Theory of Double Parton Scattering.

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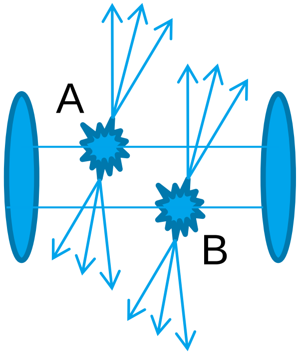
Outline

I will briefly review various topics in DPS that have received attention in recent research work:

- Total and differential cross section for DPS. 2pGPDs, 2pGTMDs, and relation between the two.
- What graphs contribute to the total and differential cross section for DPS?
- Approximation of 2pGPDs as a product of single GPDs and it’s validity.
- Interference and correlated parton contributions to DPS. Sudakov suppression of colour interference contributions.

Double Parton Scattering



Double Parton Scattering (DPS) = when you have two separate hard interactions in a single proton-proton collision

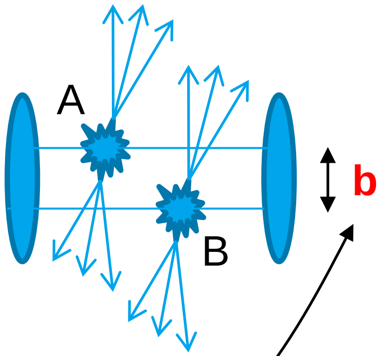
In terms of total cross section for production of AB, DPS is power suppressed with respect to single parton scattering (SPS) mechanism: $\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \frac{\Lambda^2}{Q^2}$

Why then should we study DPS?

1. DPS can compete with SPS if SPS process is suppressed by small/multiple coupling constants (same sign WW, H+W production). JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65 Del Fabbro, Treleani, Phys. Rev. D61 (2000) 077502 Bandurin, Golovanov, Skachkov, JHEP 1104 (2011) 054
2. DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_A$, $\mathbf{q}_B$ – competitive with SPS in this region.
3. DPS becomes more important relative to SPS as the collider energy grows, and we probe smaller *x* values where there is a larger density of partons.
4. DPS reveals new information about the structure of the proton – in particular, correlations between partons in the proton.

Total Cross Section for DPS

Assuming only the factorisation of the hard processes A and B, the total DPS cross section may be written as:



Symmetry factor

Two-parton generalised PDF (2pGPD)

$$\sigma_D^{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_h^{ik}(x_1, x_2, \mathbf{b}; Q_A, Q_B) \Gamma_h^{jl}(x_1', x_2', \mathbf{b}; Q_A, Q_B) \times \hat{\sigma}_{ij}^A(x_1, x_1') \hat{\sigma}_{kl}^B(x_2, x_2') \mathrm{d}x_1 \mathrm{d}x_1' \mathrm{d}x_2 \mathrm{d}x_2' \mathrm{d}^2\mathbf{b}$$

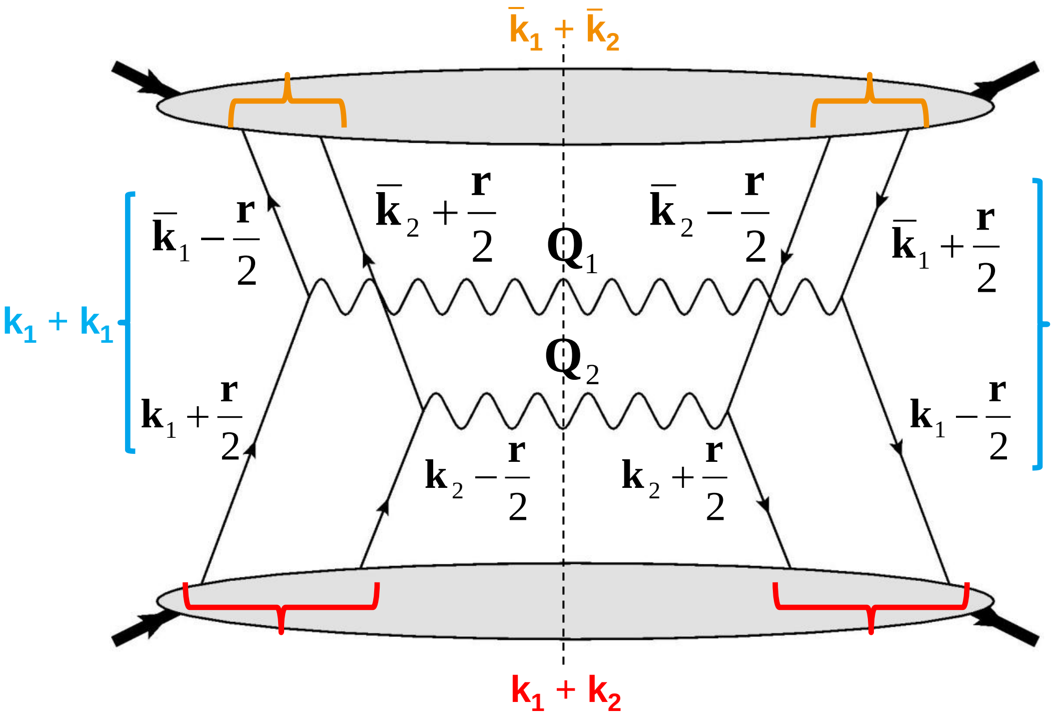
Parton level cross sections

N. Paver, D. Treleani, Nuovo Cim. A70 (1982) 215.
M. Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

b = separation in transverse space between the two partons

In this formula the two 2pGPDs are integrated over a common **b** – cannot express DPS cross section in terms of parton distributions independently integrated over their impact parameter arguments, as in single scattering case.

DPS – transverse momentum picture



Key point: transverse momentum of partons does not have to be equal in amplitude and conjugate!

Most general transverse momentum configuration of partons entering hard scatters

r = momentum imbalance of a parton line between amplitude and conjugate

$$\sigma = \int \frac{d^2\mathbf{r}}{(2\pi)^2} \int \frac{d^2\mathbf{k}_1}{(2\pi)^2} \frac{d^2\mathbf{k}_2}{(2\pi)^2} D_h^{p_1p_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{r}) \int \frac{d^2\bar{\mathbf{k}}_1}{(2\pi)^2} \frac{d^2\bar{\mathbf{k}}_2}{(2\pi)^2} D_h^{p_3p_4}(x_1, x_2, \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, -\mathbf{r})$$

$$= \int \frac{d^2\mathbf{r}}{(2\pi)^2} D_h^{p_1p_2}(x_1, x_2, \mathbf{r}) D_h^{p_3p_4}(x_1, x_2, -\mathbf{r})$$

Fourier transform of **b**-space 2pGPD wrt **b**

Differential Cross Section for DPS for q_T << Q

For small final state transverse momentum ($\mathbf{q}_i$ << Q), differential DPS cross section should have the following form:

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

2pGTMD

$$\frac{d\sigma_D^{(A,B)}}{d^2\mathbf{q}_1 d^2\mathbf{q}_2} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_h^{ik}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{b})\Gamma_h^{jl}(x_1', x_2', \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, \mathbf{b})$$
$$\times \hat{\sigma}_{ij}^A(x_1, x_1')\hat{\sigma}_{kl}^B(x_2, x_2')\mathrm{d}x_1\mathrm{d}x_1'\mathrm{d}x_2\mathrm{d}x_2'\mathrm{d}^2\mathbf{b}$$
$$\times \prod_{i=1,2}\int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i \delta(\mathbf{k}_i + \bar{\mathbf{k}}_i - \mathbf{q}_i)$$

(Neglecting a possible soft factor + dependence of the 2pGTMDs on rapidity regulator)

Differential cross section can also be expressed in terms of r space 2pGTMDs – as in total cross section, one makes the replacement:

$$\int \Gamma_h^{ik}(\mathbf{b})\Gamma_h^{jl}(\mathbf{b})d^2\mathbf{b} \to \int \Gamma_h^{ik}(\mathbf{r})\Gamma_h^{jl}(-\mathbf{r})\frac{d^2\mathbf{r}}{(2\pi)^2}$$

Operator Definitions of 2pGPDs and 2pGTMDs

From a tree-level analysis: Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

2pGPD:

$$F^{jk}(x_i,\mathbf{b}) = \int \frac{dz_2^-}{2\pi} e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi} e^{ix_1 z_1^- p^+} \times 2p^+ \int db^- \langle p | O^j(-\tfrac{1}{2}z_2, \tfrac{1}{2}z_2) O^k(b-\tfrac{1}{2}z_2, b+\tfrac{1}{2}z_2) | p \rangle \Big|_{\substack{z_i^+=b^+=0 \\ \mathbf{z_i}=\mathbf{0}}}$$

O = same bilocal gluon or quark operator that is used in definition of single PDFs

For $\mathbf{b} \neq \mathbf{0}$, the bilocal operators are evaluated at different transverse positions.

2pGTMD:

$$F^{jk}(x_i,\mathbf{k}_i,\mathbf{b}) = \int \frac{dz_2^- d\mathbf{z}_2}{(2\pi)^3} e^{ix_2 z_2^- p^+ - i\mathbf{z}_2\mathbf{k}_2} \int \frac{dz_1^- d\mathbf{z}_1}{2\pi} e^{ix_1 z_1^- p^+ - i\mathbf{z}_1\mathbf{k}_1} \times 2p^+ \int db^- \langle p | O^j(-\tfrac{1}{2}z_2, \tfrac{1}{2}z_2) O^k(b-\tfrac{1}{2}z_2, b+\tfrac{1}{2}z_2) | p \rangle \Big|_{z_i^+=b^+=0}$$

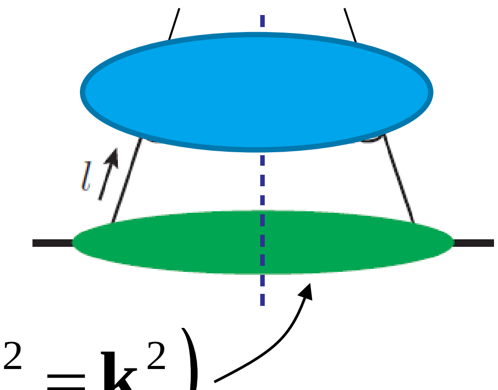
Relation between 2pGPDs and 2pGTMDs for $q_T \gg \Lambda$

SPS:

If $|\mathbf{q}| \gg \Lambda$ (but still $\ll Q$), then TMD can be written in terms of collinear PDFs and a perturbative factor.

Collins, Soper, Sterman , Nucl.Phys. B250 (1985) 199
Collins, pQCD book, Ch. 13

$$F_h(x,\mathbf{k}) = T(x,\mathbf{k}) \otimes D_h(x,\mu^2 = \mathbf{k}^2)$$



Collinear (single) PDF

Indeed, at double leading logarithmic order, we obtain the DDT formula for the differential SPS cross section for $|\mathbf{q}| \gg \Lambda$:

$$\frac{d\sigma}{dq^2\, dq_\perp^2} = \frac{d\sigma_{\text{tot}}}{dq^2} \times \frac{\partial}{\partial q_\perp^2}\left\{ D_a^q\left(x_1, q_\perp^2\right) D_b^{\bar{q}}\left(x_2, q_\perp^2\right) S_q^2\left(q^2, q_\perp^2\right) \right\}.$$

Sudakov factor

We expect there to be a similar relation between 2pGPDs and 2pGTMDs. At the double leading log level, it has been shown that the Sudakov factor for DPS is the product of Sudakov factors for SPS:

$$\pi^2 \frac{d\sigma^{(4\to 4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial \delta_{13}^2} \frac{\partial}{\partial \delta_{24}^2} \left\{ {}_{[2]}D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \times {}_{[2]}D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \times S_1\left(Q^2, \delta_{13}^2\right) S_3\left(Q^2, \delta_{13}^2\right) \times S_2\left(Q^2, \delta_{24}^2\right) S_4\left(Q^2, \delta_{24}^2\right) \right\}.$$

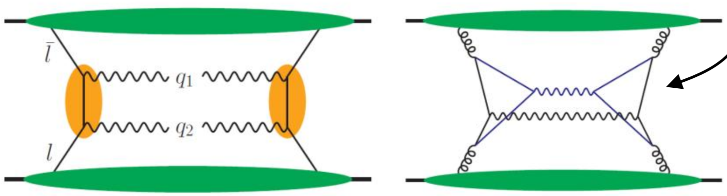
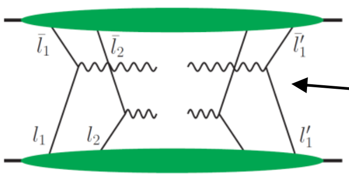
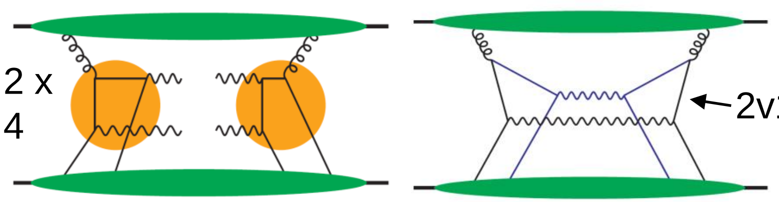
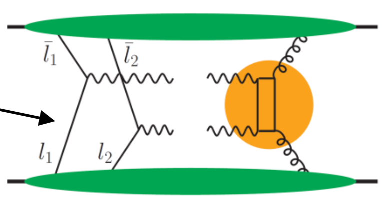
➔ for $|\mathbf{q}| \gg \Lambda$ there is a portion of the DPS differential σ that resembles the total σ

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
BDFS, Eur.Phys.J. C72 (2012) 1963

Power Counting

Some relevant diagrams:

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Diagrams	$s\sigma$	$\left.\frac{s d\sigma}{d^2\mathbf{q}_1 d^2\mathbf{q}_2}\right\|_{\|\mathbf{q}_1\|\sim\|\mathbf{q}_2\|\sim\Lambda}$
 1v1 Diagram	1	$\frac{1}{\Lambda^2 Q^2}$
 2v2 Diagram	$\frac{\Lambda^2}{Q^2}$	$\frac{1}{\Lambda^2 Q^2}$
Twist 2 x Twist 4  2v1 Diagram	$\frac{\Lambda^2}{Q^2}$	$\frac{1}{Q^4}$
(Twist 3)^2 (in total cross section) 	$\frac{\Lambda^2}{Q^2}$	$\frac{1}{\Lambda^2 Q^2}$

Everything except SPS power suppressed

'DPS' and 'SPS' both leading power!

Many twist 3 distributions suppressed due to helicity nonconservation in associated diagrams

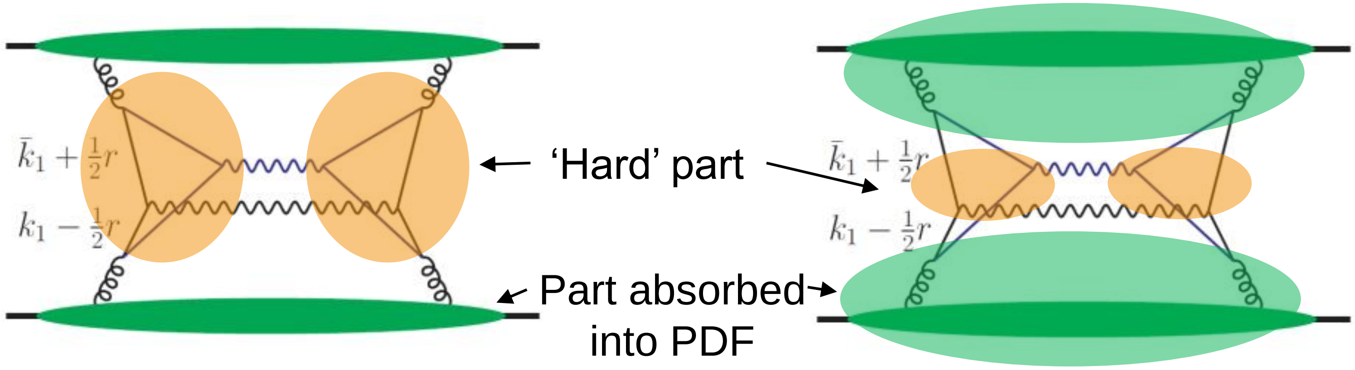
Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201
Qiu, Sterman, Nucl.Phys. B353 (1991) 105-136

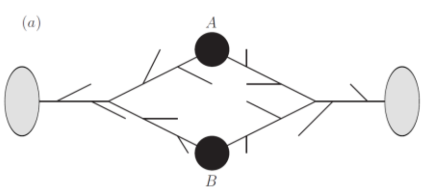
‘1v1’ or ‘Double Perturbative Splitting’ Diagrams

At level of inclusive cross section:

Regard just as SPS?

Or can/should we regard a portion of this graph as DPS?





There is no natural power suppressed ($\propto \frac{\Lambda^2}{Q^4}\left[\alpha_s \log\left(\frac{Q^2}{\Lambda^2}\right)\right]^n$) part of the 1v1 graph that we can separate off as DPS ➔ regard all of these graphs as SPS?

JG and Stirling, JHEP 1106 048 (2011) & arXiv:1202.3056

Has the advantage that we avoid double counting between DPS and SPS!

‘1v1’ or ‘Double Perturbative Splitting’ Diagrams

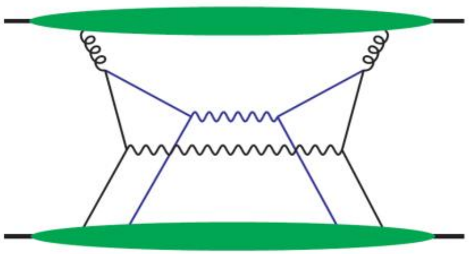
Closely related statements:

- There are no power corrections in massless perturbation theory. e.g. Collins, pQCD book, Chapter 8
- Computation of 1v1 graph using collinear approx for loop partons gives quadratically divergent scaleless integral, which vanishes in dimreg. Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.
- Differential cross section for 1v1 graph does not have a Q^2/Λ^2 power enhancement in back to back region. BDFS, Eur.Phys.J. C72 (2012) 1963

No scale Λ in massless perturbation theory!

‘2v1’ or ‘Single Perturbative Splitting’ Diagrams

Calculation reveals that 2v1 graph does contain a natural power suppressed piece $\propto \frac{\Lambda^2}{Q^4}\left[\alpha_s \log\left(\frac{Q^2}{\Lambda^2}\right)\right]^1$ that can be regarded as DPS.



Large logarithm

$$\sigma_{1v2}(s) = \sum_{s_i s_i' t_i t_i'} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}^{s_1,t_1;s_1',t_1';\mu_1}_{\bar{q}q\to\gamma*}(\hat{s} = x_1 y_1 s)\hat{\sigma}^{s_2,t_2;s_2',t_2';\mu_2}_{q\bar{q}\to\gamma*}(\hat{s} = x_2 y_2 s)$$

$$\Gamma_A^{s_1 s_2, s_1' s_2'}(x_1, x_2; \mathbf{b} = \mathbf{0}) \left[\frac{\alpha_s}{2\pi} P_{g\to q\bar{q}}^{\lambda\to t_2 t_1, t_2' t_1'}(y_2)\,\delta(1 - y_1 - y_2) \int_{\Lambda^2}^{Q^2} \frac{d\boldsymbol{J}_1{}^2}{\boldsymbol{J}_1{}^2}\right]$$

2pGPD of NP parton pair evaluated at **b = 0**

$1 \to 2$ splitting function

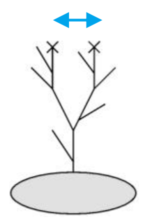
Summing up leading logs of all 2v1 graphs:

$$\sigma^{D,1v2}_{(A,B)}(s) = 2 \times \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s)$$

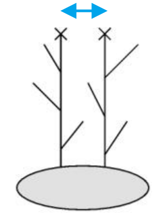
$$\times \breve{D}_p^{ij}(x_1, x_2; Q^2)\Gamma^{kl}_{p,indep}(y_1, y_2, \mathbf{b} = \mathbf{0}; Q^2)$$

$$\int d^2\mathbf{b}$$

b



b=0



BDFS, Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
JG, [arXiv:1207.0480]

'2v1' or 'Single Perturbative Splitting' Diagrams

It is necessary that the **b** space 2pGPD be smooth on scales << R_P (or the **r** space 2pGPD be sharply cut off for |**r**| values > Λ_{QCD}) for this result to be obtained.

Naively might think that this is a 'twist 4 x twist 2' contribution, since results involve 2pGPD of NP pair probed at **b = 0**. However, actually the result corresponds to b values that are << R_p^2 but >> $1/Q^2$

Geometrical prefactor for 2v1 graphs is different to that from 2v2 or zero perturbative splitting graphs. If one makes the assumption $\Gamma^{ij}_{p,indep}(x_1, x_2, \boldsymbol{b}; Q^2) = \tilde{D}^{ij}(x_1, x_2; Q^2) F(\boldsymbol{b})$:

$$\frac{1}{\sigma_{eff,2v2}} \equiv \int d^2\boldsymbol{b}[F(\boldsymbol{b})]^2$$

$$\frac{1}{\sigma_{eff,1v2}} \equiv F(\boldsymbol{b} = \boldsymbol{0})$$

Naive Gaussian for F(**b**) ➔ factor of two enhancement for 2v1.

BDFS, Eur.Phys.J. C72 (2012) 1963
JG, [arXiv:1207.0480]

Can be crosstalk between NP ladders in 2v1 diagram, at scales lower than 1→2 splitting. For not too low x values the effect of crosstalk should be numerically small due to preference of 1→2 splitting for large x values and small Q. JG, [arXiv:1207.0480]

Total Cross Section for DPS

Following suggestions from previous slides, should calculate total DPS cross section as the sum of 2v1 and 2v2 contributions only (no 1v1 contribution). There are several important (perhaps concerning?) implications that go along with this prescription:

1. The cross section can no longer be written as parton level cross sections convolved with overall 2pGPD factors for each hadron.

Original expression written down on slide 4

2v2 1v2 2v1

$$\sigma^{DPS} \propto \cancel{\int d^2\mathbf{b}\, \Gamma_a(\mathbf{b})\Gamma_b(\mathbf{b})} \rightarrow \int d^2\mathbf{b}\, \Gamma_{a,NP}(\mathbf{b})\Gamma_{b,NP}(\mathbf{b}) + D_{a,P}\Gamma_{b,NP}(\mathbf{b}=\mathbf{0}) + \Gamma_{a,NP}(\mathbf{b}=\mathbf{0})D_{b,P}$$

BDFS, Eur.Phys.J. C72 (2012) 1963
Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201

$$(A+B)^2 \neq A^2 + AB + BA$$

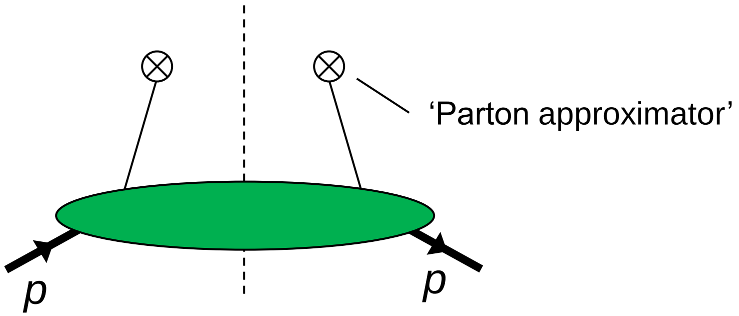
Discussed at MPI@TAU workshop

Total Cross Section for DPS

In particular, there is no concept of the 2pGPD for an individual hadron, with an associated operator definition and evolution equation – this implication discussed in Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201).

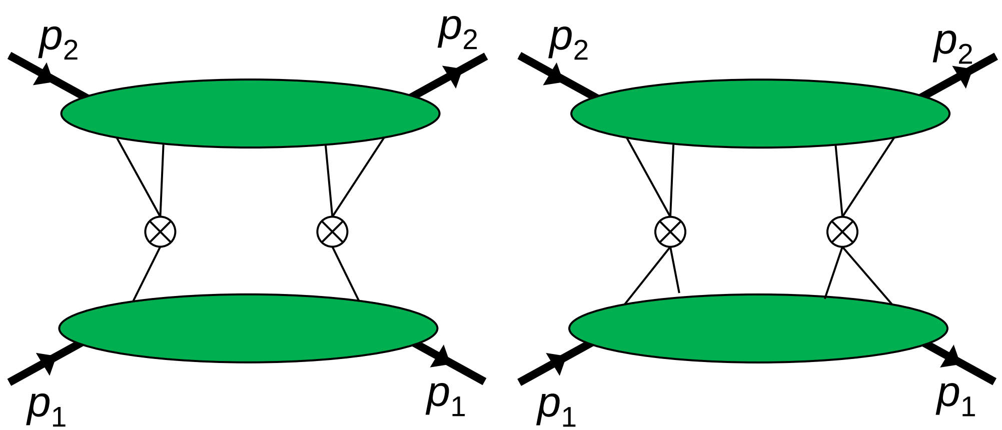
Appropriate hadronic operators in DPS apparently involve both incoming hadrons at once!

SPS hadronic operators



PDF!

DPS hadronic operators



‘2v1 matrix element’ ‘2v2 matrix element’

(Adapted from figures 9 and 10 of MW paper)

Total Cross Section for DPS

2. There are nonperturbatively and perturbatively generated parton pairs with somewhat different behaviour - **b** distribution for former is ~exp(-$\mathbf{b}^2/R_p^2$)/R_p^2, whilst that for latter is power law 1/$\mathbf{b}^2$. There must be some scale $Q_0 \sim \Lambda_{QCD}$ below which one can regard all parton pairs in the proton as nonperturbatively generated in this sense.

Y. Dokshitzer, MPI@TAU 2012:

***Uncertainty** due to choice of the* Q_0 *parameter !*

IMPORTANT ! *there is no contradiction.*

Q_0 *is not a formal parameter but a physical one:*

The parton resolution scale at which $F_g^2(\Delta^2)$ *has **no** PT tail* $1/\Delta^2$ *!*

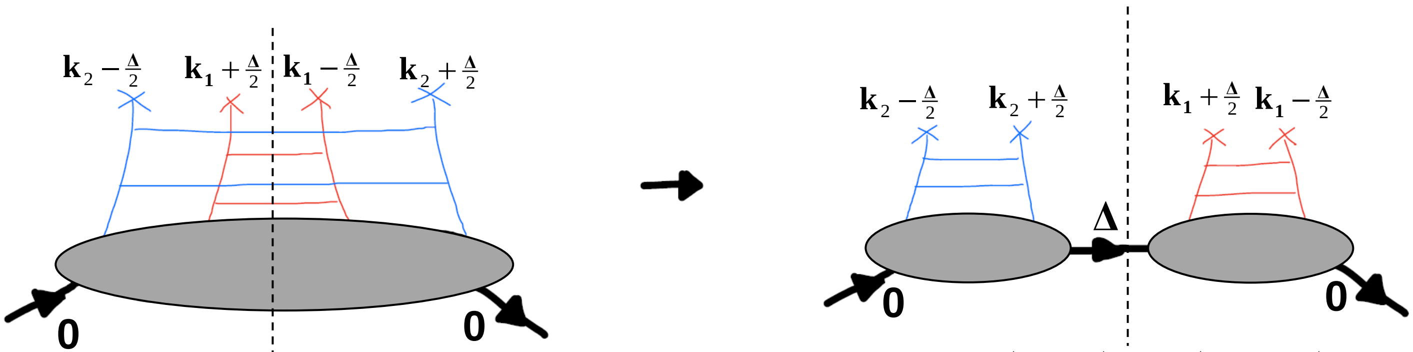
Approximation of 2pGPDs as products of GPDs

Often used to construct input 2pGPD distributions.

Basis of approximation: Insert a complete set of states $\sum_X |X\rangle\langle X|$ between two bilocal operators in 2pGPD, then assume that proton dominates in this sum

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Pictorially:



$D_p^{ij}(x_1, x_2; \mathbf{\Delta})$ $\quad\rightarrow\quad$ $D_p^{i}(x_1; \mathbf{\Delta})\, D_p^{j}(x_2; -\mathbf{\Delta})$ GPD

TMD case: $D_p^{ij}(x_1, x_2; \mathbf{k}_1, \mathbf{k}_2, \mathbf{\Delta})$ $\quad$ $D_p^{i}\left(x_1; \mathbf{k}_1 - x_1 \frac{\mathbf{\Delta}}{2}, \mathbf{\Delta}\right) D_p^{j}\left(x_2; \mathbf{k}_2 - x_2 \frac{\mathbf{\Delta}}{2}, -\mathbf{\Delta}\right)$ GTMD

Approximation of 2pGPDs as products of GPDs

$D_p^{ij}(x_1, x_2; \mathbf{\Delta}) \approx D_p^i(x_1; \mathbf{\Delta})\, D_p^j(x_2; -\mathbf{\Delta})$, or equivalently $D_p^{ij}(x_1, x_2; \mathbf{b}) = \int d^2\tilde{\mathbf{b}} D_p^i(x_1; \tilde{\mathbf{b}} + \mathbf{b}) D_p^j(x_2; \tilde{\mathbf{b}})$

corresponds to neglecting correlations between partons.

Common 'lore': approximately valid at low *x*, due to the large population of partons at such x values.

Further approximation that is often made: $D_p^i(x_1; \tilde{\mathbf{b}}) = D_p^i(x_1)\, F(\tilde{\mathbf{b}})$

$\Rightarrow \quad D_p^{ij}(x_1, x_2; \mathbf{b}) = D_p^i(x_1) D_p^j(x_1) \int d^2\tilde{\mathbf{b}} F(\tilde{\mathbf{b}} + \mathbf{b}) F(\tilde{\mathbf{b}})$

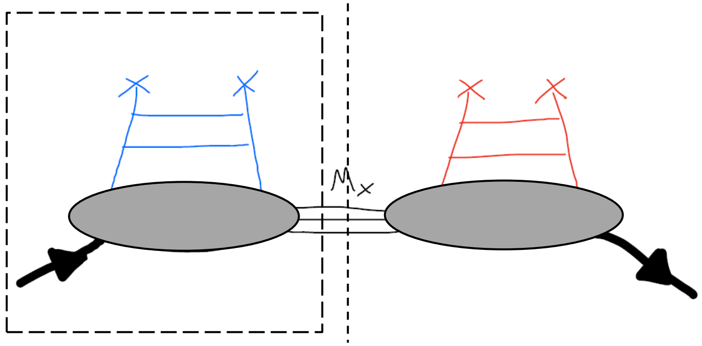
Several MCs (PYTHIA, HERWIG) use these approximations to model MPI

$\Rightarrow \quad \sigma_D^{(A,B)} = \dfrac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$

Some refinements – e.g. *x* dependent proton size Corke, Sjöstrand, JHEP 05 (2011) 009)

[arXiv:1206.5594]: Blok *et al.* extend this approximation by adding an inelastic term (good approx if amplitude is approx imaginary)

Inelastic diffraction – measured at HERA

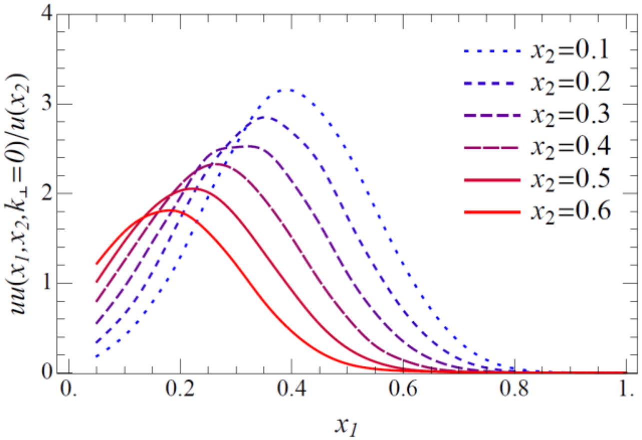


According to [arXiv:1206.5594], inclusion of this term enhances DPS cross section by a factor of 2 for $x < 10^{-3}$.

Model Calculations of 2pGPDs

[arXiv:1210.5434]: Manohar *et al.* calculated valence 2pGPDs from a flexible wall bag model, and showed that these do not factorise into GPDs.

This ratio should be independent of x_2 if factorised ansatz is a good approximation



[arXiv:1210.5434]: Diehl and Kasemets calculated valence 2pGPDs and 2pGTMDs from the 3-parton LC wavefunction: $\Psi(x_i, \boldsymbol{b}_i - \boldsymbol{b}) = \Phi(x_i)\exp\left[-\frac{1}{4a^2}\sum_{i=1}^{3} x_i(\boldsymbol{b}_i - \boldsymbol{b})^2\right]$

Some interesting differences between DPS cross sections calculated using full 2pGPDs/2pGTMDs and those calculated using products of GPDs/GTMDs in this model:

e.g. $$\frac{d\sigma}{d^2\mathbf{q}_1 d^2\mathbf{q}_2} \propto \exp\left\{-a^2\left[\boldsymbol{q}_1^2\, C_{11}(x_i,\bar{x}_i) + 2\boldsymbol{q}_1\boldsymbol{q}_2\, C_{12}(x_i,\bar{x}_i) + \boldsymbol{q}_2^2\, C_{22}(x_i,\bar{x}_i)\right]\right\}$$

C_{12} always positive in full model ➔ final state vector bosons prefer opposite hemispheres, but can have either sign depending on x under GTMD approximation.

➔ Factorised ansatz not good for partons at large x!

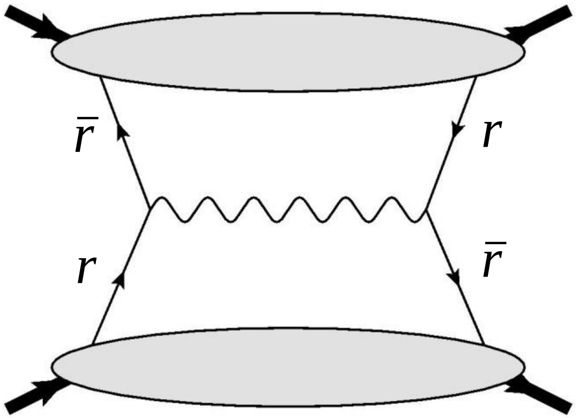
Interference contributions to proton-proton DPS

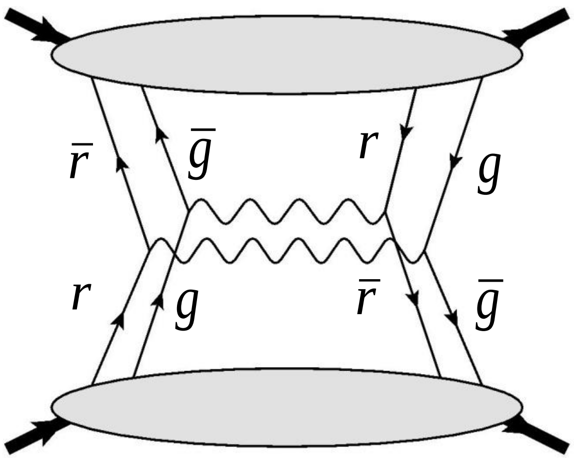
Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

SPS: One parton per proton 'leaves', interacts and 'returns'.

To reform proton, parton must return with same quantum numbers.

No interference contributions to SPS cross section.





DPS: Here we have two partons per proton interacting.

Interference contributions to total cross section in which quantum numbers are swapped between parton legs. Complementary swap is required in other proton.

Can get interference contributions in colour, spin, flavour, and quark number.

Correlated parton contributions to DPS

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g.

$$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

When considering the differential DPS cross section and 2pGTMDs, there are even more distributions to consider:

e.g. $D_p^{q\Delta q}\left(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{b}\right)$

Measures correlation between one of the quark spins and $\mathbf{k}_1$ x $\mathbf{k}_2$ (for example).

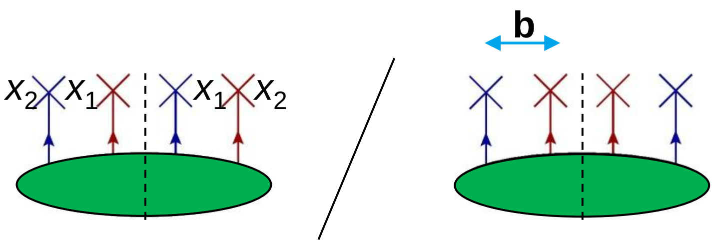
Allowed types of interference/correlated parton 2pGPDs and 2pGTMDs for p-p DPS in quark sector are catalogued in Diehl, Ostermeier and Schafer (JHEP 1203 (2012)) - also see talk by Kasemets for more detail concerning spin-correlated 2pGPDs.

In p-A collisions further types of interference contributions are possible (e.g. interference contributions in *x*) Blok, Strikman, Wiedemann [arXiv:1210.1477], Treleani, Calucci, Phys.Rev. D86 (2012) 036003

Sudakov Suppression of Colour Interference Distributions

For the 2pGPD with finite **b**, every distribution which does not have the partons with the same lightcone mtm fractions paired up into colour singlets is Sudakov suppressed:

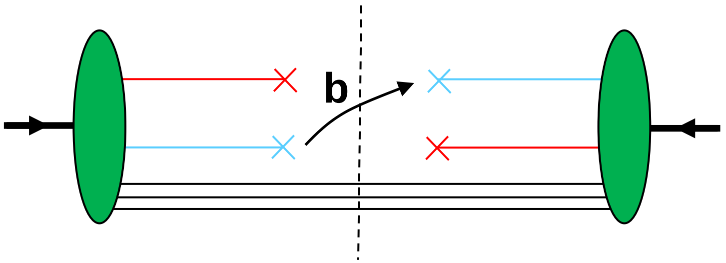
Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089)
Manohar and Waalewijn, Phys.Rev. D85 (2012) 114009



$$\sim \exp\left(\frac{\alpha_s}{2\pi} \underbrace{\left(C_R^I - C_V^I\right)}_{<0} \ln^2\left(\mathbf{b}^2 Q^2\right) \right)$$

Physical explanation: Movement of colour by large transverse distance **b** in hadron between amplitude and conjugate.

Manohar and Waalewijn, Phys.Rev. D85 (2012) 114009

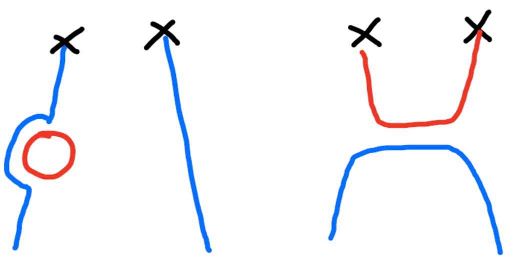


Sudakov Suppression of Colour Interference Distributions

At level of diagrams: Noncancellation of real and virtual diagrams in colour interference distributions.

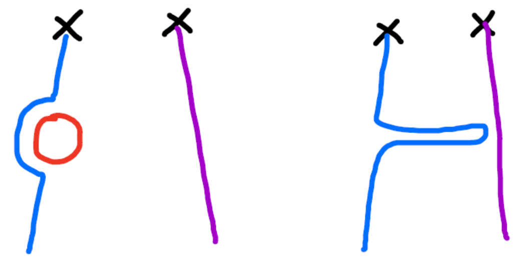
Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622

Colour singlet



$C_V = C_F$ $\qquad$ $C_R = C_F$

Colour interference

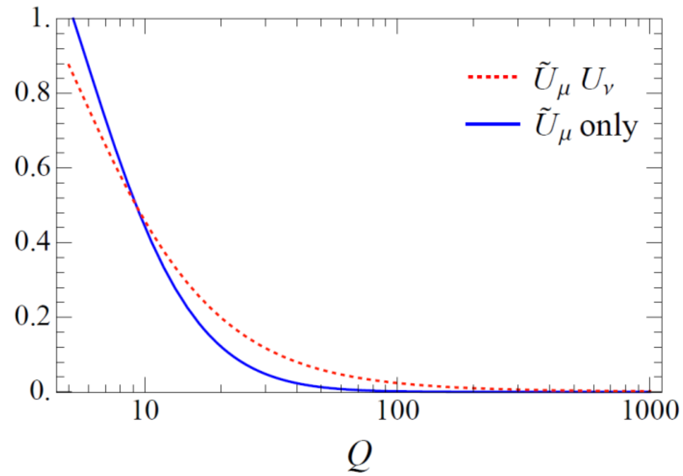


$C_V = C_F$ $\qquad$ $C_R = -\dfrac{1}{2N}$

Numerical evaluation of Sudakov factor including single logarithmic terms:

[Lower cutoff in Sudakov factor taken to be Λ = 1.4 GeV]

Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

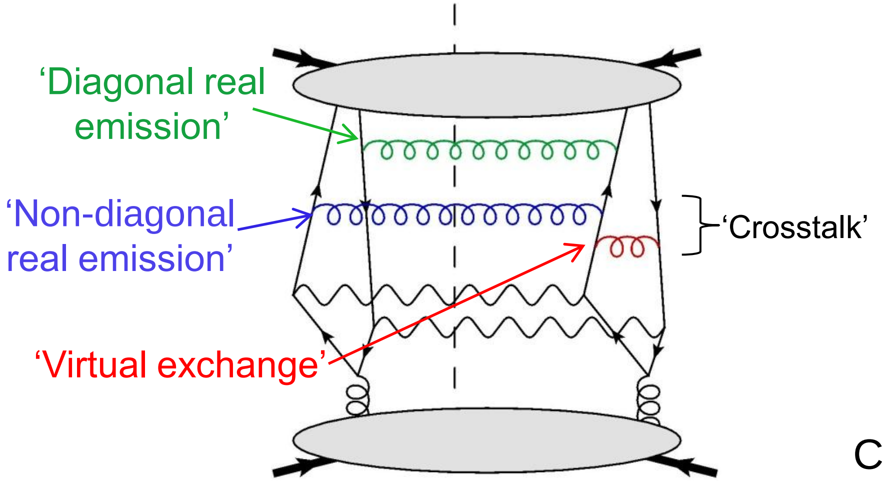


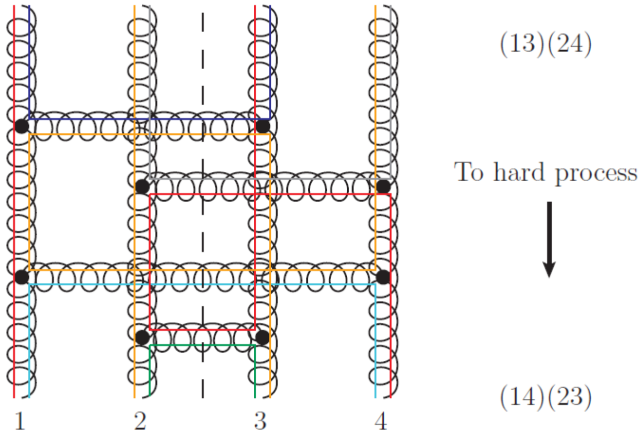
Summary

- Total DPS σ is described in terms of 2pGPDs, whilst $d^2\sigma/d^2\mathbf{q}_A d^2\mathbf{q}_B$ is described in terms of 2pGTMDs. Relation between the two for $|\mathbf{q}| \gg \Lambda$
- Calculations indicate that 2v1 graphs should be included in DPS cross section, whilst 1v1 graphs should be regarded as SPS. Some interesting (concerning?) features in this prescription?
- Making certain approximations one can write a 2pGPD as a product of single GPDs. Indications that approximation may not be good for either large or small *x* values!
- There are interference and correlated parton contributions to DPS in colour, flavour and spin space. Colour interference contributions to DPS are Sudakov suppressed.

Backup Slides

Crosstalk in 2v1 graphs





JG, [arXiv:1207.0480]

In the 2v1 graphs, crosstalk interactions between the two nonperturbatively generated ladders are allowed at the leading log level, provided that these occur at a lower scale than k^2, the scale of the 1→2 splitting on the other side.

Crosstalk interactions alter the colour structure of two NP ladders – are such interactions Sudakov suppressed?
A: No – two ladders are effectively on top of one another for scales < k^2, and soft longwave gluons can only detect overall singlet colour.

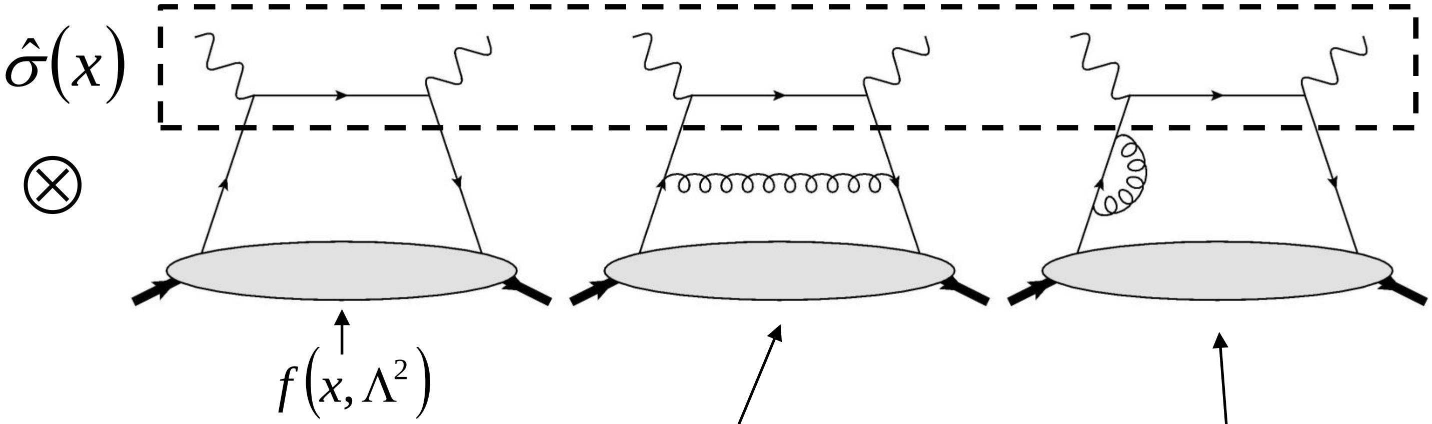
Nevertheless, there is some colour suppression of crosstalk interactions. Most likely crosstalk at low x is this recombination interaction – suppressed by $1/(N_C^2 - 1)$ as it is nonplanar.

Precise numerical impact of crosstalk interactions on σ_{2v1} remains to be worked out.

Sudakov Suppression of Colour Interference Distributions

Illustrate using just one ladder (DIS):

Artru and Mekhfi, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009



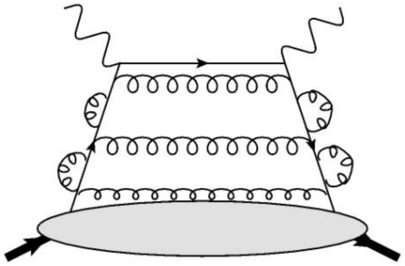
Ellis, Stirling, Webber, Chapter 5

$$+\frac{\alpha_s}{2\pi} C_R \ln\left(\frac{Q^2}{\Lambda^2}\right)\int_0^{1-\Lambda^2/Q^2} \frac{dx'}{x'}\frac{1+x'^2}{1-x'} f\left(\frac{x}{x'},\Lambda^2\right) \quad -\frac{\alpha_s}{2\pi} C_V \ln\left(\frac{Q^2}{\Lambda^2}\right) f(x,\Lambda^2)\int_0^{1-\Lambda^2/Q^2} dx' \frac{1+x'^2}{1-x'}$$

$$+\frac{\alpha_s}{\pi} C_R \ln^2\left(\frac{Q^2}{\Lambda^2}\right) f(x,\Lambda^2) \quad -\frac{\alpha_s}{\pi} C_V \ln^2\left(\frac{Q^2}{\Lambda^2}\right) f(x,\Lambda^2)$$

Sudakov Suppression of Colour Interference/Correlation Distributions

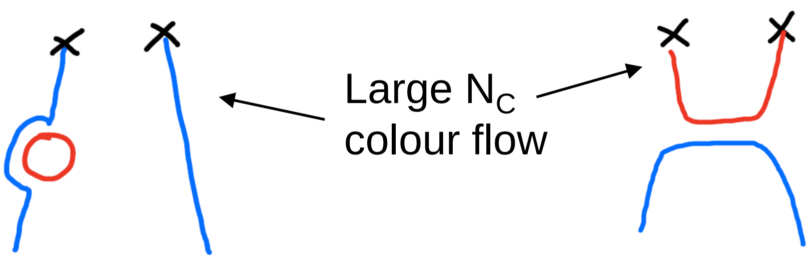
Resum arbitrary number of real & virtual emissions to double log order:



Sudakov factor

$$\sigma = \hat{\sigma}(x) \otimes f\left(x, \Lambda^2\right) \exp\left(\frac{\alpha_s}{\pi}(C_R - C_V)\ln^2\left(\frac{Q^2}{\Lambda^2}\right)\right)$$

Quark legs are in colour singlet:



Large N_C colour flow

$C_V = C_F$

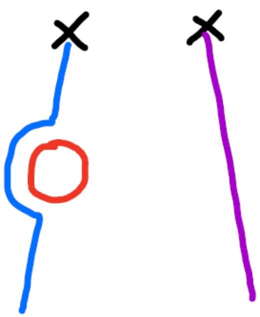
$C_R = C_F$

$C_R = C_V$

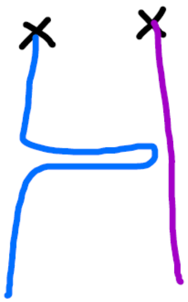
There is no Sudakov suppression!

Sudakov Suppression of Colour Interference/Correlation Distributions

Quark legs are in colour octet (as occurs in colour interference/correlation distributions):



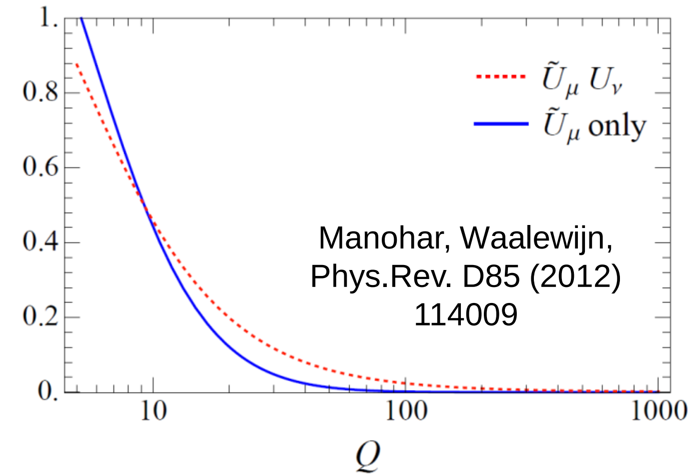
$$C_V = C_F$$



$$C_R = \left(C_F - \frac{1}{2} C_A \right) = -\frac{1}{2N}$$

$$C_R + C_V < 0$$

➔ Sudakov suppression:



Physical explanation: Movement of colour by large transverse distance **b** in hadron.

Soft gluons with wavelengths larger than 1/**b** can't resolve colour transfer – cut off in Sudakov factor should really be 1/**b**^{2}.