Diffraction & MPI

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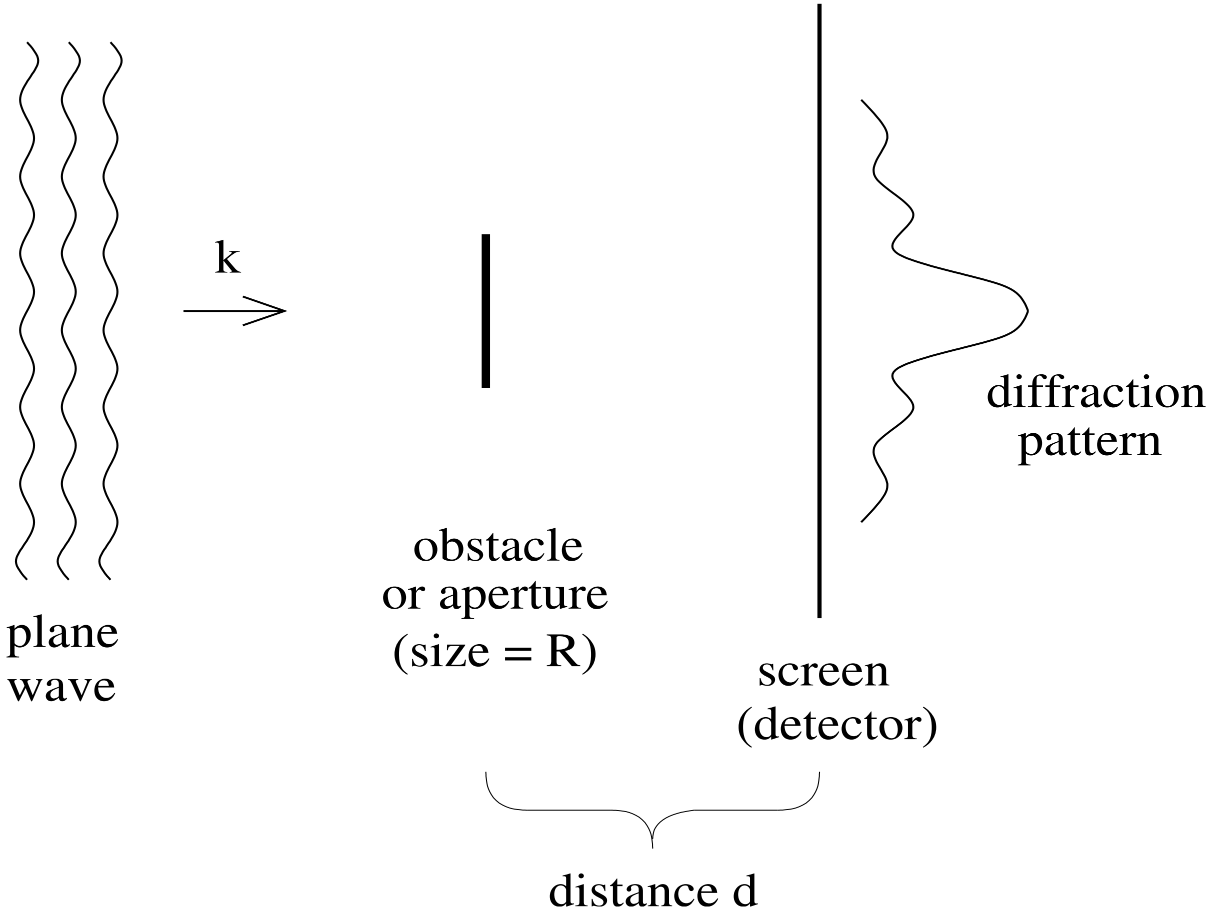
MPI @ CERN, December 3-7 , 2012

Diffraction: general concepts.

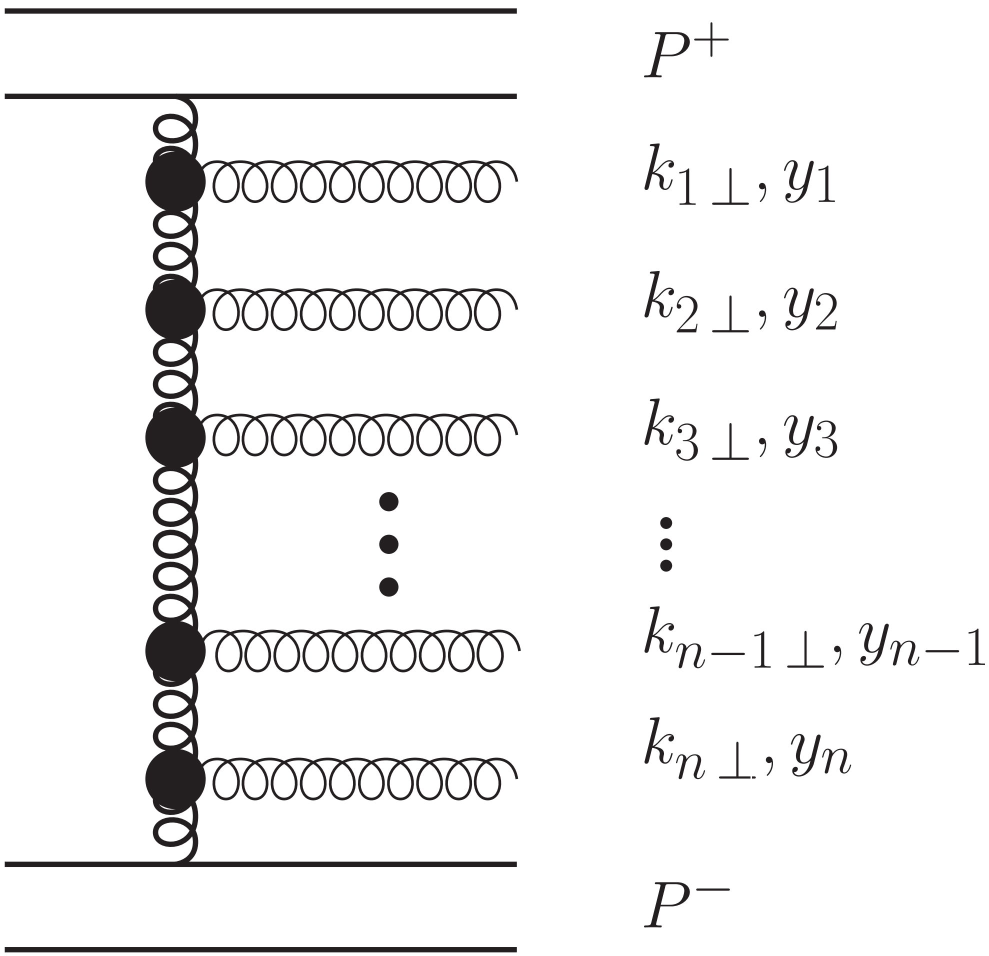
A reminder(diffraction in optics):



A reminder (multiparticle (partons) production):

- $\Delta\phi_i\, \Delta N \;\approx\; 1$

For $N \;\gg\; 1$ the multiparticle production can be considered as typical classical (semiclassical) process



General remarks:

- **Diffraction** $\Longrightarrow$ **wave nature of hadrons**
  $\Longrightarrow$ **QM;**

- **MPI** $\Longrightarrow$ **multipartical production**
  $\Longrightarrow$ **almost classical (probabilistic);**

- **Interrelation** $\Longrightarrow$ **unitarity constraints**

$$2\,A^{ii}_{el}\,(s,b) \;=\; |A^{ii}_{el}\,(s,b)\,|^2 + G^{ii}\,(s,b)$$

i $\equiv$ DOF ???

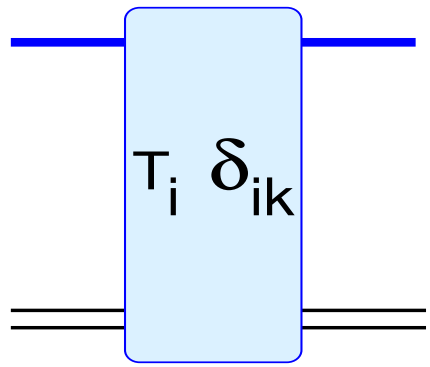
$G^{ii}\,(s,b)$ $\Longrightarrow$ **contribution of all inelastic (MPI) processes**

Diffraction in QM:

Feinberg and Pomeranchuk (1956), Good and Walker (1960)

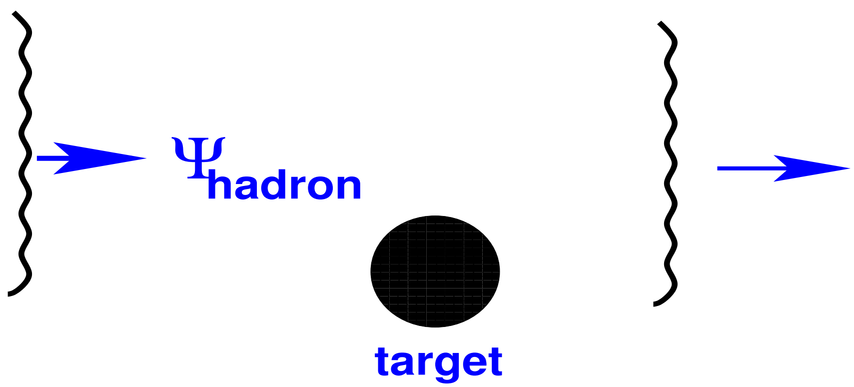
initial state — final state

hadron $\Longrightarrow \Psi_h = \sum_i \Psi_i$



$\sum_i T_i \Psi_i$ $\overset{\text{hadrons}}{\Longrightarrow}$ $\sum_h \Psi_h$

elastic + diffractive



$\Psi_{\text{fin}} = \alpha \Psi_{\text{hadron}} + \beta \Psi_{\text{diffraction}}$

- $\sigma_{diff} \propto \sum T_i^2 - \left(\sum T_i \right)^2 = \langle T^2 \rangle - \langle T \rangle^2$

What are correct DOF?

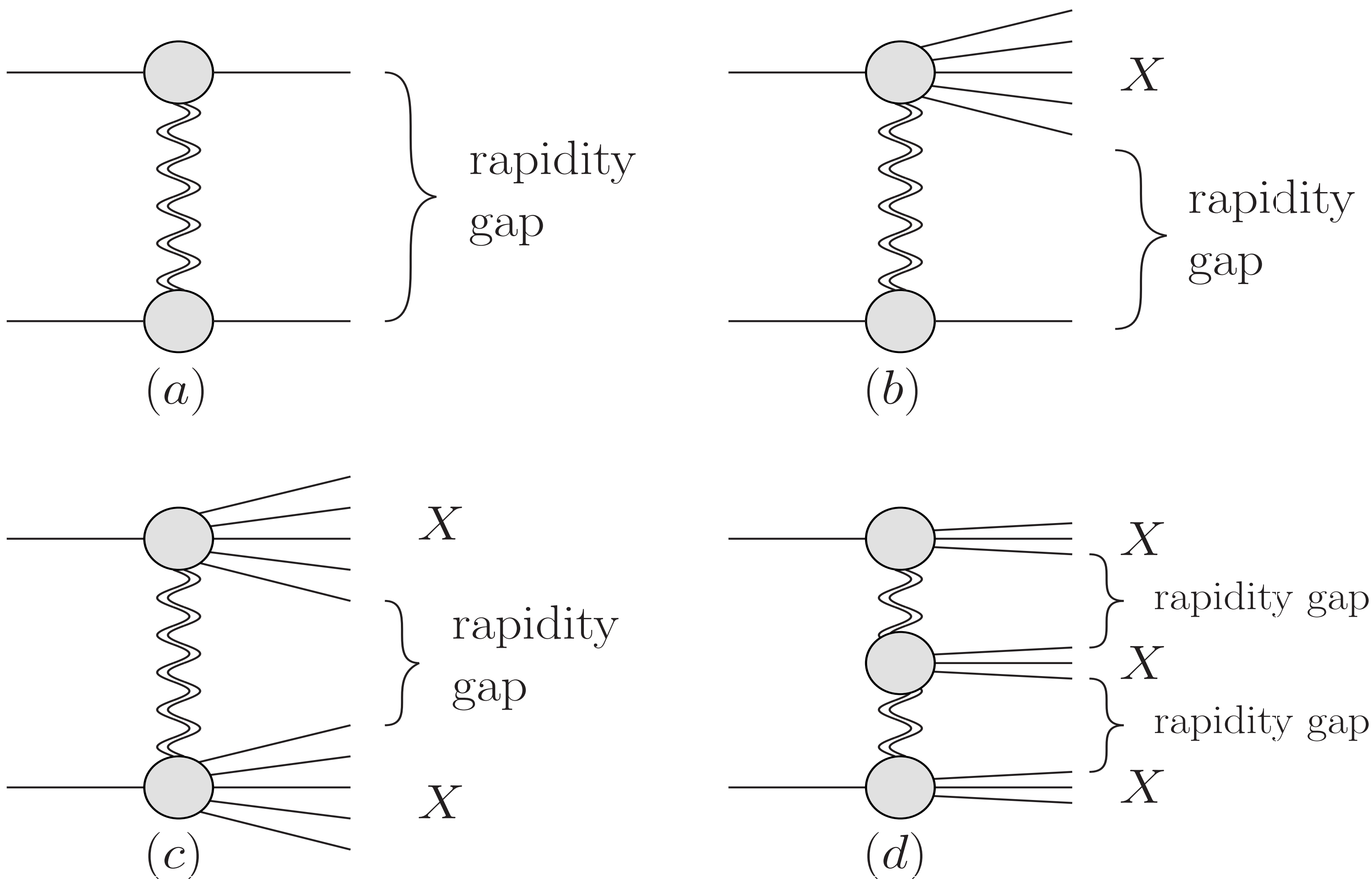
Unfortunately two answers:

1. For soft diffraction we do not know DOF

Candidates: constituent quarks, current quarks and gluons, colorless dipoles and so on

2. For hard diffraction we do know DOF: colorless dipolles (Mueller (1994))

Bookkeeping



Outline:

Unfortunately, I have to divide the talk in two parts:

1. **Soft diffraction : searching the correct degrees of freedom for soft interaction.**

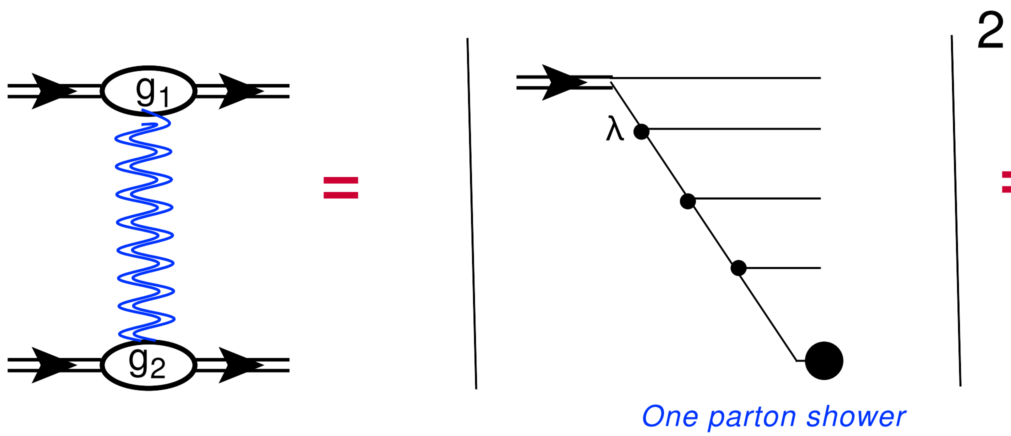
2. **Hard diffraction: the lessons from theory**

Soft D: high energy phenomenology

Soft Pomeron:

1. Parton model = $\lambda\phi^3$ theory with $< p_T > \approx \lambda$

Gribov, Feynmann (1967))



$$= \Sigma\, \lambda^{2n} \ln^n s / n! = s^{\Delta}$$

For $t \neq 0$ $\qquad \sigma \propto s^{\Delta(t = -q_T^2)}$

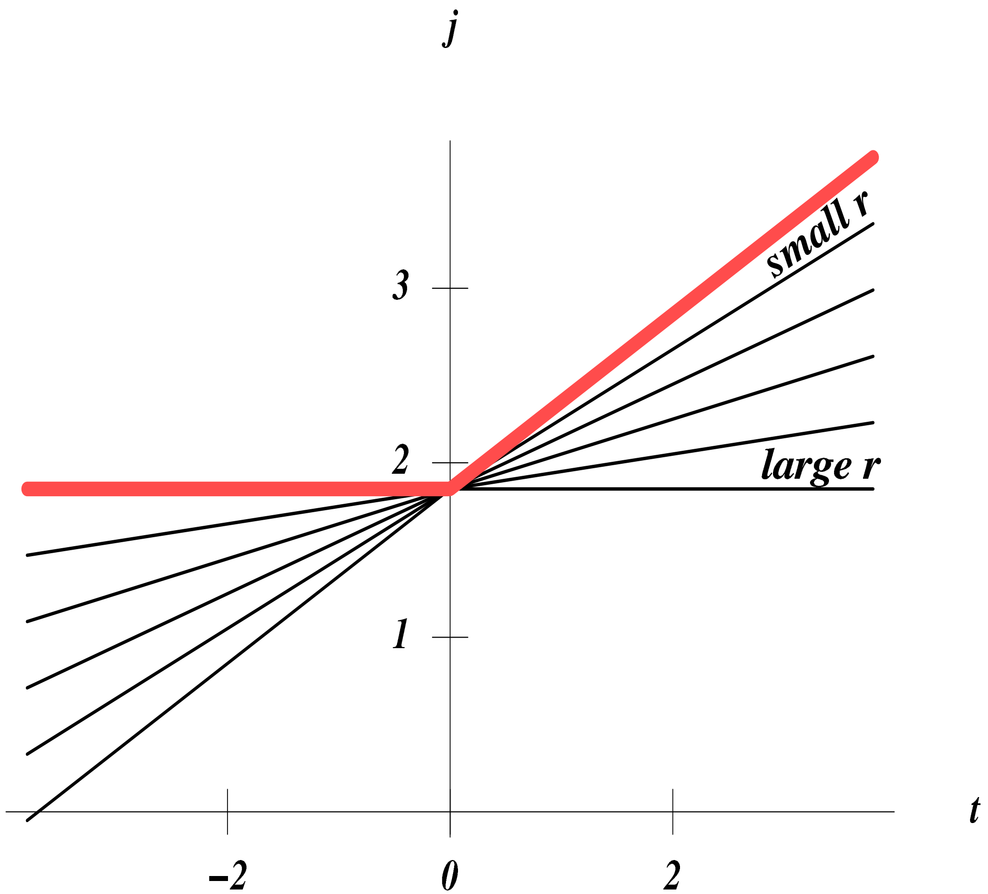
Pomeron trajectory: $\alpha(t) = 1 + \Delta + \alpha'_P\, t$

with $\alpha'_P \propto 1/\lambda^2$

2. Pomeron appears in CFT-AdS correspondence with

$$\Delta \approx 1; \qquad \alpha'_P = 0$$

Brower, Polchinski, Strassler and C. I. Tan, (2007)



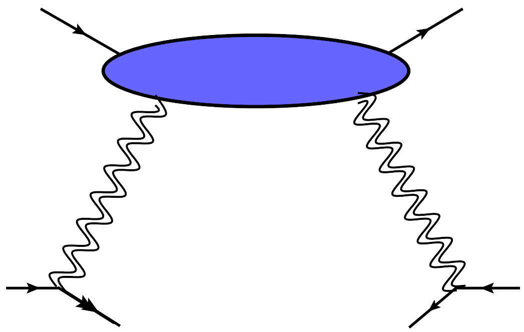
3. Pomeron phenomenology is able to describe the experimental data with

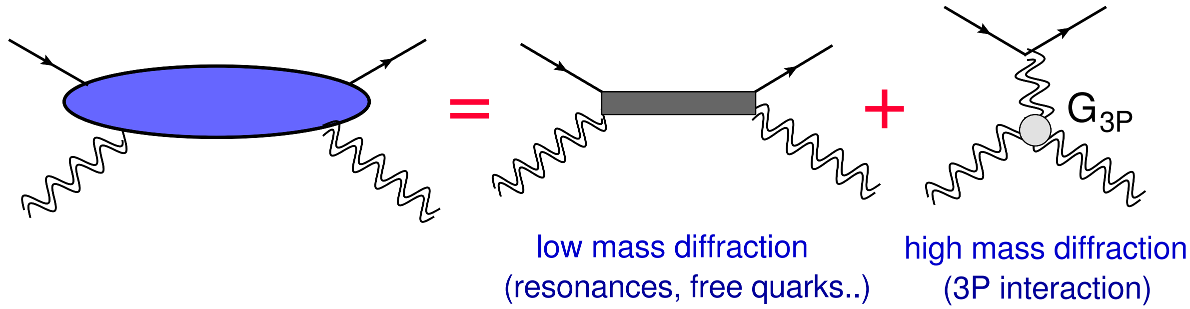
$$\Delta = 0.23 \qquad \text{and} \qquad \alpha'_P \approx 0.02\, GeV^{-2}$$

Durham group and Tel Aviv group (2000 - present)

W = $\sqrt{s}$ TeV	1.8	7	8	14	57
σ_{tot} mb	79.2	98.6	101	109	130
σ_{el} mb	18.5	24.6	25.2	27.9	34.8
$\sigma_{sd}(M^2 < 0.05s) mb$	8.2 + [2.07]	10.7 + [4.18]	10.9 + [4.3]	11.5 + [5.81]	13 + [8.68]
σ_{dd} mb	5.12 + [0.38]	6.2 + [1.24]	6.32 + [1.29]	6.78 + [1.59]	7.95 + [5.19]
$B_{el}\ GeV^{-2}$	17.4	20.2	20.4	21.6	24.6
σ_{inel} mb	60.7	74	75.6	81.1	95.2
$\frac{d\sigma}{dt}\vert_{t=0}\, mb/GeV^2$	326.34	506.4	530.7	608.11	879.2

Soft D: two different regions





$$\bullet \quad \frac{M^2 d\sigma_{diff}}{dM^2} = g_P^2(t) \left(\frac{s}{M^2}\right)^{2\Delta_P + 2\,\alpha'_P\,t} \sigma_{P-hadron}$$

$$\xrightarrow{M^2 \gg M_0^2} \quad g_P^2(t)\, G_{3P} \left(\frac{s}{M^2}\right)^{2\Delta_P + 2\,\alpha'_P\,t} \left(\frac{M^2}{M_0^2}\right)^{\Delta_P}$$

$$\int dM^2 \frac{d\sigma_{diff}}{dM^2} \propto \frac{1}{\Delta_P} \left(\ln s \text{ for } \Delta_P = 0 \right)$$

Low mass diffraction: Resonance contribution, secondary reggeons + ... , $\sigma_{LM} \propto 1/M^4$

High mass diffraction: Pomerons interaction,

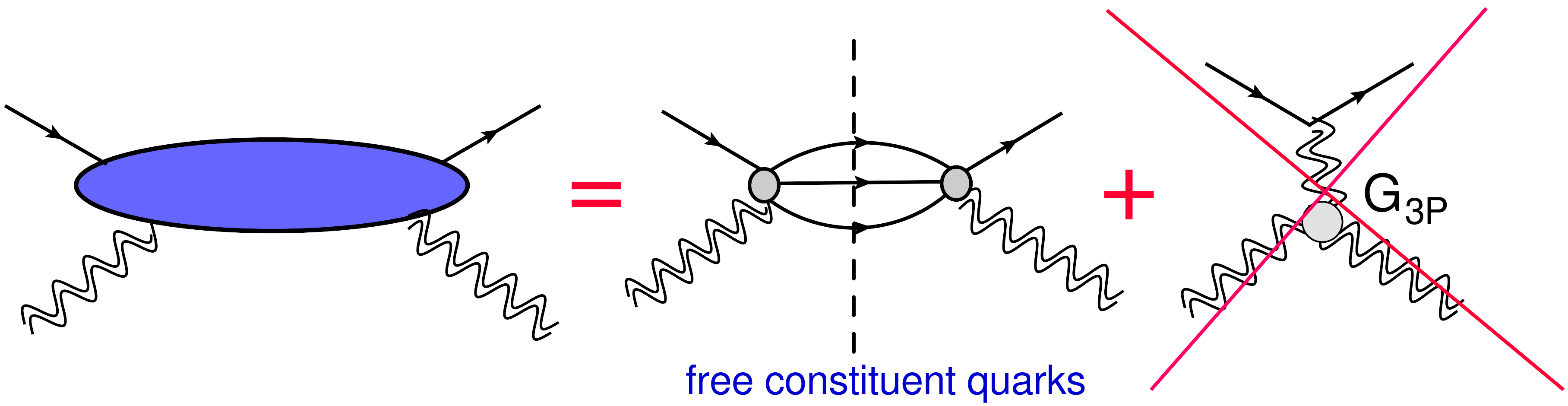
$$\sigma_{HM} \propto \left(1/M^2\right)^{1+\Delta}$$

- The division is only historic (Gustafson (2012)) but useful if you fit the value of Δ_P.
- Typical mass in high mass diffraction $M^2 \approx 150 - 200\, GeV^2$ ($\ln M^2/s_0 \approx\ 1/\Delta_P$)

Soft D: checking DOF

0. Hadrons are NOT correct DOF

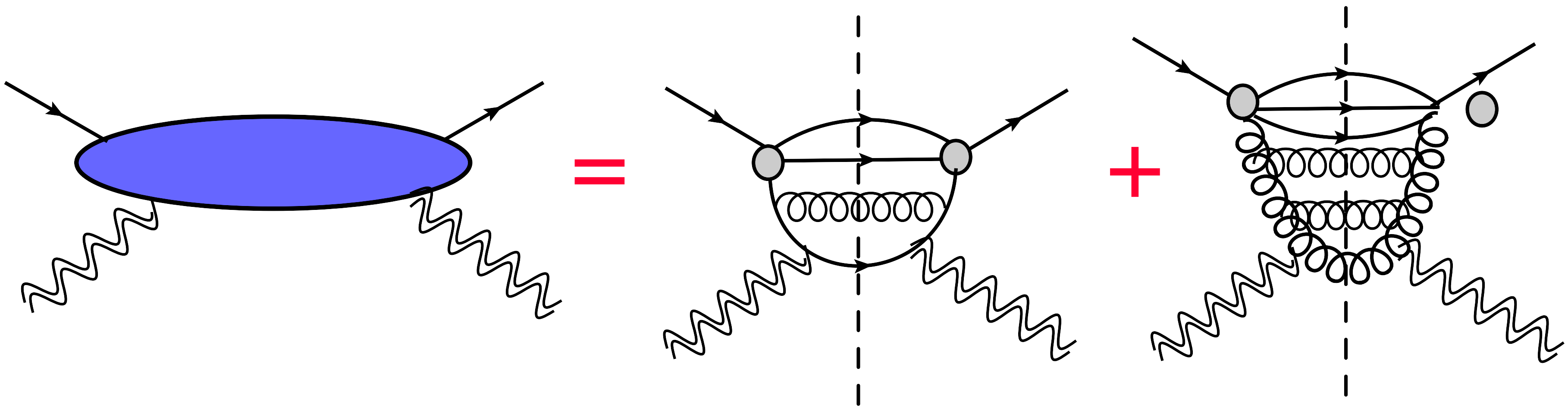
1. Constituent quarks:



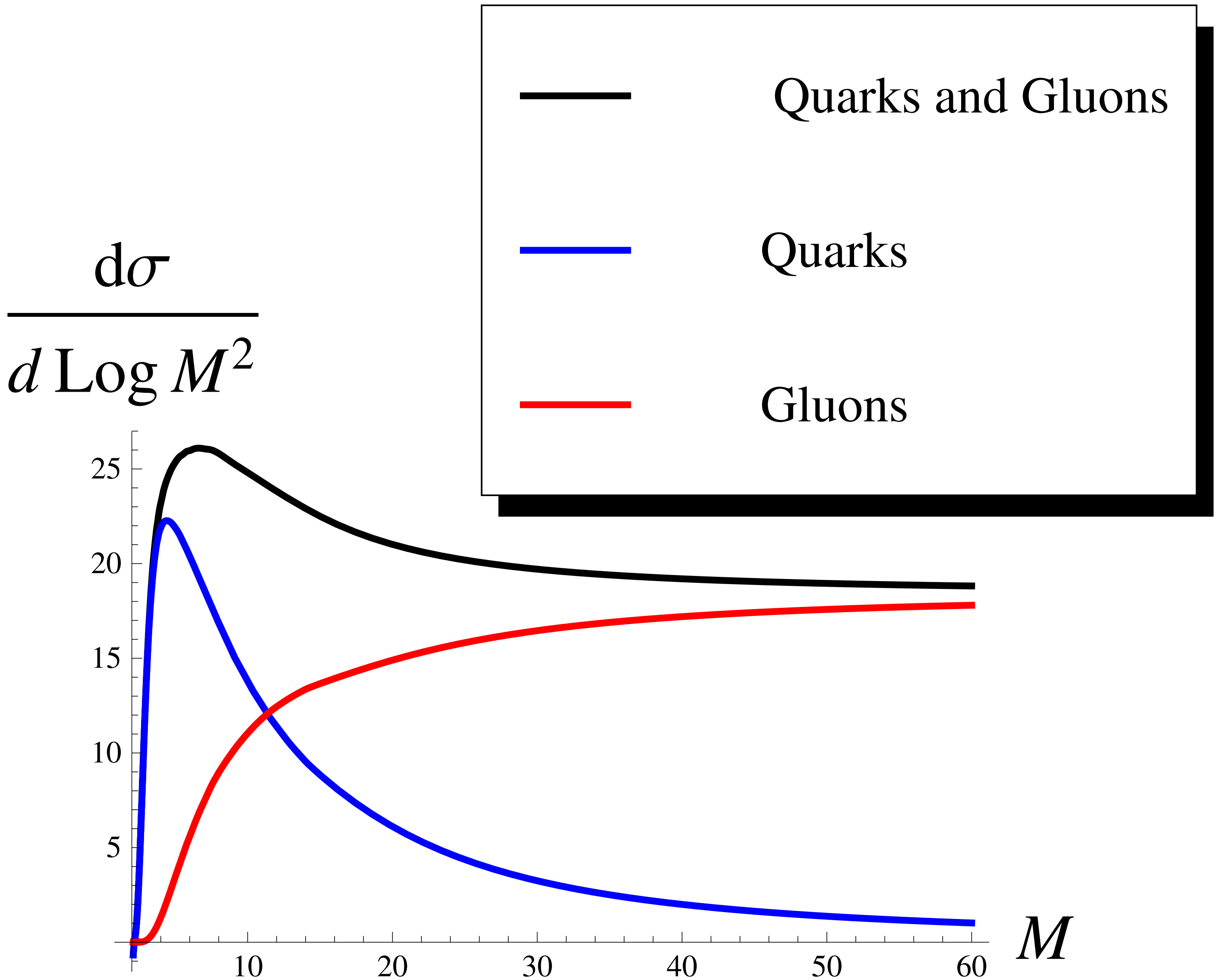
Pomeron probes the structure of the constituent quark ? !

2. Current quarks and gluons:

Pomeron as a hard probe



free quarks and gluons



- **Using GLM Low mass/high mass ratio**

3. Colourless dipoles:

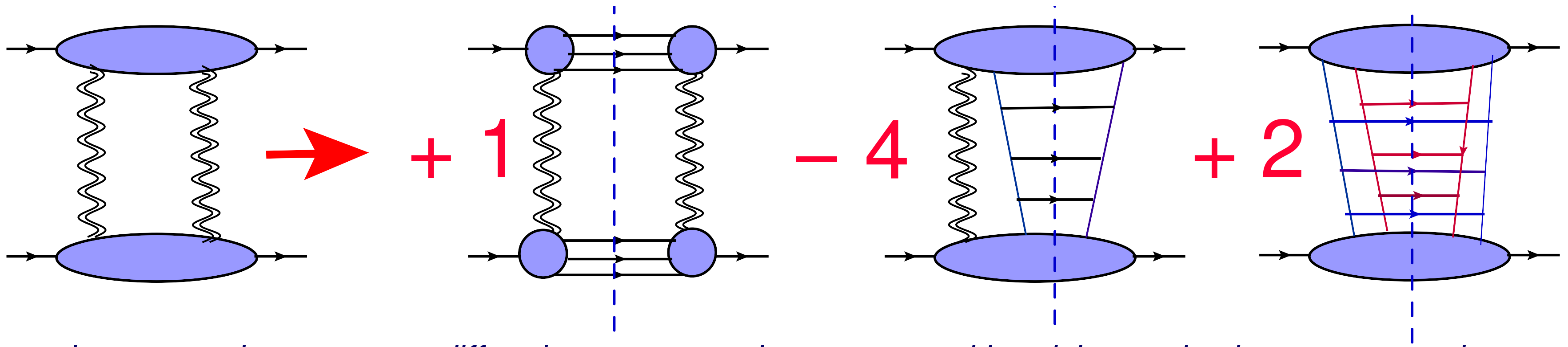
Kopeliovich + . . . (2000 - present)

+ : Colorless dipoles are correct DOF for high energy in pQCD:

They lead to similar picture at gluons for triple Pomeron term;

- : Low mass diffraction ?

Soft D: AGK cutting rules and relation between diffraction and two parton showers production



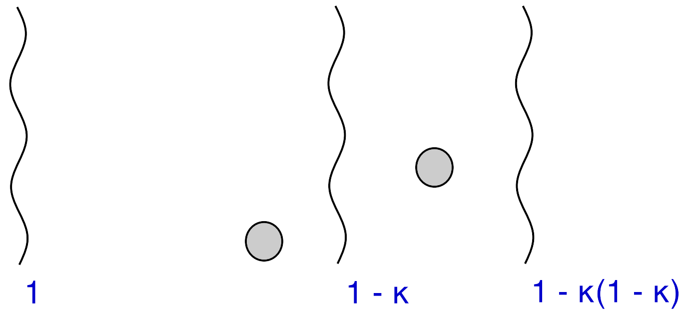
total cross section | *diffraction cross section* | *multiparticle production cross sections*

multiplicity < n > | *multiplicity 2 < n >*

$< n >$ - **average multiplicity**

These AGK rules are correct for QCD

Proof:



Unitarity constraint:

$2\text{Im}A(b) = |A(b)|^2 + G(b)$

For nucleon:

G(b) $= \kappa$ and $\sigma_{in} = \kappa S$ and

$\sigma_{el} = (\kappa/2)^2 S$

For deuteron:

$\sigma^D_{in} = (\kappa + \kappa(1-\kappa))\, S = 2\sigma^N_{in} - \kappa^2 S$

$\sigma^D_{el} = (2\kappa/2)^2 S$

$$\sigma^D_{tot} = 2\sigma^N_{tot} - \Delta\sigma$$

- $\Delta\sigma = \Delta\sigma_{el} + \Delta\sigma_{in} = -\frac{\kappa^2}{2} S = -\boxed{\frac{\left(\sigma^N_{tot}\right)^2}{2\pi R^2}}$

$\Delta\sigma_{el} = \frac{\kappa^2}{2} S$ **(1)** $\qquad \Delta\sigma_{in}\,(2 < n >) = \kappa^2 S$ **(2)**

$\Delta\sigma_{in}\,(< n >) = -\, 2\kappa^2 S$ **(-4)**

Soft D: Unitarity constraints and eikonal approach

- **Unitarity constraints** (i,l - set of quantum numbers for DOF):

$$2\,\mathrm{Im}A^{el}_{il}el\left(s,b\right) = |A^{el}_{il}\left(s,b\right)|^2 + G_{il}\left(s,b\right)$$

where $G_{il}\left(s,b\right)$ is the sum of all inelastic processes.

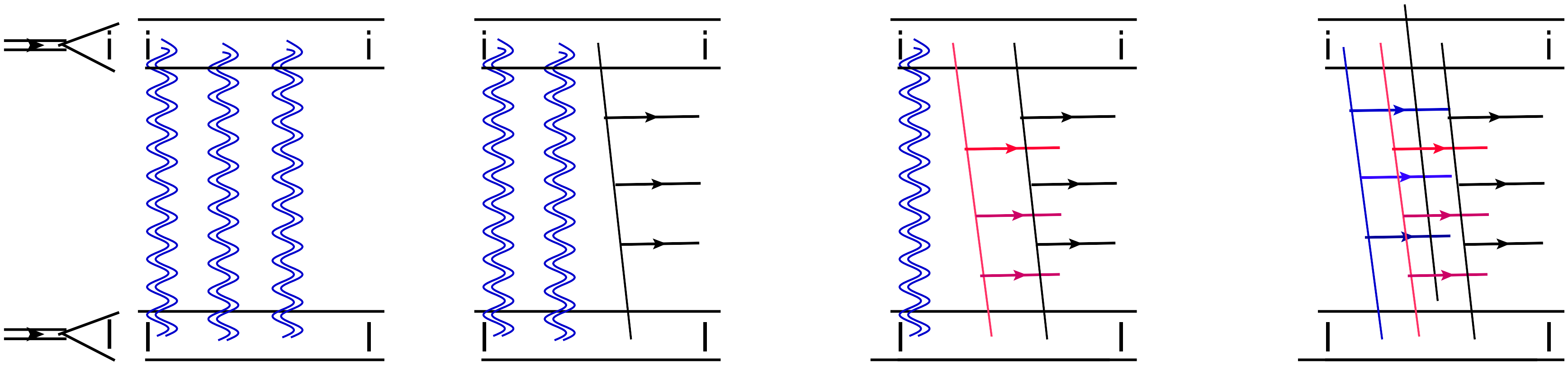
Solution for high energy (pure imaginary amplitudes):

$$A^{el}_{il} = i\left(1 - e^{-\Omega_{il}(s,b)/2}\right); \qquad G_{il}\left(s,b\right) = 1 - e^{-\Omega_{il}(s,b)};$$

$\exp\left(-\Omega_{il}\left(s,b\right)\right) \Longleftarrow$ probability to have only elastic scattering

$$\bullet \quad \Omega\left(s,b\right) \quad = \quad \int d^2b' g_i\left(\vec{b}-\vec{b}'\right)\, g_l\left(\vec{b}'\right)\, s^{\Delta}$$

$\Longleftarrow$ **one Pomeron exchange**



Diffraction + elastic | One parton shower | Two parton showers | Three parton showers

$(1 - \exp(-\Omega_{i\,l}/2))^2$ | $\Omega_{i\,l}\exp(-\Omega_{i\,l})$ | $(\Omega^2_{i\,l}/2!)\exp(-\Omega_{i\,l})$ | $(\Omega^3_{i\,l}/3!)\exp(-\Omega_{i\,l})$

Diffraction:

$$\sigma_{diff} = \sum_{i,l} < h_1|i >^2 < h_2|l >^2 \left(1 - e^{-\Omega_{il}(s,b)/2}\right)^2$$

n-parton showers productions:

$$\sigma_n = \sum_{i,l} < h_1|i >^2 < h_2|l >^2 \frac{\Omega^n_{il}(s,b)}{n!}\, e^{-\Omega_{il}(s,b)}$$

What are $|i>$???

Practice:

1. Several states with phenomenological $< h|i >$ for LMD

Example: two channel model.

$$\psi_h = \alpha\,\psi_1 + \beta\,\psi_2\,; \qquad\qquad \psi_D = -\,\beta\,\psi_1 + \alpha\,\psi_2\,;$$

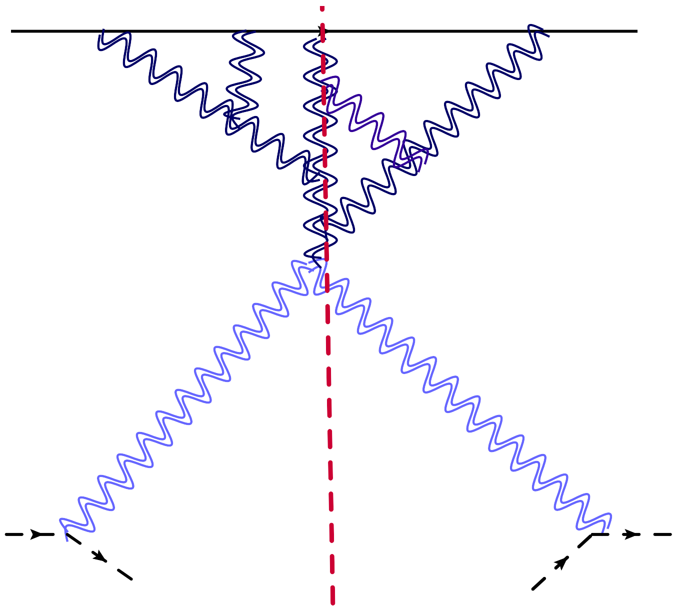
where, $\alpha^2 + \beta^2 = 1$

$$\begin{aligned} a_{el}(s,b) &= i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4\, A_{2,2}\}, \\ a_{sd}(s,b) &= i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2}\}, \\ a_{dd} &= i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\}. \end{aligned}$$

$$\sigma_{tot}(s) = 2\int d^2b\, a_{el}\,(s,b) \qquad \sigma_{el}(s) = \int d^2b\,|a_{el}\,(s,b)\,|^2,$$

$$\sigma_{sd}(s = \int d^2b\,|a_{sd}\,(s,b)\,|^2; \qquad \sigma_{dd}(s) = \int d^2b\,|a_{dd}\,(s,b)\,|^2.$$

2. Pomeron interaction for HMD

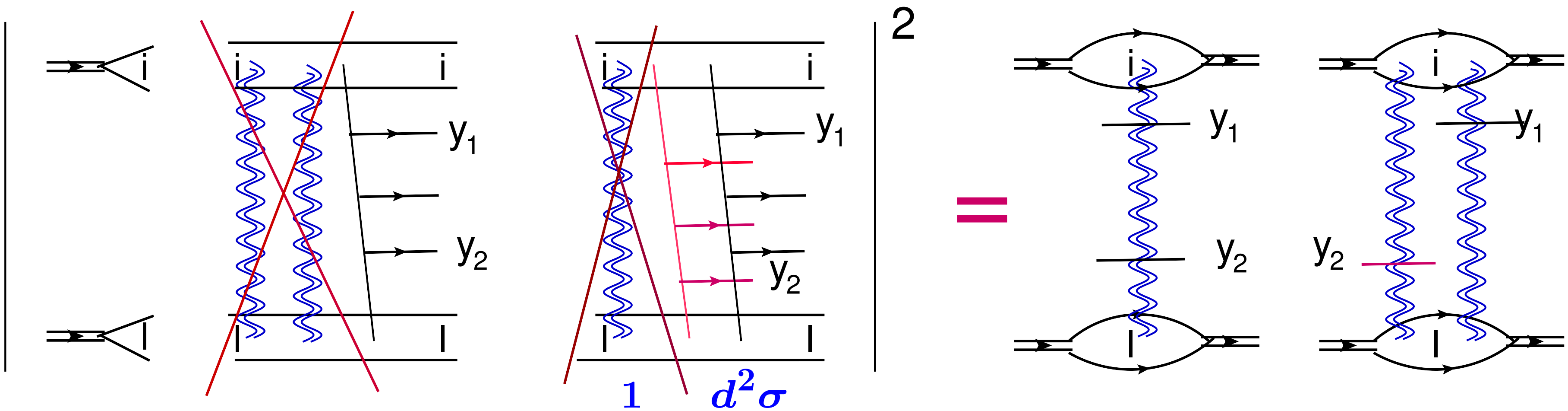


Large mass single diffraction production

Fortunately, for $\alpha'_P = 0$ we can sum all diagrams with triple Pomeron vertices

Unfortunately, we do not know why we can restrict ourselves with triple Pomeron interaction

Long Range Rapidity Correlations



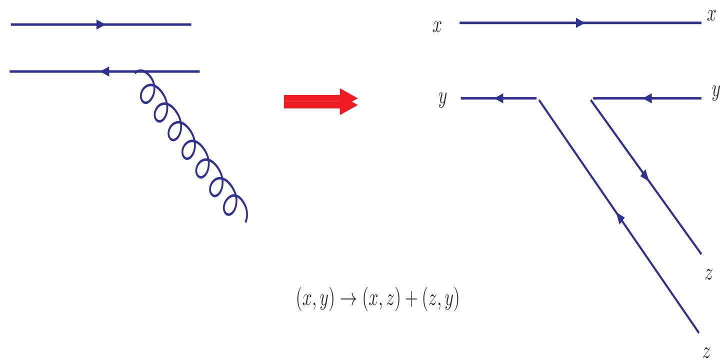
- $$R(y_1,y_2) = \frac{\frac{1}{\sigma_{in}}\frac{d^2\sigma}{dy_1 dy_2}}{\frac{1}{\sigma_{in}}\frac{d\sigma}{dy_1}\,\frac{1}{\sigma_{in}}\frac{d\sigma}{dy_2}} - 1$$

$$= \sigma_{in}\frac{\sum_{i,l} < h_1|i >^2 < h_2|l >^2 \int d^2b\,\Omega^2_{il}(s,b)}{\left(\sum_{i,l} < h_1|i >^2 < h_2|l >^2 \int d^2b\,\Omega_{il}(s,b)\right)^2} - 1$$

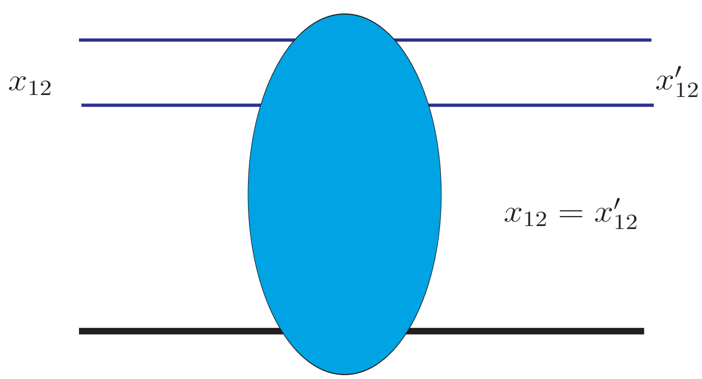
$$\approx \frac{< b^2 >_{\sigma_{in}} < b^2 >_{\Omega^2}}{\left(< b^2 >_{\Omega}\right)^2} - 1 \xrightarrow{s \gg s_0} \ln(s/s_0)$$

Hard diffraction

Correct DOF: colorless dipoles

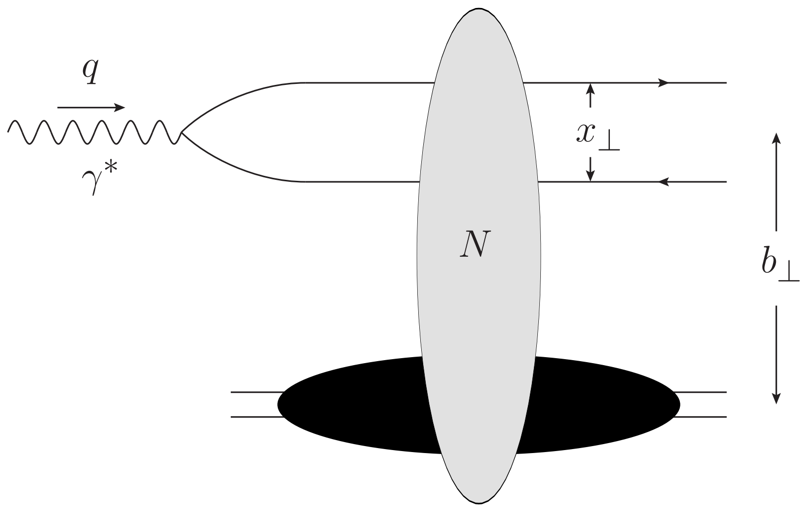


Interaction: dipole → two dipoles decay



Interaction: dipole does not change the size during scattering

Hard D: low mass



$\sigma^{\gamma^* A} =$ (Kovchegov & McLerran(1999))

$$\int \frac{d^2 x_\perp}{4\pi} d^2 b \int_0^1 \frac{dz}{z(1-z)} |\Psi^{\gamma^2 \to q\bar{q}}(x_\perp, x)|^2$$

$$\times\; N^2(x_\perp, b; Y)$$

$$\sigma^{\gamma^* A}_{tot} = \left\{ N^2 \;\to\; 2\,N \right\}$$

- $\int_0^1 \frac{dz}{z\,(1-z)} |\Psi_T^{\gamma^* \to q\bar{q}}(x_\perp, z)|^2 \approx 4\,N_c \sum_f \frac{\alpha_{EM}\,Z_f^2}{\pi}\,Q^2 \int_0^\infty dz\,z\,\left[K_1(x_\perp\,Q\,\sqrt{z})\right]^2$

$= \frac{16\,N_c}{3} \sum_f \frac{\alpha_{EM}\,Z_f^2}{\pi} \frac{1}{Q^2\,x_\perp^4}$

- $\int_0^1 \frac{dz}{z\,(1-z)} |\Psi_L^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \approx 16\,N_c \sum_f \frac{\alpha_{EM}\,Z_f^2}{\pi}\,Q^2 \int_0^\infty dz\,z^2\,\left[K_0(x_\perp\,Q\,\sqrt{z})\right]^2$

$= \frac{512}{15}\,N_c \sum_f \frac{\alpha_{EM}\,Z_f^2}{\pi} \frac{1}{Q^4\,x_\perp^6}.$

Using $N\,(x_\perp, x) \;\propto\; x_\perp^2$ we see

1. $\sigma_L^{diff} \;\leftarrow$ **short distances** $(x_\perp \propto 1/Q)$
2. $\sigma_T^{diff} \;\leftarrow$ **long distances** $(x_\perp \propto 1/\mu_{soft\ scale})$ **!!!???**
3. σ_L^{tot} **and** $\sigma_T^{tot} \;\leftarrow$ **short distances** $(x_\perp \propto 1/Q)$

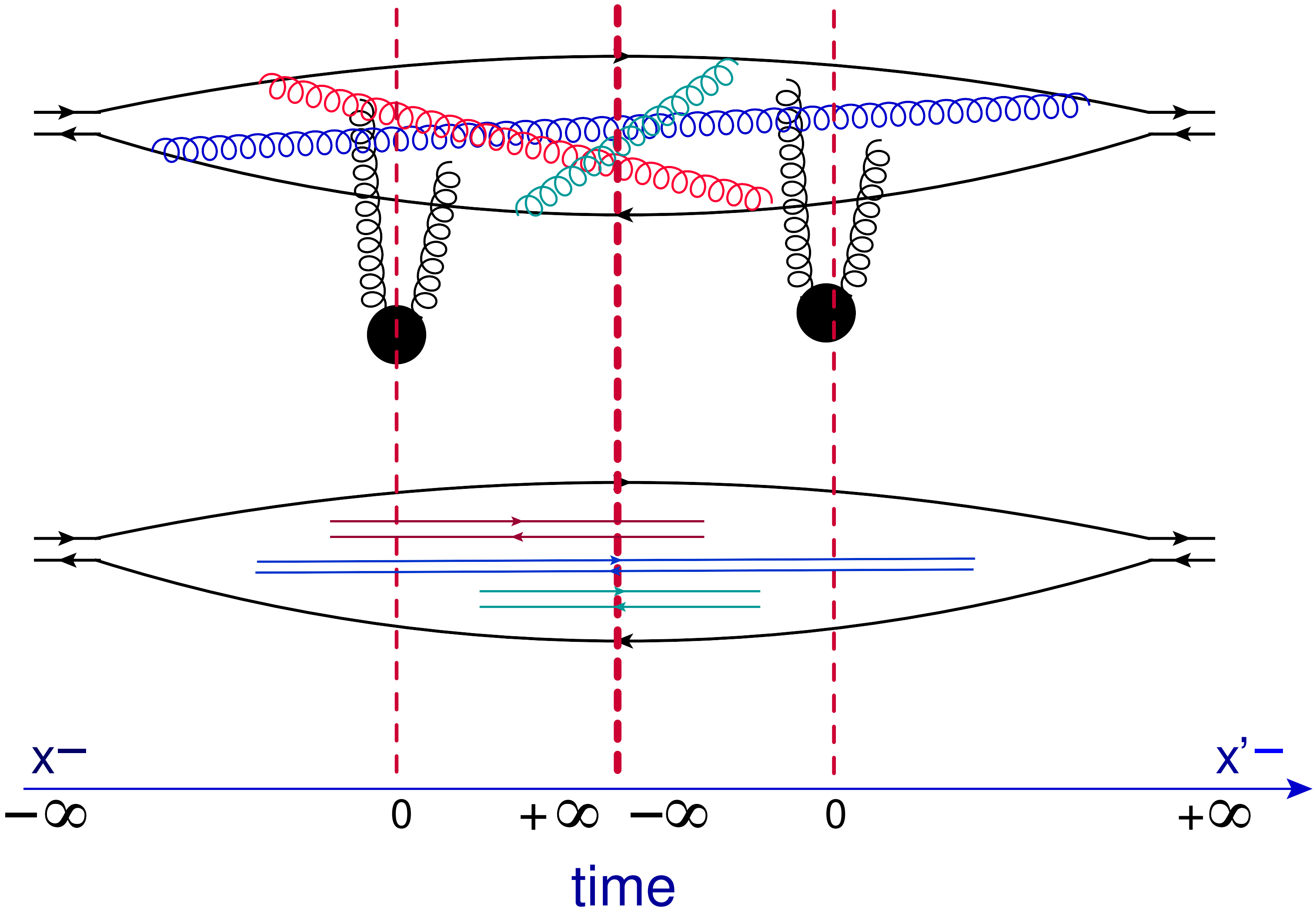
$\sigma_T^{diff} \;\propto\; 1/\mu^2_{soft\ scale} \quad \gg$ **!!!!!** $\quad \sigma_{tot} \;\propto\; 1/Q^2$

Saturation/CGC: $N\left(x_{\perp}, x\right) \propto 1$ for $x_{\perp} > 1/Q_s(x)$

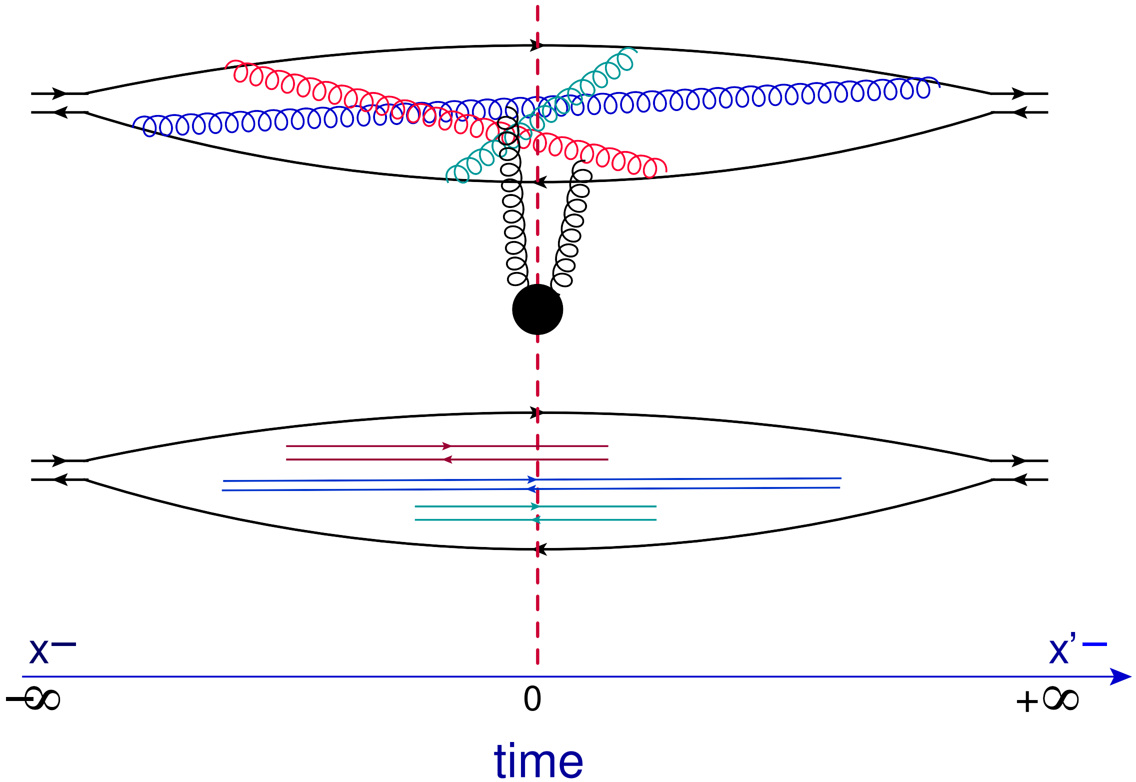
$N\left(x_{\perp}, x\right) \propto 1$ leads to both σ^{diff}_{L} and σ^{diff}_{T} $\leftarrow$ short distances $\left(x_{\perp} \leq 1/Q_s\right)$

$\sigma_{diff}/\sigma_{tot} \Longrightarrow 1/2$

Hard D: $\gamma^* \rightarrow q\bar{q} + G$



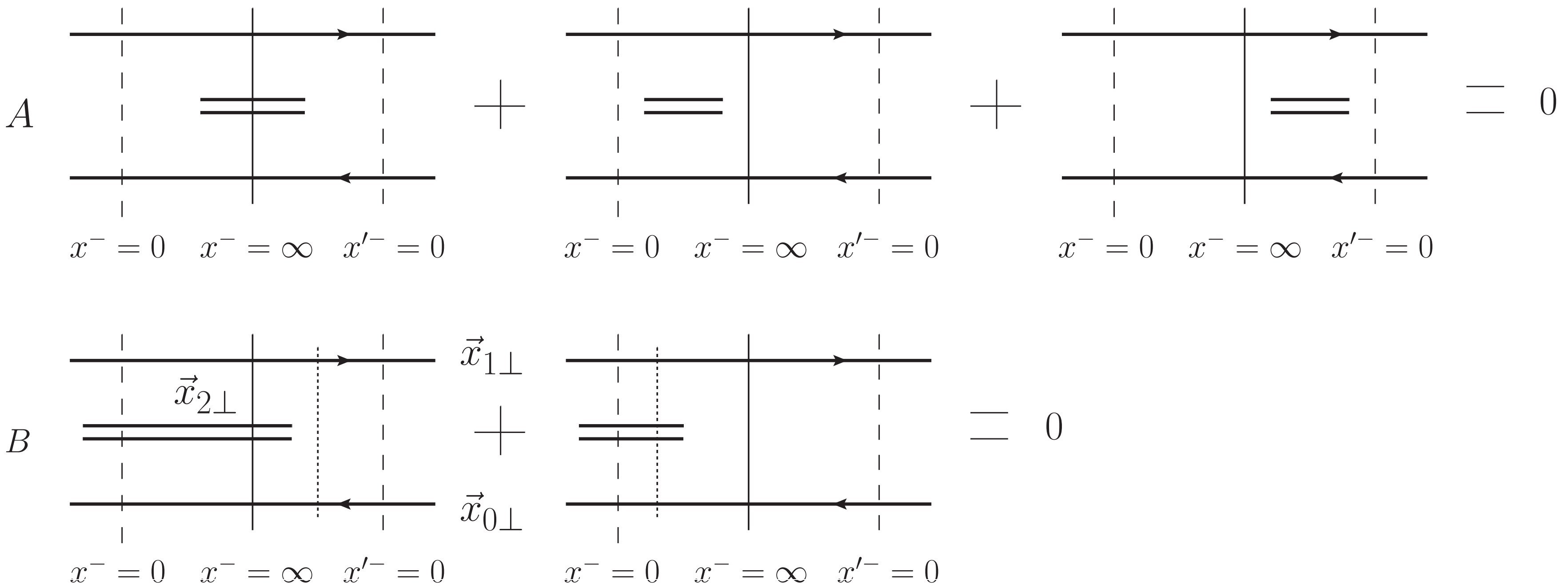
$\gamma^* \rightarrow q\bar{q} + G$ component of the total cross section



$$\sigma_{tot} = 2 \int d^2x_{02} d^2x_{01} \int_x^1 \frac{dz}{z} |\Psi_{\gamma^* \to q\bar{q}G}|^2 \sum_{i,k} N\left(x_{ik}, x\right)$$

Hard D: cancellation of the interactions in the final state

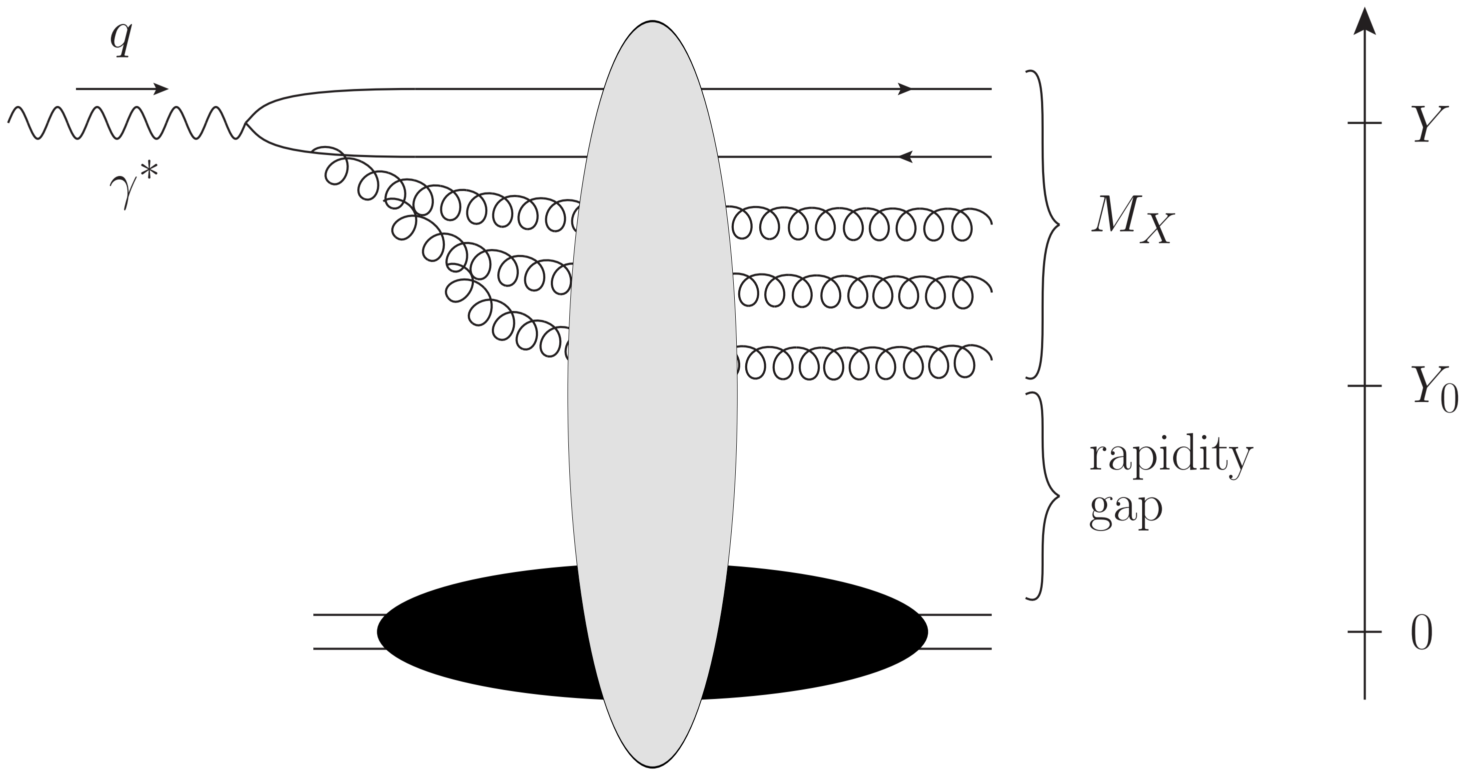
(Chen & Mueller (1995))

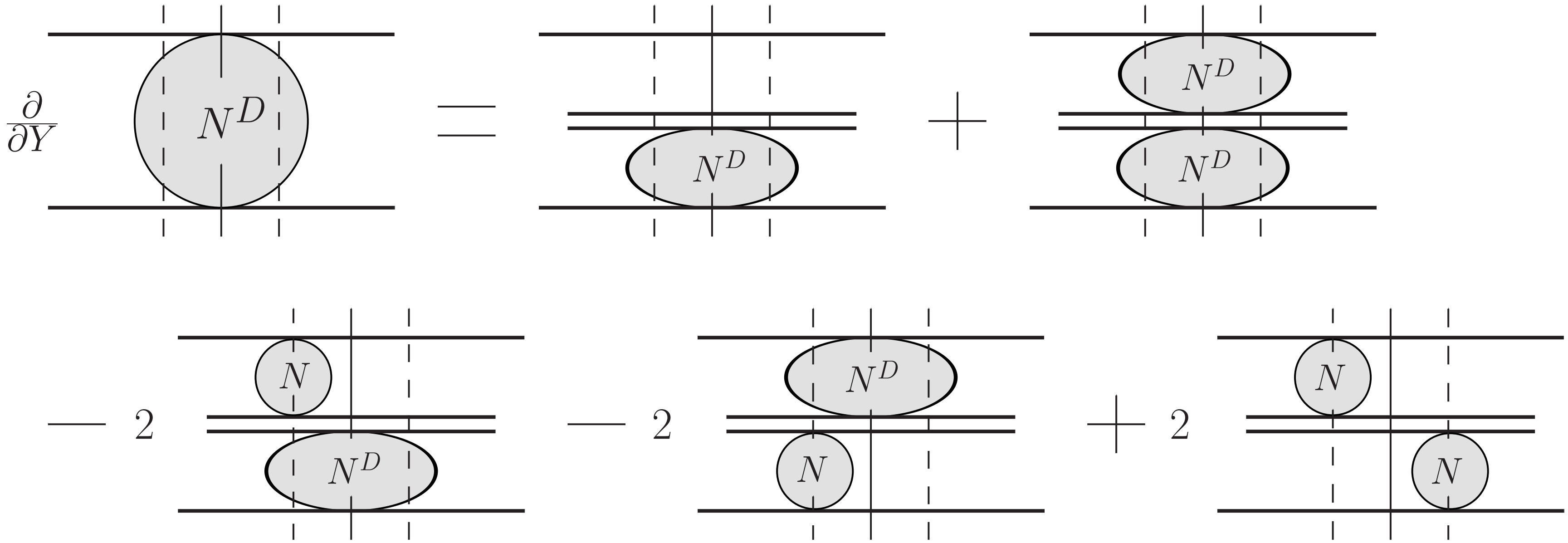


$$\sigma_{\gamma^* \to q\bar{q}G} = 2 \int d^2x_{02} d^2x_{01} \int_x^1 \frac{dz}{z} |\Psi_{\gamma^* \to q\bar{q}G}|^2 \Big(\sum_{i,k} N(x_{ik}, x) \Big)$$

Hard D: General equation

(Kovchegov & Levin (2000))





- $$\frac{\partial N^D(\vec{x}_{1\perp},\vec{x}_{0\perp},Y,Y_0)}{\partial Y} = \frac{\bar{\alpha}_S}{2\,\pi}\int d^2x_2\,\frac{x^2_{10}}{x^2_{20}\,x^2_{21}}$$

$$\Big[N^D(\vec{x}_{1\perp},\vec{x}_{2\perp},Y,Y_0) + N^D(\vec{x}_{2\perp},\vec{x}_{0\perp},Y,Y_0) - N^D(\vec{x}_{1\perp},\vec{x}_{0\perp},Y,Y_0)$$

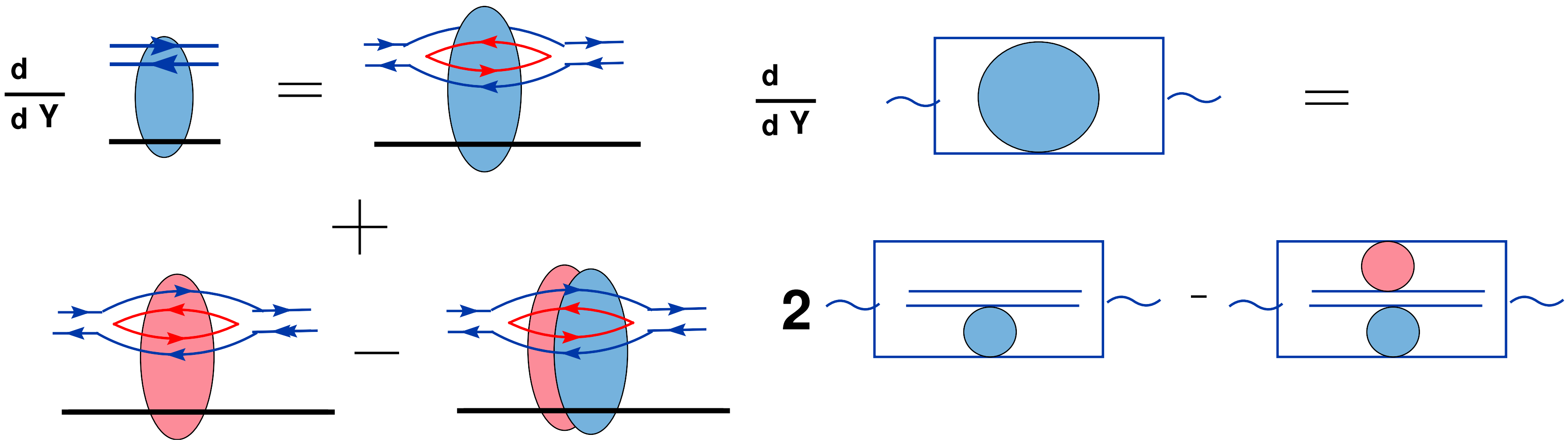
$$+ N^D(\vec{x}_{1\perp},\vec{x}_{2\perp},Y,Y_0)\,N^D(\vec{x}_{2\perp},\vec{x}_{0\perp},Y,Y_0)$$

$$-\;2\,N(\vec{x}_{1\perp},\vec{x}_{2\perp},Y)\,N^D(\vec{x}_{2\perp},\vec{x}_{0\perp},Y,Y_0)$$

$$-\;2\,N^D(\vec{x}_{1\perp},\vec{x}_{2\perp},Y,Y_0)\,N(\vec{x}_{2\perp},\vec{x}_{0\perp},Y) + 2\,N(\vec{x}_{1\perp},\vec{x}_{2\perp},Y)\,N(\vec{x}_{2\perp},\vec{x}_{0\perp},Y)\Big]$$

Initial condition: $N^D(\vec{x}_{1\perp},\vec{x}_{0\perp},Y{=}Y_0,Y_0) = N^2(\vec{x}_{1\perp},\vec{x}_{0\perp},Y{=}Y_0)$

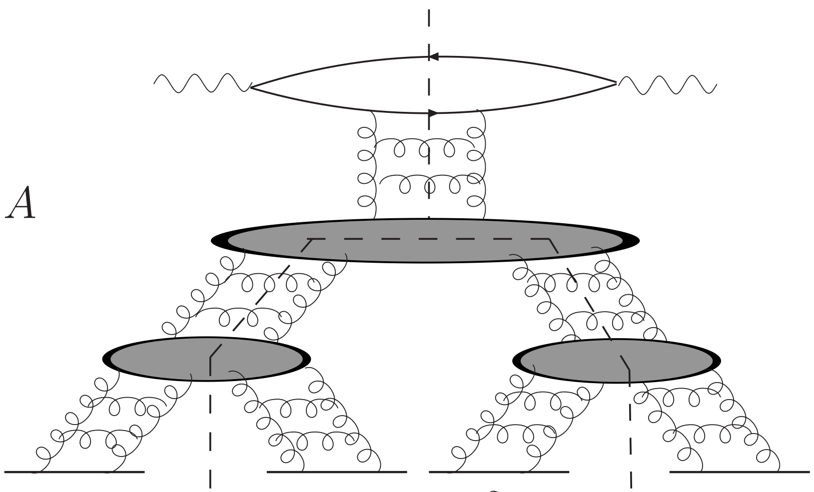
Balitsky-Kovchegov non-linear equation



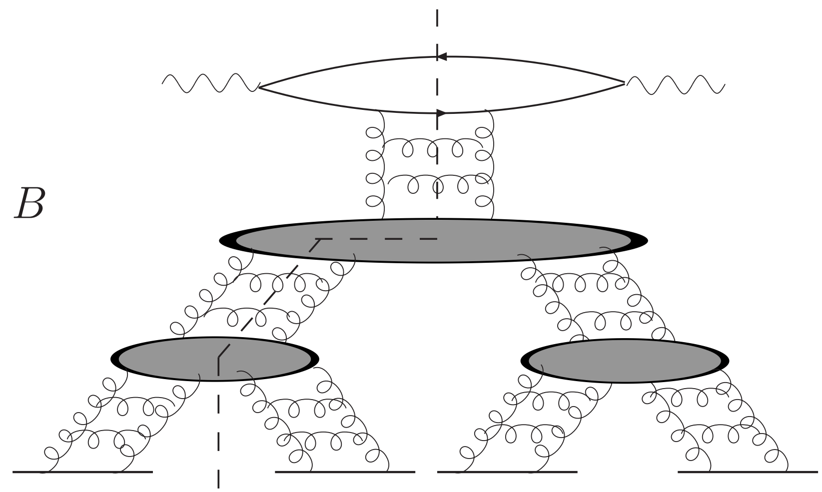
$$\frac{\partial}{\partial Y} N(\vec{x}_{10}, \vec{b}_\perp, Y) = \frac{\alpha_S N_c}{2\,\pi^2} \int d^2 x_2 \, \frac{x_{10}^2}{x_{20}^2\, x_{21}^2}$$

$$\times \left[N\left(\vec{x}_{12}, \vec{b}_\perp + \frac{\vec{x}_{20}}{2}, Y\right) + N\left(\vec{x}_{20}, \vec{b}_\perp + \frac{\vec{x}_{21}}{2}, Y\right) - N(\vec{x}_{10}, \vec{b}_\perp, Y) \right.$$

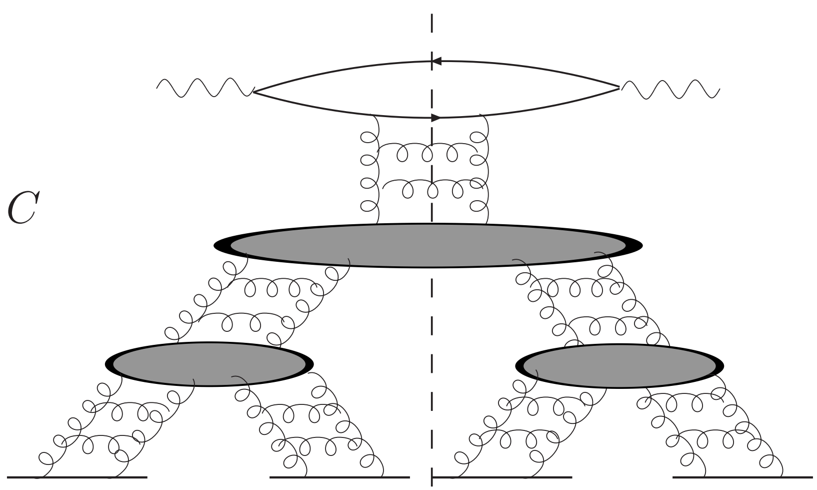
$$\left. - \; N\left(\vec{x}_{12}, \vec{b}_\perp + \frac{\vec{x}_{20}}{2}, Y\right) \; N\left(\vec{x}_{20}, \vec{b}_\perp + \frac{\vec{x}_{21}}{2}, Y\right) \right]$$



$\left(N^D\right)^2$



$-\,4\,N^D\,N_0$



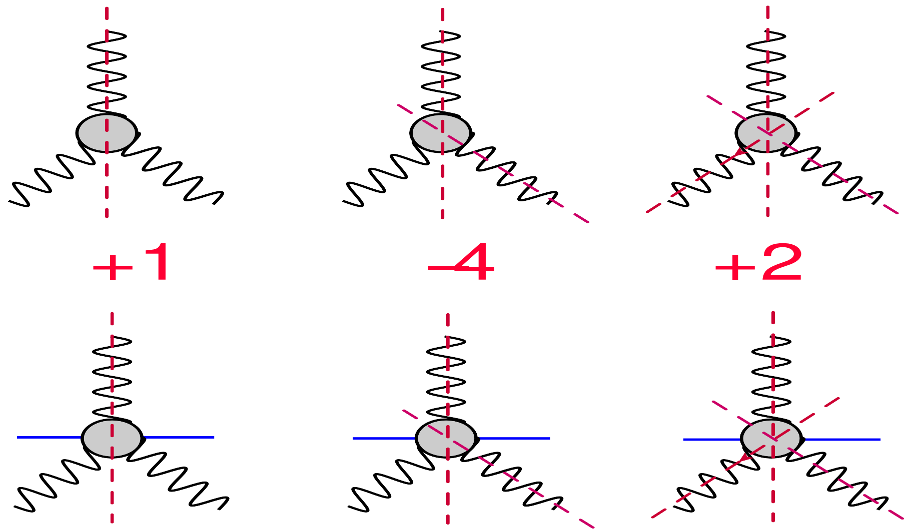
$2\,N_0^2$

BFKL Pomeron calculus
$\Longrightarrow$
t-channel unitarity

Hard D & MPI

AGK cutting rules in QCD:

(Kovchegov & Tuchin(2002), Kovchegov & Jalilian-Marian (2004),Marquet (2005), Braun(2006), Kovner & Lublinsky (2006), Hentschinski, Weigert & Schafer(2006), Gelis & Venugopalan(2007), Bartels, Salvadore & Vacca(2008), Levin &Prygarin(2008),Kormilitzin, Levin & Prygarin(2008))



No AGK cutting rules

General equation for MPI:

(Kormilitzin, Levin & Prygarin (2008)

- $$\frac{\partial M(\vec{x}_{1\perp},\vec{x}_{0\perp},Y,Y_0)}{\partial Y} = \frac{\bar{\alpha}_S}{2\,\pi}\int d^2x_2\,\frac{x^2_{10}}{x^2_{20}\,x^2_{21}}$$

$$\Big[M(\vec{x}_{1\perp},\vec{x}_{2\perp},Y,Y_0) + M(\vec{x}_{2\perp},\vec{x}_{0\perp},Y,Y_0) - M(\vec{x}_{1\perp},\vec{x}_{0\perp},Y,Y_0)$$

$$+ M(\vec{x}_{1\perp},\vec{x}_{2\perp},Y,Y_0)\,M(\vec{x}_{2\perp},\vec{x}_{0\perp},Y,Y_0)$$

$$-\;2\,N(\vec{x}_{1\perp},\vec{x}_{2\perp},Y)\,M(\vec{x}_{2\perp},\vec{x}_{0\perp},Y,Y_0)$$

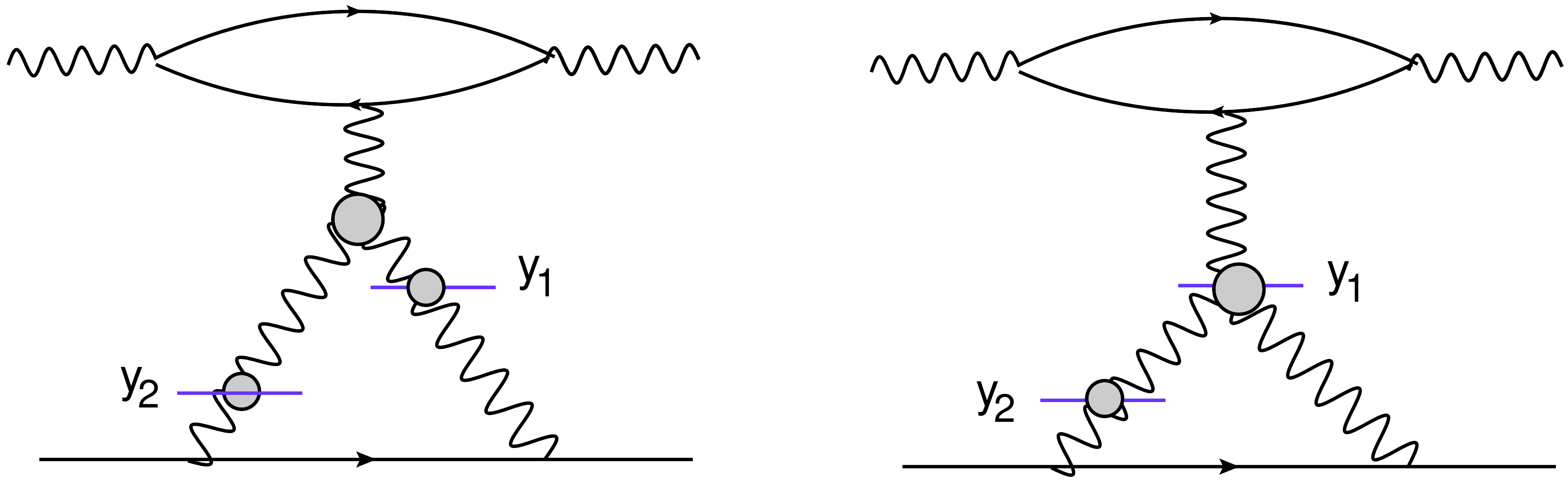
$$-\;2\,M(\vec{x}_{1\perp},\vec{x}_{2\perp},Y,Y_0)\,N(\vec{x}_{2\perp},\vec{x}_{0\perp},Y)$$

$$+\;2\,N(\vec{x}_{1\perp},\vec{x}_{2\perp},Y)\,N(\vec{x}_{2\perp},\vec{x}_{0\perp},Y)\Big]$$

Initial condition: $M\,(\vec{x}_{1\perp},\vec{x}_{0\perp},Y\!=\!Y_0,Y_0) = e^{-\Omega}\Omega^n/n!$

where $\Omega = 2\,N\,(\vec{x}_{1\perp},\vec{x}_{0\perp},Y\!=\!Y_0)$

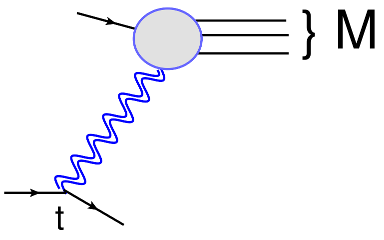
Correlations:



Correlations depend on the emission vertex and they have more complex form than from the AGK cutting rules

Experiments that could shed light on DOF

1. $\frac{d^2\sigma_{diff}}{dM^2 dt}$

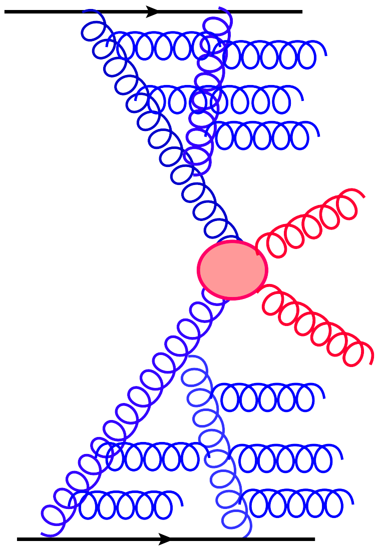


Special attention: large $t > 1\,GeV^2$

2. Production of dijets, two hadrons and their correlations in diffraction. p_T dependence from large to small p_{1T} and p_{2T};

3. Two hadron (two jet) correlations versus rapidity and p_T in central rapidity region inclusively and with fixed (large) multiplicity. p_T dependence from large to small p_{1T} and p_{2T};

4. 1 - 3 but for nuclear target. Advantage: $t \approx 1/R^2_A \to 0$, more can be calculated



5. Large p_T jet versus multiplicity of produced hadrons

Thank you