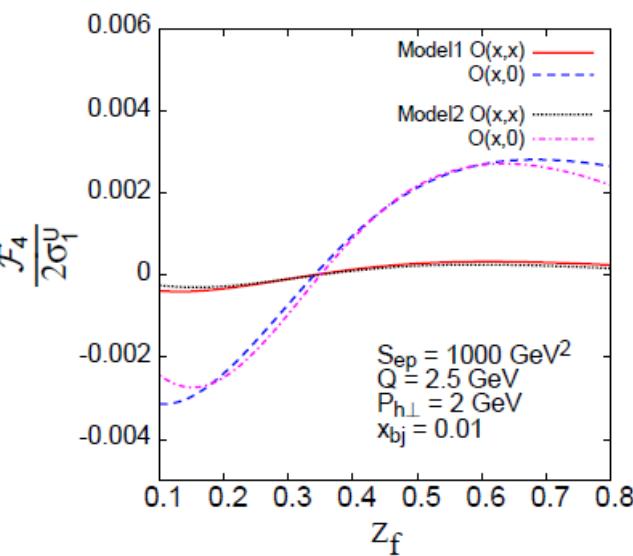
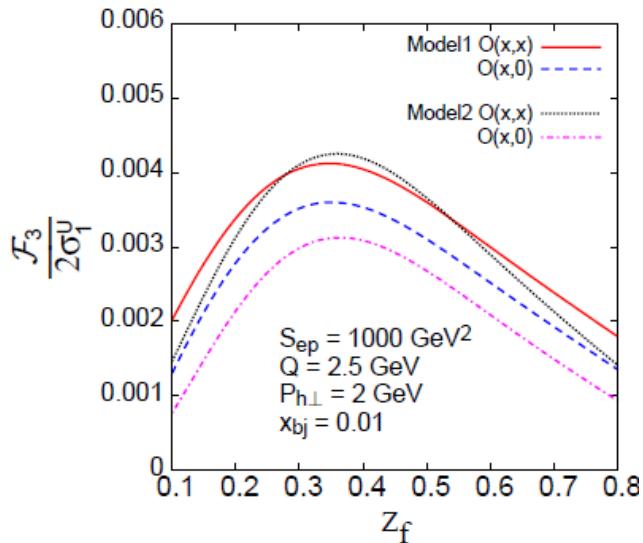
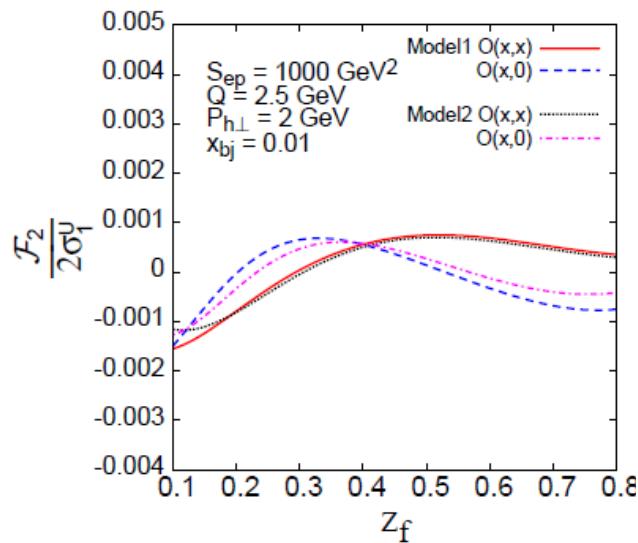
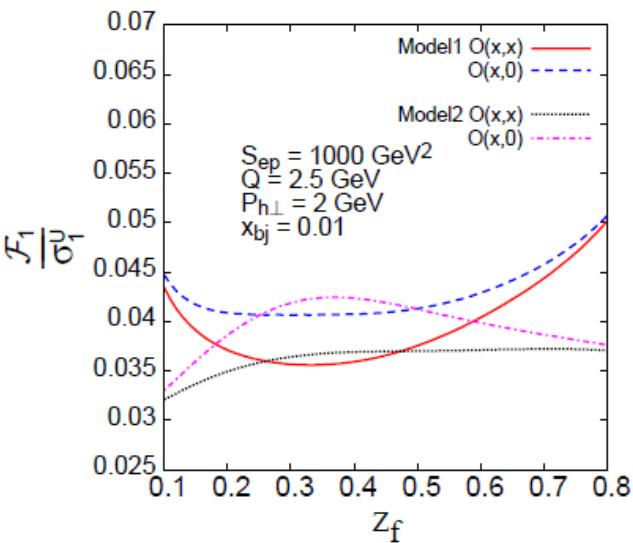


$$O(x,x) = O(x,0) = \begin{cases} 0.004 x g(x) & \text{Model1} \\ 0.001 \sqrt{x} g(x) & \text{Model2} \end{cases}$$

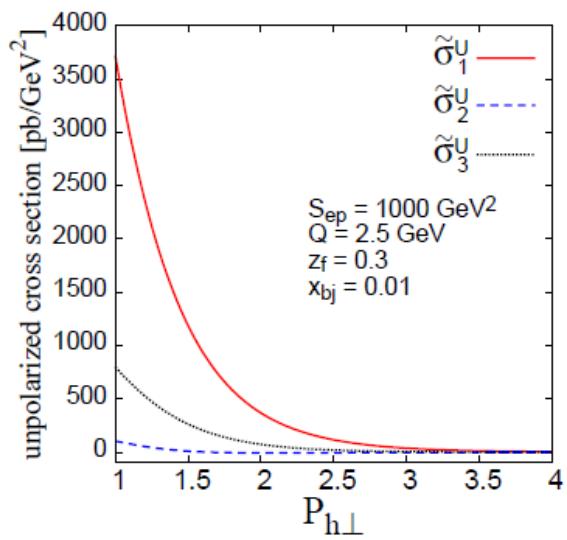
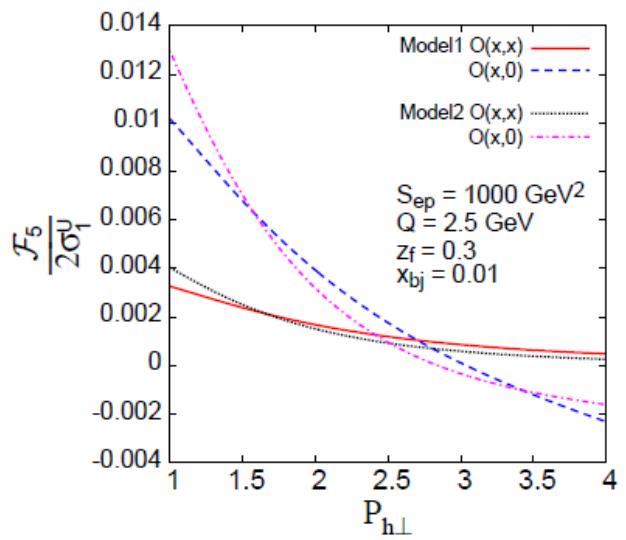
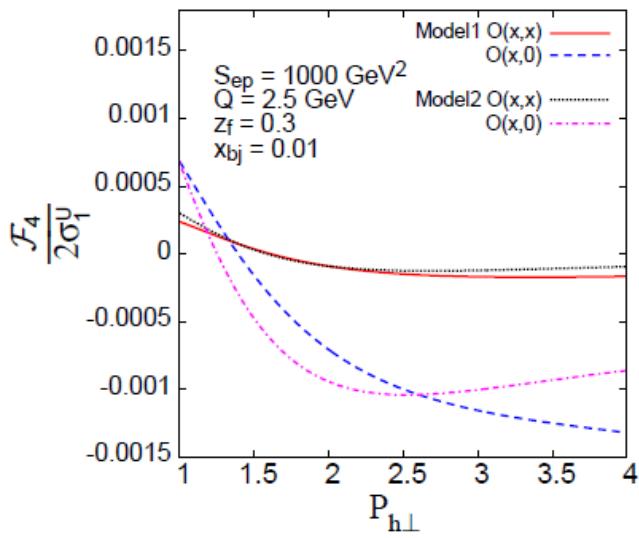
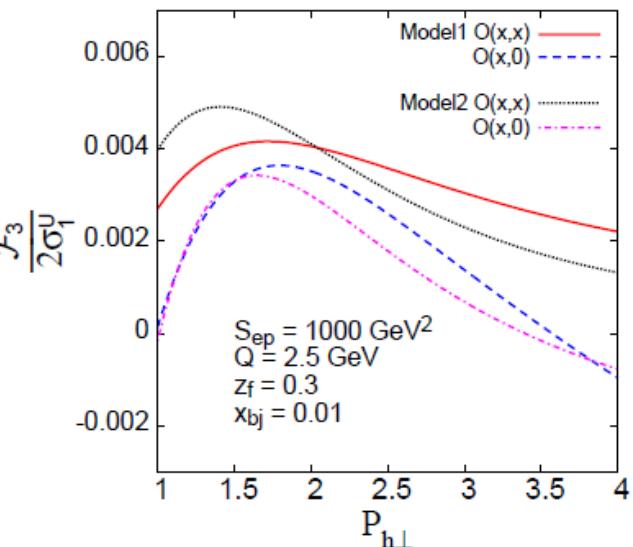
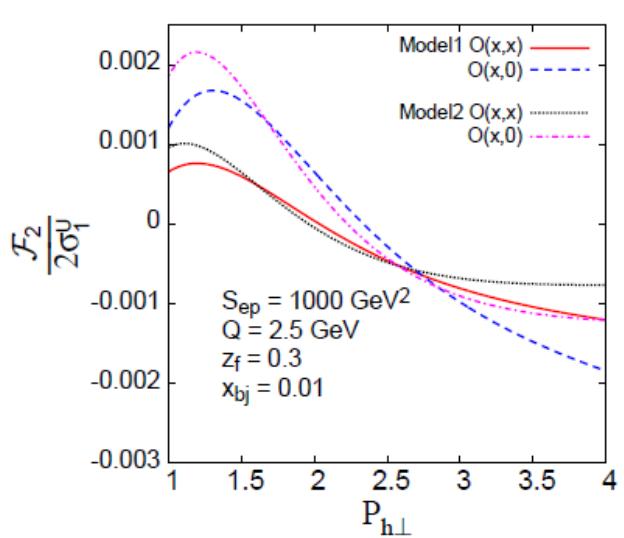
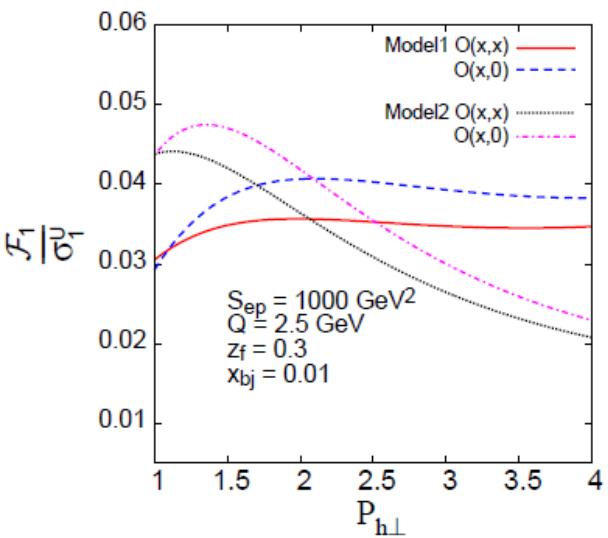
$$g(x, \mu) : \text{CTEQ6L} \quad \mu = \sqrt{Q^2 + m_c^2 + P_{h\perp}^2}$$

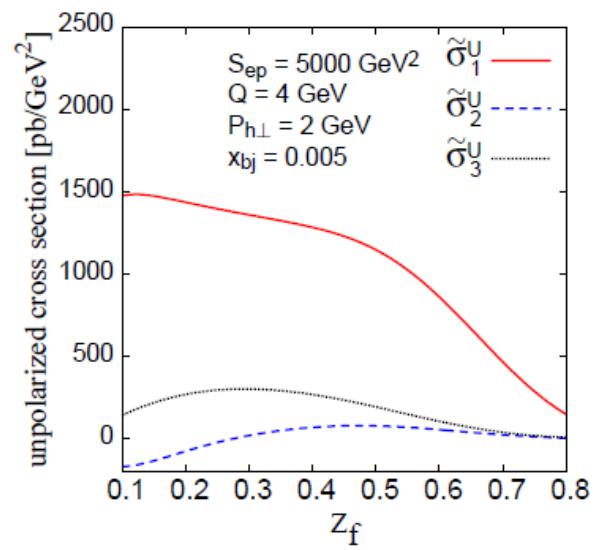
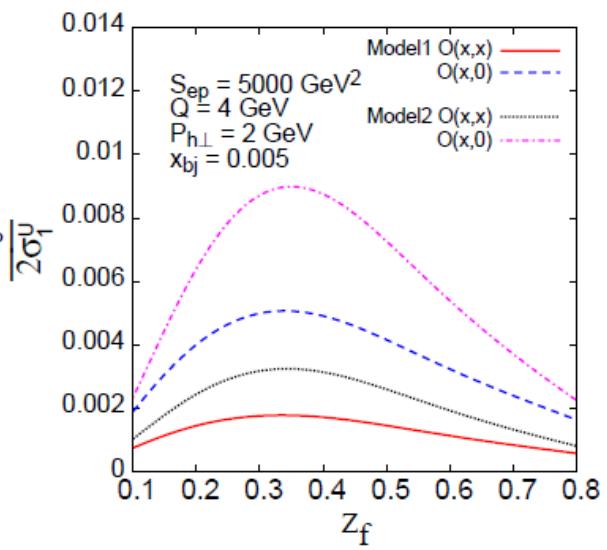
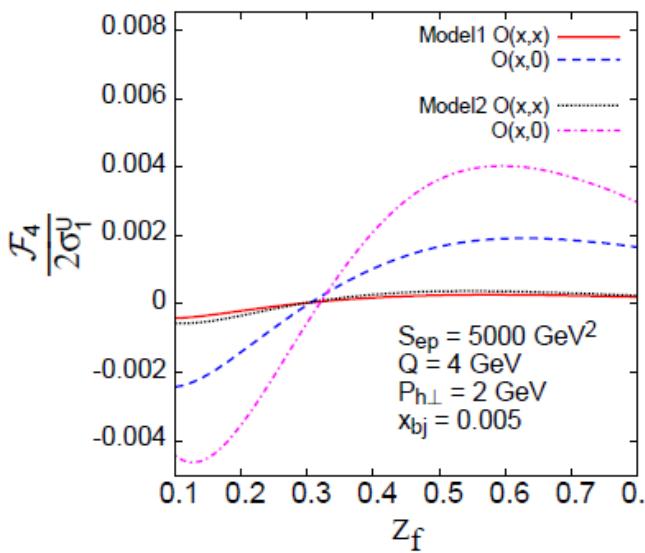
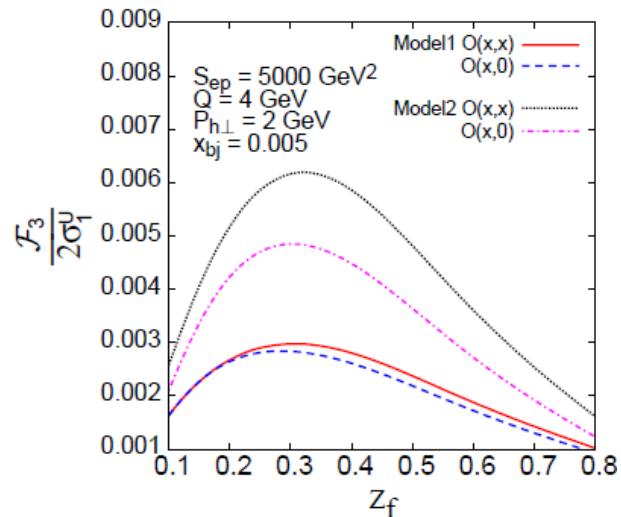
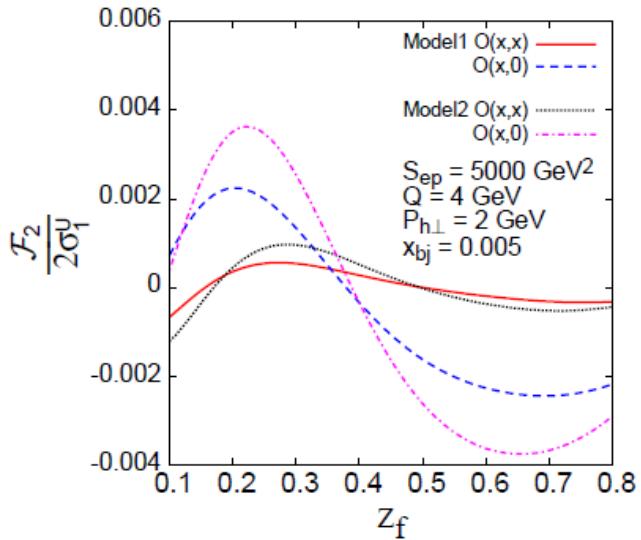
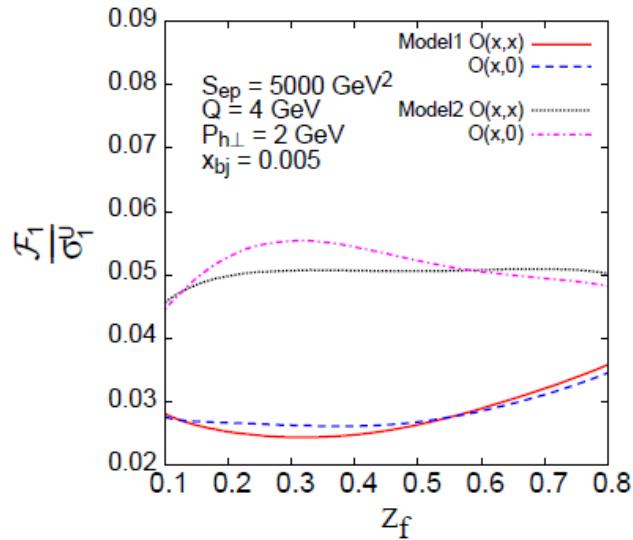
$D_c(z)$: Kneesch-Kniehl-Kramer-Schienbein08

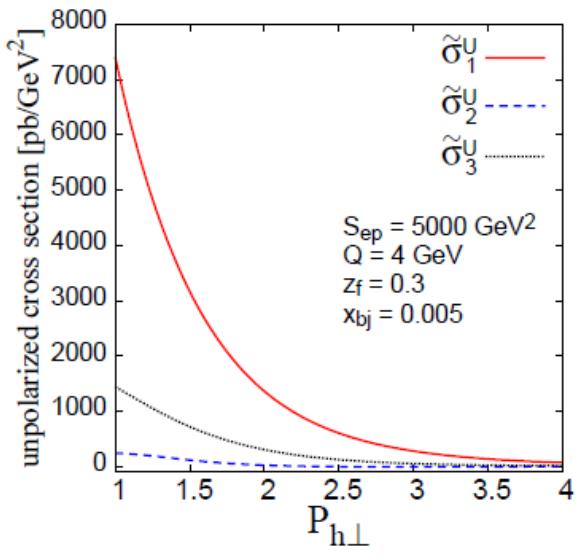
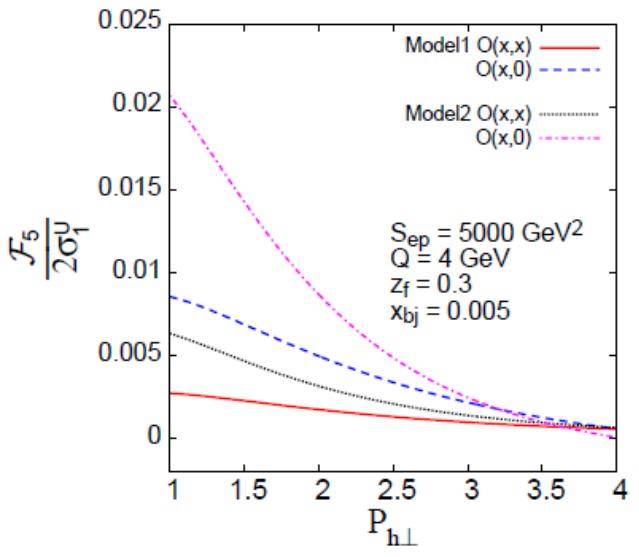
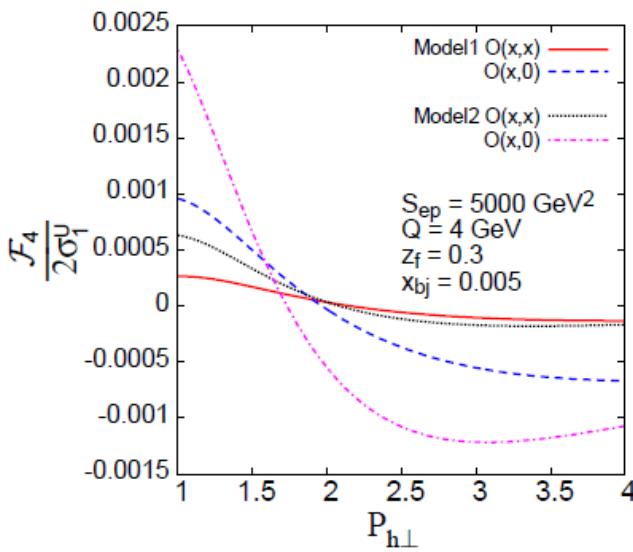
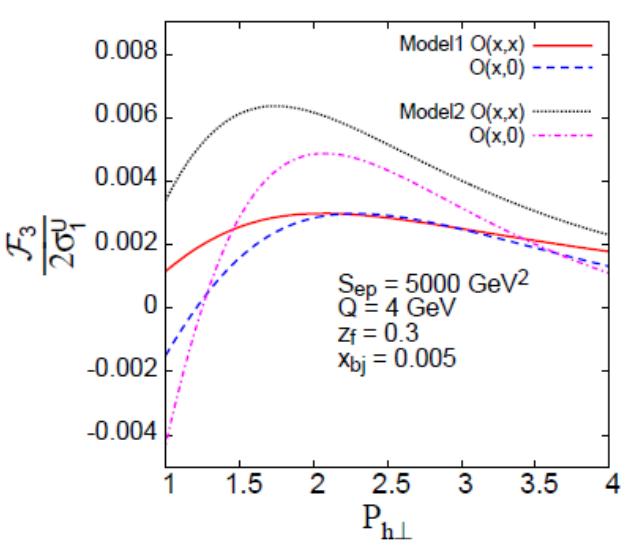
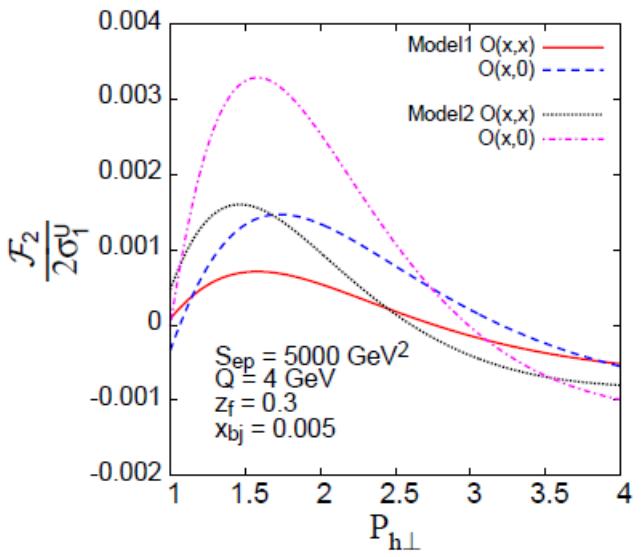
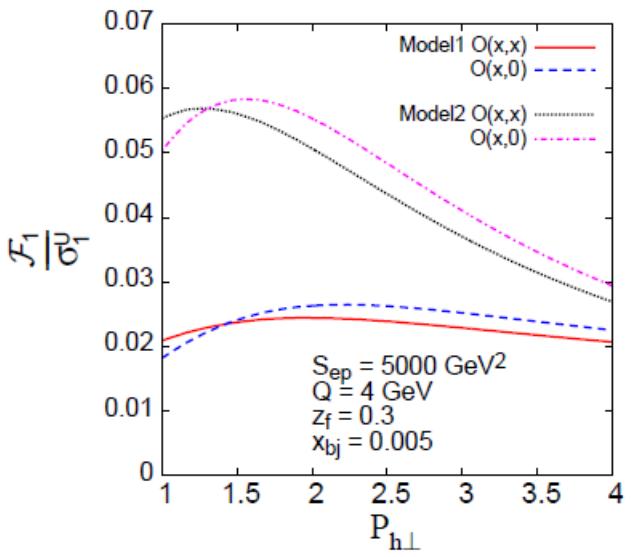
guided by
 $pp^\uparrow \rightarrow DX$
at RHIC
Koike, Yoshida,
PRD84 ('11)
014026



$$\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dx_{bj} dy dz_f dP_{h\perp}^2 d\phi_h} = \tilde{\sigma}_1^U + \tilde{\sigma}_2^U \cos \phi_h + \tilde{\sigma}_3^U \cos 2\phi_h$$



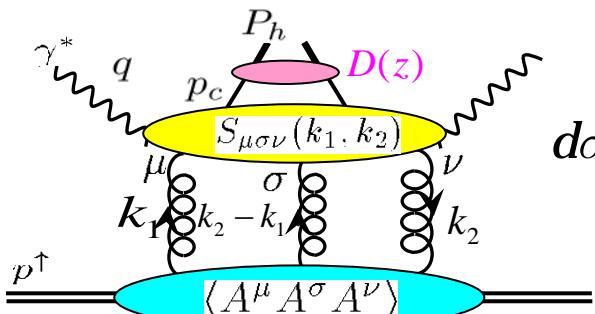




twist-3 SSA for $ep^\uparrow \rightarrow eDX$

Beppu, Koike, K.T., Yoshida,
PRD82 ('10) 054005

$$\begin{aligned} \frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\dots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c,\bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta\hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) \Delta\hat{\sigma}_k^1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) \Delta\hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta\hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta\hat{\sigma}_k^4 \right\} \right] \\ &= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h] \end{aligned}$$



$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \mathcal{S}_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \Bigg|_{k_i=x_i p} \otimes \begin{Bmatrix} N_{v\mu\sigma}(x_1, x_2) \\ O_{v\mu\sigma}(x_1, x_2) \end{Bmatrix} \otimes D(z)$$

$$n^\mu = \left(0, n^-, \mathbf{0}_\perp\right) \quad p \cdot n = p^+ n^- = 1$$

$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \left\langle p \textcolor{violet}{S}_\perp \left| F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) \right| p \textcolor{violet}{S}_\perp \right\rangle = \frac{1}{24} i \textcolor{blue}{f}^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} \textcolor{blue}{d}^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

$$i \int \frac{d\lambda}{2\pi} \frac{d\zeta}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \left\langle p \textcolor{magenta}{S}_\perp \right| F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) \left| p \textcolor{magenta}{S}_\perp \right\rangle \quad D_\perp^\sigma = \partial_\perp^\sigma - ig A_\perp^\sigma$$

$$\begin{aligned} i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p~\textcolor{violet}{S}_{\perp}\left|F_{\textcolor{violet}{a}}^{\mu n}(0)\textcolor{red}{g}F_{\textcolor{red}{c}}^{\sigma n}(\zeta n)F_{\textcolor{blue}{b}}^{\nu n}(\lambda n)\right|p~\textcolor{violet}{S}_{\perp}\right\rangle \\ =\frac{1}{24}i\textcolor{violet}{f}^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\frac{3}{40}\textcolor{blue}{d}^{abc}O^{\nu\mu\sigma}(x_1,x_2) \end{aligned}$$

$$\begin{aligned} -i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p~\textcolor{violet}{S}_{\perp}\left|F^{\mu n}(0)\textcolor{red}{D}_{\perp}^{\sigma}(\zeta n)F^{\nu n}(\lambda n)\right|p~\textcolor{violet}{S}_{\perp}\right\rangle \\ =\textcolor{blue}{M}^{\nu\mu\sigma}(x_1,x_2) \end{aligned}$$

$$i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\textcolor{violet}{S}_\perp\Big|F_{\textcolor{blue}{a}}^{\mu n}(0)\textcolor{red}{g} F_{\textcolor{brown}{c}}^{\sigma n}(\zeta n)F_{\textcolor{violet}{b}}^{\nu n}(\lambda n)\Big|p\textcolor{violet}{S}_\perp\right\rangle\\=\boxed{\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)}+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$-i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\textcolor{violet}{S}_\perp\Big|F^{\mu n}(0)\textcolor{red}{D}_\perp^\sigma(\zeta n)F^{\nu n}(\lambda n)\Big|p\textcolor{violet}{S}_\perp\right\rangle\\=M^{\nu\mu\sigma}(x_1,x_2)$$

$$i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\textcolor{violet}{S}_{\perp}\Big|F_a^{\mu n}(0)\textcolor{red}{g} F_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)\Big|p\textcolor{violet}{S}_{\perp}\right\rangle\\=\boxed{\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)}+\frac{3}{40}d^{abc}O^{\nu\mu\sigma}(x_1,x_2)$$

$$N^{\nu\mu\sigma}(x_1,x_2)=2iM_N\Big[g_{\perp}^{\nu\mu}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\sigma p n\alpha}N(x_1,x_2)-g_{\perp}^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu p n\alpha}N(x_2,x_2-x_1)-g_{\perp}^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu p n\alpha}N(x_1,x_1-x_2)\Big]$$

$$-i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p\textcolor{violet}{S}_{\perp}\Big|F^{\mu n}(0)\textcolor{red}{D}_{\perp}^{\sigma}(\zeta n)F^{\nu n}(\lambda n)\Big|p\textcolor{violet}{S}_{\perp}\right\rangle\\=M^{\nu\mu\sigma}(x_1,x_2)$$

$$M^{\nu\mu\sigma}(x_1,x_2)=2iM_N\Big[g_{\perp}^{\nu\mu}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\sigma p n\alpha}M_1(x_1,x_2)+g_{\perp}^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu p n\alpha}M_2(x_1,x_2)-g_{\perp}^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu p n\alpha}M_2(x_2,x_1)\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle\, p~S\,|\,F^{\mu n}(0)F^{\nu n}(\lambda n)\,|\, p~S\rangle = -\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu}+\Delta G(x)i\epsilon^{\mu\nu pn}\big(\textcolor{blue}{S}\!\cdot\!\textcolor{violet}{n}\big)M_{_N}+2\textcolor{red}{G_T}(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}M_{_N}+\cdots\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle\, p~S\,|\,\overline{\psi}(0)\gamma^\sigma\gamma_5\psi(\lambda n)\,|\, p~S\rangle = 2M_{_N}\Big[\Delta g(x)\big(\textcolor{blue}{S}\!\cdot\!\textcolor{violet}{n}\big)p^\sigma+\textcolor{red}{g}_T(x)S_{\perp}^\sigma+\cdots\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle \textcolor{violet}{p}~S\,|\,F^{\mu n}(0)F^{\nu n}(\lambda n)\,|\,\textcolor{violet}{p}~S\rangle\!=\!-\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu}+\Delta G(x)i\epsilon^{\mu\nu pn}\left(S\!\cdot\!\textcolor{violet}{n}\right)\textcolor{violet}{M}_N+2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}M_N+\cdots\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle \textcolor{violet}{p}~S\,|\,\overline{\psi}(0)\gamma^\sigma\gamma_5\psi(\lambda n)\,|\,\textcolor{violet}{p}~S\rangle=2M_N\Big[\Delta g(x)\left(S\!\cdot\!\textcolor{violet}{n}\right)p^\sigma+\textcolor{red}{g}_T(x)S_\perp^\sigma+\cdots\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle \textcolor{violet}{p}~\textcolor{violet}{S}_\perp\,|\,F^{\mu+}(0)F^{-+}(\lambda n)\,|\,\textcolor{violet}{p}~\textcolor{violet}{S}_\perp\rangle=x\mathcal{G}_T(x)i\epsilon^{\mu-+\alpha}S_{\perp\alpha}M_N$$

$$F^{-+}=-\partial^+A^-=-\frac{1}{\partial_-}\Big(D_{\perp j}F^{j+}+g\overline{\psi}t^a\gamma^+\psi t^a\Big)\qquad\qquad D_\nu F^{\mu\nu}=g\overline{\psi}t^a\gamma^\mu\psi t^a$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle \textcolor{violet}{p}\;S\,|\,F^{\mu n}(0)F^{\nu n}(\lambda n)\,|\,\textcolor{violet}{p}\;S\rangle\!=\!-\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu}+\Delta G(x)i\epsilon^{\mu\nu pn}\left(S\!\cdot\!\textcolor{violet}{n}\right)\boldsymbol{M}_{\mathcal{N}}+2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}\boldsymbol{M}_{\mathcal{N}}+\cdots\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle \textcolor{violet}{p}\;S\,|\,\overline{\psi}(0)\gamma^\sigma\gamma_5\psi(\lambda n)\,|\,\textcolor{violet}{p}\;S\rangle=2\boldsymbol{M}_{\mathcal{N}}\Big[\Delta g(x)\left(S\!\cdot\!\textcolor{violet}{n}\right)p^\sigma+\textcolor{red}{g}_T(x)\textcolor{violet}{S}_{\perp}^\sigma+\cdots\Big]$$

$$\int\!\frac{d\lambda}{2\pi}e^{i\lambda x}\langle \textcolor{violet}{p}\;\textcolor{violet}{S}_{\perp}\,|\,F^{\mu+}(0)F^{-+}(\lambda n)\,|\,\textcolor{violet}{p}\;\textcolor{violet}{S}_{\perp}\rangle=x\mathcal{G}_T(x)i\epsilon^{\mu-+\alpha}S_{\perp\alpha}\boldsymbol{M}_{\mathcal{N}}$$

$$F^{-+}=-\partial^+A^-=-\frac{1}{\partial_-}\Big(D_{\perp j}F^{j+}+g\overline{\psi}t^a\gamma^+\psi t^a\Big)\qquad\qquad\qquad D_\nu F^{\mu\nu}=g\overline{\psi}t^a\gamma^\mu\psi t^a$$

$$-i\int\!\frac{d\lambda}{2\pi}\!\int\!\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle \textcolor{violet}{p}\;\textcolor{violet}{S}_{\perp}\Big|F^{\mu n}(0)\textcolor{red}{D}_{\perp}^\sigma(\zeta n)F^{\nu n}(\lambda n)\Big|\textcolor{violet}{p}\;\textcolor{violet}{S}_{\perp}\right\rangle=\textcolor{blue}{M}^{\nu\mu\sigma}(x_1,x_2)$$

$$\textcolor{blue}{M}^{\nu\mu\sigma}(x_1,x_2)=2i\boldsymbol{M}_{\mathcal{N}}\Big[\textcolor{violet}{g}_{\perp}^{\nu\mu}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\sigma p n\alpha}M_1(x_1,x_2)+\textcolor{violet}{g}_{\perp}^{\mu\sigma}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\nu p n\alpha}M_2(x_1,x_2)-\textcolor{violet}{g}_{\perp}^{\sigma\nu}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\mu p n\alpha}M_2(x_2,x_1)\Big]$$

$$\int\!\frac{d\lambda}{2\pi}\!\int\!\frac{d\mu}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle \textcolor{violet}{p}\;\textcolor{violet}{S}_{\perp}\Big|\overline{\psi}(0)\gamma^\mu gF^{\alpha n}(\mu n)\psi(\lambda n)\Big|\textcolor{violet}{p}\;\textcolor{violet}{S}_{\perp}\right\rangle=\boldsymbol{M}_{\mathcal{N}}\textcolor{violet}{p}^\mu\epsilon^{\alpha p n\textcolor{violet}{S}_{\perp}}G_F(x_1,x_2)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p \ S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p \ S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i \epsilon^{\mu\nu p n} (\textcolor{blue}{S} \cdot \textcolor{violet}{n}) M_N + 2 \mathcal{G}_T(x) i \epsilon^{\mu\nu \alpha n} \textcolor{violet}{S}_{\perp \alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p \ S | \bar{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | p \ S \rangle = 2M_N \left[\Delta g(x) (\textcolor{blue}{S} \cdot \textcolor{violet}{n}) p^\sigma + \textcolor{red}{g}_T(x) \textcolor{violet}{S}_{\perp}^\sigma + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p \ \textcolor{violet}{S}_{\perp} | F^{\mu+}(0) F^{-+}(\lambda n) | p \ \textcolor{violet}{S}_{\perp} \rangle = x \mathcal{G}_T(x) i \epsilon^{\mu-+\alpha} \textcolor{violet}{S}_{\perp \alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-} \left(D_{\perp j} F^{j+} + g \bar{\psi} t^a \gamma^+ \psi t^a \right) \qquad \qquad D_\nu F^{\mu\nu} = g \bar{\psi} t^a \gamma^\mu \psi t^a$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p \ \textcolor{violet}{S}_{\perp} \Big| F^{\mu n}(0) \textcolor{red}{D}_{\perp}^\sigma(\zeta n) F^{\nu n}(\lambda n) \Big| p \ \textcolor{violet}{S}_{\perp} \right\rangle = \textcolor{blue}{M}^{\nu\mu\sigma}(x_1,x_2)$$

$$\textcolor{blue}{M}^{\nu\mu\sigma}(x_1,x_2) = 2iM_N \left[\textcolor{violet}{g}_{\perp}^{\nu\mu} \textcolor{violet}{S}_{\perp \alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1,x_2) + \textcolor{violet}{g}_{\perp}^{\mu\sigma} \textcolor{violet}{S}_{\perp \alpha} \varepsilon^{\nu p n \alpha} M_2(x_1,x_2) - \textcolor{violet}{g}_{\perp}^{\sigma\nu} \textcolor{violet}{S}_{\perp \alpha} \varepsilon^{\mu p n \alpha} M_2(x_2,x_1) \right]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p \ \textcolor{violet}{S}_{\perp} \Big| \bar{\psi}(0) \gamma^\mu g F^{\alpha n}(\mu n) \psi(\lambda n) \Big| p \ \textcolor{violet}{S}_{\perp} \right\rangle = M_N p^\mu \epsilon^{\alpha p n \textcolor{violet}{S}_{\perp}} G_F(x_1,x_2)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x,x') + G_F(x'-x,x') \right)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p~S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p~S\rangle = -\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn}\left(S\!\cdot\! n\right)M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}M_N + \cdots \Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p~S|\bar{\psi}(0)\gamma^\sigma\gamma_5\psi(\lambda n)|p~S\rangle = 2M_N\Big[\Delta g(x)\left(S\!\cdot\! n\right)p^\sigma + g_T(x)S_\perp^\sigma + \cdots\Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p~S_\perp|F^{\mu+}(0)F^{-+}(\lambda n)|p~S_\perp\rangle = x\mathcal{G}_T(x)i\epsilon^{\mu-+\alpha}S_{\perp\alpha}M_N$$

$$F^{-+}=-\partial^+A^-=-\frac{1}{\partial_-}\Big(D_{\perp j}F^{j+}+g\bar{\psi}t^a\gamma^+\psi t^a\Big)\qquad\qquad D_\nu F^{\mu\nu}=g\bar{\psi}t^a\gamma^\mu\psi t^a$$

$$-i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p~S_\perp\Big|F^{\mu n}(0)\textcolor{red}{D}_\perp^\sigma(\zeta n)F^{\nu n}(\lambda n)\Big|p~S_\perp\right\rangle=\textcolor{blue}{M}^{\nu\mu\sigma}(x_1,x_2)$$

$$M^{\nu\mu\sigma}(x_1,x_2)=2iM_N\Big[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma p n\alpha}M_1(x_1,x_2)+g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu p n\alpha}M_2(x_1,x_2)-g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu p n\alpha}M_2(x_2,x_1)\Big]$$

$$\int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p~S_\perp\Big|\bar{\psi}(0)\gamma^\mu gF^{\alpha n}(\mu n)\psi(\lambda n)\Big|p~S_\perp\right\rangle=M_Np^\mu\epsilon^{\alpha p n}S_\perp G_F(x_1,x_2)$$

$$\mathcal{G}_T(x)=-\frac{1}{2x^2}\int dx'\big(8M_2(x,x')-4M_2(x',x)-4M_1(x',x)+G_F(x'+x,x')+G_F(x'-x,x')\big)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle p~S\,|\,F^{\mu n}(0)F^{\nu n}(\lambda n)\,|\,p~S\rangle = -\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu}+\textcolor{blue}{\Delta G(x)i\epsilon^{\mu\nu pn}\left(S\!\cdot\! n\right)M_N}+2\textcolor{red}{\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}}\textcolor{magenta}{S_{\perp\alpha}M_N}+\cdots\Big]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p|S\rangle = -\frac{x}{2} \left[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn} (\textcolor{blue}{S}\cdot\textcolor{violet}{n})M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^\nu \frac{\partial}{\partial z^\nu} F^{\mu z}(0) \tilde{F}_\mu^\rho(z) - z^\nu \frac{\partial}{\partial z_\rho} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_\mu^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_\mu^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_\mu^z(uz) \right. \\ & \quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_\mu^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_\mu^z(uz) \right. \\ & \quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^\rho D_\lambda F^{\lambda \sigma}(uz) + F^{\mu z}(0) \tilde{D}_\mu \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^\mu \tilde{F}_\mu^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^\rho D_\lambda F^{\lambda \sigma}(z) + F^{\mu z}(0) \tilde{D}_\mu \tilde{F}^{\rho z}(z) \end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p|S\rangle = -\frac{x}{2} \left[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn} (\textcolor{blue}{S}\cdot\mathbf{n})M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^\nu \frac{\partial}{\partial z^\nu} F^{\mu z}(0) \tilde{F}_\mu^\rho(z) - z^\nu \frac{\partial}{\partial z_\rho} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_\mu^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_\mu^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_\mu^z(uz) \right. \\ & \quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_\mu^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_\mu^z(uz) \right. \\ & \quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^\rho D_\lambda F^{\lambda \sigma}(uz) + F^{\mu z}(0) \tilde{D}_\mu \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^\mu \tilde{F}_\mu^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^\rho D_\lambda F^{\lambda \sigma}(z) + F^{\mu z}(0) \tilde{D}_\mu \tilde{F}^{\rho z}(z) \end{aligned}$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p|S_\perp|F_a^{\mu n}(0) \textcolor{red}{g} F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n)|p|S_\perp\right\rangle = \frac{1}{24} i \textcolor{violet}{f}^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \dots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2i M_N \left[\textcolor{violet}{g}_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} N(x_1, x_1 - x_2) \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p|S\rangle = -\frac{x}{2}\Big[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn}\left(S\cdot n\right)M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}M_N + \cdots \Big]$$

$$\begin{aligned}&z^\nu\frac{\partial}{\partial z^\nu}F^{\mu z}(0)\tilde{F}_{\mu}^{\;\rho}(z)-z^\nu\frac{\partial}{\partial z_\rho}F^{\mu z}(0)\tilde{F}_{\mu\nu}(z)\qquad\qquad z^2\rightarrow 0\\&=i\int_0^1dtF^{\mu z}(0)tgF^{\rho z}(tz)\tilde{F}_{\mu}^{\;z}(z)-i\int_0^1dtF^{\mu z}(0)gF_{\mu}^{\;z}(tz)\tilde{F}^{\rho z}(z)+\int_0^1du\Big(i\int_0^1dtF^{\mu z}(0)guF^{\rho z}(tuz)\tilde{F}_{\mu}^{\;z}(uz)\\&-i\int_0^1dtF^{\mu z}(0)guF_{\mu}^{\;z}(tuz)\tilde{F}^{\rho z}(uz)-i\int_0^1dtF^{\rho z}(0)guF^{\mu z}(tuz)\tilde{F}_{\mu}^{\;z}(uz)\\&-F^{\mu z}(0)\epsilon_{\mu z\sigma}^{\;\rho}D_{\lambda}F^{\lambda\sigma}(uz)+F^{\mu z}(0)\tilde{D}_{\mu}\tilde{F}^{\rho z}(uz)+F^{\rho z}(0)D^{\mu}\tilde{F}_{\mu}^{\;z}(uz)\Big)-F^{\mu z}(0)\epsilon_{\mu z\sigma}^{\;\rho}D_{\lambda}F^{\lambda\sigma}(z)+F^{\mu z}(0)\tilde{D}_{\mu}\tilde{F}^{\rho z}(z)\end{aligned}$$

$$\begin{aligned}&i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p|\textcolor{violet}{S}_{\perp}\left|F_a^{\mu n}(0)\textcolor{red}{g}F_c^{\sigma n}(\zeta n)F_b^{\nu n}(\lambda n)\right|p|\textcolor{violet}{S}_{\perp}\right\rangle=\frac{1}{24}if^{abc}N^{\nu\mu\sigma}(x_1,x_2)+\cdots\\&N^{\nu\mu\sigma}(x_1,x_2)=2iM_N\left[g_{\perp}^{\nu\mu}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\sigma pn\alpha}N(x_1,x_2)-g_{\perp}^{\mu\sigma}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\nu pn\alpha}N(x_2,x_2-x_1)-g_{\perp}^{\sigma\nu}\textcolor{violet}{S}_{\perp\alpha}\varepsilon^{\mu pn\alpha}N(x_1,x_1-x_2)\right]\\&D_\nu F^{\mu\nu}=g\overline{\psi}t^a\gamma^\mu\psi t^a\qquad\qquad\qquad D_\mu\tilde{F}^{\mu\nu}=0\\&\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\left\langle p|\textcolor{violet}{S}_{\perp}\left|\overline{\psi}(0)\gamma^\mu gF^{\alpha n}(\mu n)\psi(\lambda n)\right|p|\textcolor{violet}{S}_{\perp}\right\rangle=M_Np^\mu\epsilon^{\alpha pn}\textcolor{violet}{S}_{\perp}G_F(x_1,x_2)\end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p \ S \ | \ F^{\mu n}(0)F^{\nu n}(\lambda n) \ | \ p \ S \rangle = -\frac{x}{2} \left[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn} (\textcolor{blue}{S}\cdot\mathbf{n})M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n}S_{\perp\alpha}M_N + \dots \right]$$

$$\begin{aligned} & z^\nu \frac{\partial}{\partial z^\nu} F^{\mu z}(0) \tilde{F}_\mu^\rho(z) - z^\nu \frac{\partial}{\partial z_\rho} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_\mu^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_\mu^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_\mu^z(uz) \right. \\ & \quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_\mu^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_\mu^z(uz) \right. \\ & \quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^\rho D_\lambda F^{\lambda \sigma}(uz) + F^{\mu z}(0) \tilde{D}_\mu \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^\mu \tilde{F}_\mu^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^\rho D_\lambda F^{\lambda \sigma}(z) + F^{\mu z}(0) \tilde{D}_\mu \tilde{F}^{\rho z}(z) \end{aligned}$$

$$\begin{aligned} i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p \ \textcolor{violet}{S}_\perp \left| F_a^{\mu n}(0) \textcolor{red}{g} F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) \right| p \ \textcolor{violet}{S}_\perp \right\rangle &= \frac{1}{24} i \textcolor{violet}{f}^{abc} N^{\nu \mu \sigma}(x_1, x_2) + \dots \\ N^{\nu \mu \sigma}(x_1, x_2) &= 2i M_N \left[\textcolor{violet}{g}_\perp^{\nu \mu} \textcolor{violet}{S}_{\perp \alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - \textcolor{violet}{g}_\perp^{\mu \sigma} \textcolor{violet}{S}_{\perp \alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - \textcolor{violet}{g}_\perp^{\sigma \nu} \textcolor{violet}{S}_{\perp \alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right] \\ D_\nu F^{\mu \nu} &= g \bar{\psi} t^a \gamma^\mu \psi t^a \quad \quad \quad \textcolor{blue}{D}_\mu \tilde{F}^{\mu \nu} = 0 \\ \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \left\langle p \ \textcolor{violet}{S}_\perp \left| \bar{\psi}(0) \gamma^\mu g F^{\alpha n}(\mu n) \psi(\lambda n) \right| p \ \textcolor{violet}{S}_\perp \right\rangle &= M_N p^\mu \epsilon^{\alpha p n \textcolor{violet}{S}_\perp} G_F(x_1, x_2) \end{aligned}$$

$$\begin{aligned} 2x^2 \frac{\partial}{\partial x} \mathcal{G}_T(x) + x \Delta G(x) &= \int dx' P \frac{1}{x-x'} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) (2N(x, x-x') - 2N(x', x'-x)) \\ &\quad + \int dx' P \frac{1}{x-x'} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) (4N(x, x') - 6N(x', x'-x) - 6N(x, x-x')) \\ &\quad + \int dx' P \frac{1}{x-x'} P \frac{1}{x} (8N(x', x'-x) + 8N(x, x-x')) + \int dx' \left(P \frac{1}{x} - \frac{\partial}{\partial x} \right) (G_F(x'+x, x') + G_F(x'-x, x')) \end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p|S\rangle = -\frac{x}{2} \left[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn} (\textcolor{magenta}{S}\cdot\textcolor{violet}{n}) M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x,x') + G_F(x'-x,x') \right)$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\ &\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &\quad + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\ &\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p|S\rangle = -\frac{x}{2} \left[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn} (\textcolor{blue}{S}\cdot\textcolor{violet}{n})M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x,x') + G_F(x'-x,x') \right)$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \quad \epsilon(x) = \frac{x}{|x|} \\ &- \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &+ P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\ &\left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1} \quad \text{Hatta, K.T., Yoshida, JHEP1302 ('13) 003}$$

$$\begin{aligned} M_2(x_1, x_2) &= \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) x_1^2 \left[\int_{x_1}^{\epsilon(x_1)} dx' \int dx'' \frac{1}{x'^3} \left\{ P \frac{2x' - x''}{(x' - x'')^2} (N(x', x' - x'') \right. \right. \\ &\left. \left. - N(x'', x'' - x')) + P \frac{1}{x' - x''} (2N(x', x'') - N(x'', x'' - x') - N(x', x' - x'')) \right\} \right. \\ &\left. - \frac{1}{2} \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^3} (G_F(x'' + x', x'') + G_F(x'' - x', x'')) - \frac{1}{4} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \right] \end{aligned}$$

$$\left\langle p \textcolor{blue}{S}_\perp \left| \overline{\psi}(0) \textcolor{red}{g} F^{\mu+}(\zeta n) \psi(\lambda n) \right| p \textcolor{blue}{S}_\perp \right\rangle = \frac{M_{_N}}{4} \not{p} \textcolor{blue}{S}_{\perp \alpha} p_\beta \varepsilon^{\alpha \beta \mu +} \textcolor{magenta}{G}_{\textcolor{violet}{F}} + i \frac{M_{_N}}{4} \gamma_5 \not{p} \textcolor{blue}{S}_\perp^\mu \widetilde{\textcolor{violet}{G}}_{\textcolor{violet}{F}}$$

$$\textcolor{red}{D}_\perp^\mu(\zeta n)\qquad\qquad\qquad \textcolor{violet}{G}_{\textcolor{violet}{D}}\qquad\qquad\qquad \widetilde{\textcolor{violet}{G}}_{\textcolor{violet}{D}}$$

$$G_D(x_1,x_2) ~=~ P\frac{1}{x_1-x_2}G_F(x_1,x_2),$$

$$\widetilde{G}_D(x_1,x_2) ~=~ \delta(x_1-x_2)\widetilde{g}(x_1)+P\frac{1}{x_1-x_2}\widetilde{G}_F(x_1,x_2)$$

$$\widetilde{g}(x)=-x\int_x^{\epsilon(x)}dx_1\left[\frac{2\Delta q(x_1)}{x_1}+\frac{1}{x_1^2}P\int_{-1}^1dx_2\left\{\frac{G_F(x_1,x_2)}{x_1-x_2}+(3x_1-x_2)\,\frac{\widetilde{G}_F(x_1,x_2)}{(x_1-x_2)^2}\right\}\right]$$

$$\textcolor{brown}{Eguchi},\,\textcolor{brown}{Koike},\,\textcolor{brown}{K.T.},\,\textcolor{brown}{NPB752}\,(06)\,1$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|S|F^{\mu n}(0)F^{\nu n}(\lambda n)|p|S\rangle = -\frac{x}{2} \left[G(x)g_{\perp}^{\mu\nu} + \Delta G(x)i\epsilon^{\mu\nu pn} (\textcolor{blue}{S}\cdot\textcolor{violet}{n})M_N + 2\mathcal{G}_T(x)i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x,x') - 4M_2(x',x) - 4M_1(x',x) + G_F(x'+x,x') + G_F(x'-x,x') \right)$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \quad \epsilon(x) = \frac{x}{|x|} \\ &- \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &+ P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\ &\left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1} \quad \text{Hatta, K.T., Yoshida, JHEP1302 ('13) 003}$$

$$\begin{aligned} M_2(x_1, x_2) &= \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) x_1^2 \left[\int_{x_1}^{\epsilon(x_1)} dx' \int dx'' \frac{1}{x'^3} \left\{ P \frac{2x' - x''}{(x' - x'')^2} (N(x', x' - x'') \right. \right. \\ &\left. \left. - N(x'', x'' - x')) + P \frac{1}{x' - x''} (2N(x', x'') - N(x'', x'' - x') - N(x', x' - x'')) \right\} \right. \\ &\left. - \frac{1}{2} \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^3} (G_F(x'' + x', x'') + G_F(x'' - x', x'')) - \frac{1}{4} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \right] \end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p~S \, | \, F^{\mu n}(0)F^{\nu n}(\lambda n) \, | \, p~S \rangle = -\frac{x}{2} \Big[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} \left(S \cdot \textcolor{violet}{n} \right) M_N + 2 \mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \cdots \Big]$$

$$\begin{aligned}\mathcal{G}_T(x)=&\frac{1}{2}\int_x^{\epsilon(x)}dx'\frac{\Delta G(x')}{x'}\\&-\int_x^{\epsilon(x)}dx'\int dx''\frac{1}{2x'^2}\Bigg[P\frac{1}{x'-x''}(\frac{\partial}{\partial x'}-\frac{\partial}{\partial x''})(2N(x',x'-x'')-2N(x'',x''-x'))\\&+P\frac{1}{x'-x''}(\frac{\partial}{\partial x'}+\frac{\partial}{\partial x''})(4N(x',x'')-6N(x'',x''-x')-6N(x',x'-x''))\\&+P\frac{1}{x'-x''}P\frac{1}{x'}(8N(x'',x''-x')+8N(x',x'-x''))+\Bigg(P\frac{1}{x'}-\frac{\partial}{\partial x'}\Bigg)(G_F(x''+x',x'')+G_F(x''-x',x''))\Bigg]\end{aligned}$$

$$\begin{aligned}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p~S \, | \, \bar{\psi}(0)\gamma^\sigma\gamma_5\psi(\lambda n) \, | \, p~S \rangle = &2M_N \Big[\Delta g(x) \left(S \cdot \textcolor{violet}{n} \right) p^\sigma + g_T(x) S_\perp^\sigma + \cdots \Big] \\ g_T(x)=&\int_x^{\epsilon(x)}\frac{dx_1}{x_1}\Bigg[\Delta q(x_1)+\frac{1}{2}P\int_{-1}^1dx_2\,\frac{1}{x_1-x_2}\Bigg\{\Bigg(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\Bigg)G_F(x_1,x_2)+\Bigg(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\Bigg)\tilde{G}_F(x_1,x_2)\Bigg\}\Bigg]\end{aligned}$$

WW

genuine twist-3

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p | S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p | S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i \epsilon^{\mu\nu p n} (\textcolor{blue}{S} \cdot \textcolor{violet}{n}) M_N + 2 \mathcal{G}_T(x) i \epsilon^{\mu\nu\alpha n} \textcolor{violet}{S}_{\perp\alpha} M_N + \dots \right]$$

$$\mathcal{G}_T(x) = \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \quad \text{WW}$$

$$\begin{aligned} & - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ & + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\ & \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned} \quad \text{genuine twist-3}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p | S | \bar{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | p | S \rangle = 2M_N \left[\Delta g(x) (\textcolor{blue}{S} \cdot \textcolor{violet}{n}) p^\sigma + g_T(x) \textcolor{violet}{S}_{\perp}^\sigma + \dots \right]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1} \left[\Delta q(x_1) + \frac{1}{2} P \int_{-1}^1 dx_2 \frac{1}{x_1 - x_2} \left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \tilde{G}_F(x_1, x_2) \right\} \right]$$

WW

genuine twist-3

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p | S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p | S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i \epsilon^{\mu\nu p n} (\textcolor{blue}{S \cdot n}) M_N + 2 \mathcal{G}_T(x) i \epsilon^{\mu\nu\alpha n} \textcolor{magenta}{S}_{\perp\alpha} M_N + \dots \right]$$

$$\mathcal{G}_T(x) = \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \quad \text{WW}$$

$$\begin{aligned} & - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ & + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\ & \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned} \quad \text{genuine twist-3}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p | S | \bar{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | p | S \rangle = 2M_N \left[\Delta g(x) (\textcolor{blue}{S \cdot n}) p^\sigma + g_T(x) \textcolor{magenta}{S}_{\perp}^\sigma + \dots \right]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1} \left[\Delta q(x_1) + \frac{1}{2} P \int_{-1}^1 dx_2 \frac{1}{x_1 - x_2} \left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \tilde{G}_F(x_1, x_2) \right\} \right]$$

WW

genuine twist-3

$$\int dx \mathbf{g}_T(x) = \int dx \Delta q(x) \equiv \Delta q \quad \int_{-1}^1 dx \mathcal{G}_T(x) = \int_0^1 dx \Delta G(x) = a_0 \equiv \Delta G$$

$$\langle \textcolor{blue}{p}~\textcolor{violet}{S}_\perp | -\!\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) \,|\, p~\textcolor{violet}{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma}\! \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\begin{aligned} \langle p\;S_{\perp}\;|-\int d\lambda\epsilon(\lambda)F^{\beta n}(0)F^{\alpha n}(\lambda n)\;|\;p\;S_{\perp}\rangle &= 2\epsilon^{n\alpha\beta\sigma}S_{\perp\sigma}\int dx\mathcal{G}_T(x)=2\epsilon^{n\alpha\beta\sigma}S_{\perp\sigma}\Delta G \\ \sim \langle p\;S_{\perp}\;|\;\mathcal{M}_{g\text{-spin}}^{+\alpha\beta}\;|\;p\;S_{\perp}\rangle\qquad\qquad\qquad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} &= F^{\mu\beta}A^{\alpha}-F^{\mu\alpha}A^{\beta} \\ A^{\alpha}(0) &= \frac{1}{2}\int d\lambda\epsilon(\lambda)F^{\alpha n}(\lambda n) \end{aligned}$$

$$\begin{aligned}
& \langle p | S_{\perp} | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p | S_{\perp} \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G \\
& \sim \langle p | S_{\perp} | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p | S_{\perp} \rangle \quad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^{\alpha} - F^{\mu\alpha} A^{\beta} \\
& \frac{1}{2} = L + \frac{1}{2} \Delta \Sigma + \Delta G \quad \text{transverse spin sum rule} \quad A^{\alpha}(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)
\end{aligned}$$

$$\langle p | S_{\perp} | - \int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p | S_{\perp} \rangle = 2 \epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2 \epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p | S_{\perp} | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p | S_{\perp} \rangle \quad \quad \quad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^{\alpha} - F^{\mu\alpha} A^{\beta}$$

$$\frac{1}{2} = L + \frac{1}{2} \Delta \Sigma + \Delta G \quad \quad \quad A^{\alpha}(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

transverse spin sum rule
 $L \stackrel{?}{=} L_q + L_g$

$$\langle p\; \textcolor{blue}{S}_\perp\; | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n)\; |\; p\; \textcolor{blue}{S}_\perp\rangle = 2\epsilon^{n\alpha\beta\sigma} \textcolor{blue}{S}_{\perp\sigma}\int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} \textcolor{blue}{S}_{\perp\sigma}\Delta G$$

$$\sim \langle p\; \textcolor{blue}{S}_\perp\; | \; \mathcal{M}_{g\text{-spin}}^{+\alpha\beta}\; |\; p\; \textcolor{blue}{S}_\perp\rangle \qquad \qquad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta}=F^{\mu\beta}A^\alpha-F^{\mu\alpha}A^\beta$$

$$\frac{1}{2}=L+\frac{1}{2}\Delta\Sigma+\Delta G \qquad \textbf{transverse spin sum rule}$$

$$L \stackrel{?}{=} L_q + L_g$$

$$J_{_q}+J_{_g}=\frac{1}{2}$$

$$W^\mu=-\frac{1}{2}\epsilon^\mu_{~\nu\rho\sigma}p^\nu\int d^3x\mathcal{M}^{+\rho\sigma}=W^\mu_q+W^\mu_g$$

$$\frac{\langle p\; \textcolor{blue}{S}_\perp\; | W^j_{q,g}\; |\; p\; \textcolor{blue}{S}_\perp\rangle}{2p^+(2\pi)^3\delta^3(0)}\equiv J_{q,g}\textcolor{blue}{S}_\perp^j$$

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})$$

$$\textcolor{brown}{Ji,~Xiong,~Yuan,~PLB717~('12)~214}$$

$$\mathcal{M}_{q,g}^{\lambda\mu\nu}=x^\mu T_{q,g}^{\lambda\nu}-x^\nu T_{q,g}^{\lambda\mu} \qquad \qquad \overline{p}^\mu=\frac{p^\mu+p'^\mu}{2} \quad \Delta^\mu=p'^\mu-p^\mu$$

$$\langle p'\, S' | T_{q,g}^{\mu\nu} | \, p\; S\rangle = \overline{u}(p',S')\Big[A_{q,g}\gamma^{(\mu}\overline{p}^{\nu)}+B_{q,g}\,\frac{\overline{p}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}+C_{q,g}\,\frac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{M}+\overline{C}_{q,g} Mg^{\mu\nu}\Big]u(p,S)$$

$$\langle p\; \textcolor{blue}{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | \; p\; \textcolor{blue}{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} \textcolor{blue}{S}_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} \textcolor{blue}{S}_{\perp\sigma} \Delta G$$

$$\sim \langle p\; \textcolor{blue}{S}_\perp | \; \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | \; p\; \textcolor{blue}{S}_\perp \rangle$$

$$\mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta}=F^{\mu\beta}A^\alpha-F^{\mu\alpha}A^\beta$$

$$\frac{1}{2}=L+\frac{1}{2}\Delta\Sigma+\Delta G$$

$$\textcolor{red}{?}$$

$$L \doteq \overset{\bullet}{L_q} + L_g$$

$$\textbf{transverse spin sum rule}$$

$$J_{_q}+J_{_g}=\frac{1}{2}$$

$$W^\mu=-\frac{1}{2}\epsilon^\mu_{~\nu\rho\sigma}p^\nu\int d^3x\mathcal{M}^{+\rho\sigma}=W^\mu_q+W^\mu_g$$

$$\frac{\langle p\; \textcolor{blue}{S}_\perp | W^j_{q,g} | \; p\; \textcolor{blue}{S}_\perp \rangle}{2p^+(2\pi)^3\delta^3(0)}\equiv J_{q,g}\textcolor{blue}{S}_\perp^j$$

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})$$

$$\textcolor{brown}{Ji,~Xiong,~Yuan,~PLB717~('12)~214}$$

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})\textcolor{red}{+\frac{p^3}{2(p^0+M_N)}\bar{C}_{q,g}}\quad \textcolor{brown}{Hatta,~K.T.,~Yoshida,~JHEP1302~('13)~003}$$

$$\mathcal{M}_{q,g}^{\lambda\mu\nu}=x^\mu T_{q,g}^{\lambda\nu}-x^\nu T_{q,g}^{\lambda\mu}$$

$$\overline{p}^\mu=\frac{p^\mu+p'^\mu}{2}\quad \Delta^\mu=p'^\mu-p^\mu$$

$$\langle p'\; S' | T_{q,g}^{\mu\nu} | \; p\; S \rangle = \overline{u}(p',S')\Big[A_{q,g}\gamma^{(\mu}\overline{p}^{\nu)}+B_{q,g}\frac{\overline{p}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}+C_{q,g}\frac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{M}+\overline{C}_{q,g}Mg^{\mu\nu}\Big]u(p,S)$$

$$\langle p \textcolor{blue}{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p \textcolor{blue}{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p \textcolor{blue}{S}_\perp | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p \textcolor{blue}{S}_\perp \rangle$$

$$\mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

$$\frac{1}{2}=L+\frac{1}{2}\Delta\Sigma+\Delta G$$

transverse spin sum rule

$$L \stackrel{?}{=} L_q + L_g$$

$$A^\alpha(0)=\frac{1}{2}\int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

$$J_q+J_g=\frac{1}{2}$$

$$W^\mu=-\frac{1}{2}\epsilon^\mu_{~\nu\rho\sigma}p^\nu\int d^3x\mathcal{M}^{+\rho\sigma}=W_q^\mu+W_g^\mu$$

$$\frac{\langle p \textcolor{blue}{S}_\perp | W_{q,g}^j | p \textcolor{blue}{S}_\perp \rangle}{2p^+(2\pi)^3\delta^3(0)} \equiv J_{q,g} \textcolor{blue}{S}_\perp^j$$

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})$$

Ji, Xiong, Yuan, PLB717 ('12) 214

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})+\frac{p^3}{2(p^0+M_N)}\bar{C}_{q,g} \quad \text{Hatta, K.T., Yoshida, JHEP1302 ('13) 003}$$

$$\mathcal{M}_{q,g}^{\lambda\mu\nu}=x^\mu T_{q,g}^{\lambda\nu}-x^\nu T_{q,g}^{\lambda\mu}$$

$$\bar{p}^\mu=\frac{p^\mu+p'^\mu}{2} \quad \Delta^\mu=p'^\mu-p^\mu$$

$$\langle p' S' | T_{q,g}^{\mu\nu} | p S \rangle = \bar{u}(p', S') \left[A_{q,g} \gamma^{(\mu} \bar{p}^{\nu)} + B_{q,g} \frac{\bar{p}^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + C_{q,g} \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g} M g^{\mu\nu} \right] u(p, S)$$

$$L=L_q+L_g \quad \left(\frac{1}{2}=J_q+J_g \right)$$

frame dependent!

Summary Twist-three gluonic correlations in $| p \ S_\perp \rangle$ single- and double-spin asymm.

SSA in SIDIS $ep^\uparrow \rightarrow eDX$ at $P_{h\perp} \gg \Lambda_{\text{QCD}}$

Twist-3 mechanism from three-gluon correlation inside the nucleon
photon-gluon fusion

Factorization formula for twist-3 SSA

convoluted with F-type correl. fns.

$$\left\{ \begin{array}{l} N(x,x), O(x,x), \frac{dN(x,x)}{dx}, \frac{dO(x,x)}{dx} \\ N(x,0), O(x,0), \frac{dN(x,0)}{dx}, \frac{dO(x,0)}{dx} \end{array} \right.$$

5 different azimuthal dependences

Numerical estimates of A_N : $\langle 1 \rangle$, $\langle \sin 2\phi \rangle$ ~ % level

good chance to access multi-gluon effects

Exact relation between $\langle p \ S_\perp | F^{+\perp} F^{+\perp} F^{+\perp} | p \ S_\perp \rangle$ & $\langle p \ S_\perp | F^{+\perp} D^\perp F^{+\perp} | p \ S_\perp \rangle$

$$g_T(x) = \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} + [\text{genuine tw.3}] \quad g_T(x) = \int_x^{\epsilon(x)} \frac{dx'}{x'} \Delta q(x') + [\text{genuine tw.3}]$$

$$\int_{-1}^1 dx \mathcal{G}_T(x) = \int_0^1 dx \Delta G(x) = a_0 \equiv \Delta G, \quad \int dx g_T(x) = \int dx \Delta q(x) \equiv \Delta q \quad \Rightarrow \text{transverse spin SR}$$

