Recent developments on diffraction in PYTHIA8

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MPI@LHC 2012, Workshop on Multi-Parton Interactions at the LHC,

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Outlook

- **(Reminder:) Update of PYTHIA8 wrt. PYTHIA6**

  same diffractive cross sections,

  extended framework for hadronization of diffractive states

- **Implementation of MBR (Minimum-Bias Rockefeller) model**
  - diffractive cross sections “from CDF”

    Central Diffraction (Double-Pomeron Exchange) simulated in PYTHIA for the first time in

    PYTHIA8.165

  - test of PYTHIA8 framework for diffractive hadronization

    - comparison to a data-driven MBR hadronization model

- **Implementation of Central Diffraction for all diffractive models in**

  PYTHIA8.170

Main processes contributing to the total *pp* cross section

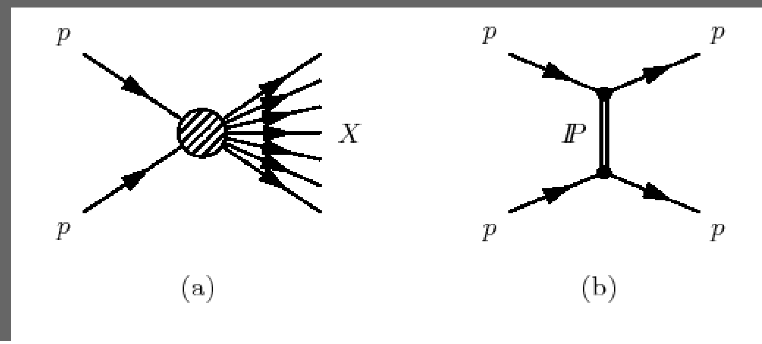
Non-diffractive:

pp → X

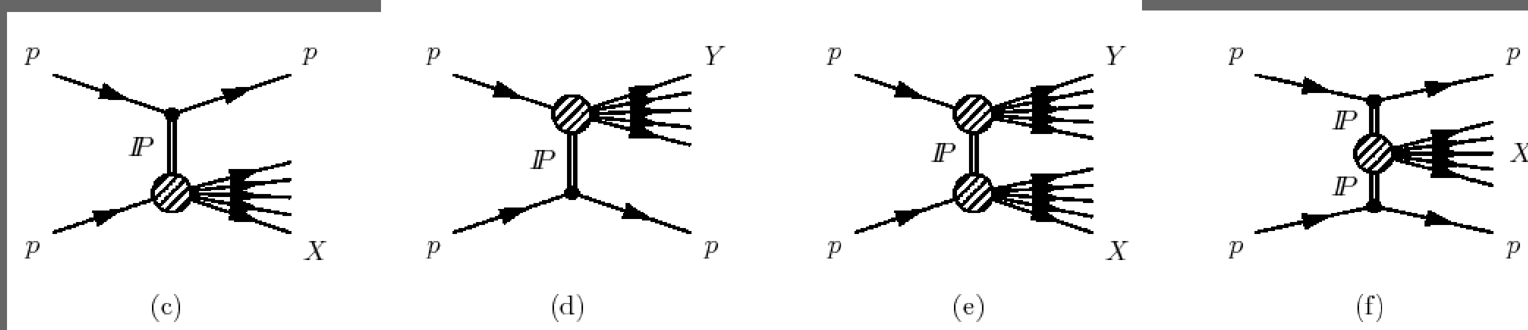
(exponentially-suppressed rapidity gap)

:Elastic

pp → pp



Diffractive:



Single dissociation (SD),

pp → Xp , pp → pY

Double dissociation (DD),

pp → XY

Central dissociation (CD)

pp → pXp

or double-Pomeron exchange (DPE)

(a) and (c)-(f) contribute to the total-inelastic cross section

Diffraction in PYTHIA 6

Diffractive Cross Section Formulæ:

$$\frac{d\sigma_{sd(AX)}(s)}{dt\,dM^2} = \frac{g_{3\mathbb{P}}}{16\pi}\,\beta_{A\mathbb{P}}^2\,\beta_{B\mathbb{P}}\,\frac{1}{M^2}\,\exp(B_{sd(AX)}t)\,F_{sd}\,,$$

$$\frac{d\sigma_{dd}(s)}{dt\,dM_1^2\,dM_2^2} = \frac{g_{3\mathbb{P}}^2}{16\pi}\,\beta_{A\mathbb{P}}\,\beta_{B\mathbb{P}}\,\frac{1}{M_1^2}\,\frac{1}{M_2^2}\,\exp(B_{dd}t)\,F_{dd}\,.$$

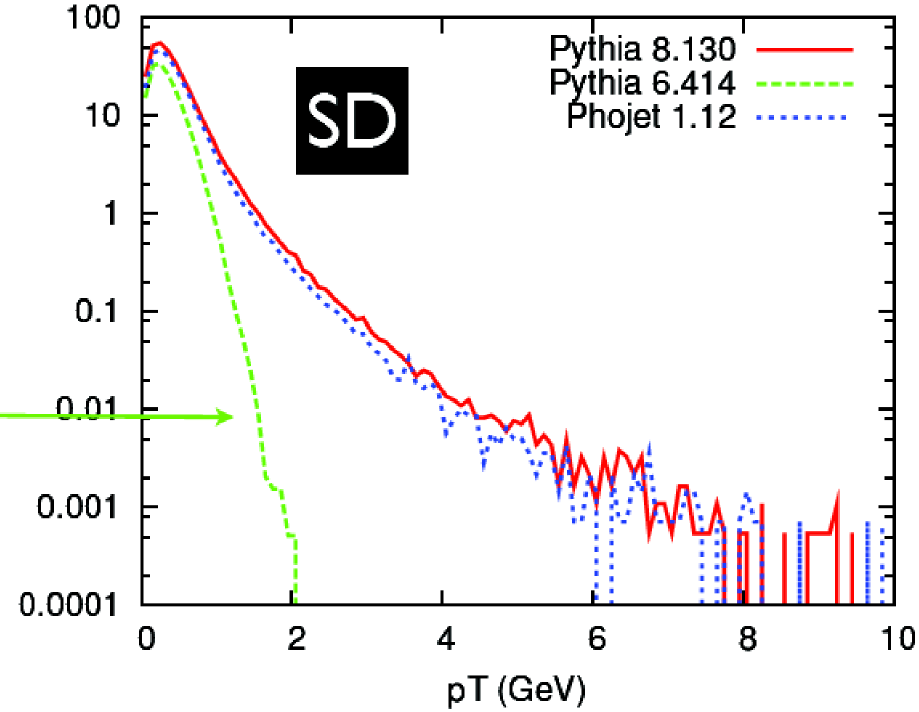
Spectra:

$2\,m_{pi} < M_D < 1$ GeV: 2-body decay

$M_D > 1$ GeV : string fragmentation

Partonic Substructure in Pomeron:

Only in POMPYT addon (P. Bruni, A. Edin, G. Ingelman) ► high-p_T "jetty" diffraction absent



Very soft spectra without POMPYT

PYTHIA 6: Supported, but not actively developed

Diffraction in PYTHIA 8

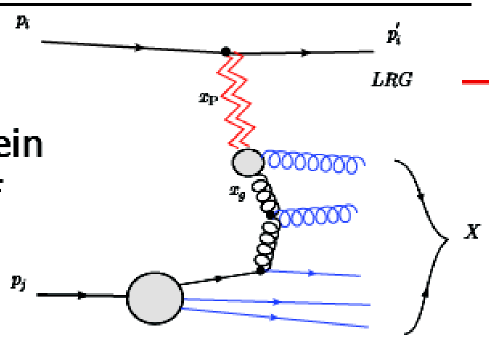
Diffractive Cross Section Formulæ:

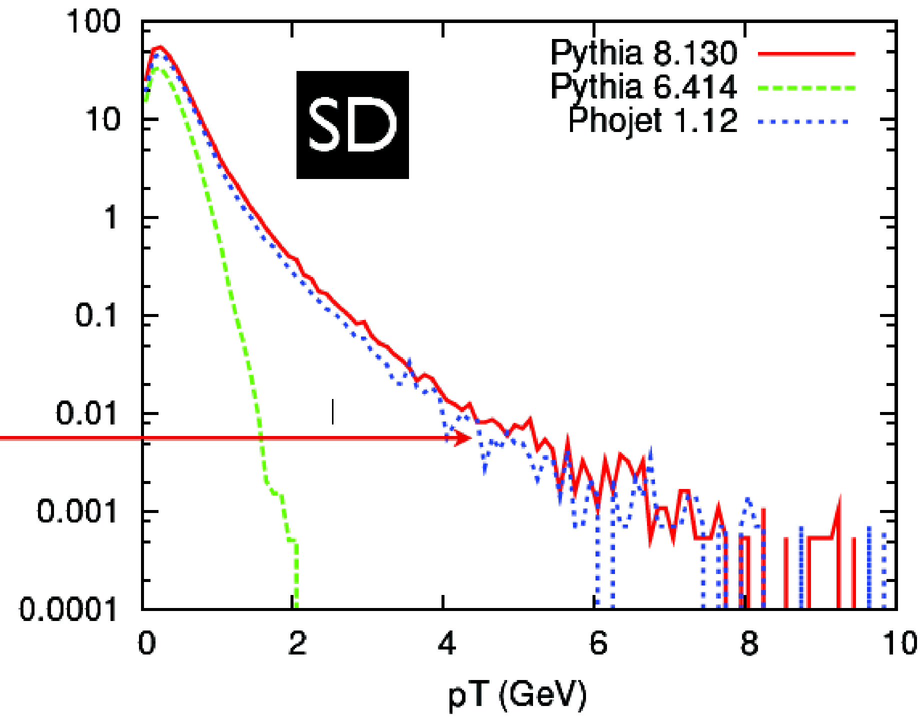
$$\frac{d\sigma_{sd(AX)}(s)}{dt\,dM^2} = \frac{g_{3\mathbb{P}}}{16\pi}\,\beta_{A\mathbb{P}}^2\,\beta_{B\mathbb{P}}\,\frac{1}{M^2}\,\exp(B_{sd(AX)}t)\,F_{sd}\,,$$

$$\frac{d\sigma_{dd}(s)}{dt\,dM_1^2\,dM_2^2} = \frac{g_{3\mathbb{P}}^2}{16\pi}\,\beta_{A\mathbb{P}}\,\beta_{B\mathbb{P}}\,\frac{1}{M_1^2}\,\frac{1}{M_2^2}\,\exp(B_{dd}t)\,F_{dd}\,.$$

Partonic Substructure in Pomeron:

Follows the Ingelman-Schlein approach of Pompyt





- $M_X \leq 10\,\mathrm{GeV}$: original longitudinal string description used
- $M_X > 10\,\mathrm{GeV}$: new perturbative description used (incl full MPI+showers for $\mathbb{P}$p system)

Choice between 5 Pomeron PDFs. Free parameter $\sigma_{\mathbb{P}p}$ needed to fix $\langle n_{\mathrm{interactions}}\rangle = \sigma_{\mathrm{jet}}/\sigma_{\mathbb{P}p}$.

Framework needs testing and tuning, e.g. of $\sigma_{\mathbb{P}p}$.

Navin, arXiv:1005.3894

MBR model

MBR (Minimum-Bias Rockefeller) Monte Carlo simulation - an event generator addressing the contribution from diffractive processes:

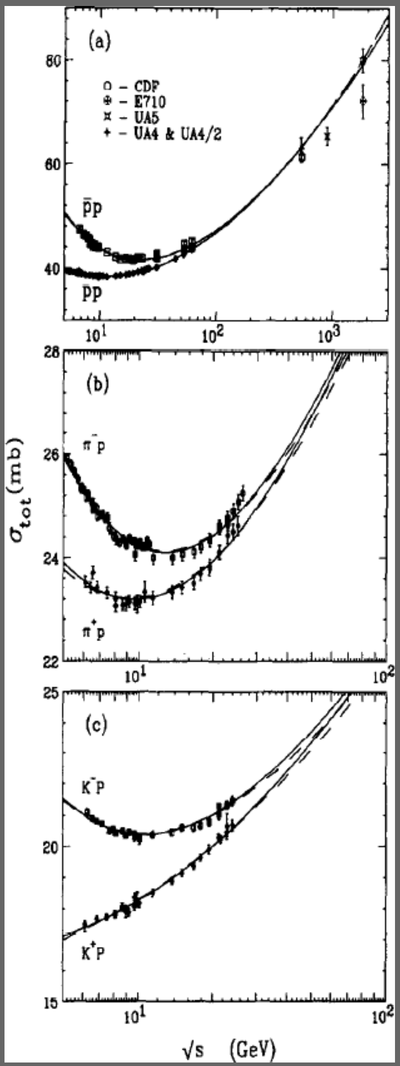
- Predicts energy dependence of the total, elastic and total-inel. cross sections.
- Fully simulates diffractive components (SD, DD, CD) of the total-inelastic xsec, based on renormalized Regge-theory model (Pomeron flux interpreted as probability of diffractive gap formation, which saturates at unity).
- Originally written for, and tested at CDF (down to lowest masses, Mx).
- Implemented in PYTHIA8.165.

Documented in **arXiv:1205.1446**

- MBR hadronization from phenomenological model based on pre-LHC and pre-Tevatron low-energy data →

can be used to test PYTHIA8 framework for hadronization of diffractive masses.

MBR - total, elastic and total-inelastic cross sections



- **Total pp cross section**

Energy dependence:

$$\sigma_{\rm tot}^{p^{\pm}p} = \begin{cases} 16.79 s^{0.104} + 60.81 s^{-0.32} \mp 31.68 s^{-0.54} & \text{for } \sqrt{s} < 1.8 \text{ TeV}, \\ \sigma_{\rm tot}^{\rm CDF} + \frac{\pi}{s_0}\left[\left(\ln\frac{s}{s_F}\right)^2 - \left(\ln\frac{s^{\rm CDF}}{s_F}\right)^2\right] & \text{for } \sqrt{s} \geq 1.8 \text{ TeV}, \end{cases}$$

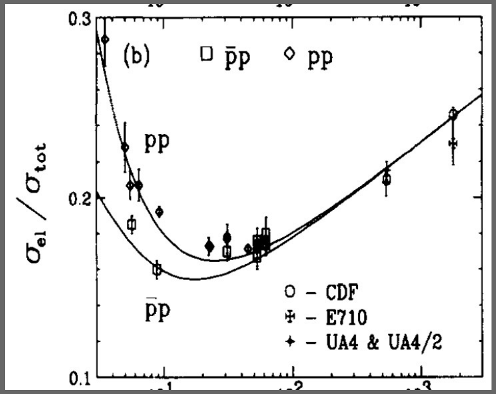
- For $\sqrt{s} < 1.8$ TeV – global fit to pre-LHC data on $p^{\pm}p$, $K^{\pm}p$, $\pi^{\pm}p$ cross-sections. **Phys.Lett B389, 176 (1996)**

- For $\sqrt{s} > 1.8$ TeV (LHC and beyond) – model based on a saturated Froissart bound. **arXiv:1105.4916**

Froissart bound with two parameters: s_F = 22 GeV, s_0 = 3.7±1.5 GeV^{-2}, normalized to the CDF measurement @1.8 TeV: 80.03 ±2.24 mb.

MBR - total, elastic and total-inelastic cross sections

Phys.Lett B389, 176 (1996)



- **Elastic cross section**

$$\sigma_{el} = r \bullet \sigma_{tot}$$

with $r = \sigma_{el}/\sigma_{tot}$ from the global fit.

Linear *log(s)* dependence at higher energies → model expected to be valid up to energies of O(50 TeV) (black-disk limit).

- **Total-inelastic cross section**

$$\sigma_{inel} = \sigma_{tot} - \sigma_{el}$$

MBR - diffractive cross sections

- Calculated based on renormalized-Regge theory.

hep-ph/0407035
arXiv:hep-ph/020314

- Differential cross sections vs. rapidity gap width, Δy, and 4-momentum transfer squared, t:

$$\frac{d^2\sigma_{SD}}{dt d\Delta y} = \frac{1}{N_{\text{gap}}(s)} \left[\frac{\beta^2(t)}{16\pi} e^{2[\alpha(t)-1]\Delta y}\right] \cdot \left\{\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right\},$$
$$\frac{d^3\sigma_{DD}}{dt d\Delta y dy_0} = \frac{1}{N_{\text{gap}}(s)} \left[\frac{\kappa\beta^2(0)}{16\pi} e^{2[\alpha(t)-1]\Delta y}\right] \cdot \left\{\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right\},$$
$$\frac{d^4\sigma_{DPE}}{dt_1 dt_2 d\Delta y dy_c} = \frac{1}{N_{\text{gap}}(s)} \left[\Pi_i \left[\frac{\beta^2(t_i)}{16\pi} e^{2[\alpha(t_i)-1]\Delta y_i}\right]\right] \cdot \kappa \left\{\kappa\beta^2(0)\left(\frac{s'}{s_0}\right)^{\epsilon}\right\},$$

$$\Delta y = \Delta y_1 + \Delta y_2$$

DD: y_0 – center of rapidity gap, DPE: y_c – rapidity of dissociated system

$$\alpha(t) = 1 + \epsilon + \alpha' t = 1.104 + 0.25\ (\text{GeV}^{-2}) \cdot t$$

$$\beta^2(t) = \beta^2(0) F^2(t) \qquad \kappa \equiv g(t)/\beta(0)$$

$$\xi = e^{-\Delta y} \qquad \xi_{SD} = M^2/s$$

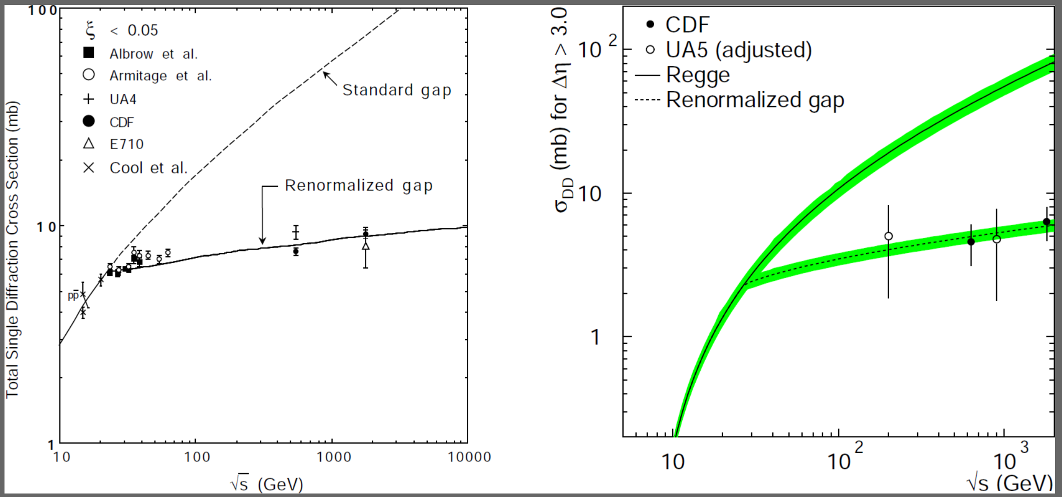
$$\xi_{DD} = M_1^2 M_2^2/(s \cdot s_0)$$

$$\text{DPE}\ \xi = \xi_1 \xi_2 = M^2/s$$

- Term in { } brackets: total Pomeron-p cross section at a reduced energy s'=s•e^{-dy}.
- Term in [ ] brackets: Pomeron flux.
- $N_{gap}(s)$: renormalization factor: *min(1,f)*, with *f* := integral of Pomeron flux
  → allows to interpret the flux as (diffractive) gap-formation probability.

MBR - diffractive cross sections

- Flux renormalization procedure brings the standard Regge theory predictions in agreement with the CDF data.



from arXiv:hep-ph/020314

- **Small gap widths, diffractive limit**

Cross section formulae are used to generate events with large (diffractive) rapidity gaps. Small gaps are suppressed by convoluting the formulae with the error function (cumulative Gauss distribution) centered at $\Delta y_s = 2$ with a width of $\sigma_S = 0.5$:

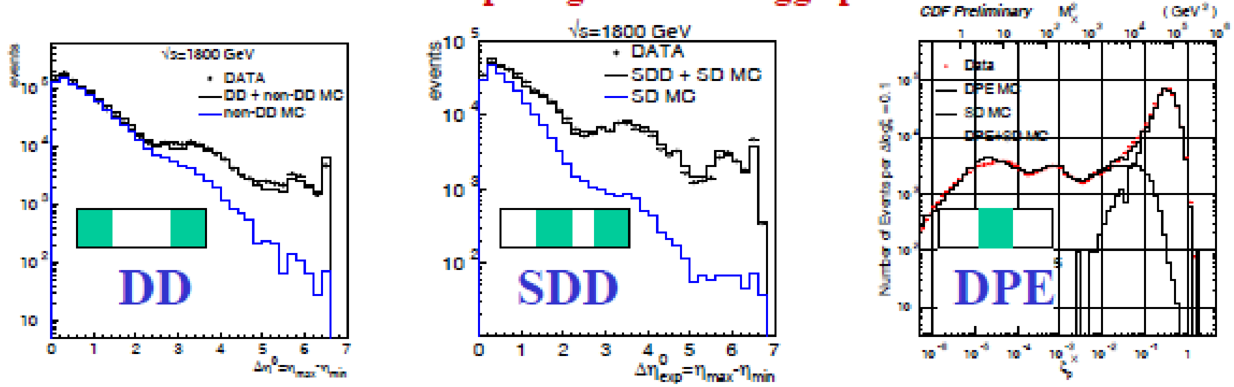
$$S = \frac{1}{2}\left[1 + erf\left(\frac{\Delta y - \Delta y_S}{\sigma_S}\right)\right]$$

SD events: coherence limit ($\xi \leq 0.135$)
DD events: arbitrary choice, because small gaps in DD and ND events cannot be unambiguously distinguished.
CD events: suppression applied on total gap width, $\Delta y = \Delta y_1 + \Delta y_2$.

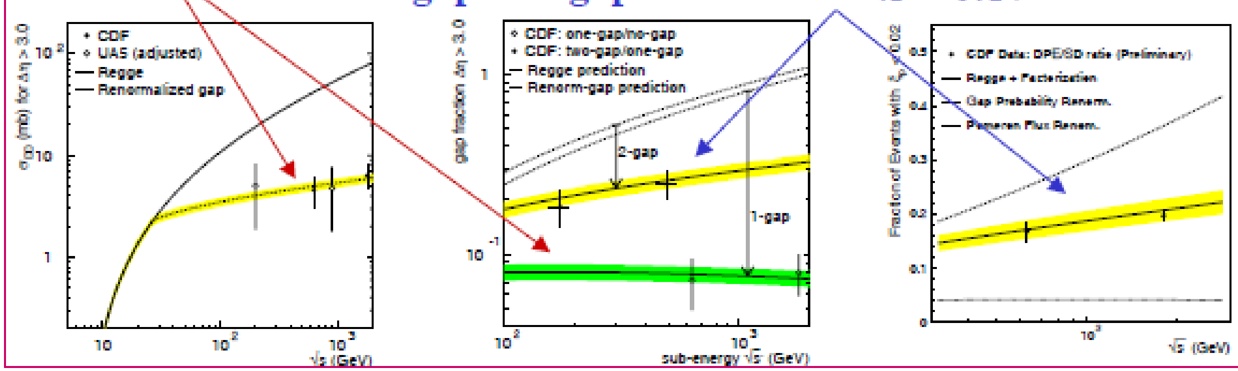
MBR vs. CDF data

Central & Double-Gap CDF Results

Differential shapes agree with Regge predictions

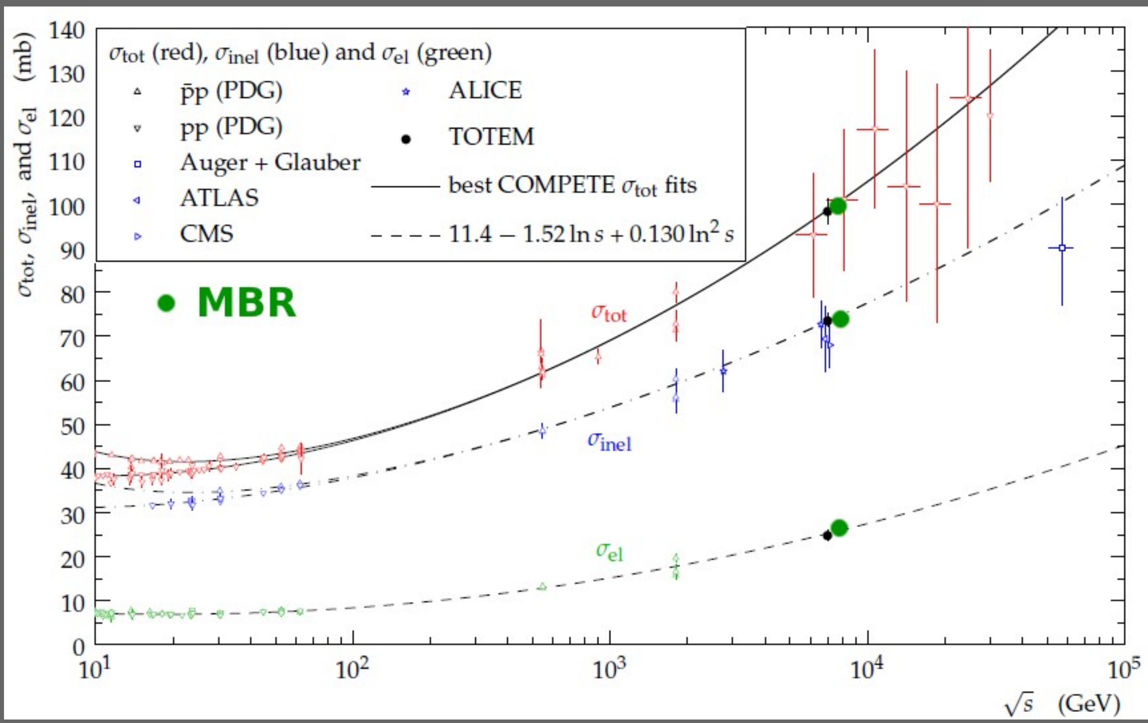


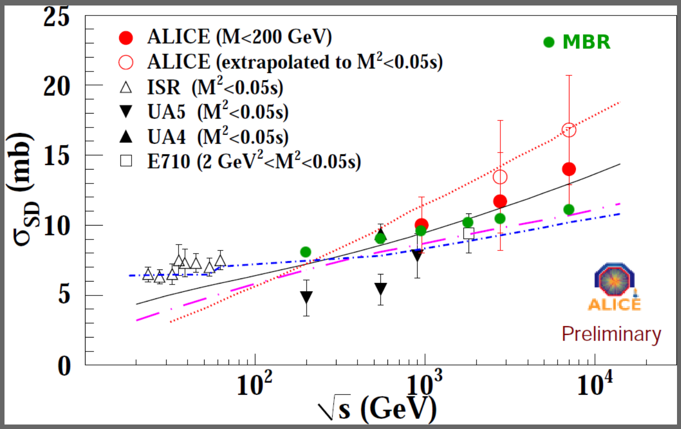
- One-gap cross sections are suppressed
  - Two-gap/one-gap ratios are $\approx \kappa = 0.17$

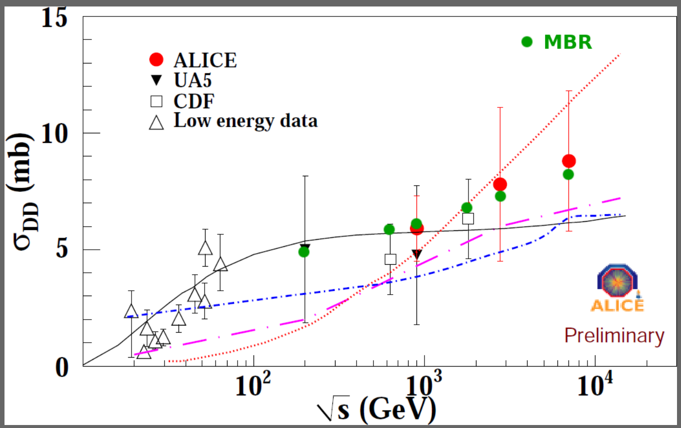


La Thuile07, 2-20 March | Diffraction and Exclusive (Higgs?) Production from CDF to LHC | K. Goulianos | 30

MBR vs. LHC data

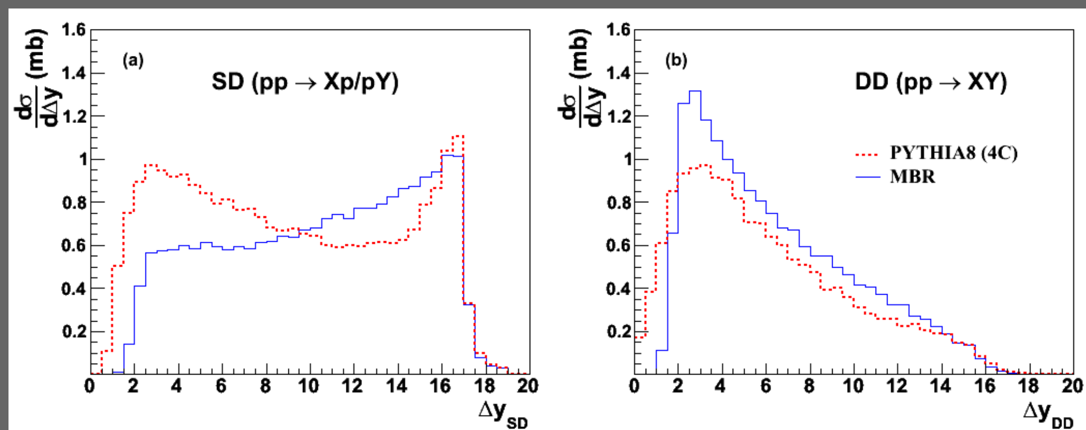






MBR - implementation in PYTHIA8.165

PYTHIA8-MBR simulation activated with Diffraction:PomFlux = 5.



SD, DD processes

MBR code added to the already existing simulation of processID =103/104, 105. Comparison with default Pythia8-4C simulation (rescaled Schuler&Sjostrand model, Diffraction:PomFlux=1) above.

- **CD (DPE) process**

  Implemented in PYTHIA for the first time. ProcessID=106, **SoftQCD:centralDiffractive = on.** Possible to extend for other CD processes such as exclusive di-jet or di-hadron production.

- **Hadronization of diffractive system based on PYTHIA8-4C tune (default).**

MBR at CDF - phenomenological model for hadronization

- In the original hadronization model in MBR at CDF the diffractive system of mass Mx hadronized as the (non-diffractive) pp collision at √s = Mx. Only one hadronic system in the game!
- Toy model: mainly π+, π- and π0 → γγ in the final state.

Track multiplicities:

Parameterisation of particle multiplicities follows a Modified Gamma Distribution described in **PLB 193, 151 (1987)**. This parameterization was tested using existing pre-LHC and pre-Tevatron pp data in a wide range of √s :

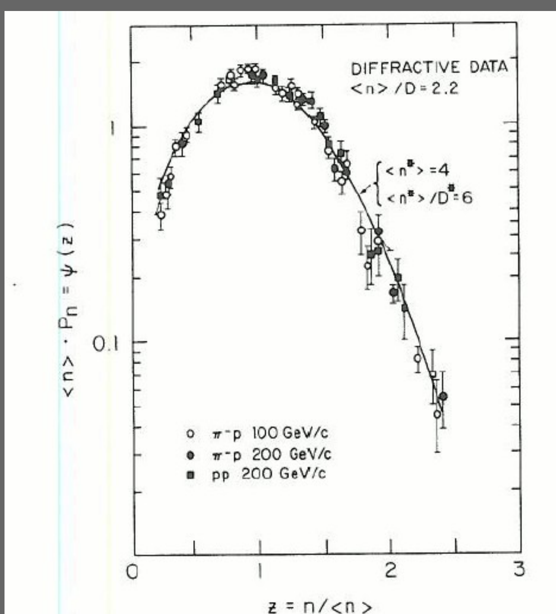


Fig. 1. The diffractive data of ref. [3] fitted with the modified gamma function.

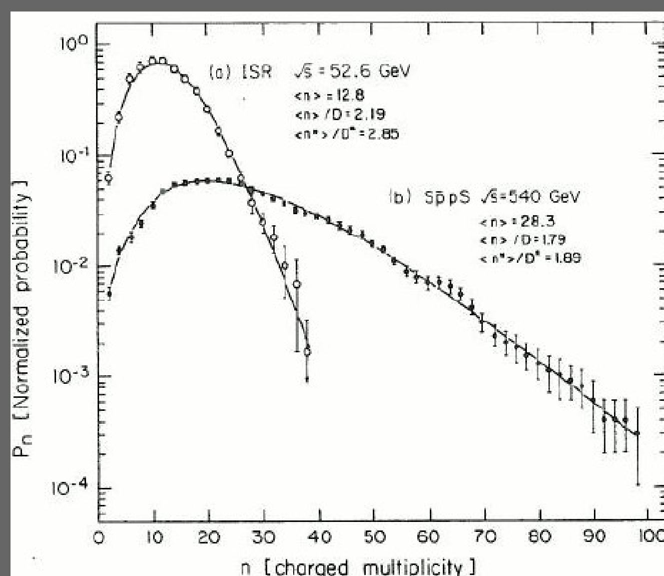


Fig. 2. Full phase space inelastic non-single-diffractive data fitted with the modified gamma function: (a) ISR data [5] at $\sqrt{s} = 52.6$ GeV and (b) collider data [7] at $\sqrt{s} = 540$ GeV.

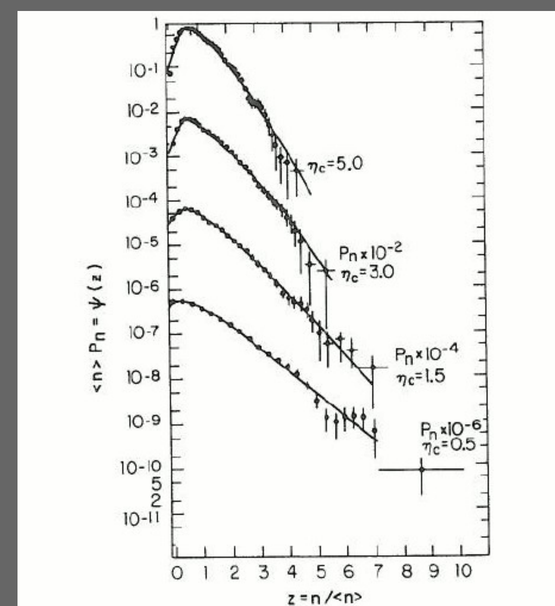


Fig. 3. Fits with the modified gamma function of charged-particle multiplicity distributions in restricted pseudorapidity intervals, defined by $|\eta| < \eta_c$, at $\sqrt{s} = 540$ GeV (data from ref. [9]).

MBR at CDF - phenomenological model for hadronization

pT spectra:

From MBR user guide at CDF

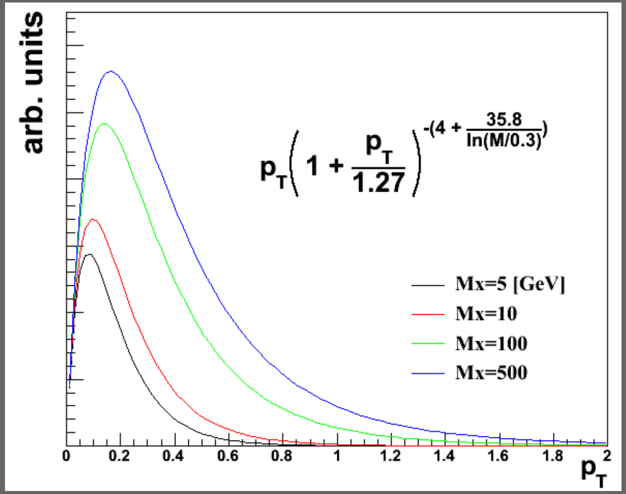
6.2 Transverse momentum distribution

For diffractive mass clusters with 2 or 3 particles, the direction of the nucleon is chosen from the angular distribution $dP/d\cos\theta \sim 1 + \cos^2\theta$, and the momentum is balanced by the remaining pion(s). For $n > 3$, the transverse momentum of the particles is taken from the empirically determined distribution [15]

$$\frac{d\sigma}{dp_T} \sim p_T \left(1 + \frac{p_T}{1.27\ \text{GeV}}\right)^{-4-35.8/\ln(M/0.3\ \text{GeV})} \quad (6.2)$$

for available mass M. The total p_T is then balanced, which changes the p_T of the particles only slightly.

[15] G. Arnison *et al.*, (UA1 Collaboration), Phys. Lett. B **118**, 167 (1982).



Mx-dependent pT spectra (UA1).

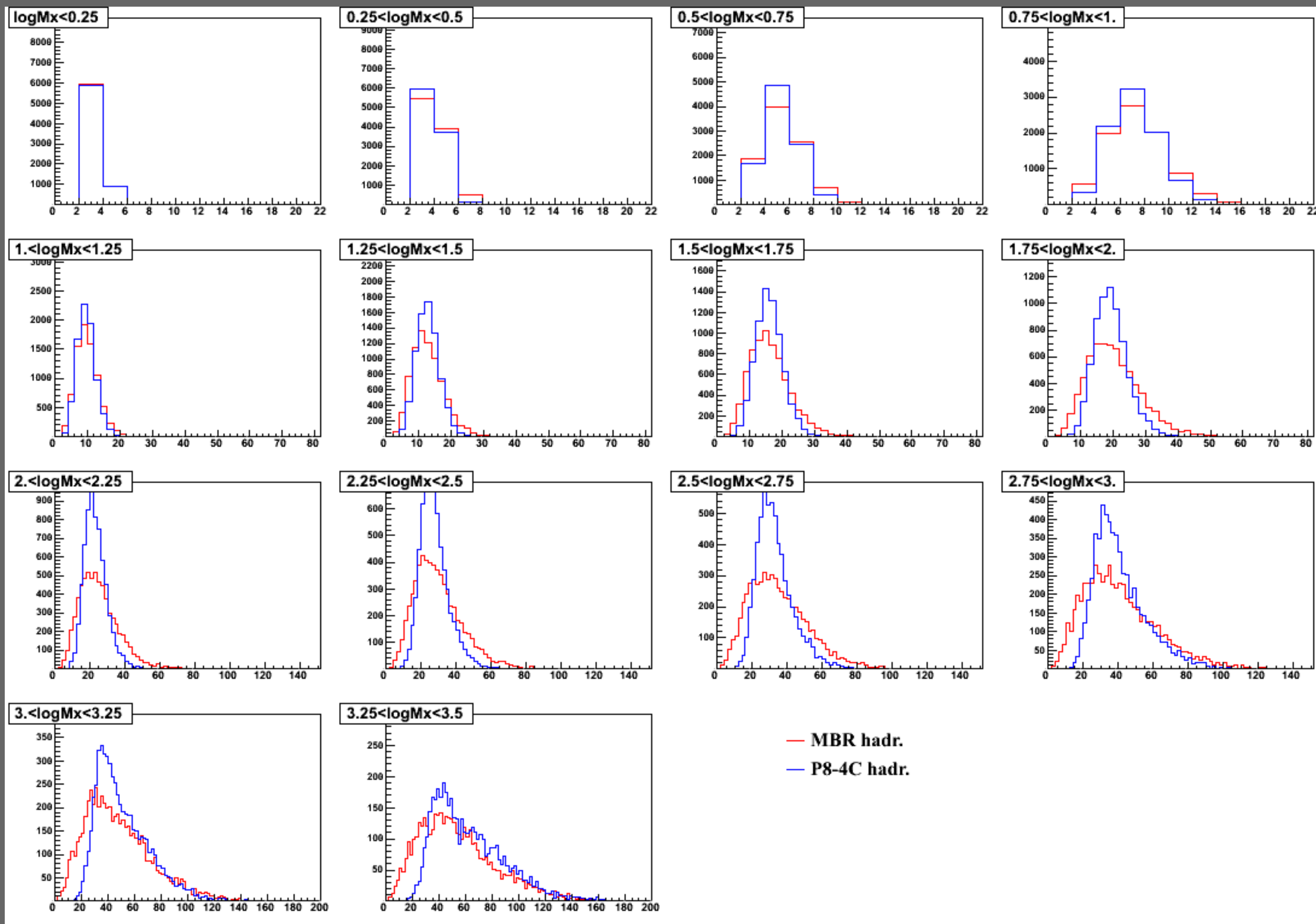
→ **Test PYTHIA8-4C framework for Mx hadronization using MBR model.**

→ **Tune critical parameters of Diffraction (sigmaPomP, pickQuarkNorm/Power) and StringPT classes to bring multiplicity and pT spectra of Mx system close to MBR hadronization model.**

PYTHIA8 vs. MBR hadroniation

Charge-particle multiplicities in bins of Mx (SD process):

14 bins of logMx (width=0.25)

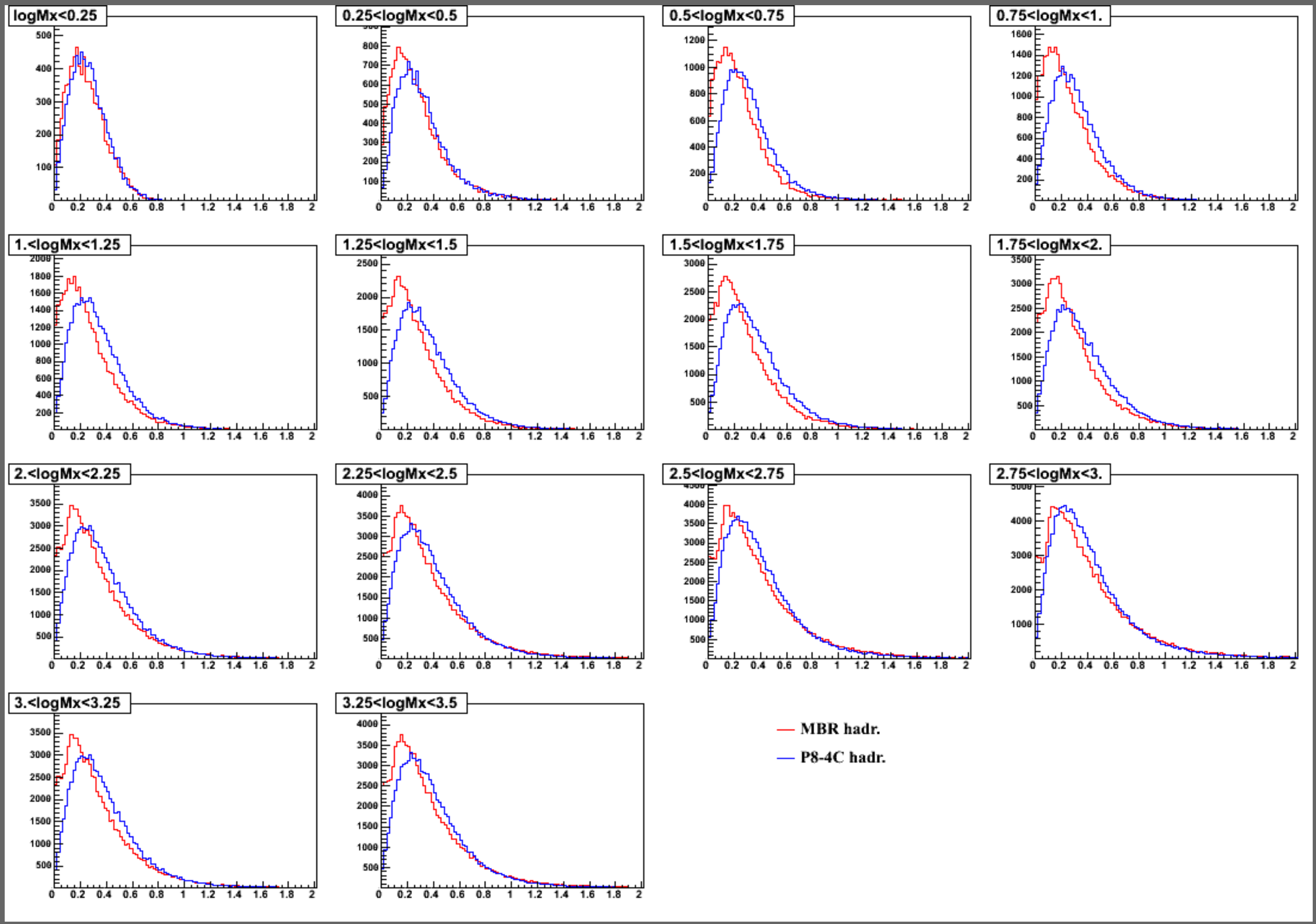


Similar average multiplicities in PYTHIA8-4C and MBR hadr. model.
Significant difference in width of distributions.

PYTHIA8 vs. MBR hadroniation

Transverse-momentum distributions in bins of Mx (SD process):

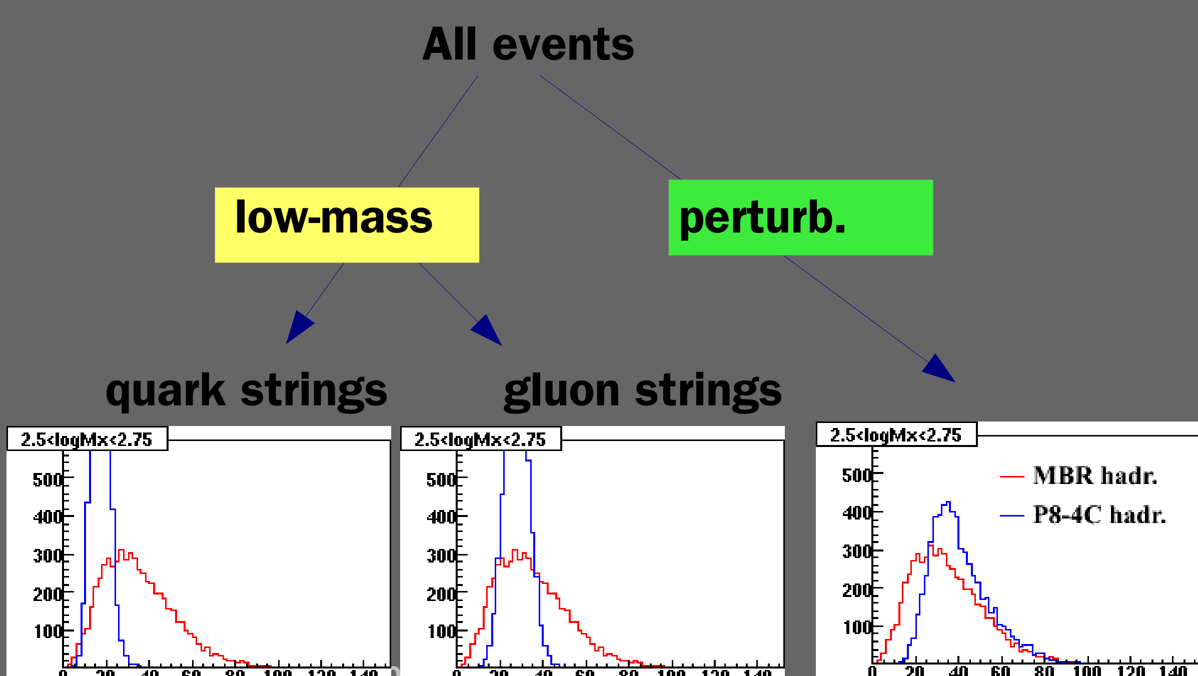
14 bins of logMx (width=0.25)

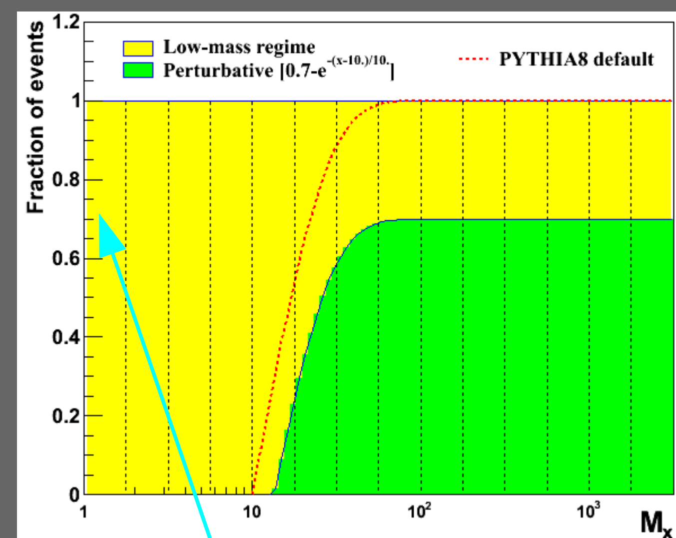


Similar high-pT tails in PYTHIA8-4C and MBR had. models. Low-pT spectra shifted towards higher values in PYTHIA8 (global set of parameters – the same for non- and diffractive events, tuned to NSD MinBias data)

Hadronization tune

- Multiplicity spectra – an interplay between low-mass/perturbative hadronization regimes (slide 5)
  - Higher multiplicities (perturbative regime): check energy dependence of the sigmaPomP parameter (constant by default); introduce low-mass regime for fraction of events. $n_{ave} = \frac{\sigma_{QCD}}{\sigma_{IPp}}$
  - Lower miltiplicities (low-mass regime) : tune the ratio of a quark to gluon induced strings, driven by pickQuarkNorm/Power parameters. Quarks give lower multiplicities than gluons. (NB. these are q/g of a Lund-string model, not to be confused or interpreted as the q/g ratio in the proton)
- pT spectra – tune the parameters of the StringPT class.

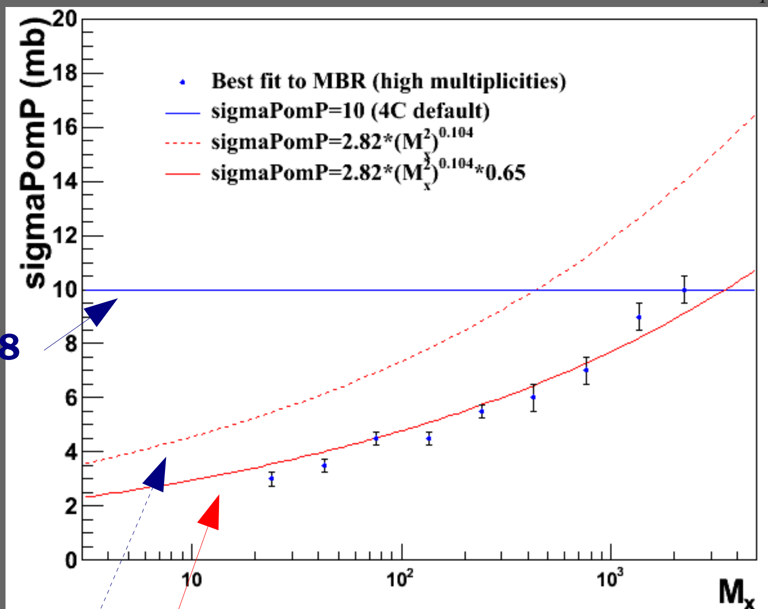




Fraction of low-mass events in the perturbative regime is given by (1-ProbMaxPert), with ProbMaxPert=0.7 (default=1)

Hadroniation tune

Diffraction:SigmaPomP parameter $n_{ave} = \frac{\sigma_{QCD}}{\sigma_{IPp}}$

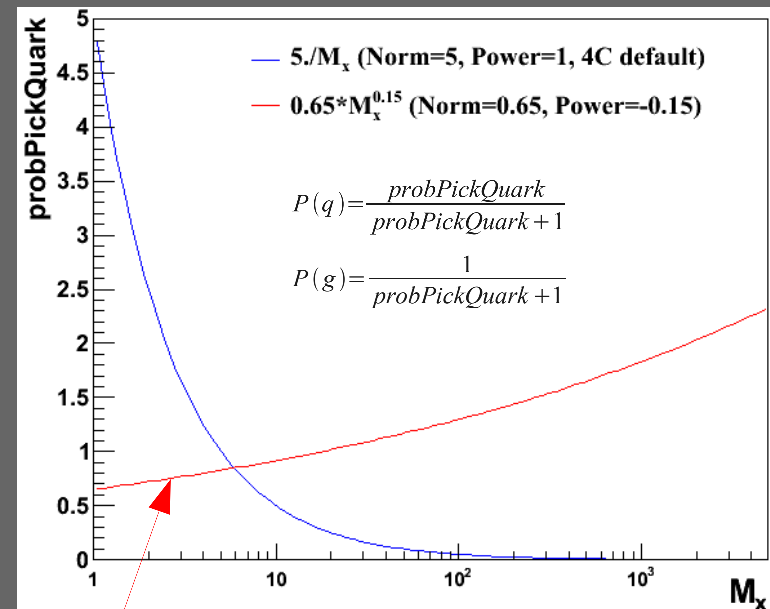


PYTHIA8 default

$\sigma^{IP\,p}(s)$ expected from Regge phenomenology for s_0=1 GeV^2 and DL t-dependence.

Red line - parameterization giving the best Pythia8 fit to MBR multiplicity distributions (in bins of Mx, fits to higher tails only, default pT spectra)

Diffraction:pickQuarkNorm/Power parameter



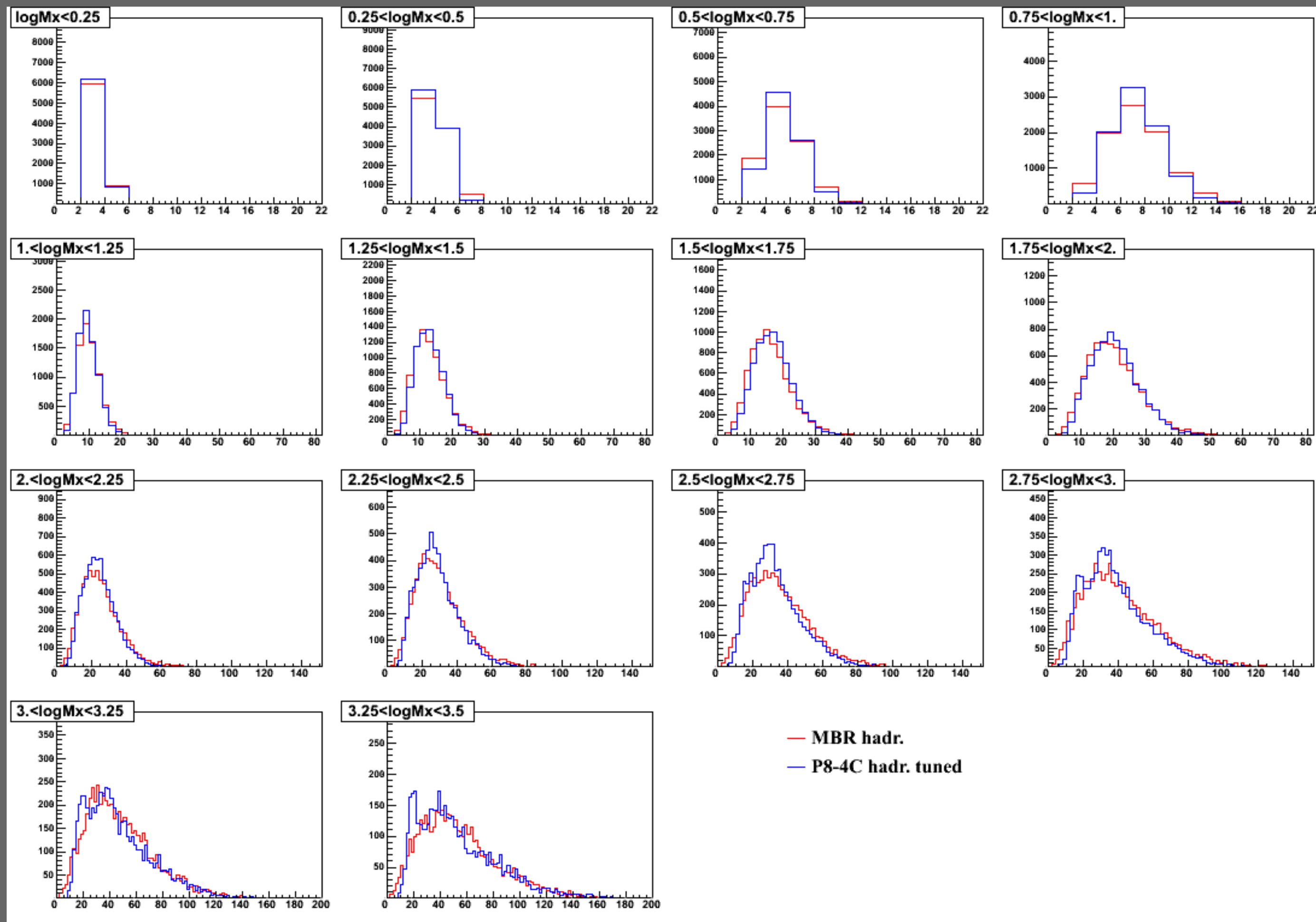
From comparison to MBR multiplicities:
- **Low-Mx bins need lower probPickQuark**
- **High-Mx bins need higher probPickQuark**

Red line - parameterization giving good description of low-multiplicity tails.

PYTHIA8 vs. MBR hadroniation - results of the tune

Charge-particle multiplicities in bins of Mx (SD process):

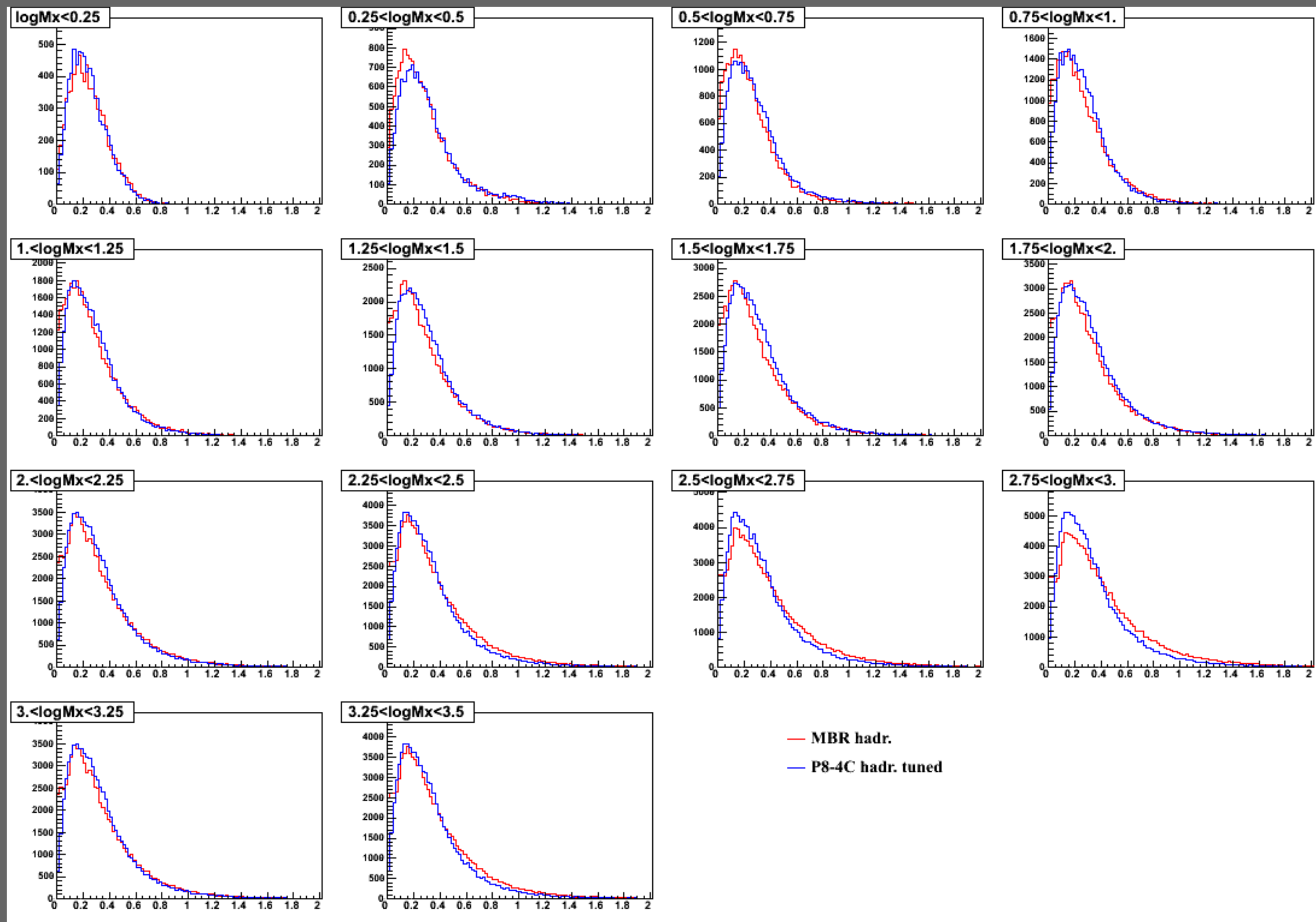
14 bins of logMx (width=0.25)



PYTHIA8 vs. MBR hadroniation - results of the tune

Transverse-momentum distributions in bins of Mx (SD process):

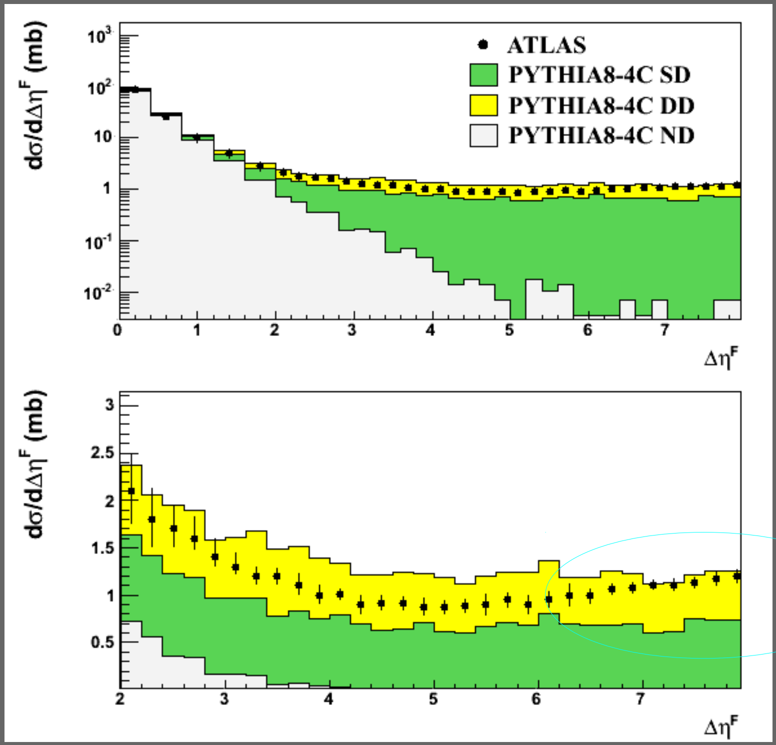
14 bins of logMx (width=0.25)



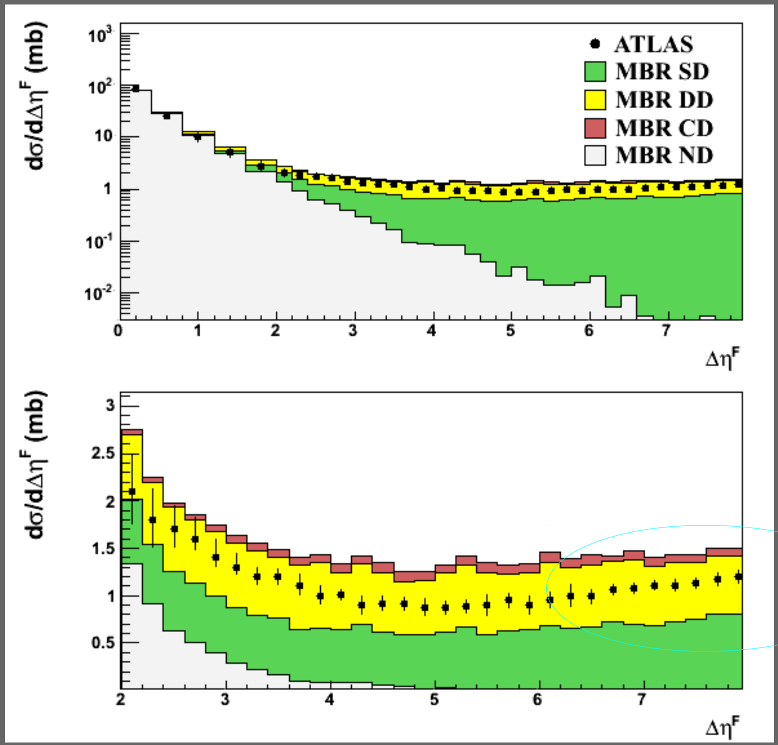
A set of: **StringPT:sigma=0.09, StringPT:enhancedWidth=5, StrangPT:enhancedFraction=0.2** gives good description of diffractive, but is not expected to describe non-diffractive events.

PYTHIA8-4C/MBR vs. ATLAS rapidity-gap data @7 TeV

PYTHIA8-4C



PYTHIA8-MBR (with "diffractive" tune)

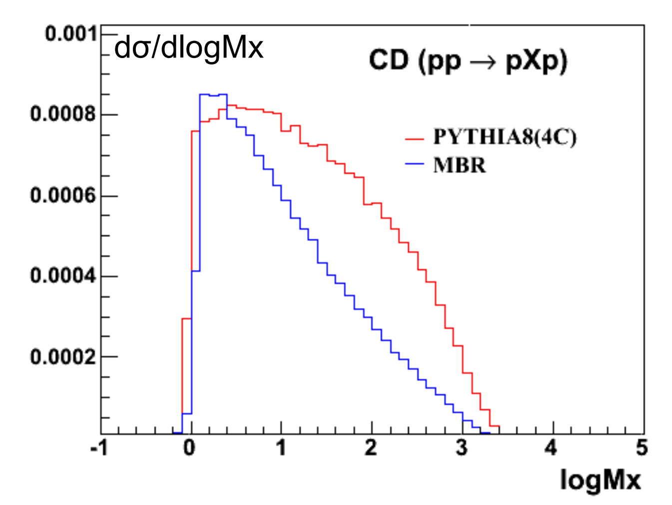


- Both predictions overestimate the data by a similar amount:
  - PYTHIA8-4C tuned to non-single-diffractive (central) MinBias data at LHC enegies
  - MBR extrapolated from CDF energies (no additional tuning, CDF systematic errors ignored).
- Expect larger differences between PYTHIA8-4C and MBR at higher rapidity-gap values (lower Mx).

Central Dissociation

- Included for the first time in PYTHIA8.165 (within MBR, PomFlux=5).
- Extended to all the diffractive models in PYTHIA8.170.
- Cross section behaves roughly like $\sim \ln^{1.5} s$ (PomFlux=1-4), and amounts to σ=1.192 mb @7 TeV (compared to σ=0.802 mb from MBR).

  Can be rescaled by changing **SigmaTotal:sigmaAXB2TeV** (default = 1.5 mb, the CD cross section @2 TeV).



- Simulated as a part of **SoftQCD:MinBias** sample, or explicitly with **SoftQCD:centralDiffractive = on**, However, for most of the tunes (e.g. the default 4C, performed before this update) the generation of CD process is automatically switched off with **SigmaTotal:zeroAXB = true** flag.
- So, in order to generate CD events one needs to switch the flag **SigmaTotal:zeroABX = false** after defining **Tune:pp**.

Summary

Recent developments on diffraction in PYTHIA8 have been reviewed:

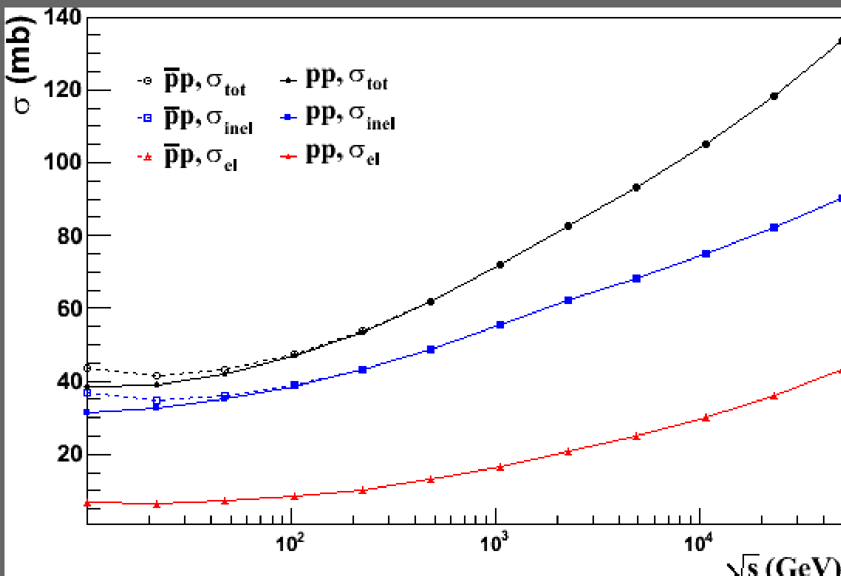
- Recalled PYTHIA8 vs. PYTHIA6 differences.
- MBR (Minimum-Bias Rockefeller) simulation, developed and successfully tested at CDF, implemented since Pythia8.165
- Test of PYTHIA8 framework for hadronization of diffractive events using data-driven MBR hadronization model.
- Central-dissociation (double-Pomeron exchange) process included for all diffractive models since PYTHIA8.170.

Thank you for your attention!

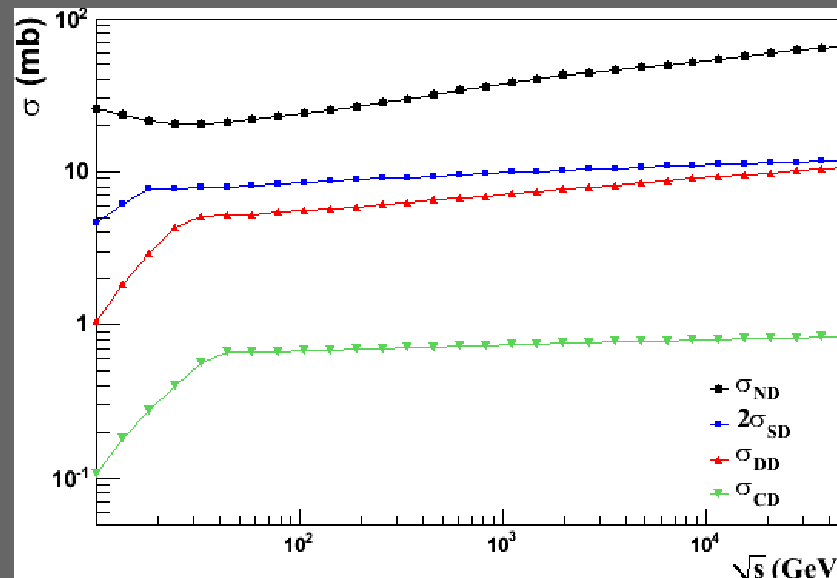
Backup

MBR - cross section predictions

Total, elastic, total-inel. xsecs



Components of total-inel. xsec: diffractive (SD,DD,CD) and non-diffractive (ND) xsecs



Cross section values [mb] vs. energy

$\sqrt{s}$ (TeV)	0.3	0.9	1.96	2.76	7	8	14
$\sigma_{\rm tot}$	56.50	69.87	81.03	85.25	98.29	100.35	109.49
$\sigma_{\rm el}$	11.28	15.83	19.97	21.70	27.20	28.09	32.10
$\sigma_{\rm inel}$	45.23	54.04	61.06	63.55	71.10	72.26	77.39
σ_{ND}	29.19	36.50	42.41	44.39	50.57	51.54	55.84
σ_{2SD}	9.10	9.76	10.22	10.41	10.91	10.98	11.26
σ_{DD}	6.21	7.03	7.67	7.97	8.82	8.94	9.47
σ_{CD}	0.718	0.746	0.766	0.776	0.800	0.804	0.818

MBR - event generation, SD

3.1 Single-diffractive events

Events are generated by first choosing the rapidity-gap width, Δy, according to Eq. (3) integrated over t:

$$\frac{d\sigma_{SD}}{d\Delta y} \sim e^{\epsilon \Delta y} \cdot \left(\frac{a_1}{b_1 + 2\alpha' \Delta y} + \frac{a_2}{b_2 + 2\alpha' \Delta y} \right) \cdot S. \tag{9}$$

The range of the generation is defined by $\Delta y_{min} = 0$ and $\Delta y_{max} = -\ln M_0^2/s$, where $M_0^2 =$MBRm2Min. The term:

$$S = \frac{1}{2}\left[1 + erf\left(\frac{\Delta y - \texttt{MBRdyminSD}}{\texttt{MBRdyminSigSD}}\right)\right], \tag{10}$$

is added to suppress events at low values of Δy, as explained in Sec. 2.2.

A value of t is then chosen according to:

$$\frac{d\sigma_{SD}}{dt} \sim F^2(t) \cdot e^{2\alpha' \Delta y t}, \tag{11}$$

where $F^2(t)$ is given by Eq. (7) and the integration is performed up to $t_{max} = -m_p^2 \cdot \frac{\xi^2}{1-\xi}$, with $\xi = e^{-\Delta y}$. The diffractive mass is calculated as $M = \sqrt{s\xi}$. The four-momenta of the outgoing proton and the dissociated mass system are calculated using Mandelstam variables for a two-body scattering process, as implemented in PYTHIA8 for other `Diffraction:PomFlux` options.

MBR - event generation, DD

3.2 Double-diffractive events

Events are generated by first choosing the rapidity-gap width according to Eq. (4) integrated over t. Eq. (4) is divergent as $\Delta y \to 0$. In order to remove the divergence, the integration over t is performed within the limits from $t_{min} = -e^{\Delta y}$ to $t_{max} = -e^{-\Delta y}$. Then, Δy is chosen from the distribution:

$$\frac{d\sigma_{DD}}{d\Delta y} \sim e^{\epsilon \Delta y} \cdot \frac{\ln \frac{s s_0}{M_0^4} - \Delta y}{2\alpha' \Delta y} \left(e^{-2\alpha' \Delta y e^{-\Delta y}} - e^{-2\alpha' \Delta y e^{\Delta y}} \right) \cdot S, \tag{12}$$

and the range of the generation is defined by $\Delta y_{min} = 0$ and $\Delta y_{max} = -\ln M_0^4/(s s_0)$, where $M_0^2 =$`MBRm2Min` and $s_0 = 1$ GeV^2. To further suppress events at low values of Δy the term:

$$S = \frac{1}{2}\left[1 + erf\left(\frac{\Delta y - \texttt{MBRdyminDD}}{\texttt{MBRdyminSigDD}}\right)\right], \tag{13}$$

is used as explained in Sec. 2.2.

The variable t is chosen according to:

$$\frac{d\sigma_{DD}}{dt} \sim e^{2\alpha' \Delta y t}, \tag{14}$$

in the range from $t_{min} = -e^{\Delta y}$ to $t_{max} = -e^{-\Delta y}$.

Then, the center of the rapidity gap, y_0, is selected uniformly within the limits:

$$-\frac{1}{2}\left(\ln \frac{s s_0}{M_0^4} - \Delta y\right) < y_0 < \frac{1}{2}\left(\ln \frac{s s_0}{M_0^4} - \Delta y\right), \tag{15}$$

and the diffractive masses are calculated as:

$$M_1^2 = \sqrt{s \cdot e^{-\Delta y - y_0}}, \tag{16}$$

$$M_2^2 = \sqrt{s \cdot e^{-\Delta y + y_0}}. \tag{17}$$

The four-momenta of the outgoing dissociated mass systems are calculated using Mandelstam variables for a two-body scattering process, as implemented in PYTHIA8 for other options of `Diffraction:PomFlux` .

MBR - event generation, CD (DPE)

3.3 Central-diffractive (DPE) events

Events are generated by first choosing the total rapidity gap width, Δy, according to Eq. (5), integrated over t_1 and t_2:

$$\frac{d\sigma_{CD}}{d\Delta y} \sim e^{\epsilon\Delta y} \int_{-\Delta y/2+y_0}^{\Delta y/2-y_0} dy_0 \; f_- \cdot f_+ \cdot S_1 S_2, \tag{18}$$

where:

$$f_\pm = \left(\frac{a_1}{b_1 + \alpha'\Delta y \pm 2\alpha' y_0} + \frac{a_2}{b_2 + \alpha'\Delta y \pm 2\alpha' y_0} \right), \tag{19}$$

and the integration is performed from $\Delta y_{min} = 0$ to $\Delta y_{max} = -\ln M_0^2/s$, where $M_0^2 =$MBRm2Min. For events at low values of Δy we suppress individual gaps with the factor:

$$S = \frac{1}{2}\left[1 + erf\left(\frac{\Delta y - \texttt{MBRdyminCD}/2}{\texttt{MBRdyminSigCD}/\sqrt{2}}\right)\right]. \tag{20}$$

Then, the direction of the centrally-produced hadronic system, y_c, is selected uniformly within the region:

$$-\frac{1}{2}(\Delta y - \Delta y_{min}) < y_c < \frac{1}{2}(\Delta y - \Delta y_{min}), \tag{21}$$

and rapidity gaps corresponding to each of the two Pomerons are calculated as:

$$\Delta y_1 = \Delta y/2 + y_0, \tag{22}$$

$$\Delta y_2 = \Delta y/2 - y_0. \tag{23}$$

The four-momentum transfers squared at each proton vertex, t_1 and t_2, are generated according to:

$$\frac{d\sigma_{CD,i}}{dt} \sim F^2(t_i) \cdot e^{2\alpha'\Delta y_i t_i}, \tag{24}$$

up to $t_{max,i} = -m_p^2 \cdot \frac{\xi_i^2}{1-\xi_i}$, where $\xi_i = e^{-\Delta y_i}$ and $i = 1, 2$. Then, the p_T and p_z of outgoing protons are calculated as $p_{T,i}^2 = (1-\xi_i)|t_i| - m_p^2\xi_i^2$ and $|p_{z,i}| = p(1-\xi_i)$, where $p = \sqrt{s/4 - m_p^2}$ is the incoming proton momentum.

Finally, the four-momentum of the hadronic system is calculated from the sum of the four-momenta of the Pomerons, each calculated as a difference between the incoming and outgoing proton four-vectors.

MBR - implementation in PYTHIA8.165

When option 5 is selected, the following parameters of the MBR model [Cie12] are used:

parm **Diffraction:MBRepsilon** (default = **0.104**; minimum = 0.02; maximum = 0.15)

parm **Diffraction:MBRalpha** (default = **0.25**; minimum = 0.1; maximum = 0.4)
the parameters of the Pomeron trajectory.

parm **Diffraction:MBRbeta0** (default = **6.566**; minimum = 0.0; maximum = 10.0)

parm **Diffraction:MBRsigma0** (default = **2.82**; minimum = 0.0; maximum = 5.0)
the Pomeron-proton coupling, and the total Pomeron-proton cross section.

parm **Diffraction:MBRm2Min** (default = **1.5**; minimum = 0.0; maximum = 3.0)
the lowest value of the mass squared of the dissociated system.

parm **Diffraction:MBRdyminSDflux** (default = **2.3**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminDDflux** (default = **2.3**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminCDflux** (default = **2.3**; minimum = 0.0; maximum = 5.0)
the minimum width of the rapidity gap used in the calculation of *Ngap(s)* (flux renormalization).

parm **Diffraction:MBRdyminSD** (default = **2.0**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminDD** (default = **2.0**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminCD** (default = **2.0**; minimum = 0.0; maximum = 5.0)
the minimum width of the rapidity gap used in the calculation of cross sections, i.e. the parameter *dy_S*, which suppresses the cross section at low *dy* (non-diffractive region).

parm **Diffraction:MBRdyminSigSD** (default = **0.5**; minimum = 0.001; maximum = 5.0)

parm **Diffraction:MBRdyminSigDD** (default = **0.5**; minimum = 0.001; maximum = 5.0)

parm **Diffraction:MBRdyminSigCD** (default = **0.5**; minimum = 0.001; maximum = 5.0)
the parameter *sigma_S*, used for the cross section suppression at low *dy* (non-diffractive region).