MPI@LHC, CERN
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Angular modulations in double parton scattering

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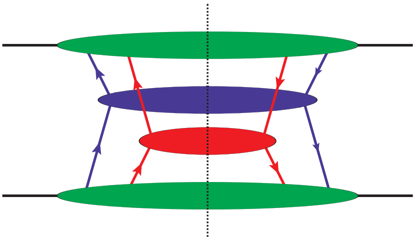
Work in collaboration with Markus Diehl

arXiv:1210.5434

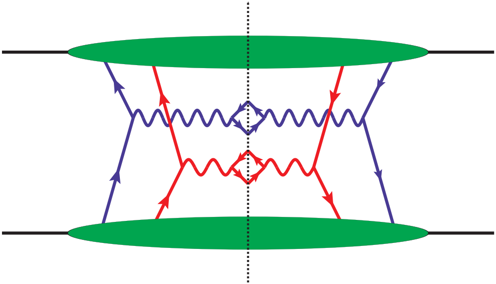


What is Double-Parton Scattering?

- Process where two partons (in each proton) interact in two separate hard interactions

Why study Double-Parton Scattering?

- Proton structure
- Background to other signals
- Multiparton interactions and Monte Carlos
- No systematic theoretical treatment

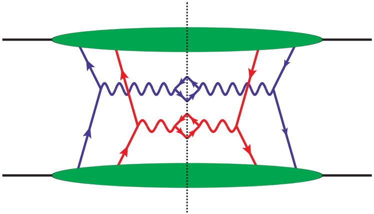


- Simple assumption of uncorrelated partons ⇒ predictive and easy
- Interferences in color, flavor, fermion number and spin correlations
- How does correlations affect the predictions from uncorrelated description?
- Study correlations between the hard interactions
- Double Drell-Yan type of process ($W^{\pm}$, Z, γ^*)

(J. R. Gaunt, C.H. Kom, A. Kulesza, W.J. Stirling, 2010, 2011; M. Myska, 2011)

Double-parton Cross section

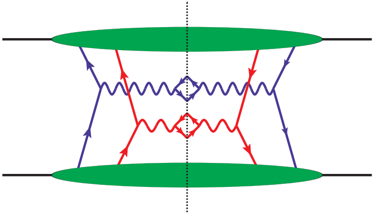
- Double-parton cross section without spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



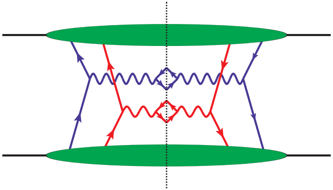
$$d\sigma \sim \hat{\sigma}_{q_1\bar{q}_1}(Q_1^2)\hat{\sigma}_{q_2\bar{q}_2}(Q_2^2) \int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$
$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2} \int d^2\boldsymbol{y} F_{q_1q_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) F_{\bar{q}_1\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$

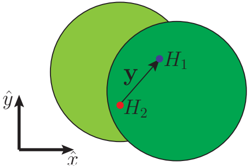
Double-parton Cross section

- Double-parton cross section with spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



Double Parton Distributions





- Double Parton Distributions $F_{q_1 q_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$
- $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ - transverse position arguments
  Fourier conjugate to (average) transverse parton momentum
- $|\boldsymbol{y}|$ - essentially distance between hard interactions

DDY & Spin - Parton Distributions

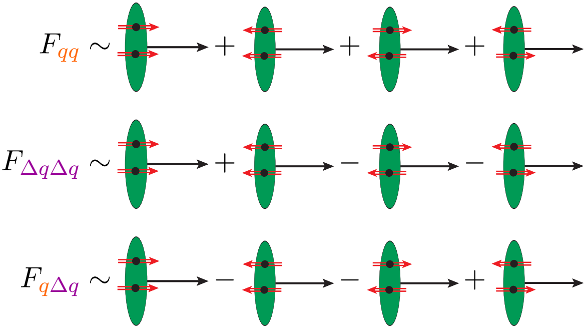
- The spin of the quarks from the same proton can be correlated
- Correlation described by parton distributions

$$F_{q_1 q_2} \sim \langle p|(\bar{q}_1 \Gamma_1 q_1)(\bar{q}_2 \Gamma_2 q_2)|p\rangle$$

- $\Gamma_{1/2} = \Gamma_q, \ \Gamma_{\Delta q}, \ \Gamma^j_{\delta q}$ projection operators for
  - Unpolarized quarks (q)
  - Longitudinally polarized quarks (Δq)
  - Transversely polarized quarks (δq)
- Transverse index $j = 1, 2$ corresponds to the transverse spin-vector

Double Parton Distributions as helicity eigenstates

- Unpolarized and longitudinally polarized quarks



- Transverse polarization $(F^{i}_{q\delta q}, F^{i}_{\Delta q\delta q}, F^{ij}_{\delta q\delta q})$ - helicity interference

- Unpolarized and longitudinally polarized quarks (f's scalar g's pseudo-scalar functions)

$$
\begin{aligned}
F_{qq} &= f_{qq}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) \\
F_{\Delta q \Delta q} &= f_{\Delta q \Delta q} \\
F_{q \Delta q} &= g_{q \Delta q}
\end{aligned}
$$

- Singly transversely polarized quarks ($\tilde{y}^i = y^j \epsilon^{ij}$)

$$
\begin{aligned}
F^i_{\Delta q \delta q} &= M \left(y^i f_{\Delta q \delta q} + \tilde{y}^i g_{\Delta q \delta q} \right) \\
F^i_{q \delta q} &= M \left(\tilde{y}^i f_{q \delta q} + y^i g_{q \delta q} \right)
\end{aligned}
$$

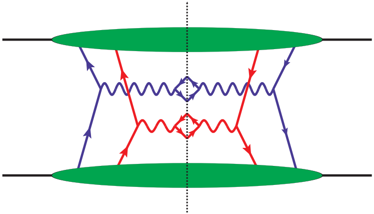
- Doubly transversely polarized quarks

$$
\begin{aligned}
F^{ij}_{\delta q \delta q} &= \delta^{ij} f_{\delta q \delta q} + \left(2 y^i y^j - y^2 \delta^{ij} \right) M^2 f^t_{\delta q \delta q} \\
&\quad + \left(y^i \tilde{y}^j + \tilde{y}^i y^j \right) M^2 g^s_{\delta q \delta q} + \left(y^i \tilde{y}^j - \tilde{y}^i y^j \right) M^2 g^a_{\delta q \delta q}
\end{aligned}
$$

(M. Diehl, D. Ostermeier, A. Schäfer, 2011)

Double-parton Cross section

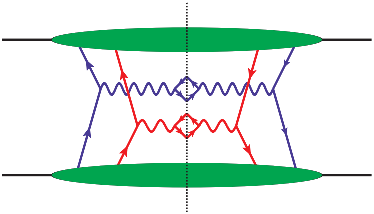
- Double-parton cross section without spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



$$d\sigma \sim \hat{\sigma}_{q_1\bar{q}_1}(Q_1^2)\hat{\sigma}_{q_2\bar{q}_2}(Q_2^2) \int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$
$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2} \int d^2\boldsymbol{y} F_{q_1q_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) F_{\bar{q}_1\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$

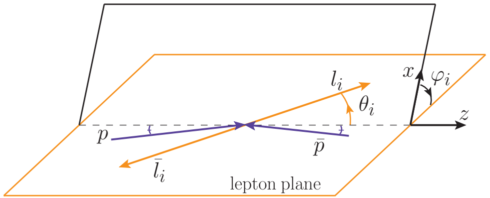
Double-parton Cross section

- Double-parton cross section with spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



$$d\sigma \sim \sum_{\substack{a_1,a_2=\{q,\Delta q,\delta q\} \\ \bar{a}_1,\bar{a}_2=\{\bar{q},\Delta\bar{q},\delta\bar{q}\}}} \hat{\sigma}^{ik}_{a_1\bar{a}_1}(Q_1^2)\hat{\sigma}^{jl}_{a_2\bar{a}_2}(Q_2^2)\int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$

$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2}\int d^2\boldsymbol{y} F^{ij}_{a_1a_2}(x_1,x_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y}) F^{kl}_{\bar{a}_1\bar{a}_2}(\bar{x}_1,\bar{x}_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})$$



- Angles defined in rest frames of bosons
- $\hat{z}$ parallel with proton momenta, for $\boldsymbol{q}_i = 0$
- $\hat{x}$ arbitrary transverse reference axis (e.g. towards center of LHC ring)
- θ_1 and θ_2 polar angles of leptons
- φ_1 and φ_2 azimuthal angles of leptons
- φ_y angle to direction between the two collisions

- Double Drell-Yan cross section,
- Low enough Q to neglect Z boson, i.e. γ^* only
- Unpolarized and longitudinally polarized quarks

(one quark flavor, no interference)

$$d\sigma^{(0)} \sim \prod_{i=1}^{2} \int d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_i\boldsymbol{q}_i} \int d^2\boldsymbol{y} \sum_{\substack{a_1\bar{a}_1=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\}\\ a_2\bar{a}_2=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\}}}$$

$$\times K_{a_1\bar{a}_1}(1+\cos^2\theta_1)\, K_{a_2\bar{a}_2}(1+\cos^2\theta_2)\, F_{a_1a_2}\bar{F}_{\bar{a}_1\bar{a}_2}$$

- K, K' coupling factors (depend of boson virtualities)
- Longitudinal polarization $\Rightarrow$ changes magnitude of cross section

- Double Drell-Yan cross section ($W^{\pm}$, γ^*/Z), unpolarized and longitudinally polarized quarks

(one quark flavor, no interference)

$$d\sigma^{(0)} \sim \prod_{i=1}^{2} \int d^2 \boldsymbol{z}_i e^{-i\boldsymbol{z}_i \boldsymbol{q}_i} \int d^2 \boldsymbol{y} \sum_{\substack{a_1\bar{a}_1=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\} \\ a_2\bar{a}_2=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\}}}$$

$$\times \left[K_{a_1\bar{a}_1}(1+\cos^2\theta_1) + 2K'_{a_1\bar{a}_1}\cos\theta_1\right]$$

$$\times \left[K_{a_2\bar{a}_2}(1+\cos^2\theta_2) + 2K'_{a_2\bar{a}_2}\cos\theta_2\right]\left[F_{a_1a_2}\bar{F}_{\bar{a}_1\bar{a}_2}\right]$$

- K, K' coupling factors (depend of boson virtualities)
- Longitudinal polarization $\Rightarrow$ change rate and angular distribution
- W-bosons, $K' = \pm K$, no transverse polarization

- Double transverse polarization

$$
\begin{aligned}
d\sigma^{(2)} \sim & \prod_{i=1}^{2} \int d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_i\boldsymbol{q}_i} \int d^2\boldsymbol{y} \sin^2\theta_1 \sin^2\theta_2 \\
\times \Big\{ & \Big[A\cos 2(\varphi_1-\varphi_2) - B\sin 2(\varphi_1-\varphi_2) \Big] (f\bar{f} - \boldsymbol{y}^4 M^4 g^a \bar{g}^a) \\
& + \Big[C\cos 2(\varphi_1+\varphi_2-2\varphi_y) - D\sin 2(\varphi_1+\varphi_2-2\varphi_y) \Big] \boldsymbol{y}^4 M^4 (f^t \bar{f}^t - g^s \bar{g}^s) \\
& + \Big[A\sin 2(\varphi_1-\varphi_2) + B\cos 2(\varphi_1-\varphi_2) \Big] \boldsymbol{y}^2 M^2 (f\bar{g}^a + g^a \bar{f}) \\
& - \Big[C\sin 2(\varphi_1+\varphi_2-2\varphi_y) + D\cos 2(\varphi_1+\varphi_2-2\varphi_y) \Big] \boldsymbol{y}^4 M^4 (f^t \bar{g}^s + g^s \bar{f}^t) \Big\}
\end{aligned}
$$

- Transverse dependence:
  - Azimuthal angle between the two leptons
  - Azimuthal angles between leptons and $\boldsymbol{y}$
- $\int d^2\boldsymbol{y}$: $\varphi_y \rightarrow$ angle to linear combination of boson momenta

- Single transverse polarization

$$d\sigma^{(1)} \sim \prod_{i=1}^{2} \int d^2 \boldsymbol{z}_i e^{-i\boldsymbol{z}_i \boldsymbol{q}_i} \int d^2 \boldsymbol{y}$$

$$\times \Big\{ \left[K_{q_1\bar{q}_1}(1+\cos^2\theta_1) + 2K'_{q_1\bar{q}_1}\cos\theta_1\right]$$

$$\times \sin^2\theta_2 \left[K_{\delta q_2 \delta\bar{q}_2}\cos(2(\varphi_2-\varphi_y)) + K'_{\delta q_2\delta\bar{q}_2}\sin(2(\varphi_2-\varphi_y))\right]$$

$$\times \left[f_{q_1\delta q}\bar{f}_{\bar{q}_1\delta\bar{q}} - g_{q_1\delta q}\bar{g}_{\bar{q}_1\delta\bar{q}} - f_{\Delta q_1\delta q}\bar{f}_{\Delta\bar{q}_1\delta\bar{q}} + g_{\Delta q_1\delta q}\bar{g}_{\Delta\bar{q}_1\delta\bar{q}}\right] + \ldots \Big\}$$

- Spin vector breaks z-axis rotation symmetry
- Transverse correlation between lepton plane and vector between hard interactions
- $\int d^2\boldsymbol{y}$: $\varphi_y \rightarrow$ angle to linear combination of boson momenta

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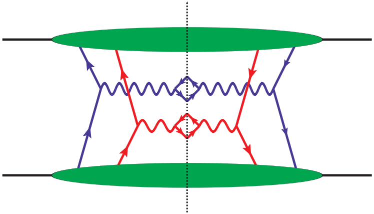
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



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$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2} \int d^2\boldsymbol{y} F^{ij}_{a_1a_2}(x_1,x_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y}) F^{kl}_{\bar{a}_1\bar{a}_2}(\bar{x}_1,\bar{x}_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})$$

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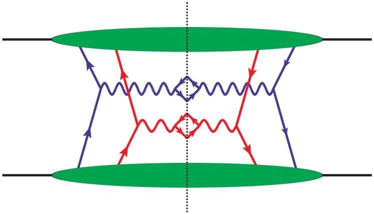

- Integrated over transverse boson momenta:

$$d\sigma \sim \sum_{\substack{a_1,a_2=\{q,\Delta q,\delta q\} \\ \bar{a}_1,\bar{a}_2=\{\bar{q},\Delta\bar{q},\delta\bar{q}\}}} \hat{\sigma}^{ik}_{a_1\bar{a}_1}(Q_1^2)\hat{\sigma}^{jl}_{a_2\bar{a}_2}(Q_2^2)$$

$$\times \int d^2\boldsymbol{y} F^{ij}_{a_1a_2}(x_1,x_2,\boldsymbol{y}) F^{kl}_{\bar{a}_1\bar{a}_2}(\bar{x}_1,\bar{x}_2,\boldsymbol{y})$$

- Reduces number of double parton distributions (parity and time reversal)
  - Unpolarized and longitudinally polarized quarks

$$F_{qq} = f_{qq}(x_1, x_2, \boldsymbol{y})$$
$$F_{\Delta q \Delta q} = f_{\Delta q \Delta q}$$
$$F_{q \Delta q} = g_{q \Delta q}$$

  - Singly transversely polarized quarks ($\tilde{y}^i = y^j \epsilon^{ij}$)

$$F^i_{\Delta q \delta q} = M \left(y^i f_{\Delta q \delta q} + \tilde{y}^i g_{\Delta q \delta q} \right)$$
$$F^i_{q \delta q} = M \left(\tilde{y}^i f_{q \delta q} + y^i g_{q \delta q} \right)$$

  - Doubly transversely polarized quarks

$$F^{ij}_{\delta q \delta q} = \delta^{ij} f_{\delta q \delta q} + \left(2 y^i y^j - y^2 \delta^{ij} \right) M^2 f^t_{\delta q \delta q} + \left(y^i \tilde{y}^j + \tilde{y}^i y^j \right) M^2 g^s_{\delta q \delta q} + \left(y^i \tilde{y}^j - \tilde{y}^i y^j \right) M^2 g^a_{\delta q \delta q}$$

- Longitudinal polarization still change both magnitude and angular modulation
- $d\sigma^{(1)} = 0$, $\boldsymbol{y}$ integral gives zero due to azimuthal dependence on φ_y.
- Azimuthal correlations remain

$$d\sigma^{(2)} \sim \sin^2\theta_1 \sin^2\theta_2 \Big[A\cos 2(\varphi_1 - \varphi_2) - B\sin 2(\varphi_1 - \varphi_2) \Big] \times \int d^2\boldsymbol{y}\, f_{\delta q_1 \delta q_2}(x_1, x_2, \boldsymbol{y}) \bar{f}_{\delta\bar{q}_1 \delta\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

(transversely polarized quarks in both interactions)

- Dependence on angles between final state particles implies dependence on invariant mass of particle pairs

Positivity bounds

- Integrated ('collinear') distributions, $f_{qq}(x_1, x_2, \boldsymbol{y})$
- Six different distributions for each combination of quark flavors
- Many distributions, poorly known
- Find constraints from general principles?
- Can be done! Thanks to probabilistic interpretation
- Project out helicity eigenstates

$$F_{q_1 q_2} \sim \langle p|(\bar{q}_1 \Gamma_1 q_1)(\bar{q}_2 \Gamma_2 q_2)|p\rangle$$

Γ_{++}, Γ_{--} for plus and minus helicity quarks and Γ_{+-}, Γ_{-+} for plus/minus interference

- Helicity matrix

$$\begin{pmatrix} f_{qq} + f_{\Delta q \Delta q} & -i|y|M f_{\delta q q} & -i|y|M f_{q \delta q} & 2y^2 M^2 f^t_{\delta q \delta q} \\ i|y|M f_{\delta q q} & f_{qq} - f_{\Delta q \Delta q} & 2 f_{\delta q \delta q} & -i|y|M f_{q \delta q} \\ i|y|M f_{q \delta q} & 2 f_{\delta q \delta q} & f_{qq} - f_{\Delta q \Delta q} & -i|y|M f_{\delta q q} \\ 2y^2 M^2 f^t_{\delta q \delta q} & i|y|M f_{q \delta q} & i|y|M f_{\delta q q} & f_{qq} + f_{\Delta q \Delta q} \end{pmatrix}$$

- Columns (rows) correspond to different quark helicities in the (conjugate) amplitudex
  - $f_{qq} + f_{\Delta q \Delta q}$: $q^+ q^+$ in amplitude, $q^+ q^+$ in conjugate amplitude
  - $2y^2 M^2 f^t_{\delta q \delta q}$ $q^- q^-$ in amplitude, $q^+ q^+$ in conjugate amplitude
- Probabilistic interpretation of arbitrary helicity state ⇒ Matrix positive semi-definite
- Can derive positivity bounds, similar to single parton distributions

(A. Bacchetta, M. Boglione, P.J. Mulders, 1999; M. Diehl, Ph. Hägler, 2005)

- Same for pure gluon and mixed quark-gluon distributions
- After rotations, put into common form

$$M = \frac{1}{4}\begin{pmatrix} f_{ab} + f_{\Delta a \Delta b} & h_{\delta a b} & h_{a \delta b} & -2h^t_{\delta a \delta b} \\ h_{\delta a b} & f_{ab} - f_{\Delta a \Delta b} & 2h_{\delta a \delta b} & h_{a \delta b} \\ h_{a \delta b} & 2h_{\delta a \delta b} & f_{ab} - f_{\Delta a \Delta b} & h_{\delta a b} \\ -2h^t_{\delta a \delta b} & h_{a \delta b} & h_{\delta a b} & f_{ab} + f_{\Delta a \Delta b} \end{pmatrix}$$

- Diagonal elements give trivial bounds

$$f_{ab} \pm f_{\Delta a \Delta b} \geq 0.$$

- Principal minors of the two dimensional subspaces

$$f_{ab} + f_{\Delta a \Delta b} \geq 2|h^t_{\delta a \delta b}|$$
$$f_{ab} - f_{\Delta a \Delta b} \geq 2|h_{\delta a \delta b}|$$
$${f_{ab}}^2 \geq (f_{ab} + f_{\Delta a \Delta b})(f_{ab} - f_{\Delta a \Delta b}) \geq h^2_{\delta a b}$$
$${f_{ab}}^2 \geq (f_{ab} + f_{\Delta a \Delta b})(f_{ab} - f_{\Delta a \Delta b}) \geq h^2_{a \delta b}.$$

- Same for pure gluon and mixed quark-gluon distributions
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$$M = \frac{1}{4}\begin{pmatrix} f_{ab} + f_{\Delta a \Delta b} & h_{\delta a b} & h_{a \delta b} & -2h^t_{\delta a \delta b} \\ h_{\delta a b} & f_{ab} - f_{\Delta a \Delta b} & 2h_{\delta a \delta b} & h_{a \delta b} \\ h_{a \delta b} & 2h_{\delta a \delta b} & f_{ab} - f_{\Delta a \Delta b} & h_{\delta a b} \\ -2h^t_{\delta a \delta b} & h_{a \delta b} & h_{\delta a b} & f_{ab} + f_{\Delta a \Delta b} \end{pmatrix}$$

- Eigenvalues give most stringent bounds

$$f_{ab} + h_{\delta a \delta b} - {h_{\delta a \delta b}}^t \pm \sqrt{(h_{\delta a b} + h_{a \delta b})^2 + (f_{\Delta a \Delta b} - h_{\delta a \delta b} - h^t_{\delta a \delta b})^2} \geq 0$$

$$f_{ab} - h_{\delta a \delta b} + {h_{\delta a \delta b}}^t \pm \sqrt{(h_{\delta a b} - h_{a \delta b})^2 + (f_{\Delta a \Delta b} + h_{\delta a \delta b} + {h_{\delta a \delta b}}^t)^2} \geq 0$$

- Bounds can aid construction of double parton distributions
- Stable under (homogeneous) leading order evolution
- Sets an upper limit on the size of spin correlations

Summary

- Spin correlations change magnitude and angular distribution
- Transversely polarized quarks ⇒ dependence on azimuthal angles:
  - between directions of leptons
  - between directions of leptons and transverse boson momenta
- Azimuthal correlations also in collinear cross section
- Should hold also for quark initiated jets
- Dependence on angles between final state particles implies dependence on invariant mass of particle pairs
- We have derived positivity bounds on distributions
- Bounds are stable under leading order evolution and set upper limits on spin correlations between partons in the proton
- Need measurements to help determine which correlations are important