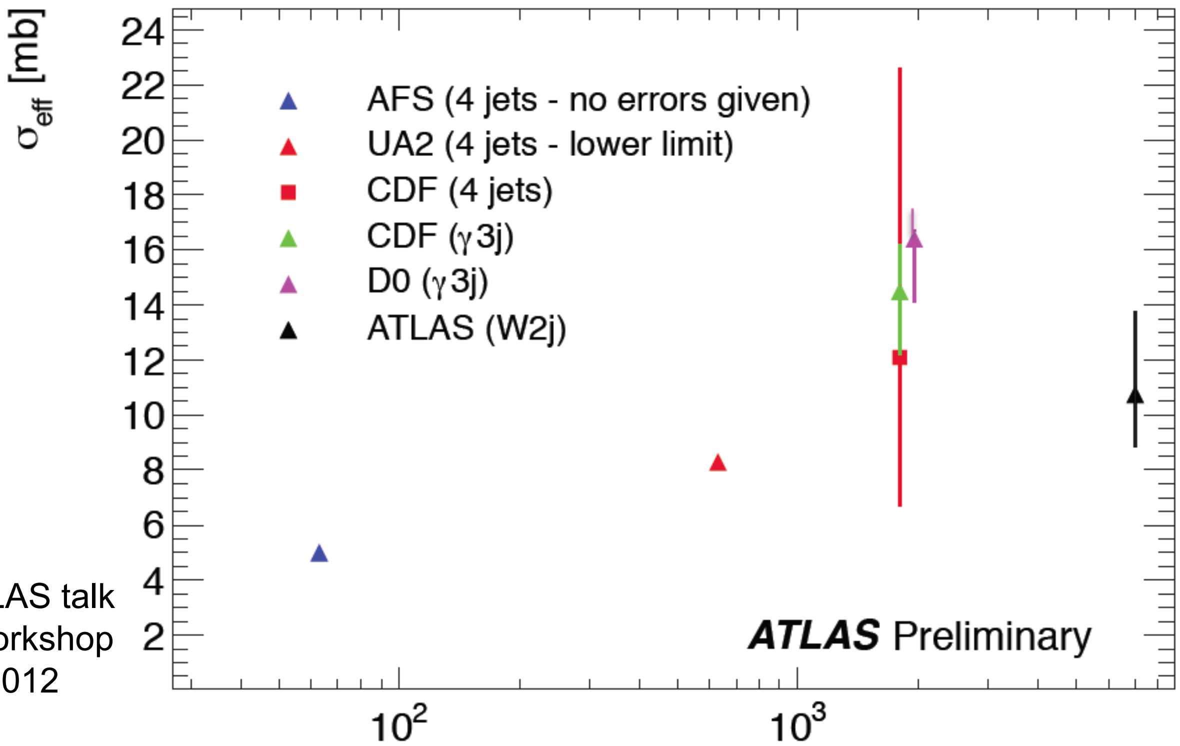
A MODEL OF MULTI PARTON INTERACTIONS

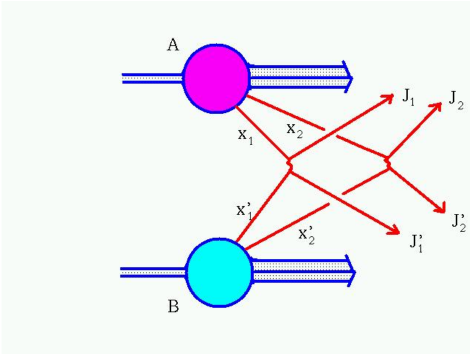
G. Calucci, S. Salvini and Daniele Treleani

- Some indications from experiments
- Connected and disconnected interactions
- A probabilistic picture of MPI. Cancellation of unitarity corrections
- Inclusive and exclusive cross sections
- DPI in pD collisions

EXPERIMENTAL EVIDENCE: σ_{eff}



From ATLAS talk at MPI workshop Tel Aviv 2012



$$\sigma^{pp}_{double}(A,B) = \frac{m}{2}\frac{\sigma_A\sigma_B}{\sigma_{eff}}$$

Experiment	$\sqrt{s}$ (GeV)	Final state	p_T^{min} (GeV)	η range	$\sigma_{\rm eff}$
AFS (pp), 1986	63	4 jets	$p_T^{\rm jet} > 4$	$\lvert\eta^{\rm jet}\rvert < 1$	~ 5 mb
UA2 ($p\bar{p}$), 1991	630	4 jets	$p_T^{\rm jet} > 15$	$\lvert\eta^{\rm jet}\rvert < 2$	> 8.3 mb (95% C.L.)
CDF ($p\bar{p}$), 1993	1800	4 jets	$p_T^{\rm jet} > 25$	$\lvert\eta^{\rm jet}\rvert < 3.5$	$12.1^{+10.7}_{-5.4}$ mb
CDF ($p\bar{p}$), 1997	1800	$\gamma + 3$ jets	$p_T^{\rm jet} > 6$	$\lvert\eta^{\rm jet}\rvert < 3.5$	
			$p_T^{\gamma} > 16$	$\lvert\eta^{\gamma}\rvert < 0.9$	$14.5 \pm 1.7^{+1.7}_{-2.3}$ mb
DØ ($p\bar{p}$), 2010	1960	$\gamma + 3$ jets	$60 < p_T^{\gamma} < 80$	$\lvert\eta^{\gamma}\rvert < 1.0$ ||	
			$15 < p_T^{\rm jet2} < 30$	$1.5 < \lvert\eta^{\gamma}\rvert < 2.5$	
				$\lvert\eta^{jet}\rvert < 3.0$	$\sigma_{eff} = 16.4 \pm 0.3({\rm stat}) \pm 2.3({\rm syst})$ mb
ATLAS (Wjj) ?	7TeV	20 on l/ν, 20 on j			$\sigma_{\rm eff}(7\ {\rm TeV}) = 11 \pm 1\ ({\rm stat.})\ ^{+3}_{-2}\ ({\rm sys.})$ mb.

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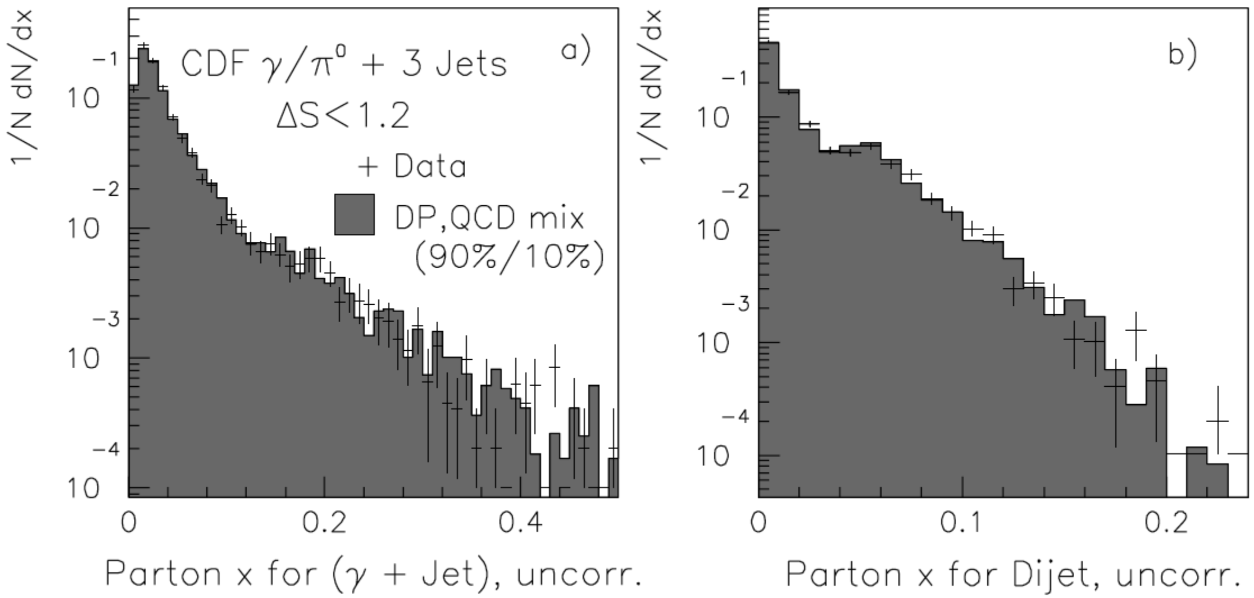
EXPERIMENTAL EVIDENCE: x DEPENDENCE

F. Abe et al. [CDF Collaboration], Phys. Rev. D 56, 3811(1997).

Dependence of σ_{eff} on x



Distributions of x are plotted in Figs. 20(a) and 20(b), along with a prediction obtained by applying the $\Delta S < 1.2$ selection to the admixture 90% MIXDP+10% PYTHIA.
No systematic deviation of the DP rate vs x, and thus no x dependence to σ_{eff} , is apparent over the x range accessible to this analysis (0.01 – 0.40 for the photon+jet scattering, 0.002–0.20 for the dijet scattering).

TP fractions

From D0 talk
at MPI workshop
Tel Aviv 2012

γ+3jet final state also can be produced by Tripple Parton interaction (TP).
In γ+3jet TP events all 3 jets should stem from 3 different parton scatterings.
To estimate the TP fraction we used results on DP+TP fractions and fractions of Type I(II) events found in our previous measurement.
TP in γ+3jet data is calculated as:

$$f_{tp}^{\gamma 3j} = f_{dp+tp}^{tp} \cdot f_{dp+tp}^{\gamma 3j}$$

The fraction of TP in MixDP can be found as:

$$f_{dp+tp}^{tp} = F_{typeII} \cdot f_{dp}^{\gamma 2j} + F_{typeI} \cdot f_{dp}^{jj}$$

$f_{dp+tp}^{\gamma 3j}$ - measured in previous DP analysis;

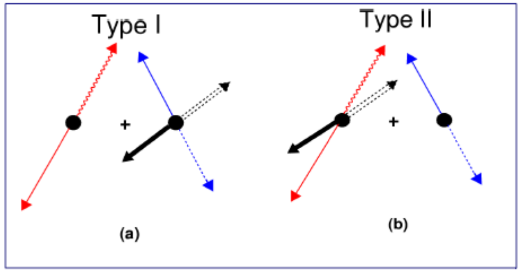
f_{dp}^{jj} - estimated using dijet cross section;

$f_{dp}^{\gamma 2j}$ - measured;

$F_{typeI(II)}$ - found from the model (MixDP).

Probability to produce another parton scattering is proportional to $R=\sigma_{ij}/\sigma_{eff}$, ratio $f_{tp}^{\gamma 3j}/f_{dp}^{\gamma 3j}$ should be proportional to R.

Types in MixDP model



TP fractions

p_T^{jet2} (GeV)	$f_{tp}^{\gamma 3j}$ (%)	$f_{tp}^{\gamma 3j} / f_{dp}^{\gamma 3j}$ (%)
15 – 20	5.5 ± 1.1	13.5 ± 3.0
20 – 25	2.1 ± 0.6	6.6 ± 2.0
25 – 30	0.9 ± 0.3	3.8 ± 1.4

F. Abe et al. [CDF Collaboration], Phys. Rev. D 56, 3811(1997).

“The TP contribution to all MIXDP events is then 17_{-8}^{+4} %”

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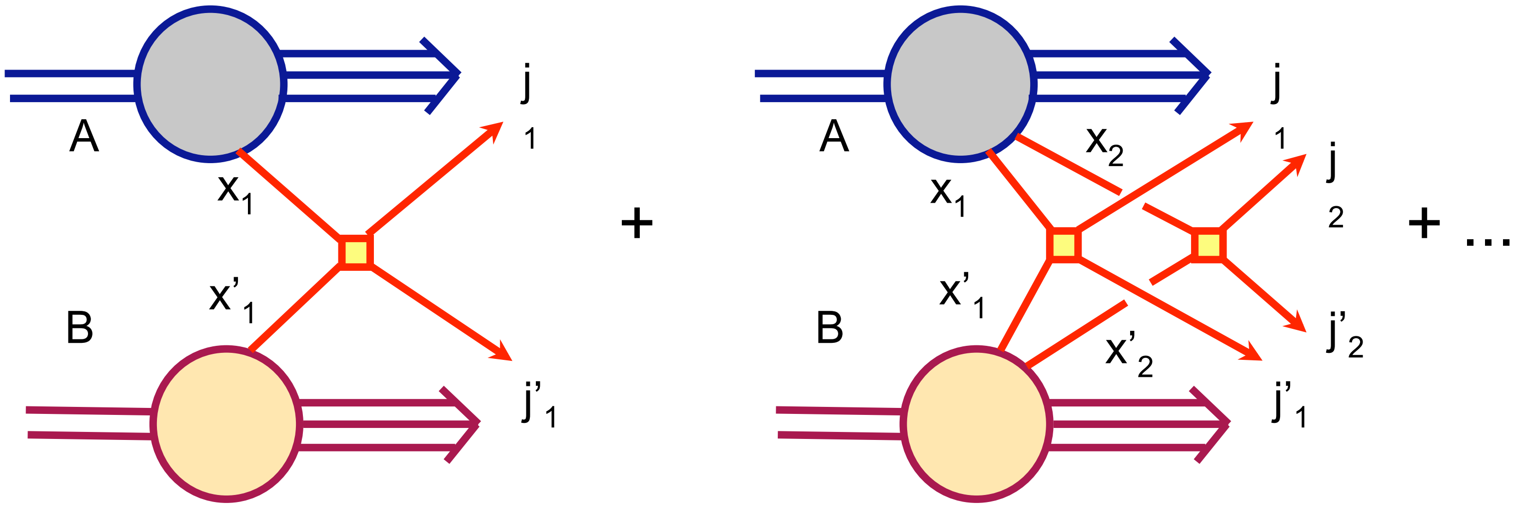
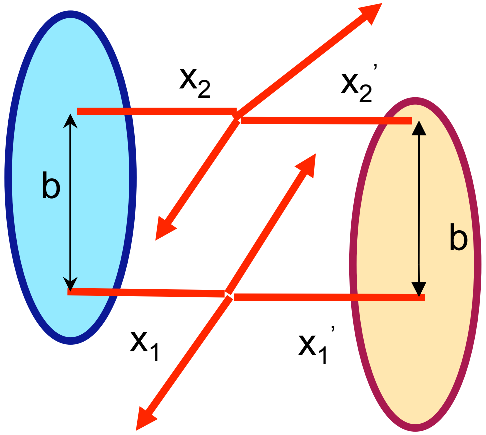
The events selected when looking for Double Parton Interactions are those where the transverse momenta are approximately balanced in pairs. Which is the dynamical mechanism behind ?

At small x the hard cross section may be larger than the total inelastic cross section.
Unitarity is restored by introducing **multiple parton interactions** which, for a given final state, are those processes that **maximize the incoming parton flux**



In such a regime, the **average multiplicity** of interactions is **larger than one** and the **inclusive cross section** is proportional to the **multiplicity of hard interactions**.

To maximize the incoming parton flux, the hard component of the interaction is disconnected. In the case of the **double parton collision** one thus obtains the geometrical picture here below where the non-perturbative components are factorized into a function which depends on two fractional momenta and on the relative transverse distance *b* between the two interaction points



The corresponding inclusive double parton-scattering cross-section, for two parton processes A and B in a pp collision, is given by

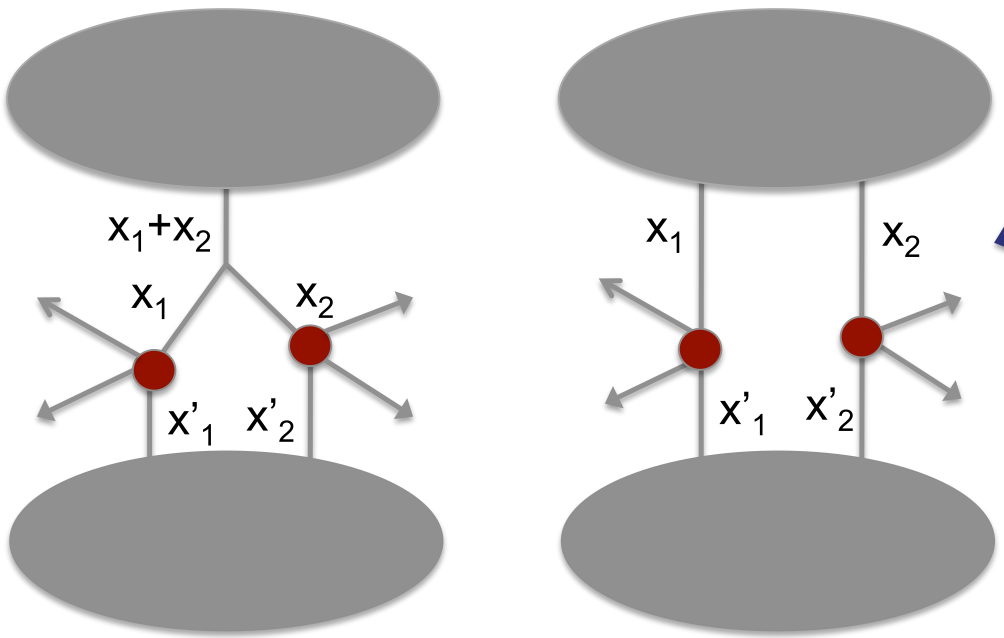
$$\sigma^D_{(A,B)} = \frac{m}{2}\sum_{i,j,k,l}\int D_{ij}(x_1,x_2;b)\hat{\sigma}^A_{ik}(x_1,x_1')\hat{\sigma}^B_{jl}(x_2,x_2')D_{kl}(x_1',x_2';b)dx_1dx_1'dx_2dx_2'd^2b$$

Which, assuming universality for the dependence on b, leads to the expression of the cross section utilized in the experimental analysis

$$\sigma^{pp\ (A,B)}_{double} = \frac{m}{2}\frac{\sigma_A\sigma_B}{\sigma_{eff}}$$

In the experimental analysis, Double Parton Interactions are recognized as the processes where the transverse momenta are approximately balanced in pairs and are uncorrelated azimuthally. Can one obtain the same final state configuration in different ways ?

The hard part of the interaction is **connected**. The whole hard process takes place in a single point in transverse space



3 -> 4

4 -> 4

The hard part of the interaction is **disconnected**. The hard process takes place in two different points in transverse space

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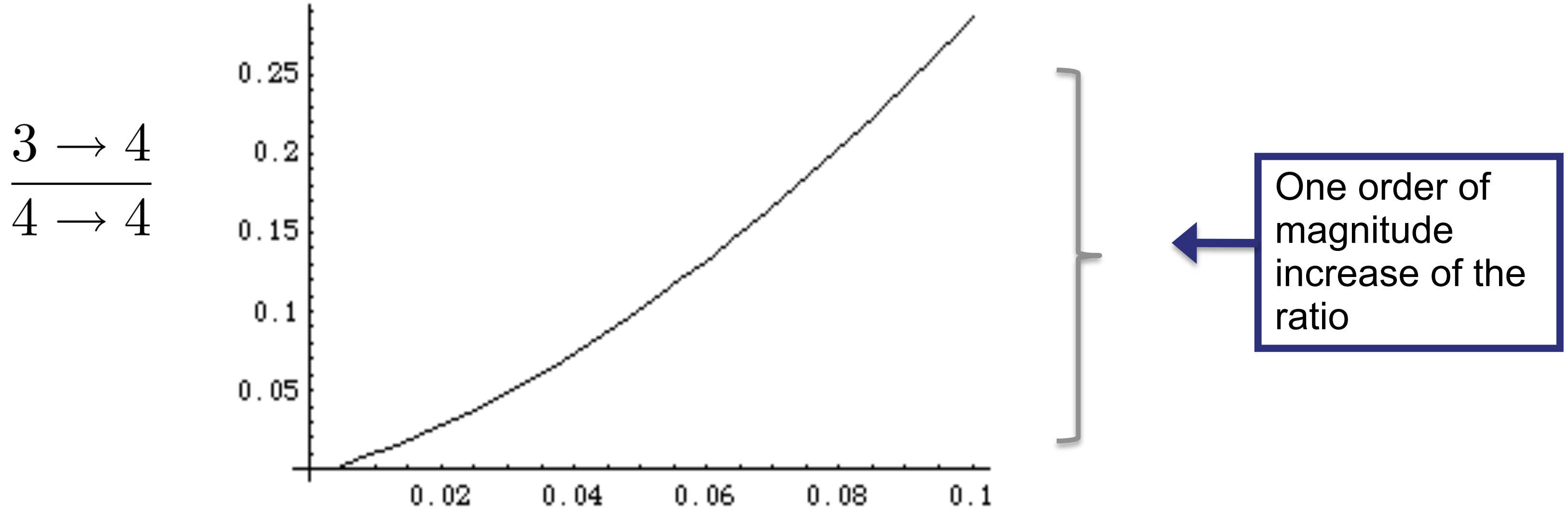
The incoming partons fractional momenta x_1, x_1', x_2, x_2' are accessible experimental quantities and the 3->4 and 4->4 processes have a very different dependence on fractional momenta.

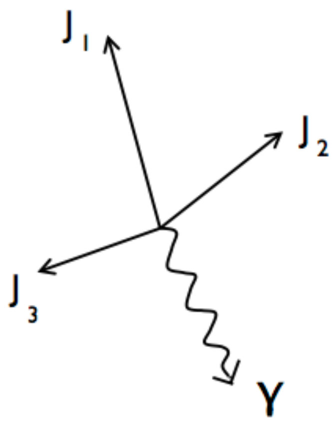
As an example, by selecting events where all fractional momenta ere equal, for the ratio (3->4)/(4->4) one obtains the behavior here below

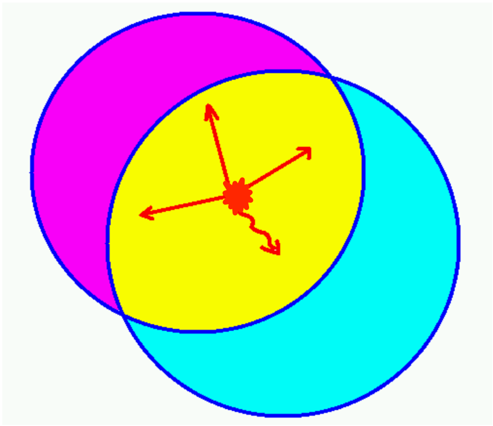


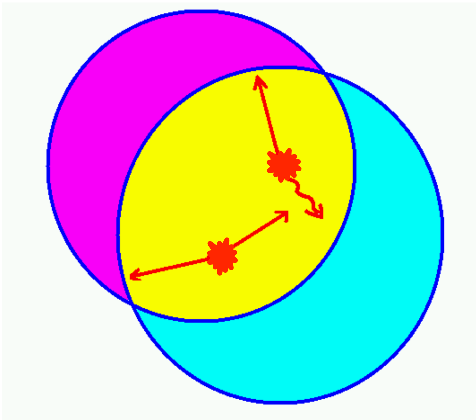
$$x = x_1 = x_1' = x_2 = x_2' \quad Q^2 = 20 \text{ GeV}^2$$

A sizable contribution of 3->4 processes will thus lead to a sizable dependence of the effective cross section on the incoming fractional momenta.
A suggestion to the experimental colleagues is then ***measure the dependence of the cross section on the incoming fractional momenta***

While a given final state may be generated by MPI terms with different numbers of incoming partons, at low *x* one is dominated by the contribution of the disconnected multiparton collisions, which maximize the incoming parton flux







2->4 or 3->4 **connected** hard interaction. Dominant at **finite *x***

(2->2)^2 **disconnected** hard interaction. Dominant at ***x* ->0**

Notice that different contributions to a connected term (e.g. 2->4 and 3->4) add coherently. On the contrary, when considering disconnected contributions, ***terms corresponding to different topologies add incoherently in the cross section***

One may thus argue that a convenient way to group all contributions to the cross section, in the regime of multiple parton interactions, is topological.

Given the different scales of the hadron size and of the large momenta exchanged, the hard component of the interaction may be disconnected, with the different hard parts linked only through soft exchanges and localized in different regions in transverse space.

Terms with different topologies can generate the same multiparton final state. ***Terms with different topologies however add incoherently***

The ***different MPI terms*** may thus be understood as the contributions to the final state due to the ***different disconnected parts of the hard component of the interaction***.

One has in this way to advantage that MPI can be discussed within a probabilistic framework

In the simplest case, in each different region the interaction may be evaluated at the lowest order in the coupling constant. In other cases one may need to keep into account higher order terms in the coupling constant and higher twists.

A Probabilistic Model for MPI in pp Collisions

Multiple Parton Interactions may hence be discussed within a ***probabilistic framework*** and a functional approach is very general. One may simplify the problem by assuming that each single interactions is approximated by a two-body parton collisions. Introducing the ***exclusive*** n-body parton ***distributions*** $W_n(u_1 \dots u_n)$, $u_i \equiv (\mathbf{b}_i, x_i)$

which represent the probability to find the hadron in a configuration with *n* partons with coordinated $u_1 \dots u_n$, where $\mathbf{b}_i$ are the transverse parton coordinates and x_i the fractional momenta. The multi-parton generating functional is defined as:

$$\mathcal{Z}[J] = \sum_n \frac{1}{n!} \int J(u_1) \dots J(u_n) W_n(u_1 \dots u_n) du_1 \dots du_n,$$

Probability conservation imposes the normalization condition $\mathcal{Z}[1] = 1.$

The exclusive distributions are the coefficients of the expansion of $\mathcal{Z}$ around 0, the many-body densities, i.e. the ***inclusive distributions*** $D_n(u_1 \dots u_n)$ are the coefficients of the expansion of $\mathcal{Z}$ around 1:

$$D_1(u) = \left.\frac{\partial \mathcal{Z}}{\partial J(u)}\right|_{J=1},$$

$$D_2(u_1, u_2) = \left.\frac{\partial^2 \mathcal{Z}}{\partial J(u_1) \partial J(u_2)}\right|_{J=1},$$

$$\dots$$

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One may introduce the logarithm of the generating functional $\mathcal{F}[J] = \ln(\mathcal{Z}[J])$

and, by expanding in the vicinity of *J*=1, one obtains the many-body ***parton correlations***

$$\mathcal{F}[J] = \int D(u)[J(u)-1]du + \sum_{n=2}^{\infty} \frac{1}{n!} \int C_n(u_1 \dots u_n)\big[J(u_1)-1\big] \dots \\ \dots \big[J(u_n)-1\big] du_1 \dots du_n$$

By making the ***simplifying assumption*** that ***each disconnected part of the hard interaction can be approximated by a two-parton initiated process***, one can thus obtain a rather general functional expression for the hard cross section

$$\sigma_H = \int d^2\beta \sigma_H(\beta) \quad \text{where} \quad \sigma_{inel} = \sigma_H + \sigma_{soft}$$

$$\sigma_H(\beta) = \Big[1 - \exp\big(-\partial \cdot \hat{\sigma} \cdot \partial'\big)\Big] \mathcal{Z}_A[J]\mathcal{Z}_B[J']\Big|_{J=J'=1}$$

Where each single partonic interaction is approximated by a two-body parton collisions. Notice that in the functional expression above:

- All disconnected collisions are included and
- <u>Multi-parton correlations</u> are kept into account <u>at all orders</u>.

The functional expression of the hard cross section can be expanded as a sum of MPI :

$$\sigma_H(\beta) = \Big[1 - \exp\big(-\partial \cdot \hat{\sigma} \cdot \partial'\big)\Big] \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \Big|_{J=J'=1}$$
$$= \sum_{N=1}^{\infty} \frac{\big(\partial \cdot \hat{\sigma} \cdot \partial'\big)^N}{N!} \, e^{-\partial \cdot \hat{\sigma} \cdot \partial'} \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \Big|_{J=J'=1}$$

Which allows to show, on rather general grounds, that in the model ***all unitarity corrections, induced by the presence of MPI, cancel in the evaluation of the average number of collisions, which is given by the single scattering inclusive cross section*** (AGK cancellation)

$$\langle N \rangle \sigma_H(\beta) = \sum_{N=1}^{\infty} \frac{N\big(\partial \cdot \hat{\sigma} \cdot \partial'\big)^N}{N!} \, e^{-\partial \cdot \hat{\sigma} \cdot \partial'} \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \Big|_{J=J'=1}$$
$$= \partial_{J_1} \cdot \hat{\sigma} \cdot \partial_{J'_1} \sum_{N=0}^{\infty} \frac{\big(\partial \cdot \hat{\sigma} \cdot \partial'\big)^N}{N!} \, e^{-\partial \cdot \hat{\sigma} \cdot \partial'} \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \Big|_{J=J'=1}$$
$$= \big(\partial_{J_1} \cdot \hat{\sigma} \cdot \partial_{J'_1}\big) \mathcal{Z}_A[J]\mathcal{Z}_B[J'] \Big|_{J=J'=1}$$
$$= \int D_A(x_1; b_1)\hat{\sigma}(x_1 x'_1) D_B(x'_1; b_1 - \beta) dx_1 dx'_1 d^2 b_1 \equiv \sigma_S(\beta)$$

One may analogously show that the similar relations hold for all inclusive cross sections:

$$\langle N\rangle\sigma_H = \sigma_S \quad \text{and} \quad \frac{1}{2}\langle N(N-1)\rangle\sigma_H = \sigma_D$$

and more in general
$$\frac{1}{K!}\langle N(N-1)\dots(N-K+1)\rangle\sigma_H = \sigma_K$$

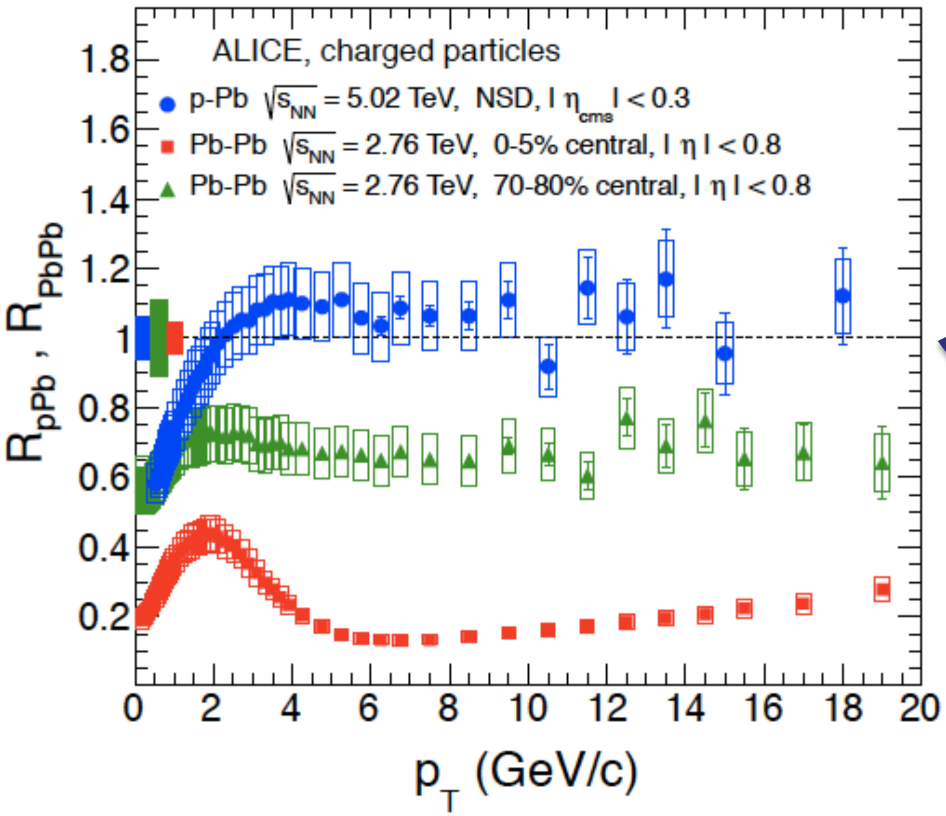
Notice that all effects of multi-parton correlations are taken into account in the derivation of the relations above

When identifying the ***MPI*** as the ***different disconnected parts of the hard component of the interaction*** and when taking into account only the two-body initiated parton collisions, one can thus conclude that

- The ***inclusive cross section*** is directly proportional to the ***average number*** of partonic collisions and that

- The ***K-partons inclusive cross section*** is analogously proportional to the ***K^{th} moment of the distribution*** in the number of partonic collisions.

Notice that the statement that all unitarity corrections induced by the presence of MPI cancel in the inclusive cross section, which is given by the single scattering term, is perfectly consistent with the recent observation of ALICE that the observed ratio R_{pPb} , of the large p_t charged particles produced in proton-lead collisions, is consistent with 1



$$\sigma_{incl}|_{pPb} = \langle N_{pPb} \rangle \sigma_H \ ; \quad \sigma_{incl}|_{pN} = \langle N_{pN} \rangle \sigma_H$$

$$\langle N_{pPb} \rangle = \langle N_{pN} \rangle \langle T(b) \rangle$$

$$R_{pPb} = \frac{\sigma_{incl}|_{pPb}}{\langle T(b) \rangle \sigma_{incl}|_{pN}} = 1$$

Of course the ratio will be very different from 1 when plotting the analogous ratio for the double parton scattering cross section, where the dominant contribution goes rather as $[T(b)]^{4/3}$

Inclusive and Exclusive Cross Sections

Both CDF and D0 report a sizable contribution of triple parton collisions in their samples of events with double parton collisions: CDF reports a fraction of about 17% and D0 a fraction ranging from 3.8% to 13.5%. In a regime of MPI it makes thus sense to discuss about ***inclusive*** and ***exclusive*** multi-parton cross sections

In model discussed above, the ***inclusive cross sections*** are the ***moments of the distribution in the number of MPI***.
The ***exclusive cross sections*** correspond, on the contrary, to the ***events where only a given number of hard collisions are present***

Inclusive and exclusive cross sections thus result from independent measurements and are related in a different way to the hadron structure.

Notice that ***the exclusive cross sections cannot be evaluated with the usual QDC-parton model expression***. While the inclusive cross sections are in fact linked directly to the multi-parton structure of the hadron the link of the "exclusive“ cross sections with the hadron structure is much more elaborate.

K-parton scattering inclusive cross section

One has:

$$\sigma_H \equiv \sum_{N=1}^{\infty} \tilde{\sigma}_N, \qquad \sigma_K \equiv \sum_{N=K}^{\infty} \frac{N(N-1)\dots(N-K+1)}{K!} \tilde{\sigma}_N$$

Contribution of the hard collisions to the inelastic cross section

$$\sigma_{inel} = \sigma_H + \sigma_{soft}$$

N-parton scattering exclusive cross sections. Only events with *N* partonic collisions are selected

Notice that the relations above represent also a set of ***sum rules connecting the inclusive and the exclusive cross sections.*** At each order in the number of hard collisions, the exclusive cross sections can therefore be expressed by a different combination of inclusive cross sections

As an example, at order $\hat{\sigma}^3$ one obtains :

$$\begin{cases} \tilde{\sigma}_1 = \sigma_S - 2\sigma_D + 3\sigma_T \\ \tilde{\sigma}_2 = \sigma_D - 3\sigma_T \\ \tilde{\sigma}_3 = \sigma_T \end{cases}$$

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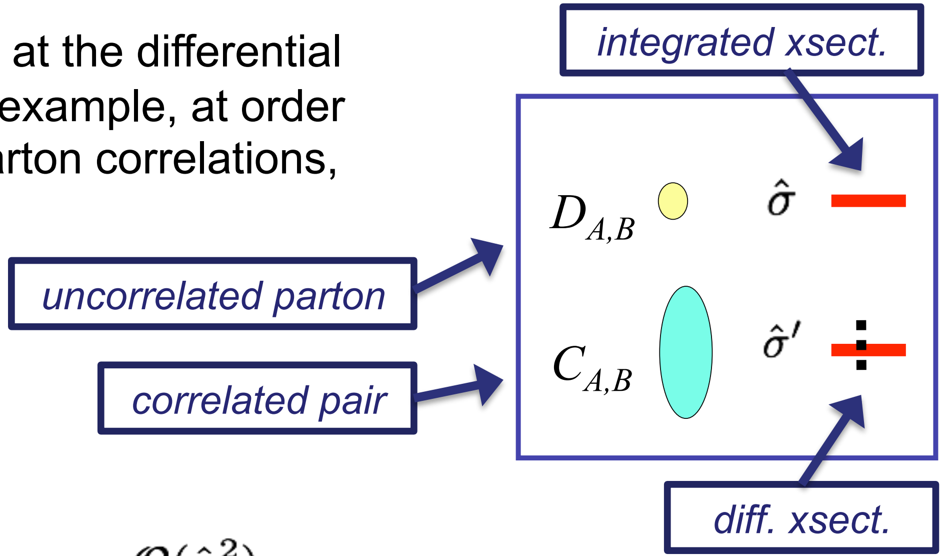
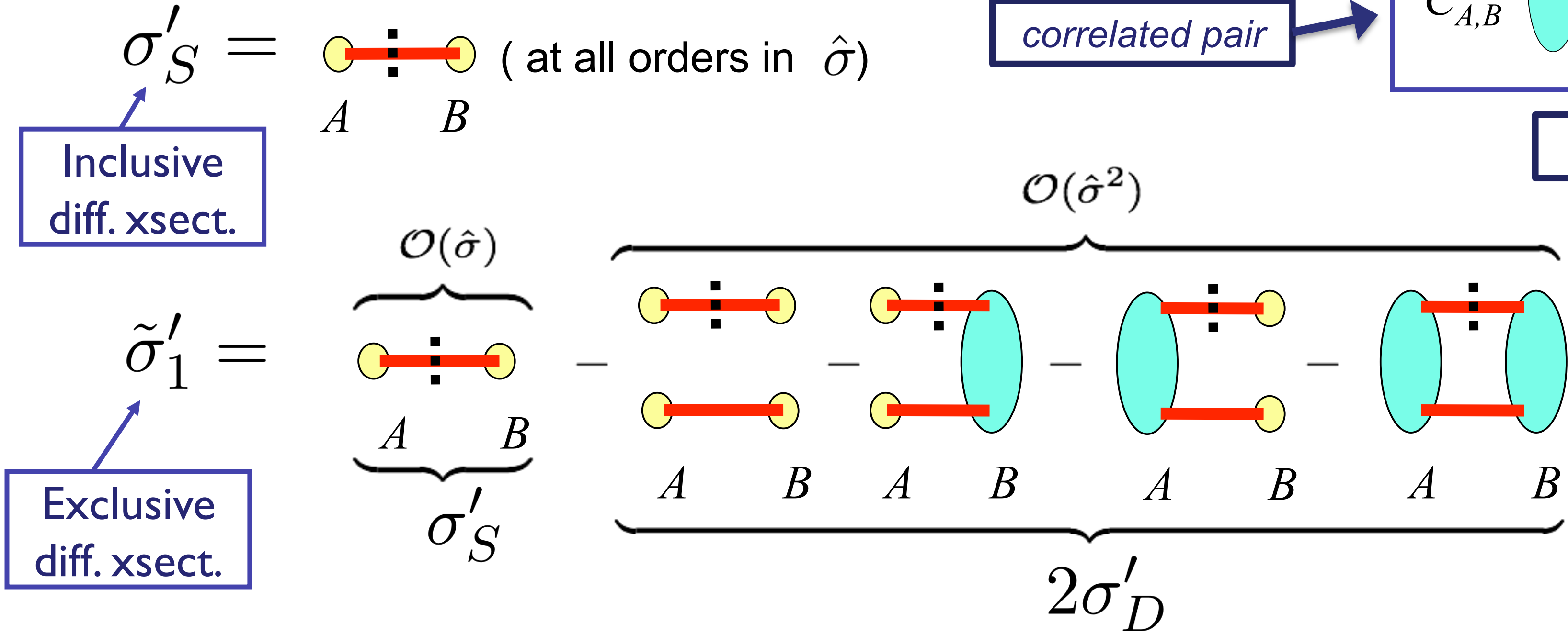
The relations above hold also when looking at the differential inclusive and exclusive cross sections. For example, at order and $\hat{\sigma}^2$, including explicitly the two-body parton correlations, one finds:





By comparing the differential inclusive and exclusive cross sections one thus obtains information on the dependence of the effective cross section on y and p_t

$$\frac{d\tilde{\sigma}_1}{dyd\mathbf{p}_t} = \frac{d\sigma_S}{dyd\mathbf{p}_t} - \frac{d\sigma_S}{dyd\mathbf{p}_t}\frac{\sigma_S}{\sigma_{eff}}$$

If, in a given phase space interval, only single and double collisions give sizable contributions, one can thus obtain the effective cross section, as a function of rapidity and momentum transfer, by measuring the difference between the inclusive and the exclusive cross sections:

$$\frac{d\sigma_S}{dyd\mathbf{p}_t} - \frac{d\tilde{\sigma}_1}{dyd\mathbf{p}_t} = \frac{d\sigma_S}{dyd\mathbf{p}_t}\frac{\sigma_S}{\sigma_{eff}} \quad \text{at } \mathcal{O}(\hat{\sigma}^2)$$

The validity of the relation can be tested by checking if the sum rules are satisfied by the measured inclusive and exclusive single and double parton cross sections

$$\sigma_S = \tilde{\sigma}_1 + 2\tilde{\sigma}_2$$

$$\sigma_D = \tilde{\sigma}_2$$

If the sum rules are not satisfied, one needs to take into account triple collisions. In such a case the double inclusive and exclusive cross sections are not equal and their difference provides information on the scale factor, which characterizes the triple parton collisions:

$$\frac{d\sigma_D}{dy_1d\mathbf{p}_{t1}dy_2d\mathbf{p}_{t2}} - \frac{d\tilde{\sigma}_2}{dy_1d\mathbf{p}_{t1}dy_2d\mathbf{p}_{t2}} = \frac{d\tilde{\sigma}_2}{dy_1d\mathbf{p}_{t1}dy_2d\mathbf{p}_{t2}}\frac{\sigma_S}{\sigma_{eff}\tau} \quad \text{at } \mathcal{O}(\hat{\sigma}^3)$$

Here the scale factor characterizing triple parton cross sections is $\tau\sigma_{eff}^2$

Double parton interactions and hadron structure

Without loss of generality one may write the double parton distribution functions as

$$\Gamma(x_1, x_2;\ b) = G(x_1, x_2) f_{x_1 x_2}(b), \qquad G(x_1, x_2) = K_{x_1 x_2} G(x_1) G(x_2)$$

where *f* is normalized to one and the transverse scales, characterizing *f*, may still depend on fractional momenta. In the simplest case one would have $K_{xx'}$=1 which, after integrating on *b*, would be the case of a Poissonian multi-parton distribution. In *pp* one thus has

$$\sigma^{pp\ (A,B)}_{double}(x_1, x_1', x_2, x_2') \ = \ \frac{m}{2} \frac{K_{x_1 x_2} K_{x_1' x_2'}}{\pi \Lambda^2(x_1, x_1', x_2, x_2')} \sigma_A(x_1, x_1') \sigma_B(x_2, x_2')$$

where
$$\int f_{x_1 x_2}(b) f_{x_1' x_2'}(b) db = \frac{1}{\pi \Lambda^2(x_1, x_1', x_2, x_2')}$$

$$\sigma_{eff}(x_1, x_1', x_2, x_2') = \frac{\pi \Lambda^2(x_1, x_1', x_2, x_2')}{K_{x_1 x_2} K_{x_1' x_2'}}$$

All new information on the hadron structure is thus summarized in the effective cross section and, Since all new information on the hadron structure is summarized by a single quantity, the effective cross section, *pp* collisions alone does not allow to discriminate between Λ and *K*.

Additional information can however be provided by looking at MPI in *pA*

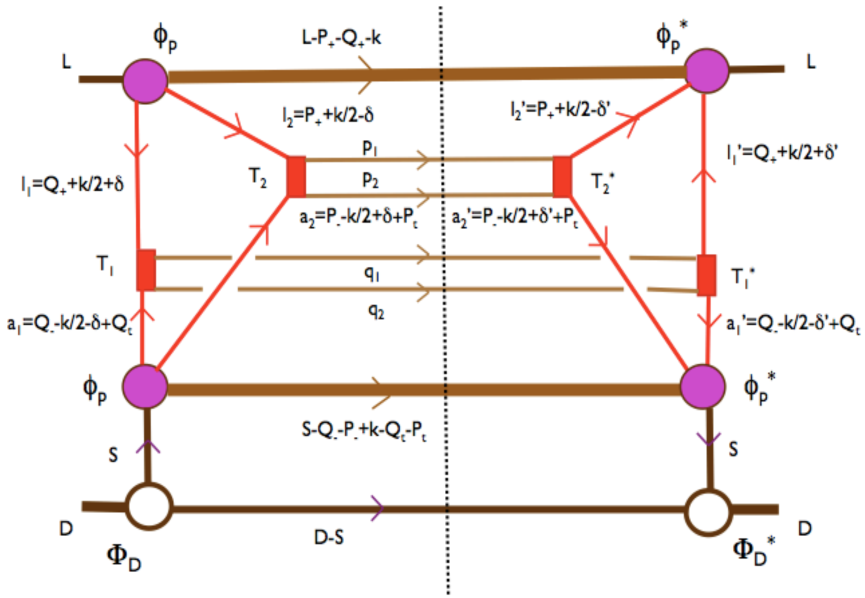
The simplest case is Double parton interactions in *pD*

There are three different contributions to the double parton scattering cross section σ^{pD}_{double}

$$\sigma^{pD}_{double} = \sigma^{pD}_{2,0} + \sigma^{pD}_{1,1;\mathcal{D}} + \sigma^{pD}_{1,1;\mathcal{I}}$$

$\sigma^{pD}_{2,0}$ is the contribution to the cross section where only a single target nucleon undergoes a double parton collision, while there is no large momentum transfer exchange with the second nucleon

$\sigma^{pD}_{2,0}$



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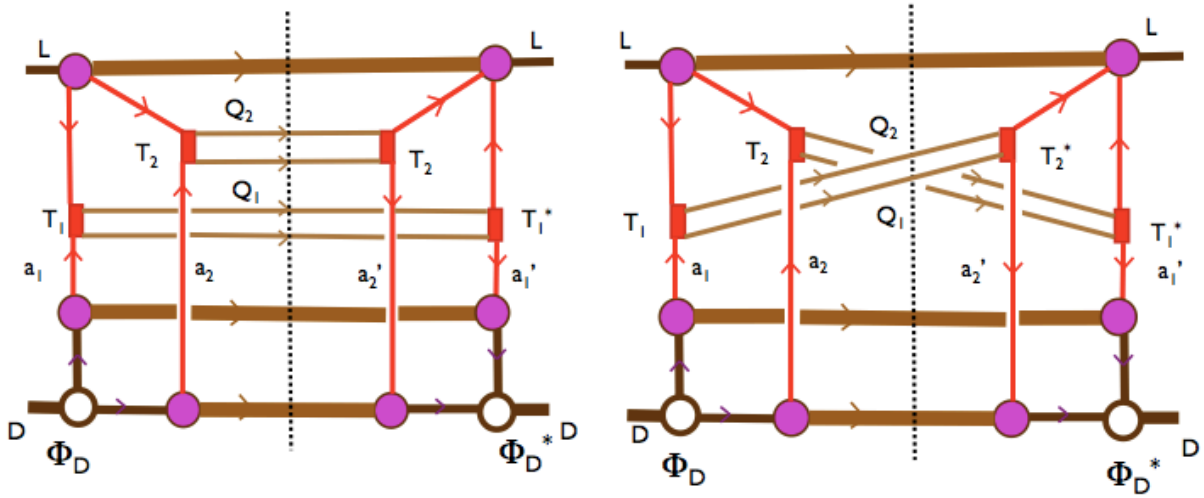
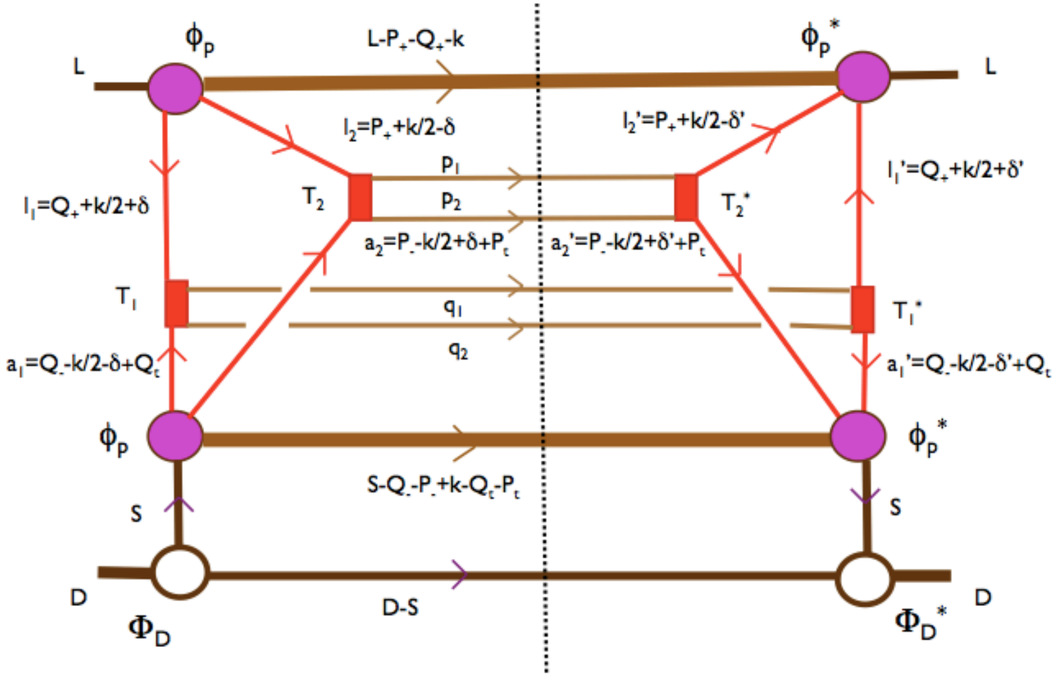
$\sigma^{pD}_{1,1;\mathcal{D}}$ is the diagonal contribution to the cross section, where both target nucleons interact with large momentum exchange

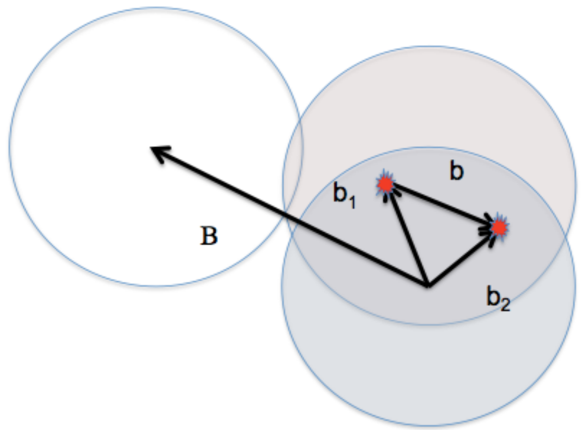
$\sigma^{pD}_{1,1;\mathcal{I}}$ is the off diagonal contribution to the cross section, where both target nucleons interact with large momentum exchange



a)   b)

Only a single target nucleon interacts with large momentun exchange





Configuration in transverse space

$$\sigma_{2,0}^{pD} = \int \Gamma(x_1, x_2;\ b)\hat{\sigma}(x_1 x_1')\hat{\sigma}(x_2 x_2')\Gamma\left(\frac{x_1'}{Z}, \frac{x_2'}{Z};\ b\right)\frac{|\Psi_D(Z,B)|^2}{Z(2-Z)} dZ\ dB\ db$$

where Z is the fractional momentum of a nucleon with respect to ½ of the Deuteron's momentum

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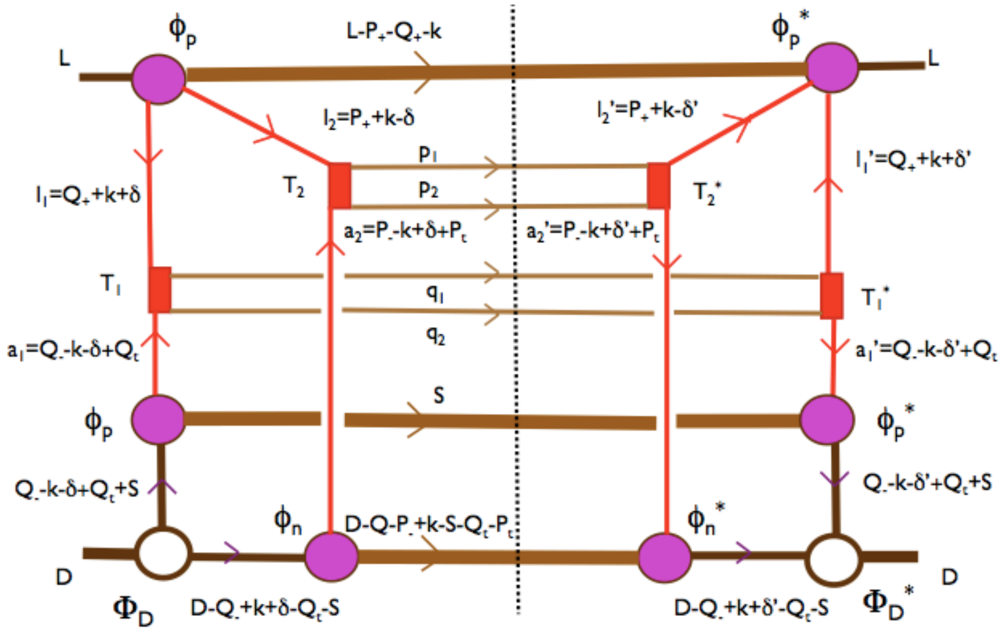
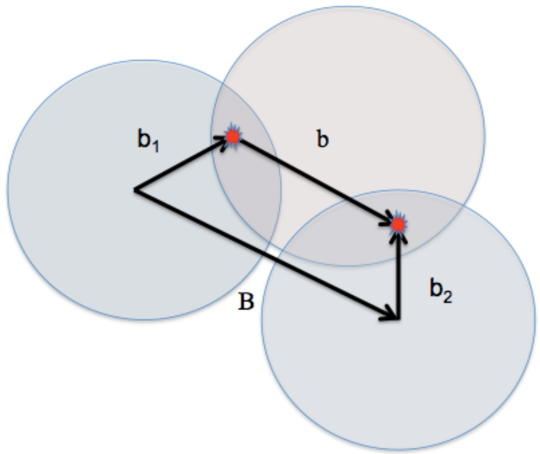
Both target nucleons interact with large momentun exchange.
Diagonal contribution:





Configuration in transverse space

$$\sigma^{pD}_{1,1;\mathcal{D}} = \int \Gamma(x_1, x_2;\ b)\hat{\sigma}(x_1 x_1')\hat{\sigma}(x_2 x_2')\Gamma\Big(\frac{x_1'}{Z};\ b_1\Big)\Gamma\Big(\frac{x_2'}{2-Z};\ b_2\Big)$$
$$\times \frac{|\Psi_D(Z,B)|^2}{Z(2-Z)}\delta(B - b_1 + b_2 - b)dZ\ dB\ db_1\ db_2\ db$$

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In general, to evaluate the cross section one needs to take into account also the interference term. In the simplest cases however the interference term is absent. An interesting case where the interference term is missing is W+JJ production.

In this case the DPI on both nucleons is directly proportional to the ***correlation in fractional momenta*** and it depends only a weakly on the correlations in the transverse coordinates:

DPI on both nucleons in a pD collision

SPI on the proton

SPI on the neutron

SPI on the second nucleon

$$\sigma^{W+JJ}_{pD;1,1}(x_1,x_1',x_2,x_2') \;=\; K_{x_1x_2}\int\Big[\sigma^W_{pp}(x_1,x_1'/Z)+\sigma^W_{pn}(x_1,x_1'/Z)\Big]\sigma^{JJ}_{pN}\big(x_2,x_2'/(2-Z)\big)$$
$$\times\left\{\frac{|\Psi_D(Z,0)|^2}{Z(2-Z)}+\frac{d}{dB^2}\,\frac{|\Psi_D(Z,B)|^2}{Z(2-Z)}\Big|_{B=0}\big(2R^2+\langle b^2\rangle\big)+\dots\right\}dZ$$

Correlation in fractional momenta

Average transverse distance between interacting parton pairs

Concluding summary

At x->0 one is dominated by disconnected interactions, which maximize the incoming parton flux. A useful information to understand the actual underlying dynamics is to ***measure the dependence of the cross section on the incoming fractional momenta***

By identifying the ***different MPI terms*** as the contributions to the final state due to the ***different disconnected parts of the hard component of the interaction*** one has the advantage that MPI can be discussed within a probabilistic framework.

There is experimental evidence of sizable contributions of triple parton scatterings in samples of events with double parton collisions. One may then introduce inclusive and exclusive cross sections.
By comparing inclusive and exclusive cross sections one may obtain information on the hadron structure and check the consistency of the picture of the interaction

MPI provide a direct information on multi parton correlations. Measuring MPI in *pp* collisions does not provide however enough information to disentangle between longitudinal and transverse correlations in the hadron structure.

To disentangle the effects of longitudinal and transverse correlation one can however measure MPI in *pA* collisions, which can provide very useful additional information.

Backup

Can we distinguish experimentally between 3->4 and 4->4 processes?

Yes, we can

The incoming partons fractional momenta x_1, x_1', x_2, x_2' are accessible experimental quantities

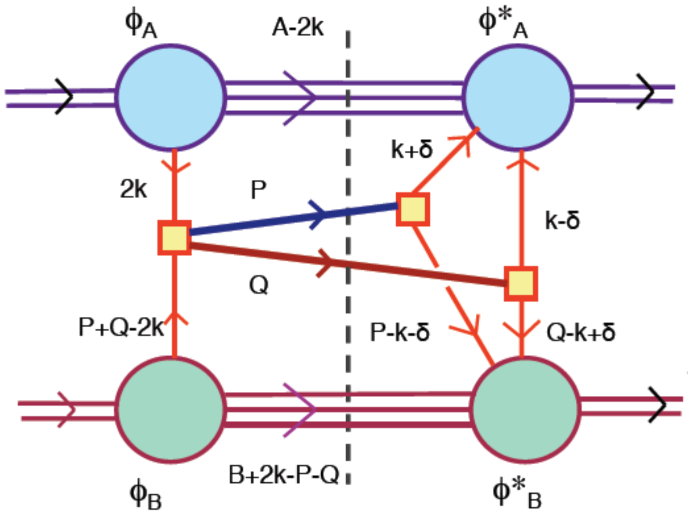
Once large p_t partons are paired, by compensating their transverse momenta, the incoming fractional momenta can be reconstructed. E.g. in the case of a photon + 3 J one has:

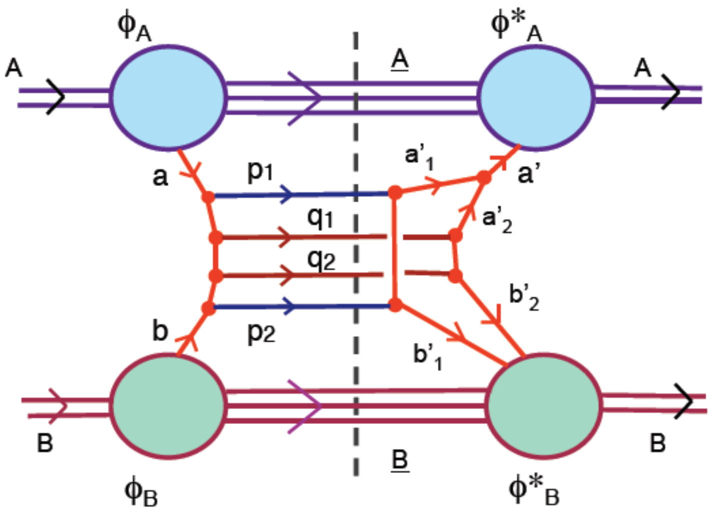
$$
\begin{aligned}
x_{pp}^{\gamma J} &= \frac{p_J^{\gamma}}{p_{\rm beam}}[e^{\pm\eta_\gamma} + e^{\pm\eta_J}] \\
x_{pp}^{JJ} &= \frac{E_t(i) + E_t(j)}{2p_{\rm beam}}[e^{\pm\eta_i} + e^{\pm\eta_j}]
\end{aligned}
$$

The dependence of the cross section on the incoming fractional momenta thus identifies the incoming parton flux, which allows to understand if the process is initiated by 3 or 4 initial state partons and the dependence on the incoming fractional momenta is very different in the two cases.

At low *x* one is thus dominated by disconnected multiparton collisions.

Although the same multiparton final state may be generated by MPI terms with different numbers of partonic collisions, disconnected hard multiparton collisions add incoherently

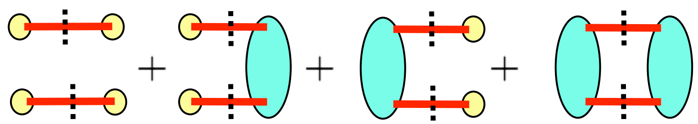




The short distance, which characterizes the left hand side of the cut, forces the two interactions in the right hand side to be localized in the same point in transverse space. The two interactions in the right hand side of the cut cannot thus be considered any more as disconnected and the non perturbative contribution in the left hand side needs to be considered together with higher order corrections in the coupling.

Double parton scattering differential "exclusive" cross sections at order $\hat{\sigma}^3$ taking into account two-body parton correlations

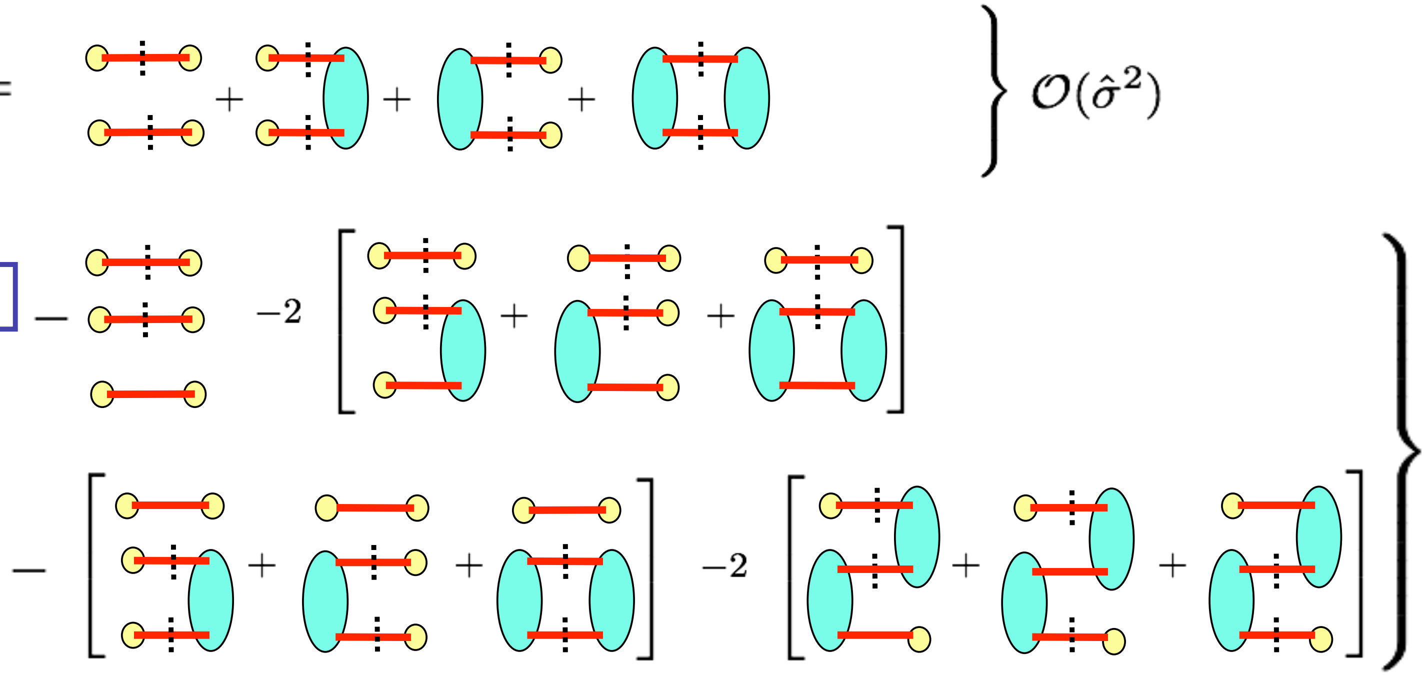
$2\sigma''_D =$



(at all orders in $\hat{\sigma}$)

inclusive

$2\tilde{\sigma}''_2 =$



$\mathcal{O}(\hat{\sigma}^2)$

$\mathcal{O}(\hat{\sigma}^3)$

exclusive

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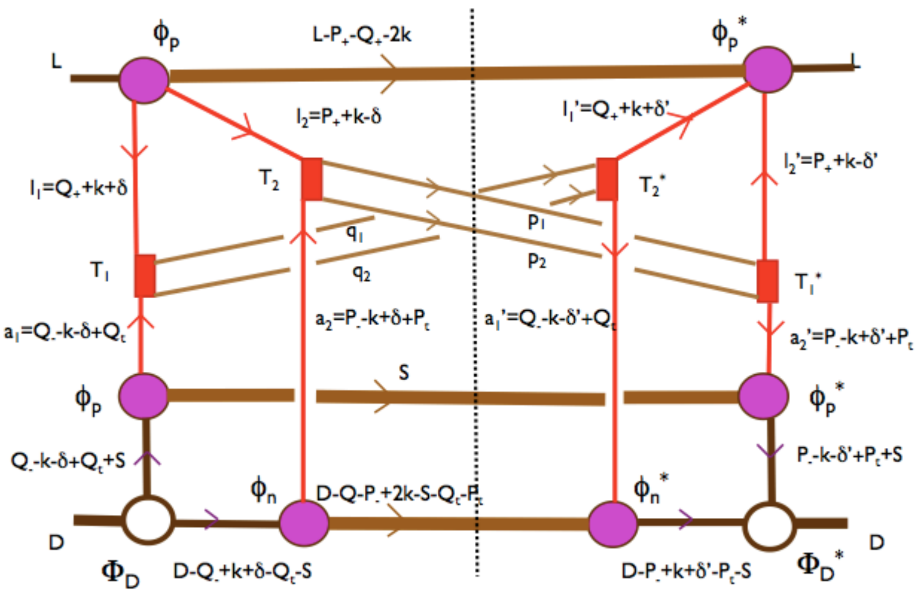
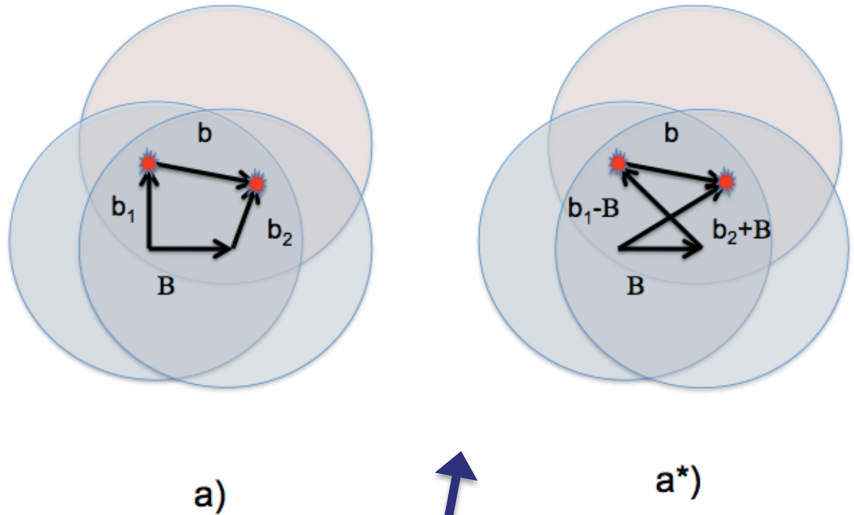
Both target nucleons interact with large momentum exchange.
Off-diagonal contribution:



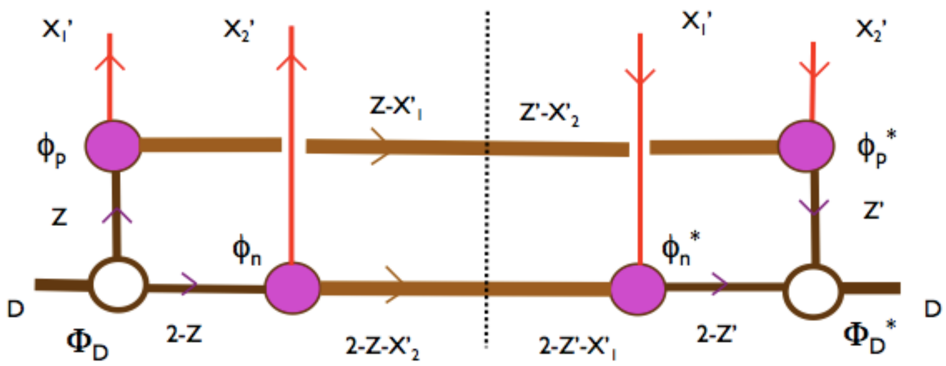


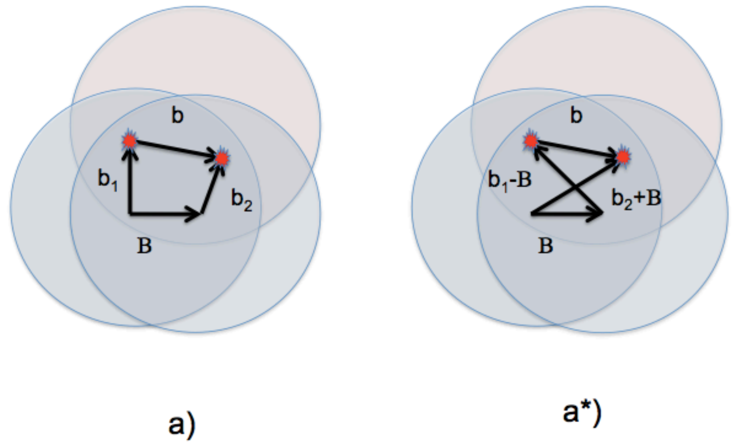
Configurations in transverse space of the two interfering amplitudes in the diagram

off-diagonal parton distributions

$$\sigma^{pD}_{1,1;\mathcal{I}} = \int \Gamma(x_1,x_2;\ b)\hat{\sigma}(x_1x_1')\hat{\sigma}(x_2x_2')\tilde{H}\Big(\frac{x_1'}{Z},\frac{x_2'}{Z'};\ b_1,b_2+B\Big)\tilde{H}\Big(\frac{x_2'}{2-Z},\frac{x_1'}{2-Z'};\ b_2,b_1-B\Big)$$
$$\times\frac{\Psi_D(Z,B)}{Z}\frac{\Psi_D^*(2-Z',B)}{2-Z'}\delta(B+b_2-b-b_1)\delta(Z'-Z+x_1'-x_2')dZ\,dZ'\,dB\,db_1\,db_2\,db$$

In the interference term the cross section depends on the off-diagonal parton distributions





In the interference term the nucleon's fractional momenta are different in the right and in the left hand side of the cut: $Z-Z'=x'_1-x'_2$

x'_1 and x'_2 are measured in the final state. When $x'_1-x'_2$ is large the contribution of the interference term is small. In many cases of interest the contribution of the interference term is however sizable.

The two interactions are localized in two points in transverse space. Given two interaction points the parton with fractional momentum x'_1 may be provided by the proton and the parton with fractional momentum x'_2 by the neutron (configuration a) or vice-versa (configuration a*)