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Splitting diagrams in double logarithm approximation of pQCD

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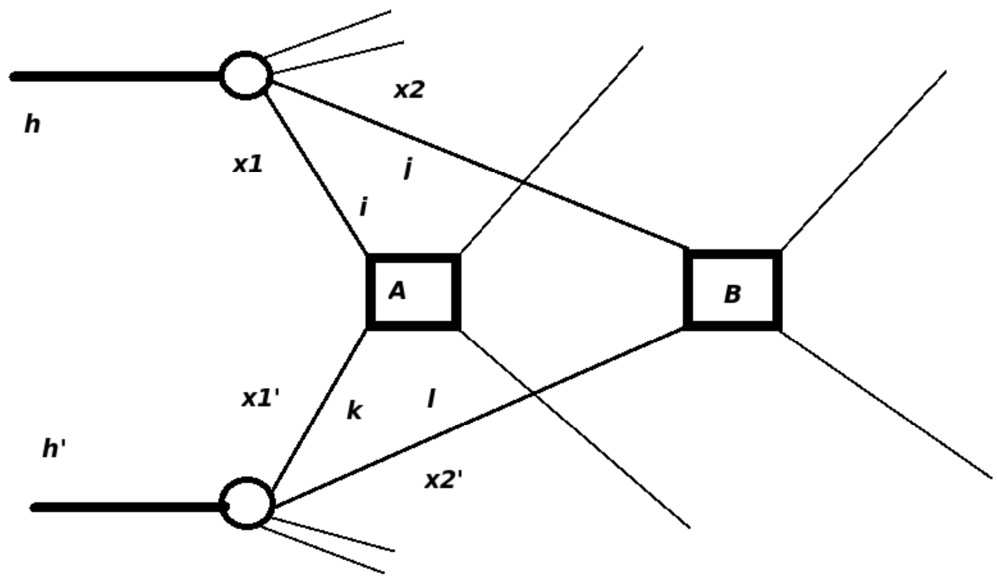
A revised formula for the inclusive cross section of a double parton scattering in a hadron collision is studied in double logarithm approximation of pQCD.

The possible phenomenological issues are discussed.

In the last years it has become obvious:

Multiple parton interactions play an important role in hadron-hadron collisions at high energies and are one of the most common, yet poorly understood, phenomenon at the LHC.

With only the assumption of factorization of the two hard parton subprocesses A and B



the inclusive cross section of a double parton scattering process in a hadron collision is written in the following form (*Paver, Treleani; Mekhfi;..., Diehl,.....*)

$$\sigma_{DPS}^{AB}=\frac{m}{2}\sum_{i,j,k,l}\int\Gamma_{ij}(x_1,x_2;\mathrm{b}_1,\mathrm{b}_2;Q_1^2,Q_2^2)\hat{\sigma}_{ik}^{A}(x_1,x_1',Q_1^2)\hat{\sigma}_{jl}^{B}(x_2,x_2',Q_2^2)$$

$$\times\Gamma_{kl}(x_1',x_2';\mathrm{b}_1-\mathrm{b},\mathrm{b}_2-\mathrm{b};Q_1^2,Q_2^2)dx_1dx_2dx_1'dx_2'd^2b_1d^2b_2d^2b,$$

where b is the impact parameter — the distance between centers of colliding (e.g., the beam and the target) hadrons in transverse plane.

$\Gamma_{ij}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)$ **are the double parton distribution functions, which depend on the longitudinal momentum fractions** x_1 **and** x_2**, and on the transverse position** $\mathbf{b_1}$ **and** $\mathbf{b_2}$ **of the two parton undergoing hard processes** A **and** B **at the scales** Q_1 **and** Q_2**.**

$\hat{\sigma}_{ik}^{A}$ **and** $\hat{\sigma}_{jl}^{B}$ **are the parton-level subprocess cross sections.**

The factor $m/2$ **appears due to the symmetry of the expression for interchanging parton species** i **and** j**.** $m=1$ **if** $A=B$**, and** $m=2$ **otherwise.**

The double parton distribution functions $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the main object of interest as concerns multiple parton interactions. In fact, these distributions contain all the information when probing the hadron in two different points simultaneously, through the hard processes A and B.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathrm{b}_1) f(\mathrm{b}_2),$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with the fixed normalization

$$\int f(\mathrm{b}_1) f(\mathrm{b}_1 - \mathrm{b}) d^2 b_1 d^2 b = \int T(\mathrm{b}) d^2 b = 1,$$

and

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function (not *calculated* in perturbative QCD).

If one makes the further assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2),$$

the cross section of double parton scattering can be expressed in the simple form

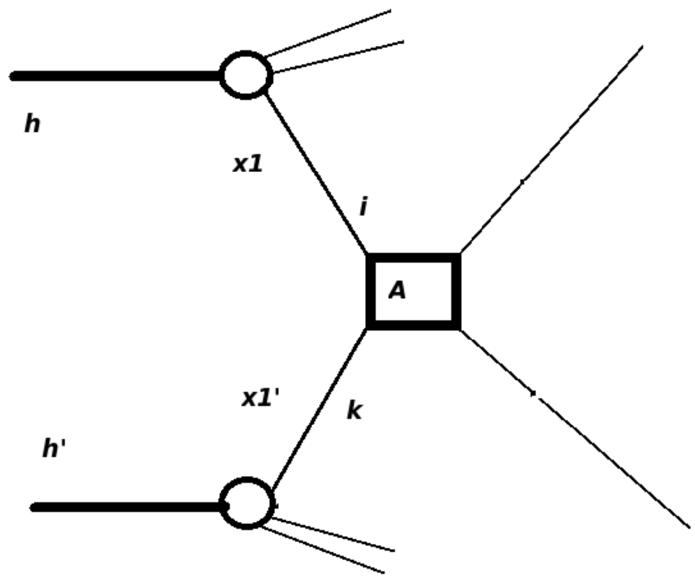
$$\sigma_{\rm DPS}^{\rm AB} = \frac{m}{2} \frac{\sigma_{\rm SPS}^A \sigma_{\rm SPS}^B}{\sigma_{\rm eff}},$$

$$\pi R_{\rm eff}^2 = \sigma_{\rm eff} = [\int d^2 b (T(\mathrm{b}))^2]^{-1}$$

is the effective interaction transverse area (effective cross section)
$R_{\rm eff}$ is an estimate of the size of the hadron

with the usual form

$$\sigma^A_{\rm SPS} = \sum_{i,k} \int D^i_h(x_1;Q_1^2) f(\mathrm{b}_1) \hat{\sigma}^A_{ik}(x_1, x_1^{'}) D^k_{h'}(x_1^{'};Q_1^2) f(\mathrm{b}_1 - \mathrm{b}) dx_1 dx_1^{'} d^2b_1 d^2b$$



$$= \sum_{i,k} \int D^i_h(x_1;Q_1^2) \hat{\sigma}^A_{ik}(x_1, x_1^{'}) D^k_{h'}(x_1^{'};Q_1^2) dx_1 dx_1^{'}$$

for the inclusive cross section of single hard scattering.

For our further goal, the momentum (*instead of the mixed (momentum and coordinate)*) representation is more convenient:

$$\sigma^{D}_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{q}; Q_1^2, Q_2^2) \hat{\sigma}^{A}_{ik}(x_1, x_1^{'}) \hat{\sigma}^{B}_{jl}(x_2, x_2^{'})$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; -\mathrm{q}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} \frac{d^2 q}{(2\pi)^2}.$$

Here the transverse vector q is equal to the difference of the momenta of partons from the wave function of the colliding hadrons in the amplitude and the amplitude conjugated. Such dependence arises because the difference of parton transverse momenta within the parton pair is not conserved.

The main problem is to make the correct calculation of the two-parton functions $\Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2)$ **WITHOUT simplifying factorization assumptions** (*which are not sufficiently justified and should be revised: Blok, Dokshitzer, Frankfurt, Strikman; Diehl, Schafer; Gaunt, Stirling; Manohar, Waalewijn; Ryskin, Snigirev*)

These functions were available in the current literature only for $\mathbf{q} = 0$ **in the collinear approximation. In this approximation the two-parton distribution functions,**$\Gamma_{ij}(x_1, x_2; \mathbf{q} = 0; Q^2, Q^2) = D_h^{ij}(x_1, x_2; Q^2, Q^2)$ **with the two hard scales set equal, satisfy the generalized Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations** (*Kirshner; Shelest, Snigirev, Zinovjev*)

The functions in question have a specific interpretation in the leading logarithm approximation of perturbative QCD: they are the inclusive probabilities that in a hadron h **one finds two bare partons of types** i **and** j **with the given longitudinal momentum fractions** x_1 **and** x_2**.**

The evolution equation for Γ_{ij} consists of two terms. The first term describes the independent (simultaneous) evolution of two branches of parton cascade: one branch contains the parton x_1, and another branch — the parton x_2.

The second term allows for the possibility of splitting one parton evolution (one branch k) into two different branches, i and j. It contains the usual splitting function $P_{k\to ij}(z)$. The solutions of the generalized DGLAP evolution equations with the given initial conditions at the reference scales μ^2 may be written in the form:

$$D_h^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= D_{h1}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) + D_{h2}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

with

$$D_{h1}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= \sum_{{j_1}'{j_2}'} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \, D_h^{{j_1}'{j_2}'}(z_1, z_2; \mu^2) D_{{j_1}'}^{j_1}(\frac{x_1}{z_1}; \mu^2, Q_1^2) D_{{j_2}'}^{j_2}(\frac{x_2}{z_2}, \mu^2, Q_2^2)$$

and

$$D_{h2}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) = \sum_{j' j_1' j_2'} \int_{\mu^2}^{\min(Q_1^2, Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \times$$

$$D_h^{j'}(z_1 + z_2; \mu^2, k^2) \frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}(\frac{z_1}{z_1 + z_2}) D_{j_1'}^{j_1}(\frac{x_1}{z_1}; k^2, Q_1^2) D_{j_2'}^{j_2}(\frac{x_2}{z_2}; k^2, Q_2^2)$$

where $\alpha_s(k^2)$ is the QCD coupling,
$D_{j_1'}^{j_1}(z; k^2, Q^2)$ are the known single distribution functions (the Green's functions) at the parton level with the specific δ-like initial conditions at $Q^2 = k^2$.

$D_h^{j_1', j_2'}(z_1, z_2, \mu^2)$ is the initial (input) two-parton distribution at the relatively low scale μ.

The one parton distribution (before splitting into the two branches at some scale k^2) is given by $D_h^{j'}(z_1 + z_2, \mu^2, k^2)$.

The first term is the solution of homogeneous evolution equation (independent evolution of two branches), where the input two-parton distribution is generally NOT known at the low scale μ. For this non-perturbative two-parton function at low z_1, z_2 one may assume the factorization $D_h^{{j_1}'{j_2}'}(z_1, z_2, \mu^2) \simeq D_h^{{j_1}'}(z_1, \mu^2) D_h^{{j_2}'}(z_2, \mu^2)$ neglecting the constraints due to momentum conservation ($z_1 + z_2 < 1$).

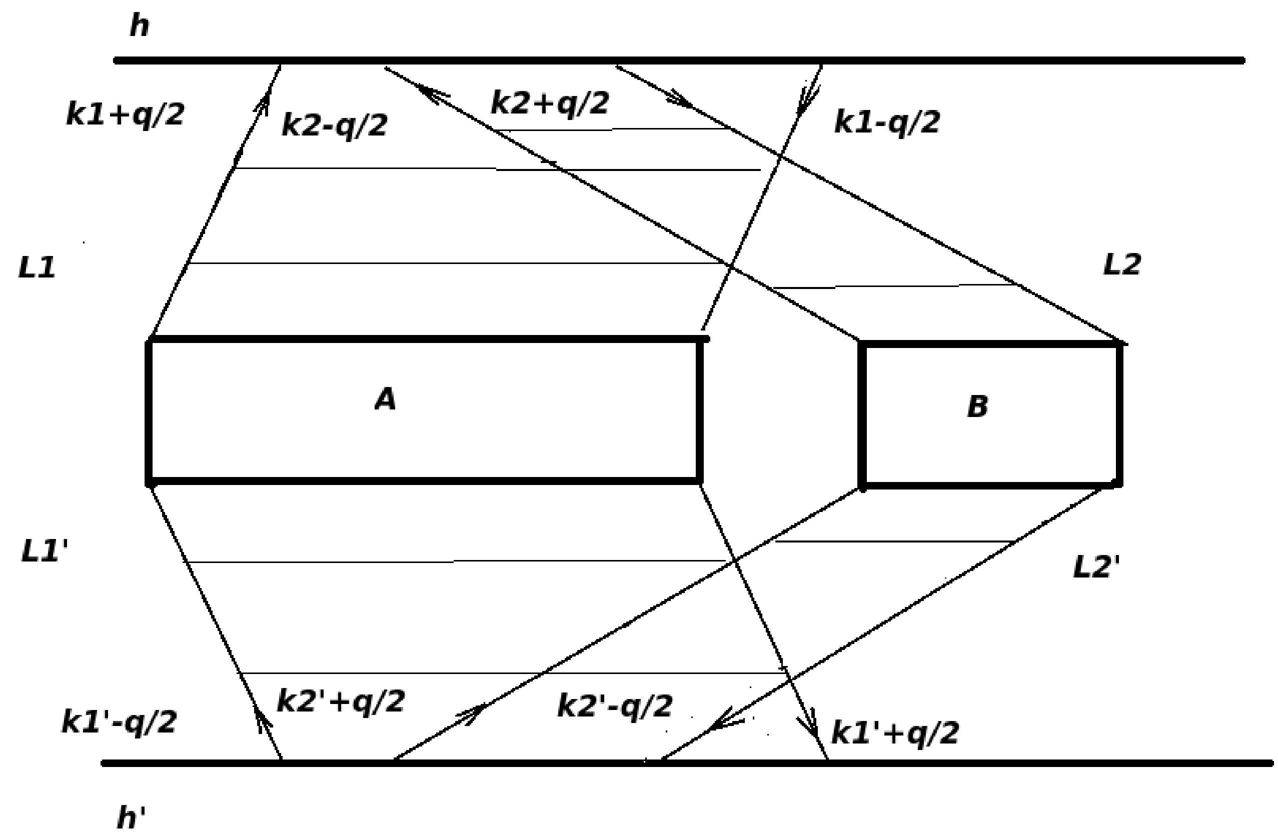
This leads to

$$D_{h1}^{ij}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) \simeq D_h^{i}(x_1; \mu^2, Q_1^2) D_h^{j}(x_2; \mu^2, Q_2^2)$$

the factorization hypothesis usually used in current estimations.

However, one should note that the input two-parton distribution $D_h^{j_1', j_2'}(z_1, z_2, \mu^2)$ may be more complicated than that given by factorization ansatz.

As a rule the multiple interactions take place at relatively low transverse momenta and low $x_{1,2}$, where the factorization hypothesis for the first term is a good approximation.



In this case the cross section for double parton scattering can be estimated, using the two-gluon form factor of the nucleon $F_{2g}(q)$ for the dominant gluon-gluon scattering mode (or something similar for other parton scattering modes)

$$\sigma^{D,1\times1}_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int D^i_h(x_1;\mu^2,Q^2_1) D^j_h(x_2;\mu^2,Q^2_2) \hat{\sigma}^A_{ik}(x_1,x'_1) \hat{\sigma}^B_{jl}(x_2,x'_2)$$

$$\times D^k_{h'}(x'_1;\mu^2,Q^2_1) D^l_{h'}(x'_2;\mu^2,Q^2_2) dx_1 dx_2 dx'_1 dx'_2 \int F^4_{2g}(q) \frac{d^2q}{(2\pi)^2}.$$

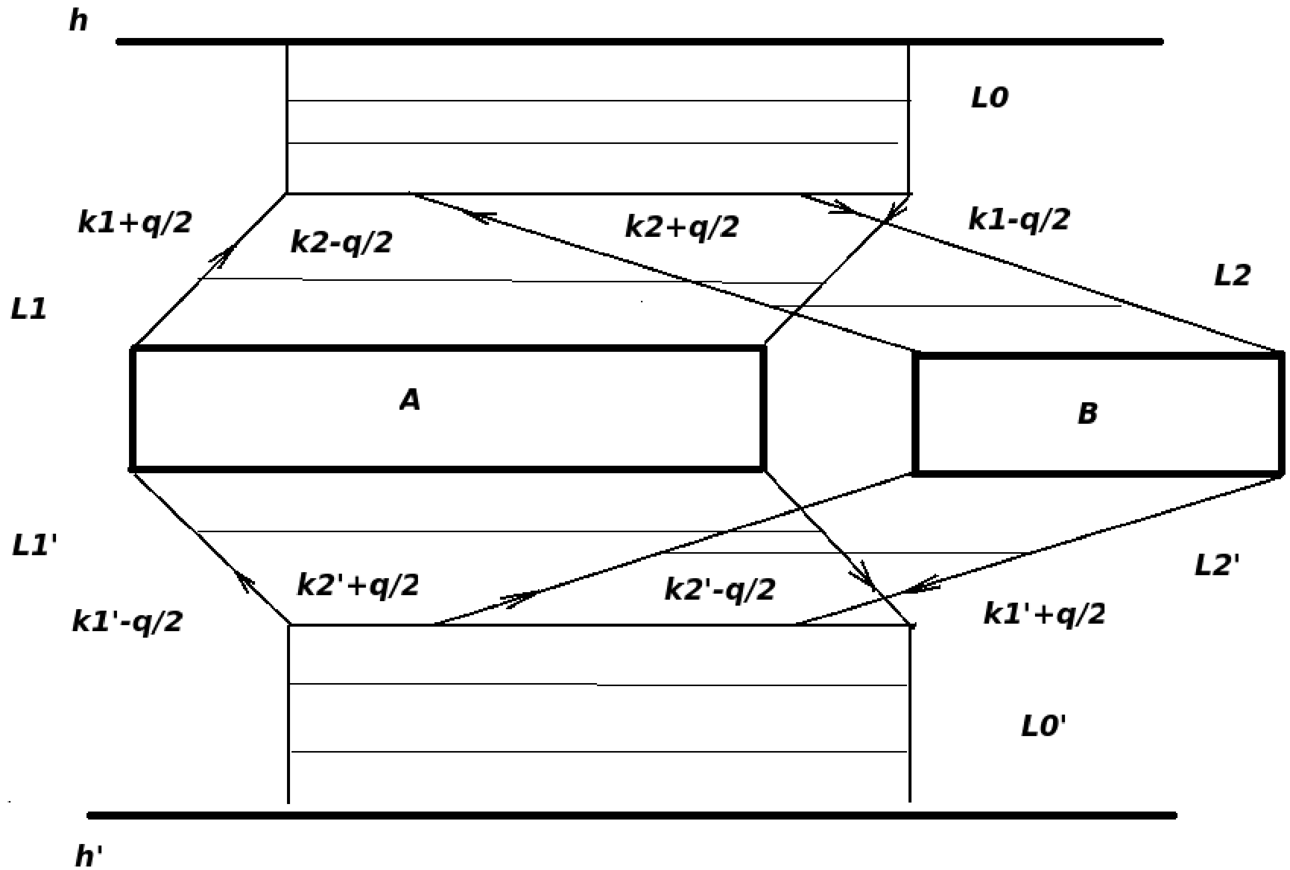
From the dipole fit $F_{2g}(q) = 1/(q^2/m_g^2 + 1)^2$ it follows that the characteristic value of q is of the order of "effective gluon mass" m_g. Thus the initial conditions for the single distributions can be fixed at some not large reference scale $\mu \sim m_g$, because of the weak logarithmic dependence of these distributions on the scale value.

In this approach

$$\int F^4_{2g}(q) \frac{d^2q}{(2\pi)^2}$$

gives the estimation of $[\sigma_{\rm eff}]^{-1}$.

The second term is the solution of complete evolution equation with the evolution originating from one "nonperturbative" parton at the reference scale. Here the independent evolution of two branches starts at the scale k^2 from a point-like parton j'.



In this case, the large q_t domain is NOT suppressed by the form factor $F_{2g}(q)$ and the corresponding contribution to the cross section reads

$$\sigma^{D,2\times 2}_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} \frac{d^2q}{(2\pi)^2} \sum_{j' j_1' j_2'} \int_{q^2}^{\min(Q_1^2,Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2}$$

$$\times \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j'}(z_1+z_2;\mu^2,k^2) \frac{1}{z_1+z_2} P_{j'\to j_1' j_2'}\left(\frac{z_1}{z_1+z_2}\right)$$

$$\times D^i_{j_1'}\left(\frac{x_1}{z_1};k^2,Q_1^2\right) D^j_{j_2'}\left(\frac{x_2}{z_2};k^2,Q_2^2\right) \hat\sigma^A_{ik}(x_1,x_1') \hat\sigma^B_{jl}(x_2,x_2')$$

$$\times \sum_{j' j_1' j_2'} \int_{q^2}^{\min(Q_1^2,Q_2^2)} dk'^2 \frac{\alpha_s(k'^2)}{2\pi k'^2} \int_{x_1'}^{1-x_2'} \frac{dz_1}{z_1} \int_{x_2'}^{1-z_1} \frac{dz_2}{z_2} D_{h'}^{j'}(z_1+z_2;\mu^2,k'^2)$$

$$\times \frac{1}{z_1+z_2} P_{j'\to j_1' j_2'}\left(\frac{z_1}{z_1+z_2}\right) D^k_{j_1'}\left(\frac{x_1'}{z_1};k'^2,Q_1^2\right) D^l_{j_2'}\left(\frac{x_2'}{z_2};k'^2,Q_2^2\right),$$

or in substantially shorter yet less transparent form:

$$\sigma^{D,2\times2}_{(A,B)}\frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1^{'} dx_2^{'} \int^{\min(Q_1^2,Q_2^2)} \frac{d^2q}{(2\pi)^2} D^{ij}_{h2}(x_1,x_2;q^2,Q_1^2,Q_2^2)$$

$$\times\hat{\sigma}^A_{ik}(x_1,x_1^{'})\hat{\sigma}^B_{jl}(x_2,x_2^{'})D^{kl}_{h'2}(x_1',x_2';q^2,Q_1^2,Q_2^2).$$

By analogy, the combined ("interference") contribution may be written as

$$\sigma^{D,1\times2}_{(A,B)} = \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1^{'} dx_2^{'} \int^{\min(Q_1^2,Q_2^2)} F^2_{2g}(q)\frac{d^2q}{(2\pi)^2}$$

$$\times[D^i_h(x_1;\mu^2,Q_1^2)D^j_h(x_2;\mu^2,Q_2^2)\hat{\sigma}^A_{ik}(x_1,x_1^{'})\hat{\sigma}^B_{jl}(x_2,x_2^{'})D^{kl}_{h'2}(x_1',x_2';q^2,Q_1^2,Q_2^2)$$

$$+D^{ij}_{h2}(x_1,x_2;q^2,Q_1^2,Q_2^2)\hat{\sigma}^A_{ik}(x_1,x_1^{'})\hat{\sigma}^B_{jl}(x_2,x_2^{'})D^k_{h'}(x_1';\mu^2,Q_1^2)D^l_{h'}(x_2';\mu^2,Q_2^2)].$$

The equations (above and below) present our solution of the problem — we obtain the estimation of the inclusive cross section for double parton scattering, taking into account the QCD evolution and basing on the well-known collinear distributions, extracted from deep inelastic scattering:

$$\boldsymbol{\sigma}^{D}_{(A,B)} = \boldsymbol{\sigma}^{D,1\times1}_{(A,B)} + \boldsymbol{\sigma}^{D,1\times2}_{(A,B)} + \boldsymbol{\sigma}^{D,2\times2}_{(A,B)}$$

Afterwards similar results were obtained also by Blok, Dokshitzer, Frankfurt, Strikman with an emphasis on the differential cross sections, then by Gaunt, Stirling **(as concerning 1×1, 1×2 components)**

2×2 **component (double splitting diagrams) is the subject of discussion and our disagreement with** *Blok, Dokshitzer, Frankfurt, Strikman;Gaunt, Stirling;Manohar, Waalewijn*, **mainly in a terminology.**

At a large final scale Q^2 the contribution of second (2×2) component should dominate being proportional to $q^2 \sim Q^2$, while the contributions of the 1×1 or 1×2 components $\sim m_g^2 \sim 1/\sigma_{eff}$ are limited by the nucleon (hadron) form factorF_{2g}.

In terms of impact parameters $\mathbf{b}$ this means that in the second (2×2) term two pairs of partons are very close to each other; $|\mathbf{b}_1 - \mathbf{b}_2| \sim 1/Q$.

We have to emphasize that the dominant contribution to the phase space integral comes from a large $q^2 \sim Q^2$ and, strictly speaking, the above reasoning makes no allowance for the collinear (DGLAP) evolution of two independent branches of the parton cascade (i.e., in the ladders $L1, L2, L1'$ and $L2'$) in the 2×2 term.

Formally in the framework of collinear approach this contribution should be considered as the result of interaction of *one* pair of partons with the $2 \to 4$ hard subprocess (*Blok, Dokshitzer, Frankfurt, Strikman;Gaunt, Stirling;Manohar, Waalewijn*).

On the boundary of phase space our formula reproduces naturally this result ($2 \to 4$) due to the specific δ-like initial conditions at $k^2 = Q^2$ for Green's functions.

Recall, however, that when estimating the phase space integral we neglect the anomalous dimension, γ, of the parton distributions $D_j^k(x/z, k^2, Q^2) \propto (Q^2/k^2)^\gamma$. In collinear approach the anomalous dimensions $\gamma \propto \alpha_s << 1$ are assumed to be small. On the other hand, in a low x region the value of anomalous dimension is enhanced by the $\ln(1/x)$ logarithm and may be rather large numerically.

So the integral over q^2 is *slowly* convergent and the major contribution to the cross section is expected to come actually from some characteristic intermediate region, $m_g^2 << q^2 << Q_1^2$ $(Q_1 < Q_2)$.

Thus we do not expect such strong sensitivity to the upper limit of q-integration as in the case of the pure phase space integral.

Therefore it makes sense to consider the quantitative contribution of the 2×2 term even within the collinear approach as applied to the LHC kinematics, where the large (in comparison with m_g) available values of Q_1 and Q_2 provide wide enough integration region for the characteristic loop momenta q.

We demonstrate this fact by direct calculation in the double logarithm approximation. Let us put down the all integrations with splitting functions separately to make the analysis more transparent

$$D_{h2}^{ij}(x_1,x_2;q^2,Q_1^2,Q_2^2)=\sum_{j'j_1'j_2'}\int\limits_{q^2}^{\min(Q_1^2,Q_2^2)}dk^2\frac{\alpha_s(k^2)}{2\pi k^2}\int\limits_{x_1}^{1-x_2}\frac{dz_1}{z_1}\int\limits_{x_2}^{1-z_1}\frac{dz_2}{z_2}$$

$$\times D_h^{j'}(z_1+z_2;\mu^2,k^2)\frac{1}{z_1+z_2}P_{j'\to j_1'j_2'}(\frac{z_1}{z_1+z_2})D_{j_1'}^{i}(\frac{x_1}{z_1};k^2,Q_1^2)D_{j_2'}^{j}(\frac{x_2}{z_2};k^2,Q_2^2).$$

In the double logarithm approximation we can restrict ourselves to the gluon main contribution only and rewrite the integral under consideration in the following form

$$D_{h2}^{gg}(x_1,x_2;q^2,Q_1^2,Q_2^2)=\int\limits_{q^2}^{\min(Q_1^2,Q_2^2)}dk^2\frac{\alpha_s(k^2)}{2\pi k^2}\int\frac{du}{u^2}D_h^g(u;\mu^2,k^2)\int\frac{dz}{z(1-z)}$$

$$\times P_{g\to gg}(z)D_g^g(\frac{x_1}{uz};k^2,Q_1^2)D_g^g\left(\frac{x_2}{u(1-z)};k^2,Q_2^2\right).$$

The Green's functions (gluon distributions at the parton level) in the double logarithm approximation read

$$xD_g^g(x,t) \simeq 4N_c t v^{-3/2} \exp\left[v - at\right]/\sqrt{2\pi},$$

where

$$v = \sqrt{8N_c t \ln(1/x)},\ a = \tfrac{11}{6}N_c + \tfrac{1}{3}n_f/N_c^2$$

$$t(Q^2) = \frac{2}{\beta}\ln\left[\frac{\ln(\frac{Q^2}{\Lambda^2})}{\ln(\frac{\mu^2}{\Lambda^2})}\right],$$

and where

$$\beta = (11N_c - 2n_f)/3$$

n_f is the number of active flavors, Λ is the dimensional QCD parameter, $N_c = 3$ is the color number and the one-loop running QCD coupling

$$\alpha_s(Q^2) = \frac{4\pi}{\beta\ln(Q^2/\Lambda^2)}$$

was used

After that the integral may be rewritten as

$$x_1 x_2 D_{h2}^{gg}(x_1, x_2; \tau, T_1, T_2)$$

$$\sim \int_{\tau}^{\min(T_1,T_2)} dt \int dz P_{g\to gg}(z) \int dy \exp\left[\sqrt{8N_c} d(t,y,z)\right],$$

where

$$d(t,y,z) = \sqrt{ty} + \sqrt{(T_1 - t)(Y_1 - y)} + \sqrt{(T_2 - t)(Y_2 - y)}$$

with

$$t = t(k^2),\ T_1 = t(Q_1^2),\ T_2 = t(Q_2^2),\ \tau = t(q^2)$$

and

$$y = \ln(1/u),\ Y_1 = \ln(1/x_1) - \ln(1/z),\ Y_2 = \ln(1/x_2) - \ln(1/(1-z)).$$

We keep the leading exponential terms only, which have the same structure both at the parton level and at the hadron one under the *smooth enough initial conditions* at the reference scale.

We are interested in the domain with large enough T_1, T_2, $\ln(1/x_1)$ and $\ln(1/x_2)$, when the exponential factors are large in comparison with 1 and where the approximations above are justified. In this case the integration over the rapidity y has the saddle point structure in the wide interval of z-integration not near the kinematic boundaries. The saddle-point equation reads

$$\frac{\sqrt{t}}{\sqrt{y_0}} - \frac{\sqrt{(T_1 - t)}}{\sqrt{(Y_1 - y_0)}} - \frac{\sqrt{(T_2 - t)}}{\sqrt{(Y_2 - y_0)}} = 0.$$

It may be solved explicitly in the simplest case of the two hard scales set equal $T_1 = T_2 = T$ and at $Y_1 \simeq Y_2 \simeq Y = \ln(1/x)$, i.e., in the z-region where $\ln(1/z) << \ln(1/x)$ and $\ln(1/(1-z)) << \ln(1/x)$ (*In spite of the large nonexponential factor like $\ln(1/x)$ (due to the singularity of the splitting function $P_{g\to gg}(z)$) the contribution from the integration region near the kinematical boundaries $z \sim x$ and $1-z \sim x$ is not dominant, since in this case the obtained exponential factor $\exp\left[\sqrt{8N_c}\sqrt{Y(T-\tau)}\right]$ is not leading***).**

Then the saddle-point is equal to

$$y_0 = Yt/(4T - 3t).$$

Thus, the splitting integrals reduce to

$$x^2 D_{h2}^{gg}(x, x; \tau, T, T) \sim \int_{\tau}^{T} dt \int_{x}^{1-x} dz P_{g\to gg}(z) \exp\left[\sqrt{8N_c}\sqrt{Y(4T-3t)}\right].$$

The t-integration is not a saddle-point type and therefore one of edges, namely $t \to \tau$, dominates. That is

$$x^2 D_{h2}^{gg}(x, x; \tau, T, T) \sim \exp\left[\sqrt{8N_c}\sqrt{Y(4T-3\tau)}\right].$$

What follows from our estimation of splitting integrals in the double logarithm approximation by the saddle-point method ?

For single splitting diagrams (1×2 contribution)

the lower limit for the t-integration may be taken at the reference scale, i.e., $\tau = t(q^2)|_{q=\mu} = 0$ due to the strong suppression factor $F^2_{2g}(q)$. The characteristic value of q being of the order of "effective gluon mass" $m_g \sim \mu$ in the further q-integration. Thus one obtains for this contribution the following estimation

$$\boldsymbol{x^2 D^{gg}_{h2}(x, x; 0, T, T) \sim \exp\left[\sqrt{8N_c}(\sqrt{YT} + \sqrt{YT})\right].}$$

It means that the splitting takes place in the "characteristic point" with the scale k^2 close to μ^2 and with the longitudinal momentum fraction $u \sim 1$ (the saddle-point $y_0 \sim t \sim \tau \sim 0$ in this case).

After splitting one has the TWO independent ladders with the well-developed BFKL and DGLAP evolution. Every ladder contributes to the cross section with the large exponential factor, $\exp\left[\sqrt{8N_c}\sqrt{YT}\right]$, which is just the *same* as for single distributions.

Therefore in the double logarithm approximation single splitting diagrams have, in fact, the factorization property if one takes the leading exponential factors into consideration only.

For double splitting (2×2) diagrams

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the leading exponential contribution arises from the lower limits of t- and either lower or upper limits of q-integrations depending on the available rapidity interval Y.

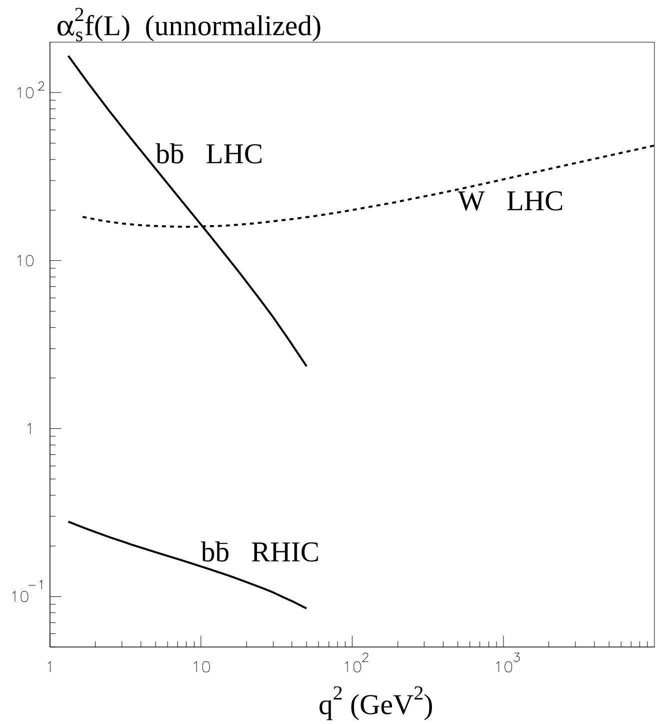
There is competition between the exponential factor caused by the evolution, which prefers a small τ, and the phase space factor in q^2-integral.

Due to the *non-logarithmic* **character of the integration over d^2q for a** *not sufficiently large Y* **the contribution from the** *upper limit* **of q** *may dominate*. **Indeed, let us consider the production of two $b\bar{b}$ pairs in a central rapidity ($\eta \sim 0$) region. That is we take $T_1 = T_2 = T$, $Y_1 = Y_2 = Y$ and keep just the leading exponential factors in the double parton distributions**

$$\boldsymbol{x^2 D_{h2}(x, x, q^2, Q^2, Q^2) \sim \exp(\sqrt{8N_c Y(4T - 3\tau)} - 2aT + a\tau).}$$

Thus the logarithmic dq^2/q^2 integral takes the form

$$\int \frac{\boldsymbol{dq^2}}{\boldsymbol{q^2}} \boldsymbol{\exp(2\sqrt{8N_c Y(4T - 3\tau)} - 4aT + 2a\tau) q^2.}$$



The q-dependence of the integrand $f(L)$ in the logarithmic scale

$$f(L) = exp\left(2\sqrt{8N_c Y(4T - 3\tau)} - 4aT + 2a\tau\right) \exp\left(\ln\left(\frac{\mu^2}{\Lambda^2}\right) e^{\beta\tau/2}\right)$$

with

$$L = \ln(q^2/\Lambda^2) = \ln(\mu^2/\Lambda^2) e^{\beta\tau/2}$$

For the DPS production of two $b\bar{b}$ pairs the major contribution comes from a low q^2 in the case of $Y = 5$ corresponding to the LHC energy $\sqrt{s} = 14$ TeV

That is the reaction may be effectively described by the 1×1 term; the formation of TWO parton branches (one to two splitting) takes place mainly at low scales.

However at the RHIC energy, when the available rapidity interval is not large ($Y = 2$), the q^2-dependence is not steep and the contribution caused by the splitting somewhere in the mid of evolution is still not negligible.

The same can be said about the DPS W-boson production at the LHC. Here the upper edge of the q^2-integral dominates. This part may be described as the collision of *one* pair of partons supplemented by a more complicated, $2 \to 4$ or $2 \to 2W$, hard matrix element. However, clearly we need to account also for contributions from the whole q^2-interval.

For the debatable double splitting diagrams, depending on the precise kinematics, we may deal:

- **either with a single parton pair collision (times the $2 \to 4$ hard subprocess)** *in accordance with Blok, Dokshitzer, Frankfurt, Strikman;Gaunt, Stirling;Manohar, Waalewijn*
- **or with the contribution of the 1×1 type where the formation of two parton branches (one to two splitting) takes place at low scales**
- **or with the 2×2 configuration where the splitting may happen EVERYWHERE (with more or less equal probabilities) during the evolution.**

In order to probe the QCD evolution of the double distribution functions better we suggest also to investigate the processes with two quite different scales, in particular, production of a $b\bar{b}$ pair (or J/ψ) with W, which was estimated at the LHC kinematical conditions using the factorized component only.

The experimental effective cross section, $\sigma_{\text{eff}}^{\text{exp}}$, which is not measured directly but is extracted by means of the normalization to the product of two single cross sections:

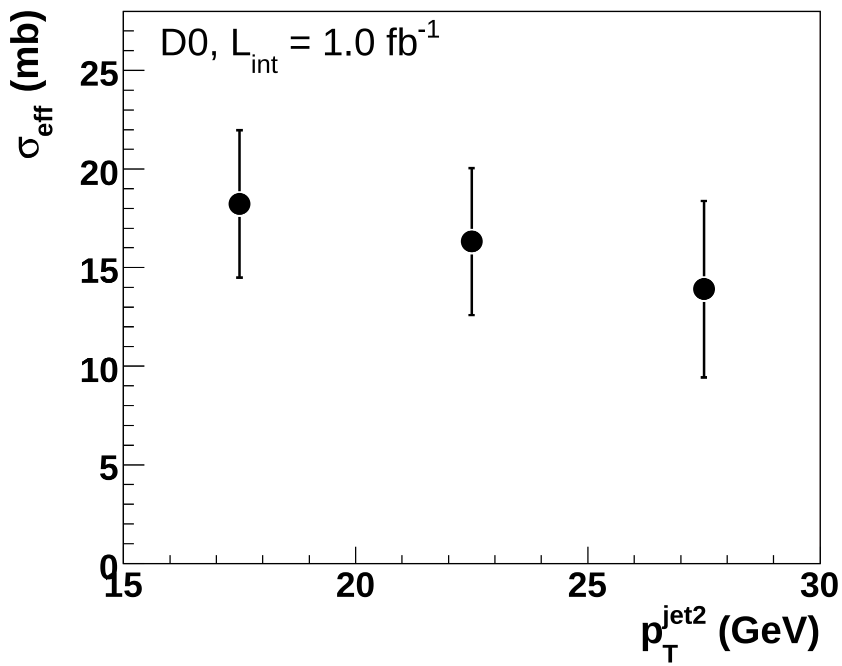
$$\frac{\boldsymbol{\sigma}_{DPS}^{\gamma+3j}}{\boldsymbol{\sigma}^{\gamma j}\boldsymbol{\sigma}^{jj}} = [\boldsymbol{\sigma}_{\text{eff}}^{\text{exp}}]^{-1},$$

appears to be dependent on the probing hard scale. It should DECREASE with increasing the resolution scale because all additional contributions to the cross section of double parton scattering are positive and increase.

In the above formula, $\sigma^{\gamma j}$ and σ^{jj} are the inclusive $\gamma+$ jet and dijets cross sections, $\sigma_{DPS}^{\gamma+3j}$ is the inclusive cross section of the $\gamma+3$ jets events produced in the double parton process.

It is worth noticing that the CDF and D0 Collaborations extract $\sigma_{\text{eff}}^{\text{exp}}$ without any theoretical predictions on the $\gamma+$ jet and dijets cross sections, by comparing the number of observed double parton $\gamma+3$ jets events in ONE $p\bar{p}$ collision to the number of $\gamma+$ **jet** and **dijets** events occurring in TWO separate $p\bar{p}$ collisions.

The recent D0 measurements represent this effective cross section, $\sigma_{\rm eff}^{\rm exp}$, as a function of the second (ordered in the transverse momentum, p_T) jet p_T, $p_T^{\rm jet2}$, which can serve as a resolution scale. The obtained cross sections reveal a tendency to be dependent on this scale.



This observation can be interpreted as the first indication to the QCD evolution of double parton distributions
(*Snigirev; Flensburg, Gustafson, Lonnblad, Ster*).

CONCLUSIONS

We suggest a practical method which makes it possible to estimate the inclusive cross section for double parton scattering, taking into account the QCD evolution and basing on the well-known collinear distributions, extracted from deep inelastic scattering

We also support the conclusion that the experimentally measured effective cross section, $\sigma_{\rm eff}^{\rm exp}$, should decrease with increasing the resolution scale Q^2 due to presence of the evolution (correlation) term in the two-parton distributions.

The structure/contribution of splitting diagrams is dependent on kinematics *available*.

OUTLOOK

It is interesting to study the double parton distribution functions beyond the leading logarithm approximation over Q^2:

- **BFKL regime** $Q^2 = \text{const}$ **,** $\ln(1/x) \to \infty$
  (*partly done, Flensburg, Gustafson, Lonnblad, Ster*)

- **colour glass condensate approach**
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- **DPS in pA** (*Strikman, Treleani;..., Blok, Wiedemann; d'Enterria, Snigirev,....*)

- **DPS with 2 different scales** (*Ryskin, Snigirev; Berger,.....*)

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