pQCD and Double Parton Interactions

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Where and how to look for double (multiple) hard interactions

A new subject

Theoretically, bound to be complicated an issue (many-body problem)

New feature : **fake on-mass-shell singularities in momentum space**

New object : **Generalized Double Parton Distributions** ${}_2\mathrm{GPD}$

New confusion : **MPI or not MPI ?..**

Perturbative QCD effects in MPI

Plentitude of *hard scales :*

jet transverse momenta

transverse momentum imbalances of jet pairs

total tr. momentum of the 4-jet system (imbalance of imbalances)

Three facets of pQCD in MPI

logarithmic evolution of parton distributions

double logarithmic form factor effects (ratios of hard scales involved)

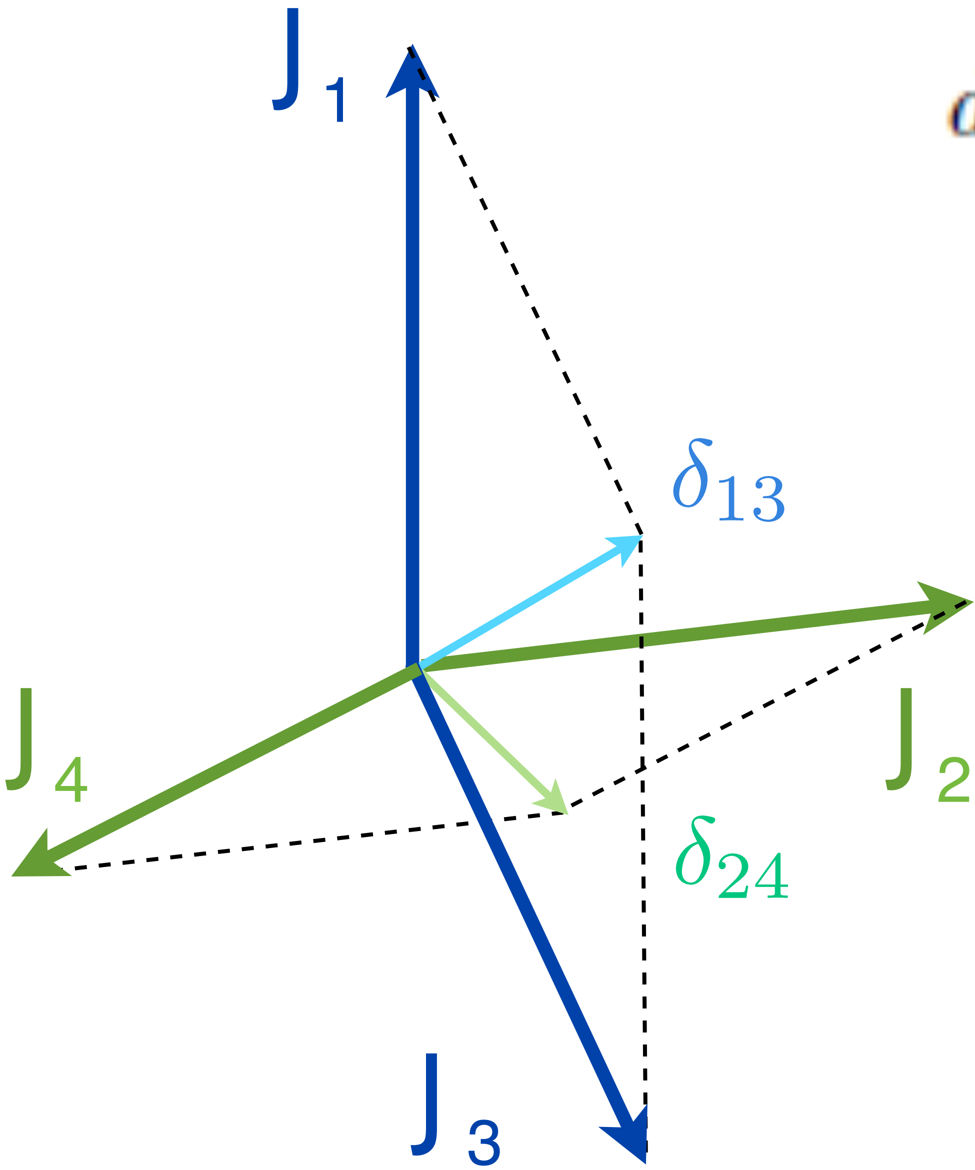
new PT mechanism - an interplay between NP a PT initial parton flows

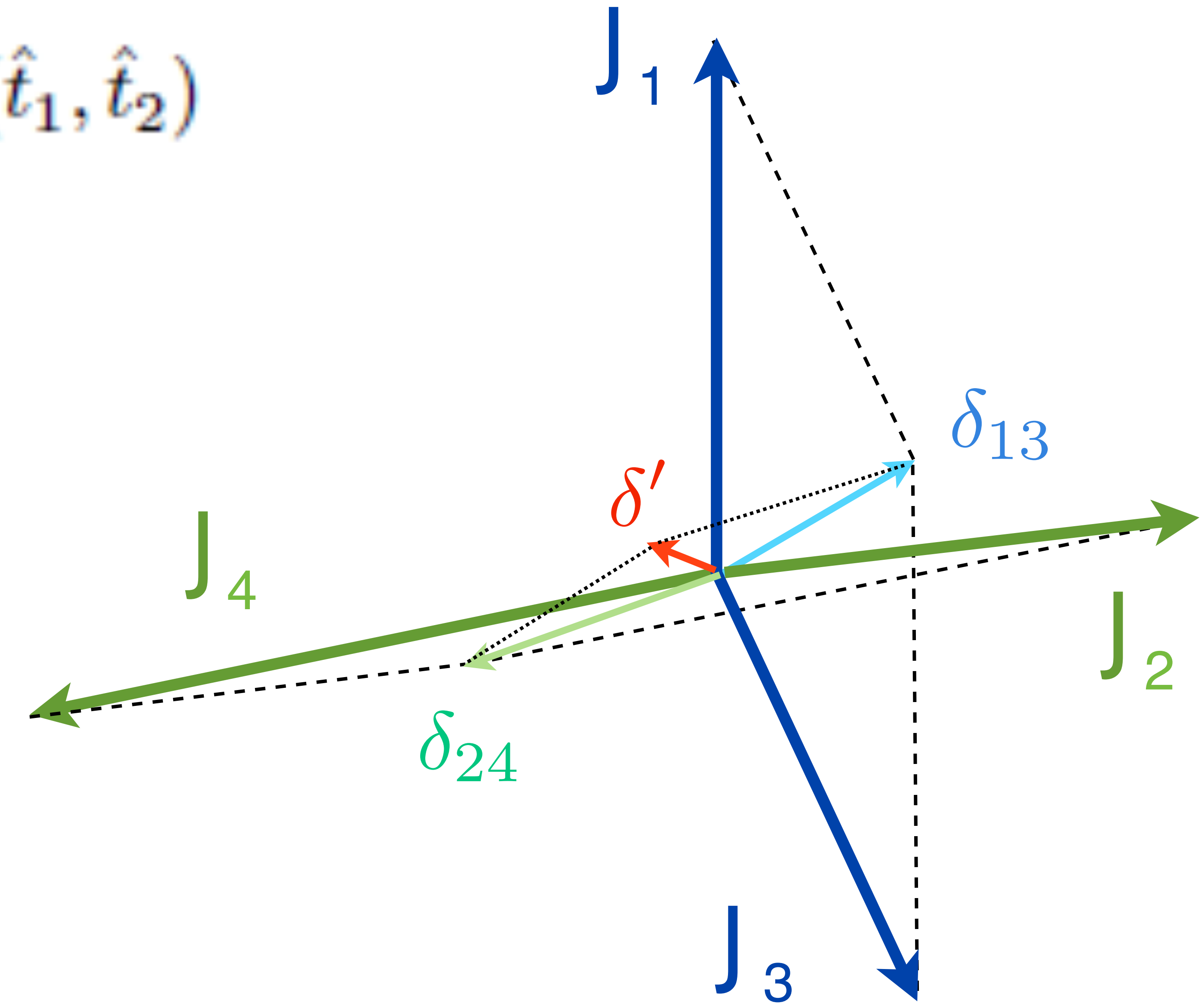
back-to-back kinematics

$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$

$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$

$d\Sigma(\hat{t}_1, \hat{t}_2)$





$$d\sigma^{(4\to4)} \propto \frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$d\sigma^{(3\to4)} \propto \frac{\alpha_s^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

2-parton collision

The standard approach to the multi-jet production is the QCD improved parton model.

It is based on the assumption that the cross section of a hard hadron–hadron interaction is calculable in terms of the convolution of parton distributions within colliding hadrons with the cross section of a hard ***two-parton collision***.

$$\sigma_2 = \int d^2\rho_1 d^2B \; f(x_1, \vec{\rho}_1, p^2) f(x_2, \vec{B} - \vec{\rho}_1, p^2) \; \frac{d\sigma^h}{d\hat{t}} d\hat{t}$$

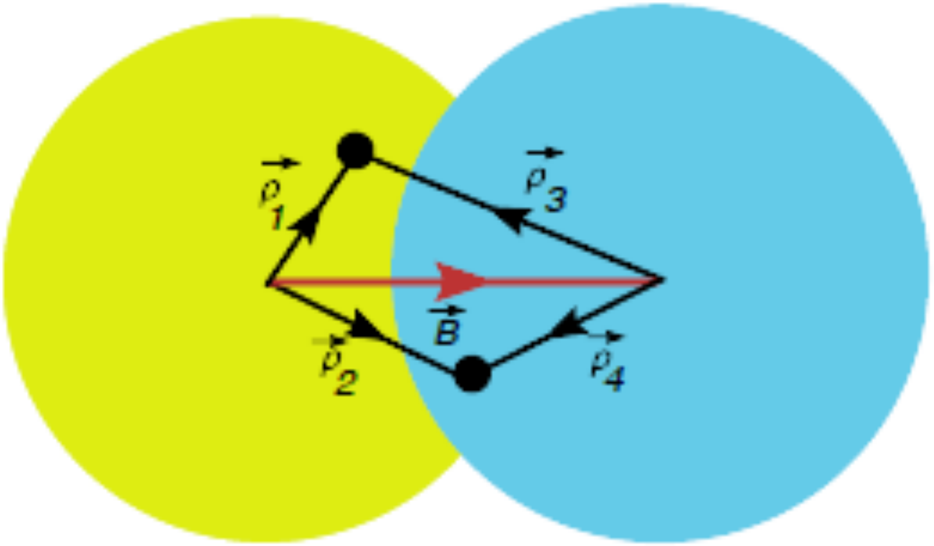
parton probability density : $f(x, \vec{\rho}, p^2) = \psi^+(x, \vec{\rho}, p^2)\psi(x, \vec{\rho}, p^2)$

Result of the impact parameter integration - squaring of the amplitude in the momentum space:

$$\int \frac{d^2k_\perp}{(2\pi)^2}\psi(x, k_\perp) \int \frac{d^2k'_\perp}{(2\pi)^2}\psi^\dagger(x, k'_\perp) \times \int d^2\rho \, e^{i\vec{\rho}\cdot(\vec{k}_\perp - \vec{k}'_\perp)} \quad = \int \frac{d^2k_\perp}{(2\pi)^2} \, \psi(x, k_\perp) \times \psi^\dagger(x, k_\perp)$$

An application of this picture to the processes with production of, e.g., ***four jets***
implies that all jets in the event are produced in a hard collision of ***two*** initial state partons.

There exists a kinematical domain
- ***double back-to-back kinematics*** -
where a more complicated mechanism becomes important :
double hard interaction
of two partons in one hadron
with two partons in the second hadron.



Let us see, what difference does it make to our formulae

Multi-parton wave function

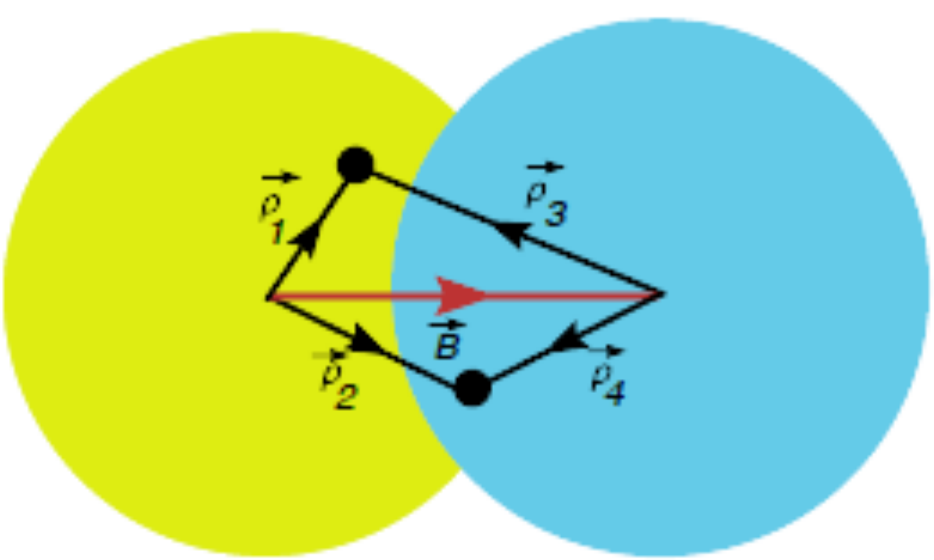
$$\psi_n \ (x_1,\vec{\rho}_1,x_2,\vec{\rho}_2,...) = \int \prod_{i=1}^{i=n} \frac{d^2k_i}{(2\pi)^2} \exp(i\sum_{i=1}^{i=n} \vec{k}_i\vec{\rho}_i) \ \psi_n(x_1,\vec{k}_1,x_2,\vec{k}_2,..)(2\pi)^2\delta(\sum \vec{k}_i)$$

Inclusive 2-parton probability distribution in the impact parameter space :

$$D \ (x_1,x_2,\vec{\rho}_1,\vec{\rho}_2) = \sum_{n=3}^{n=\infty} \int \prod_{i\geq 3}^{i=n} [dx_i d^2\rho_i] \, \psi_n(x_1,\vec{\rho}_1,x_2,\vec{\rho}_2,...x_i,\vec{\rho}_i,) \ \psi_n^+(x_1,\vec{\rho}_1,x_2,\vec{\rho}_2,...,x_i,\vec{\rho}_i,...)\delta(\sum_{i=1}^{i=n} x_i\vec{\rho}_i)$$

Independent impact parameter integration → equality of parton momenta in ψ and $\psi^\dagger$

$$k_\perp = k'_\perp$$



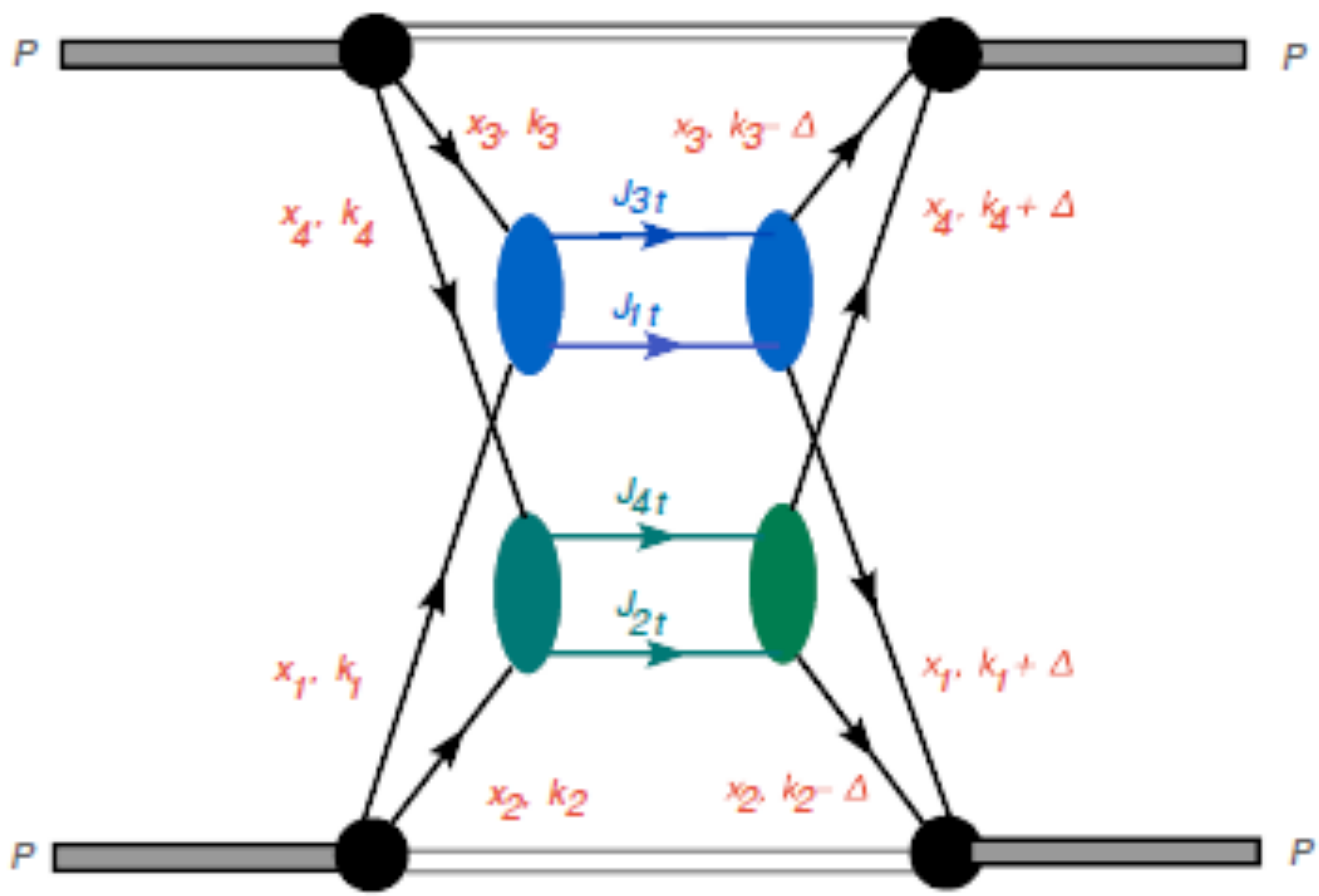
$\rho_1 + \rho_2$ → $k'_1 - k_1 = -(k'_2 - k_2) \equiv \Delta$

$\rho_3 + \rho_4$ → $k'_3 - k_3 = -(k'_4 - k_4) \equiv \widetilde{\Delta}$

$(\rho_1 - \rho_2) + (\rho_3 - \rho_4)$ → $\Delta = -\widetilde{\Delta}$

$\delta\big((\rho_1 - \rho_2) - (\rho_3 - \rho_4)\big)$ → $\vec{\Delta}$ ***arbitrary***

4-parton collision

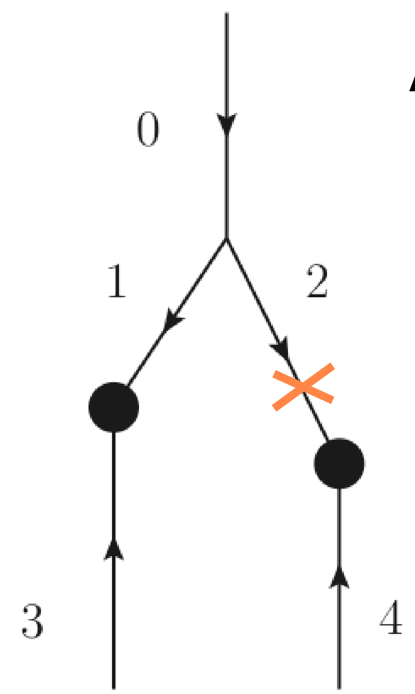


In order to be able to trace the *relative distance between the partons*, one has to use the mixed *longitudinal momentum – impact parameter* representation which, in the momentum language, reduces to introduction of a **mismatch** between the transverse momentum of the parton in the ***amplitude*** and that of the same parton in the ***amplitude conjugated***.

We have examined the ***transverse momentum*** structure of the interaction amplitude

Now, have a look at the ***longitudinal momenta*** of participating partons ...

An underwater stone of the MPI analysis

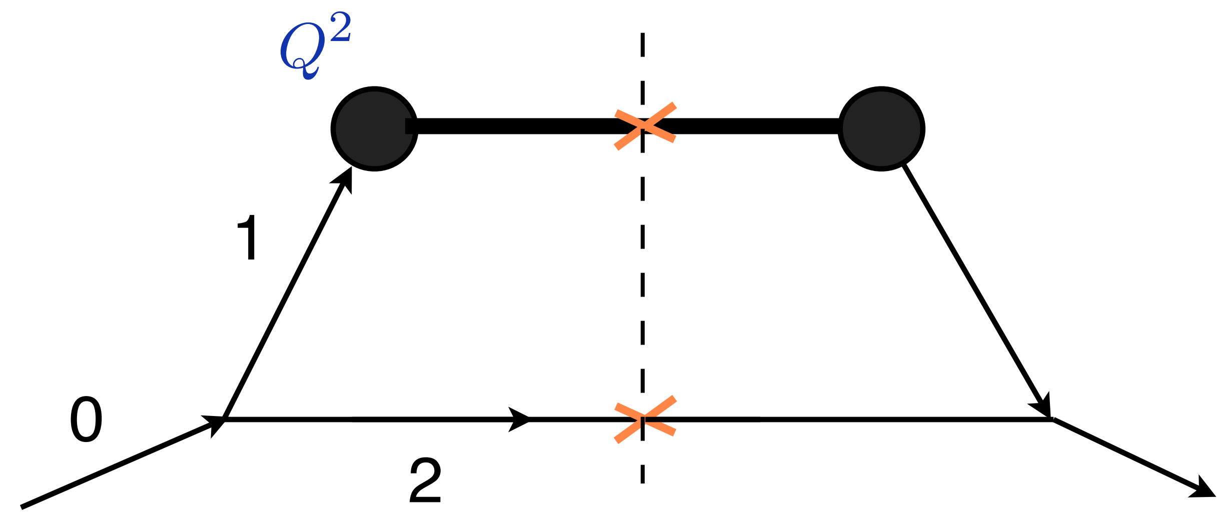


A tree Feynman diagram. Momenta of internal parton lines are fixed ...

not anymore

Singularities in the physical region of parton momenta !

Return to a good old single hard interaction picture :



In DIS we trace the fate of **1** but *integrate* over "histories" of the accompanying parton **2**.

Now we want #**2** to enter 2nd hard interaction.

In the above picture it does it "*in the next room*".

In fact, partons **3** and **4** ***cannot be*** represented by plainly independent ***plane waves***: they belong to ***one hadron***, and therefore, are ***localized within the hadron pancake***...

Remedy: introduce wave packet smearing (longitudinal momentum fraction integration).

Importantly, this has to be done at the ***amplitude level*** !

$k_{3+} + k_{4+}$ fixed by hard scattering kinematics

$k_{3+} - k_{4+}$ arbitrary

The fake singularity disappears.

The question to MC gurus hangs...

mind your head

From theory to experiment

- General formalism for DPI:

$$d\sigma^{(\mathrm{DPI})}_{Y+Z}(s) = \frac{m}{2\sigma_{\mathrm{eff}}(s)} \int dx_1\, dy_1\, dx_2\, dy_2\, [f_{i_1 j_1}(x_1, y_1, \mu_F) f_{i_2 j_2}(x_2, y_2, \mu_F)$$
$$d\sigma_{i_1 i_2 \to Y}(x_1, x_2, s)\, d\sigma_{j_1 j_2 \to Z}(y_1, y_2, s)]$$

There is NO factorization !

$$f_{i_1 j_1}(x_1, y_1, \mu_F) f_{i_2 j_2}(x_2, y_2, \mu_F)$$

$$\Longrightarrow \int d^2\Delta\ f_{i_1 j_1}(x_1, y_1, Q_x, Q_y; \vec{\Delta}) f_{i_2 j_2}(x_2, y_2, Q_x, Q_y; \vec{\Delta})$$

Generalized double parton distributions

4-parton cross section

$$\frac{1}{S} = \frac{\int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2; \vec{\Delta})\, D_b(x_3, x_4; -\vec{\Delta})}{D(x_1)D(x_2)\, D(x_3)D(x_4)}$$

S - effective parton interaction area ***(S*** - *is NOT a "cross section"* ***)***

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1\, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \times \frac{1}{S}$$

D - ***the generalized double parton distribution*** - is a new object we know little about.

Can one model it, for lack of anything better ?

Such a model has been developed by B. Bloc et al

in ***The Four jet production at LHC and Tevatron in QCD***
Phys. Rev. D83 : 071501, 2011; e-Print: **arXiv:1009.2714** [hep-ph]

pQCD Physics of Multiparton Interactions
Eur.Phys.J. C72 (2012) 1963; e-Print: **arXiv:1106.5533** [hep-ph]

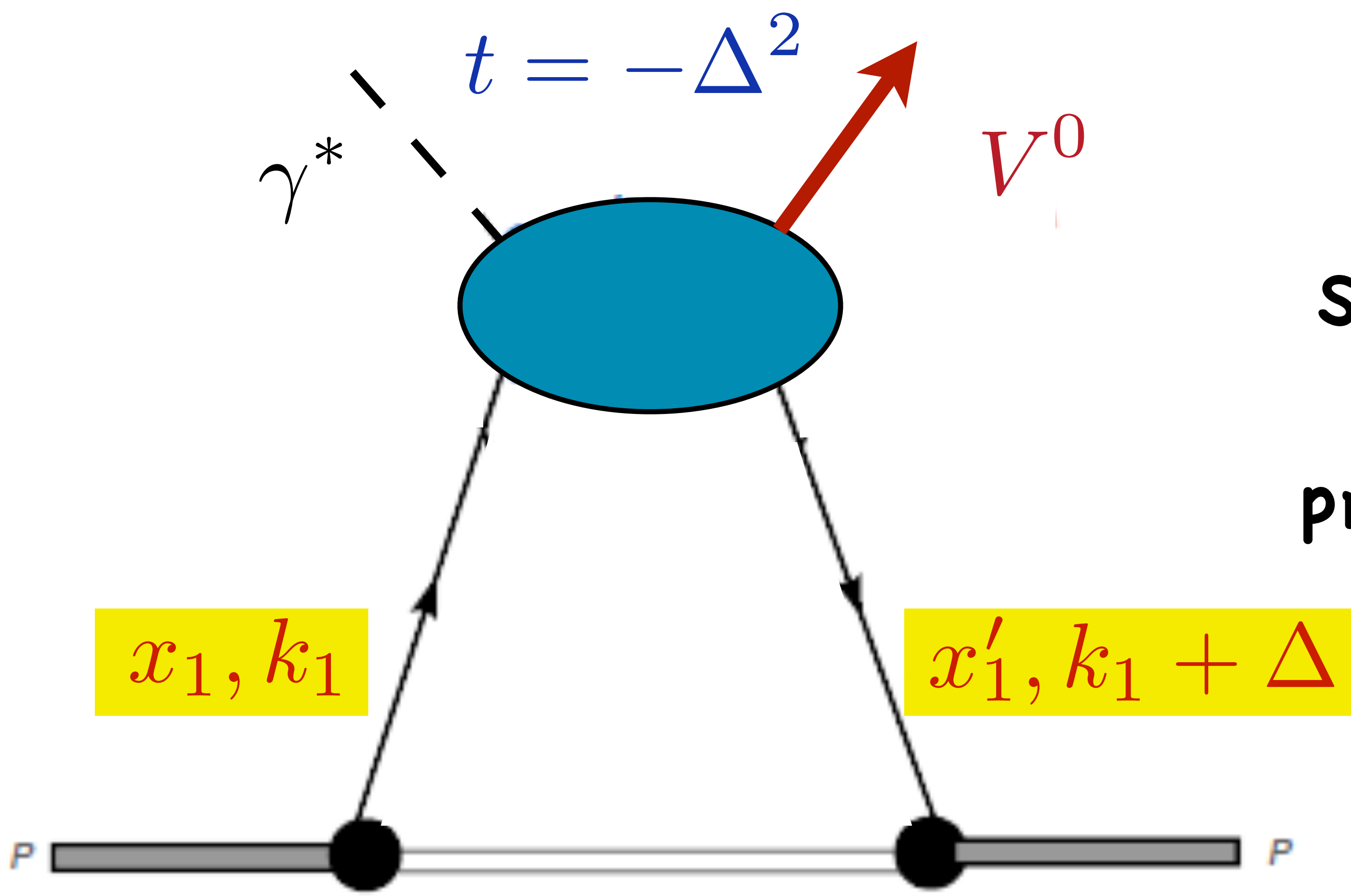
Independent parton approximation fails to explain the DPI contribution

in: ***Origins of Parton Correlations in Nucleon and Multi-Parton Collisions***
B.Bloc et al e-Print: **arXiv:1206.5594** [hep-ph]

- **demonstrated that in the Tevatron kinematics** ($0.1 > x > 0.001$)
  PT effects **can take care of the missing factor 2 enhancement**

- **gave an estimate for *non-PT* intra-hadron 2-parton correlations** ($x < 0.001$)

  based on the analysis of ***inelastic diffraction***
  in the framework of the Gribov-Regge Pomeron picture



G P D

Such an amplitude describes exclusive photo-(/electro-) production of vector mesons at HERA !

Note : the analogy is ***imperfect***. ***OK*** for high enough energies (x< 0.1): $A \simeq i\,\mathrm{Im}A$

Imaginary part of the "skewed" amplitude **vs.** that of a non-diagonal "elastic" transition ...

Generalized parton distribution :

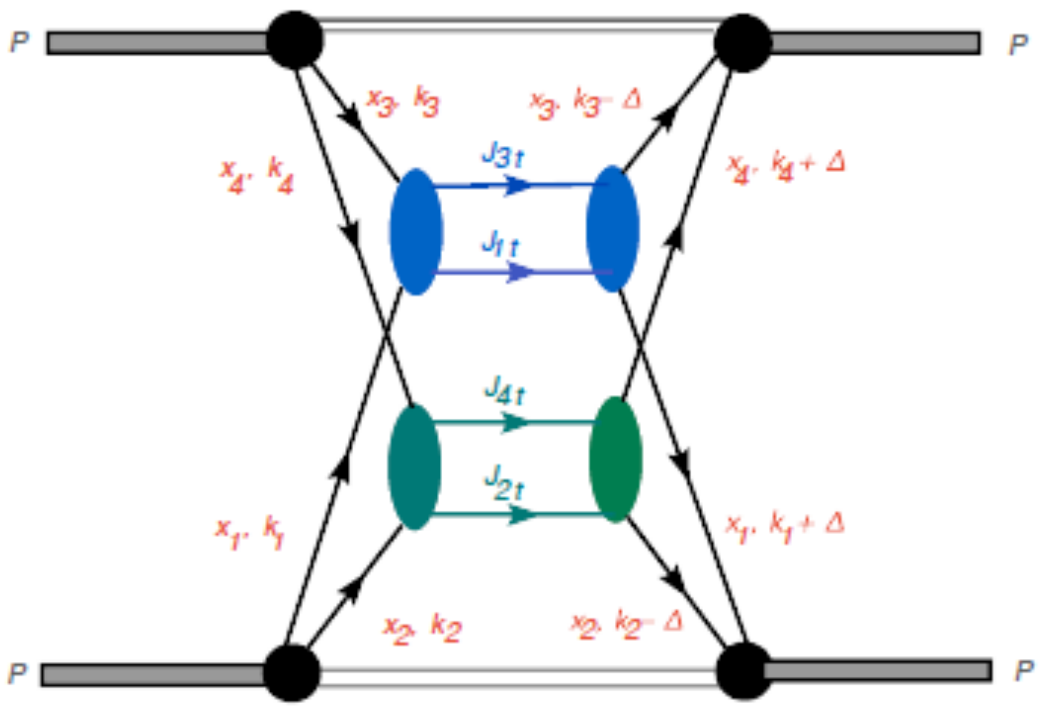
$$G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2) F_{2g}(\Delta)$$

G - the usual 1-parton distribution (determining DIS structure functions)

F - the two-gluon form factor of the nucleon

the dipole fit : $F_{2g}(\Delta) \simeq \frac{1}{\left(1 + \Delta^2/m_g^2\right)^2}$

$m_g^2(x \sim 0.03, Q^2 \sim 3\mathrm{GeV}^2)$
$\simeq 1.1\mathrm{GeV}^2$



If partons were ***uncorrelated***, we would write

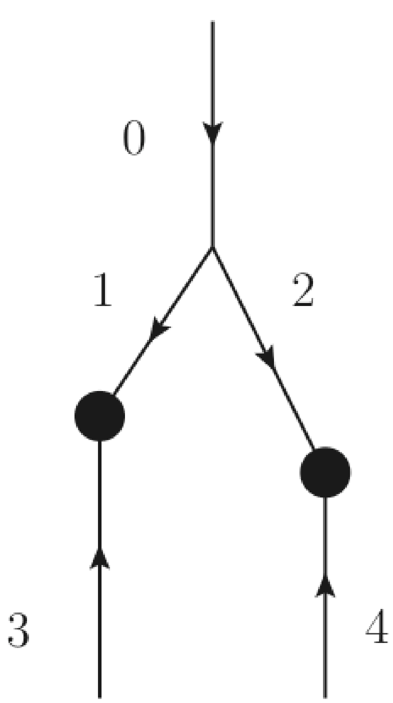
$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta}) G(x_2, p_2^2, \vec{\Delta})$$

and use the dipole fit to get the estimate

$$\frac{D(x_1, x_2, -\vec{\Delta}) D(x_3, x_4, \vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)} \simeq F_{2g}^4(\Delta)$$

The “interaction area” : ⇒

$$\int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2) \times F_g^2(\Delta^2) = \frac{m_g^2}{28\pi}$$



Another mechanism : 2 partons from a short-range PT correlation

No Δ —dependence from the upper side ! ⇒

$$\int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2) = \frac{m_g^2}{12\pi}$$

3-> 4 contribution vs. **4-> 4** is enhanced by a factor

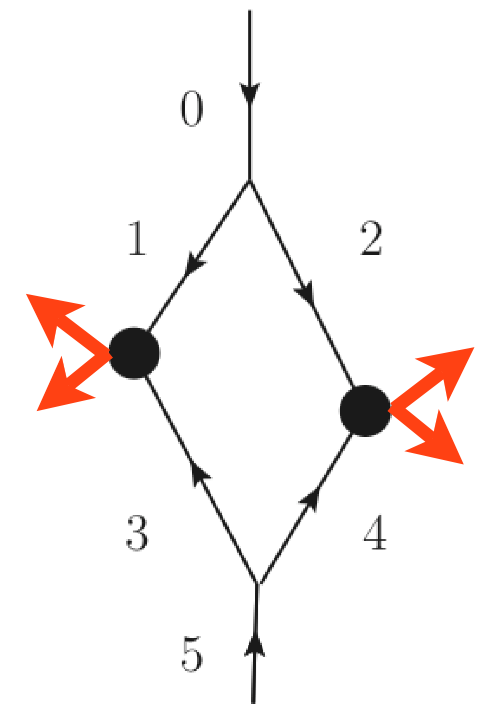
$$2 \times \frac{7}{3} \simeq 5$$

remark on the “2 -> 4” processes

What if *both parton pairs* originate from PT splittings ?

No Δ —dependence whatsoever.

$$\frac{1}{S} = \frac{\int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1,x_2;\vec{\Delta})\, D_b(x_3,x_4;-\vec{\Delta})}{D(x_1)D(x_2)\,D(x_3)D(x_4)}$$



The integral diverges...?..

This is NOT an amplitude of a *4-parton collision*
but a one-loop correction to the (“*leading twist*”) *2-parton collision*

What distinguishes “double hard collisions” is the differential jet spectrum :

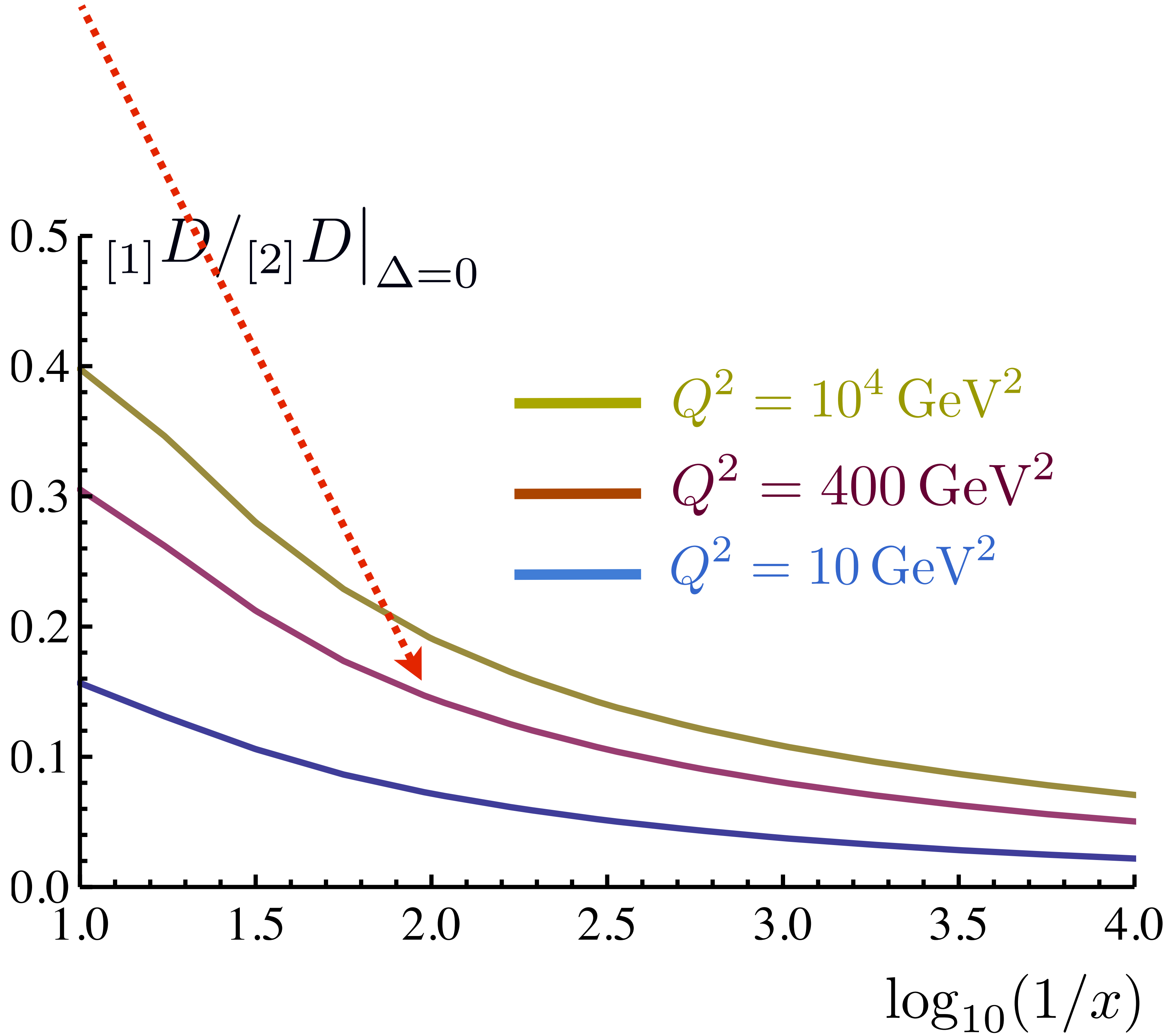
This mechanism produces 4-prickle hedgehogs *(as any 2->4)*
while *4->4* and *3->4* produces two back-to-back jet pairs

Results of
Numerical Analysis

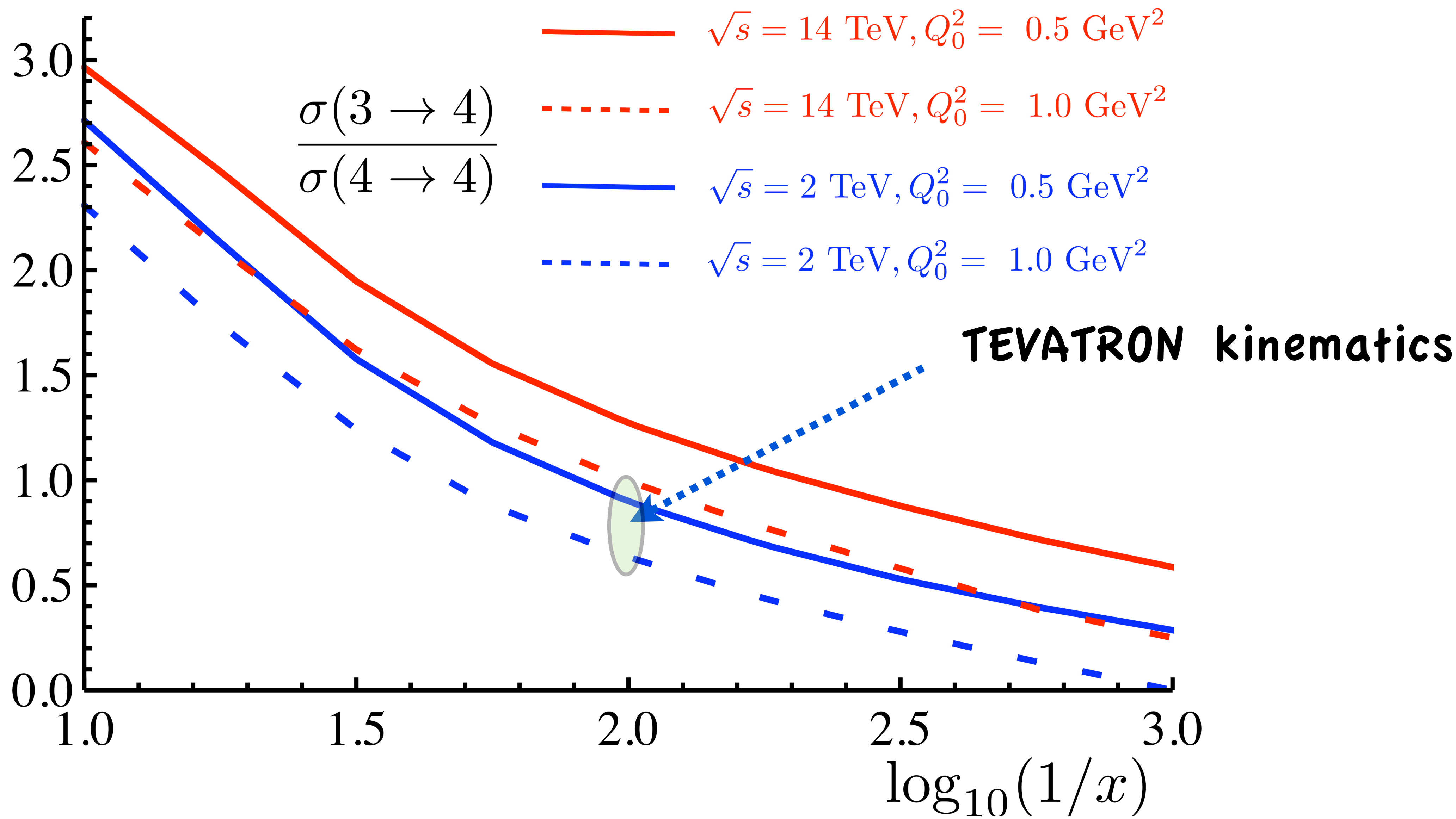
Origins of Parton Correlations in Nucleon and Multi-Parton Collisions

B.Bloc et al ***e-Print: arXiv:1206.5594 [hep-ph]***

At TEVATRON energies, the 3->4 contribution amounts to about 15-20% of the 4->4 :



the ratio of 3->4 to 4->4 contributions to the "total" Xsection



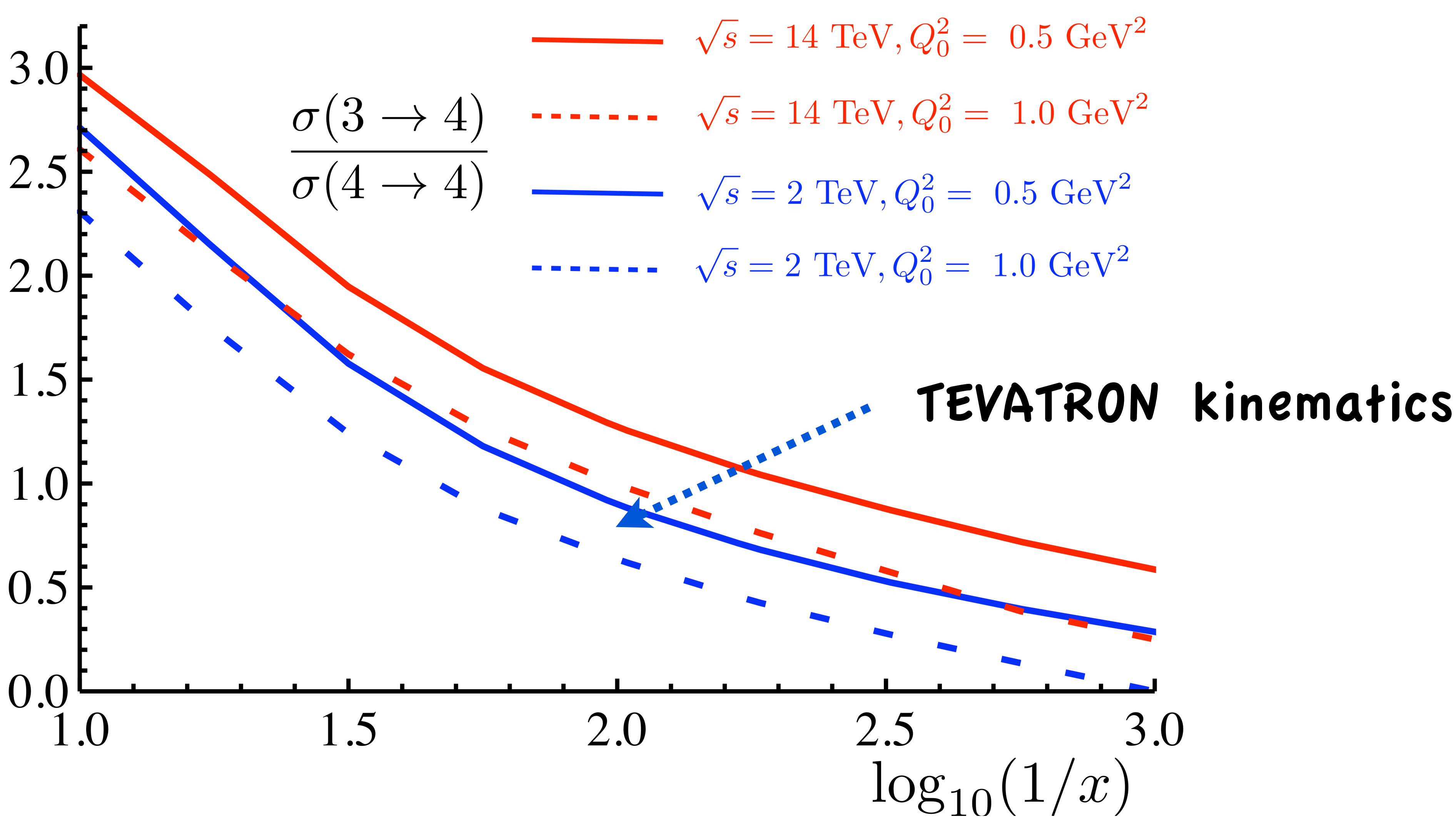
***Uncertainty* due to choice of the Q_0 parameter !**

IMPORTANT ! *there is no contradiction.*

Q_0 *is not a formal parameter but a physical one:*

*The parton resolution scale at which $F_g^2(\Delta^2)$ falls **faster** than $1/\Delta^2$!*

Daniele Treleani : you guys seem to have predicted too sharp an x-dependence of $\sigma_{\rm eff}$ (in disaccord with the CDF finding)



In this picture, Q_i^2 change together with ***x*** ...

In a realistic situation (*keeping* Q_i^2 ***fixed***) the ***x***-dependence is ***much milder***

In the CDF kinematics ($Q_1^2 = (20\,{\rm GeV})^2,\ Q_1^2 = (5\,{\rm GeV})^2$)

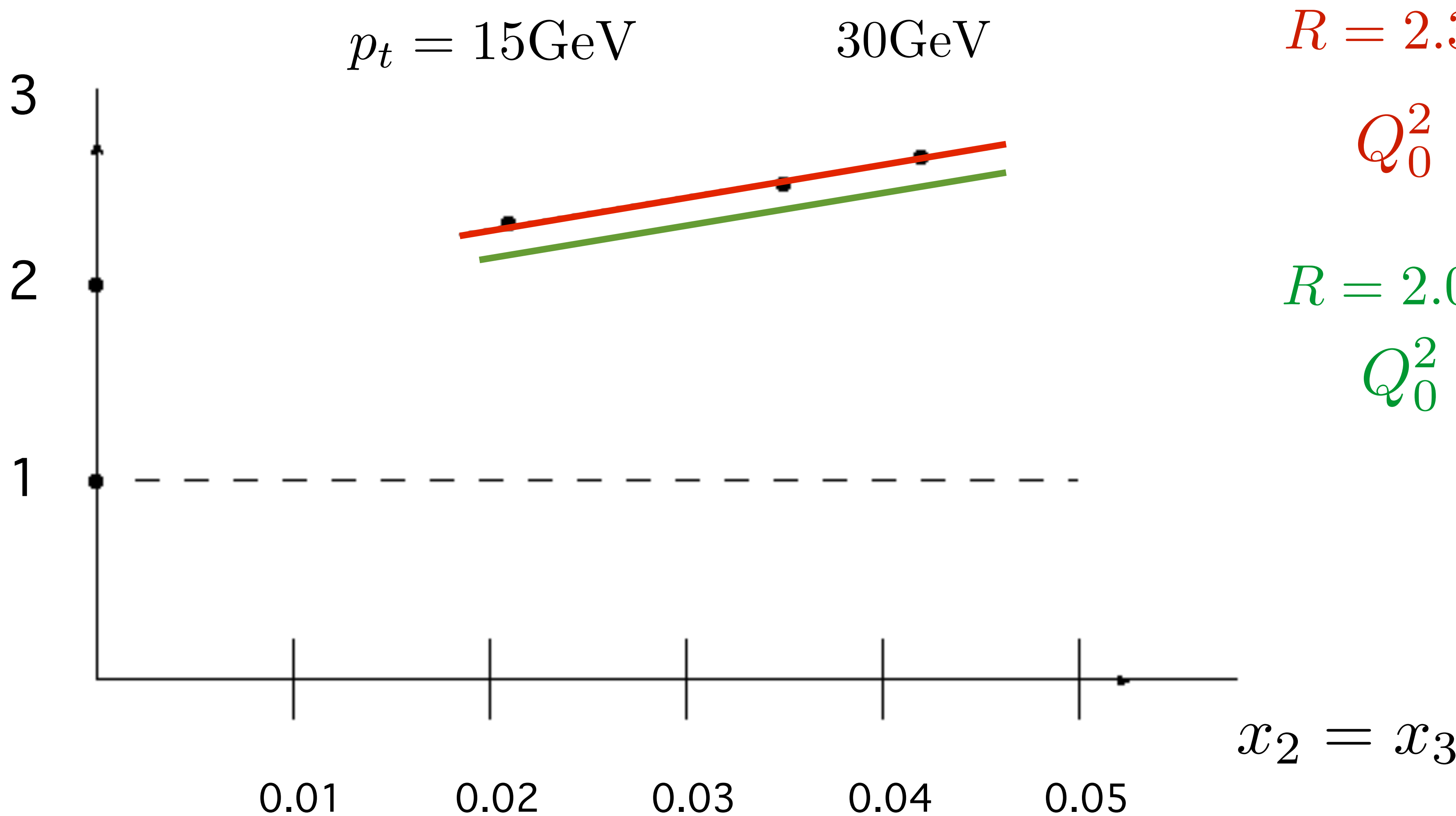
$R = 1 + \frac{\sigma_{3\to4}}{\sigma_{4\to4}}$ varies between $1.7 \div 2.0$ ($x_i = 0.001 \div 0.1$)

x-dependence

D0 kinematics (photon + 3jets)

$$x_\gamma = x_1 = 0.1$$

$$R = 1 + \frac{\sigma_{3\to4}}{\sigma_{4\to4}}$$



$R = 2.37 \div 2.65$

$Q_0^2 = 0.5\,\text{GeV}^2$

$R = 2.08 \div 2.38$

$Q_0^2 = 1\,\text{GeV}^2$

Peculiarities of the 3->4 MPI mechanism

- is bound to bring in essential ***x-dependence*** of $\sigma_{\rm eff}$

  in particular, in "pre-forward" kinemo ($x_1, x_2 \gg x_3, x_4$) where **3->4** is ***large***

- should cause ***asymmetry in rapidity*** of accompanying multiplicity density

- introduces ***specific correlation*** between jet-pair ***transverse momentum imbalances***

$$d\sigma^{(3\to 4)} \propto \frac{\alpha_s^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$\Longrightarrow \quad \delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$$

vs.

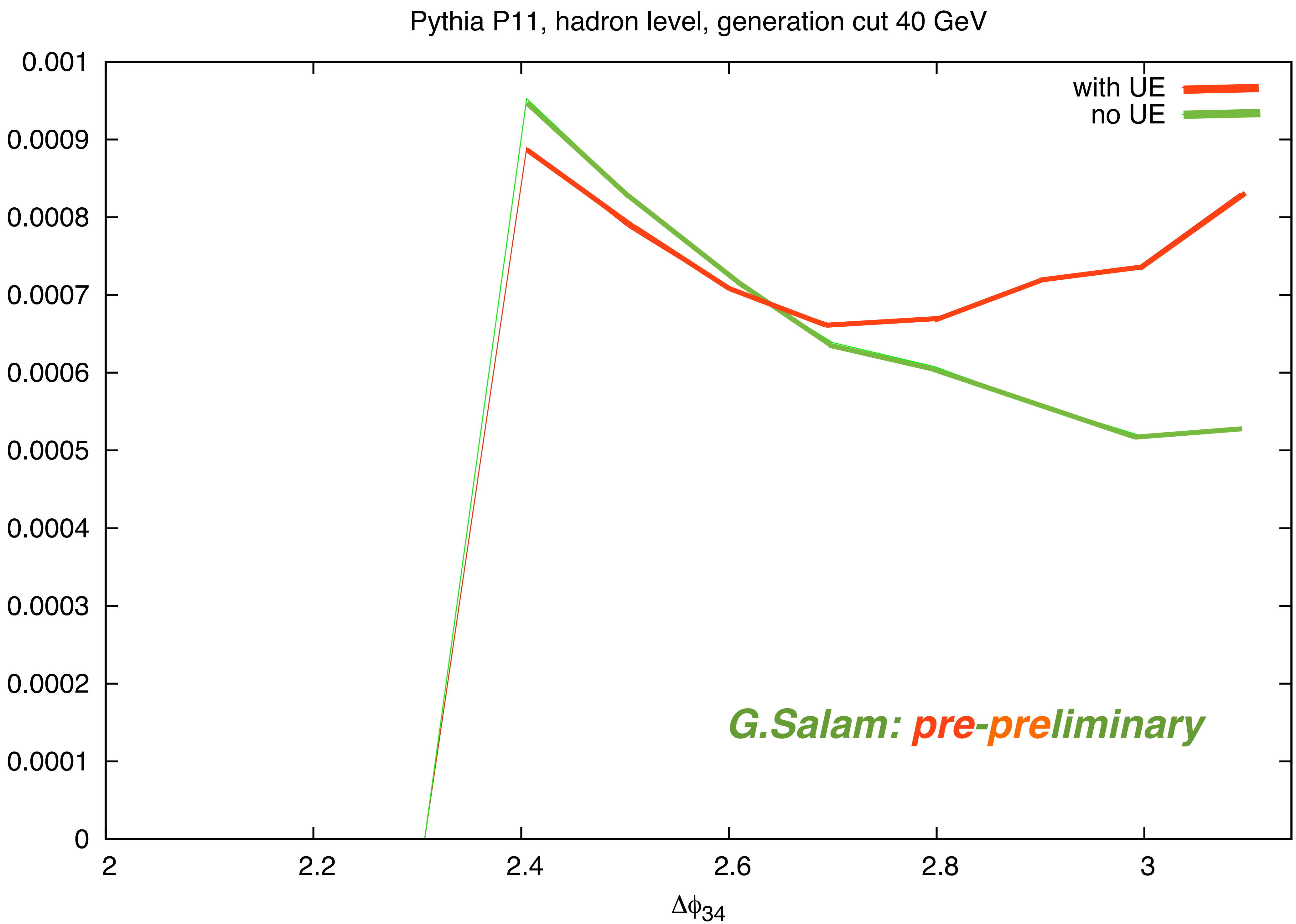
$$d\sigma^{(4\to 4)} \propto \frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

Where and how to look for double (multiple) hard interactions

Event selection cuts : $p_{1t} > p_{2t} > p_{3t} > p_{4t}$

$p_{1t} + p_{2t} > 120\,\mathrm{GeV},\ p_{2t} > 0.6 p_{1t}$ $p_{3t} + p_{4t} > 60\,\mathrm{GeV},\ p_{4t} > 0.6 p_{3t}$

$\phi_{12},\ \phi_{34} > 3\pi/4$ $\Delta\Omega_{ij} > \pi/4$ (*separation in **azimuth** and/or **rapidity***)



MPI

A new subject

Theoretically complicated

Experimentally challenging

Conclusions

Theorists : **think harder**

Q: can one get away within the *probabilistic picture*, in some approximation, or interferences (“cross-talk”) are unavoidable ?

MC builders : **think twice**

Q: how do you make sure that the two partons originate, space-time-wise, from *one and the same hadron* ?

Experimenters : **do it** (but mind your head, now and then)

A.

EXTRAS

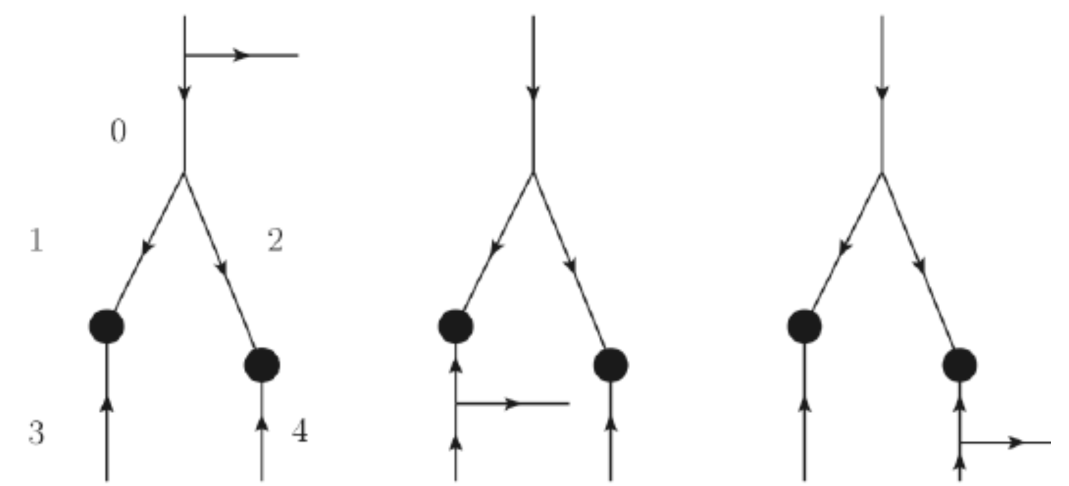
Generalization of the DDT-formula for back-to-back 4-jet production spectrum

$$\pi^2 \frac{d\sigma^{(4\to 4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial \delta_{13}^2} \frac{\partial}{\partial \delta_{24}^2} \Big\{ {}_{[2]}D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \times {}_{[2]}D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \times S_1\left(Q^2, \delta_{13}^2\right) S_3\left(Q^2, \delta_{13}^2\right) \times S_2\left(Q^2, \delta_{24}^2\right) S_4\left(Q^2, \delta_{24}^2\right) \Big\}$$

Not forgetting the Δ—integration and short-range correlations :

$$_{[2]}D_a \times {}_{[2]}D_b + {}_{[2]}D_a \times {}_{[1]}D_b + {}_{[1]}D_a \times {}_{[2]}D_b$$

Additional 3 -> 4 contribution :



$$\frac{\pi^2\, d\sigma_2^{(3\to 4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\alpha_s(\delta^2)}{2\pi\, \delta^2} \sum_c P_c^{1,2}\left(\frac{x_1}{x_1+x_2}\right)$$

$$S_1(Q^2,\delta^2)\, S_2(Q^2,\delta^2)\, \frac{\partial}{\partial \delta'^2} \left\{ S_c(\delta^2, \delta'^2) \frac{G_a^c(x_1+x_2; \delta'^2, Q_0^2)}{x_1+x_2} S_3(Q^2,\delta'^2) S_4(Q^2,\delta'^2) \times {}_{[2]}D_b^{3,4}(x_3, x_4; \delta'^2, \delta'^2) \right\}$$