Solution of KGBJS equation and resummed form of BK

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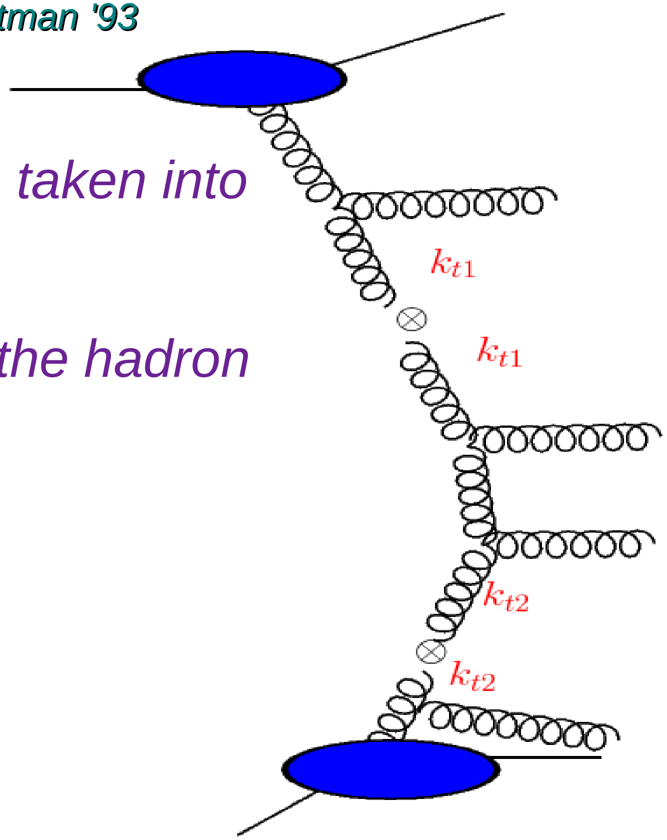
QCD at high energies

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 S)^2} \overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t})$$

$$\times \, \phi_{a/A}(x_1, k_{1t}^2, \mu^2)\, \phi_{b/B}(x_2, k_{2t}^2, \mu^2) \frac{1}{1+\delta_{cd}}$$

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

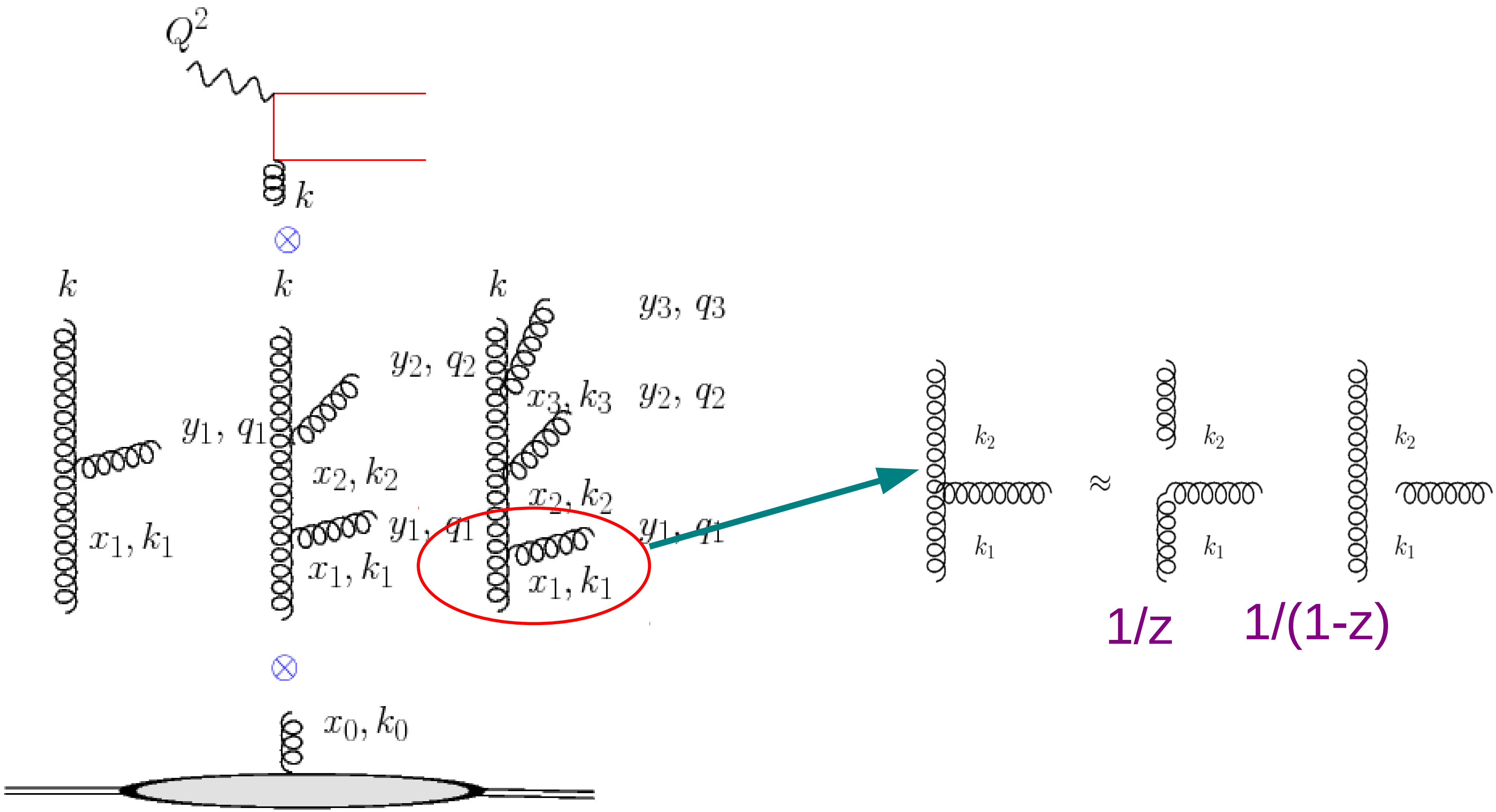


- *Longitudinal and transversal parton degrees of freedom taken into account; also hard scale*
- *Capable of taking into account finite transversal size of the hadron*
- *Realistic kinematics at lowest order*
- *Gluon density depends on k_t*
- *Gauge invariant matrix elements with off-shell gluons*

Lipatov '95

van Hameren, Kotko, Kutak '12

CCFM evolution equation



Implemented in CASCADE
Monte Carlo ***H. Jung 02***

CCFM evolution equation

For simplicity let's consider low x limit of the CCFM

$$\mathcal{A}(x,k^2,p) = \mathcal{A}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\mathbf{q}}{\pi q^2} \theta(p - zq) \Delta_{ns}(z,k,q) \mathcal{A}\left(\frac{x}{z}, k'^2, q\right)$$

p – linked to some hard scale

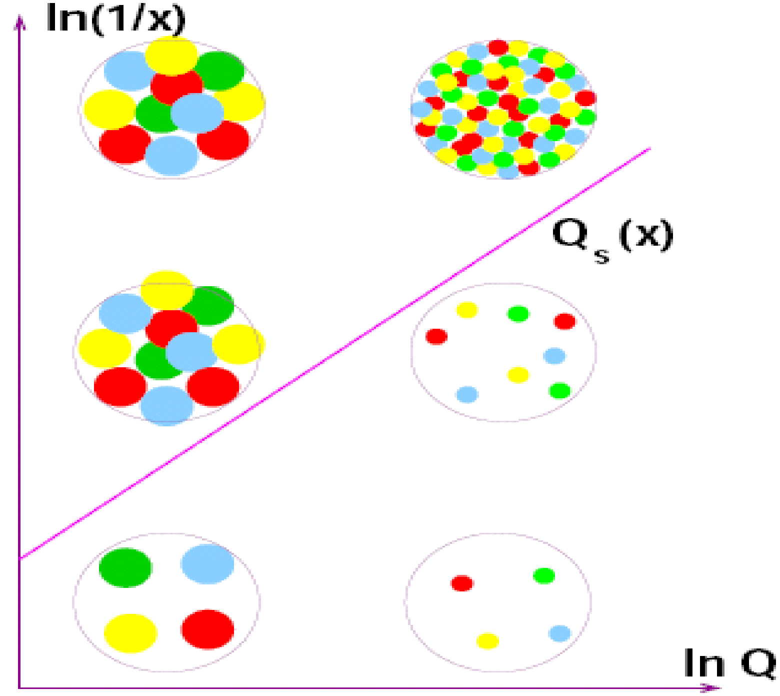
splitting

coherence

Linear equation – problems with unitarity.
Let us come back for a while to BK...

High energy factorization and saturation

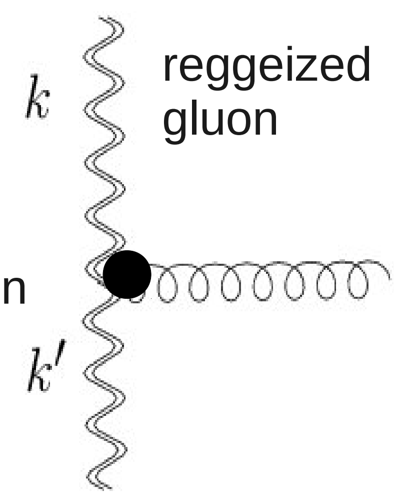
Saturation – state where number of gluons stops growing due to high occupation number.



Cross sections change their behavior from power like to logarithmic like.

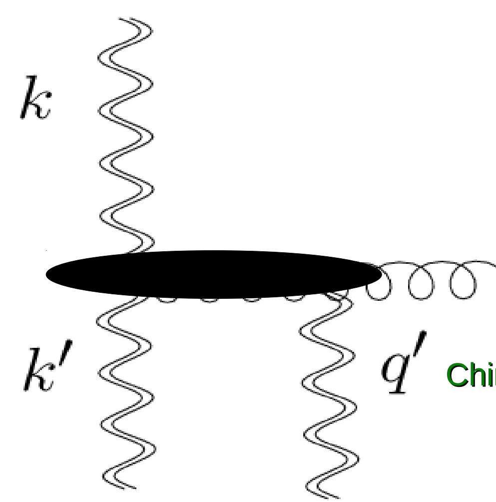
On microscopic level it means that gluon apart splitting recombine

splitting



Linear evolution equation

recombination

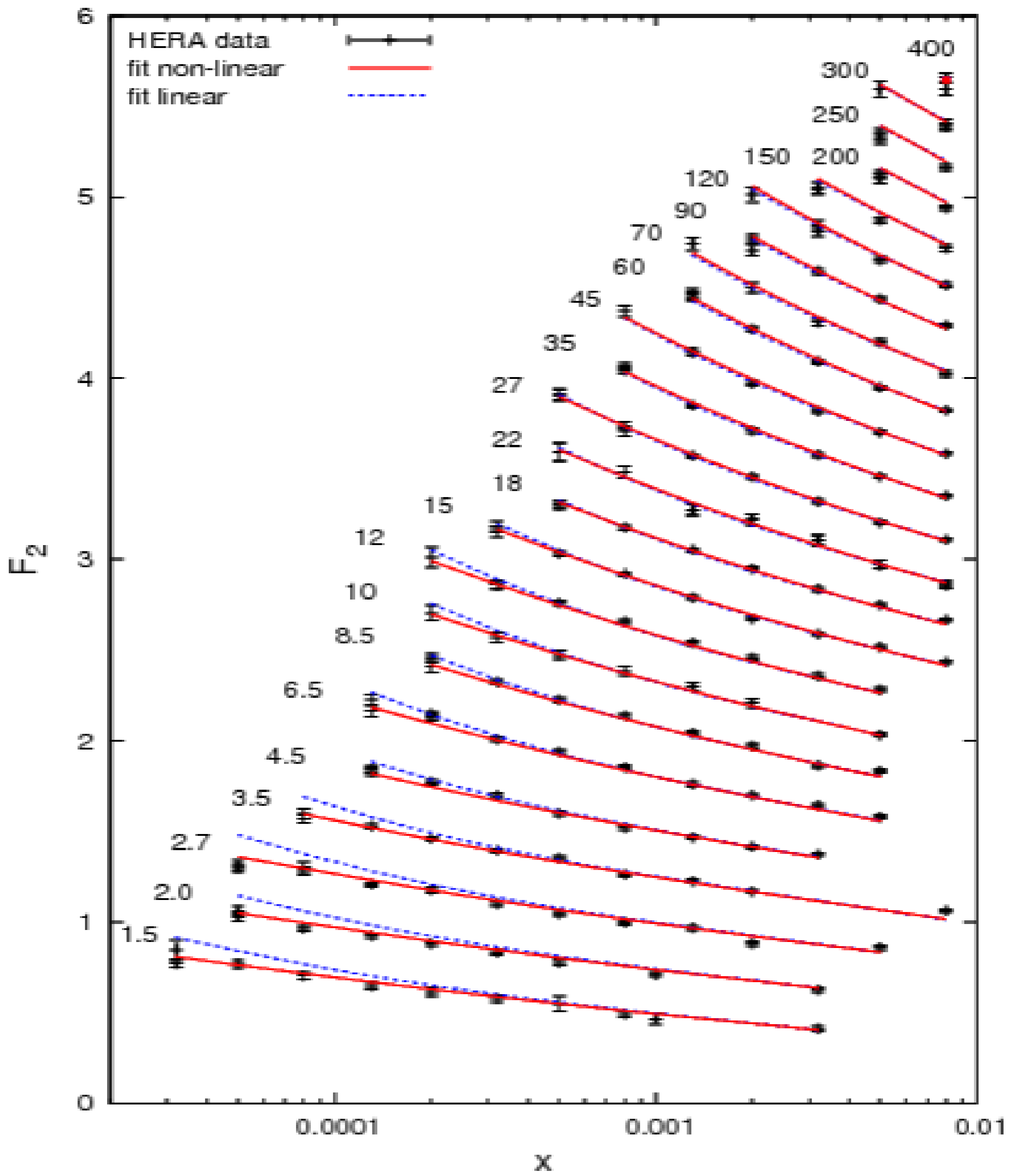


Nonlinear evolution equations
BK, JIMWLK
CGC framework
DIPSY

Bartels,Wusthoff Z.Phys. C66 (1995) 157-180

Chirilli,Szymanowski,Wallon '10

Recent hints for saturation in F_2 data



S.Sapeta. K.Kutak
PRD

Fit of BK-DGLAP and BFKL-DGLAP to combined H1-ZEUS data

Very good description with BK-DGLAP in range $Q^2 > 1.5$ GeV^2

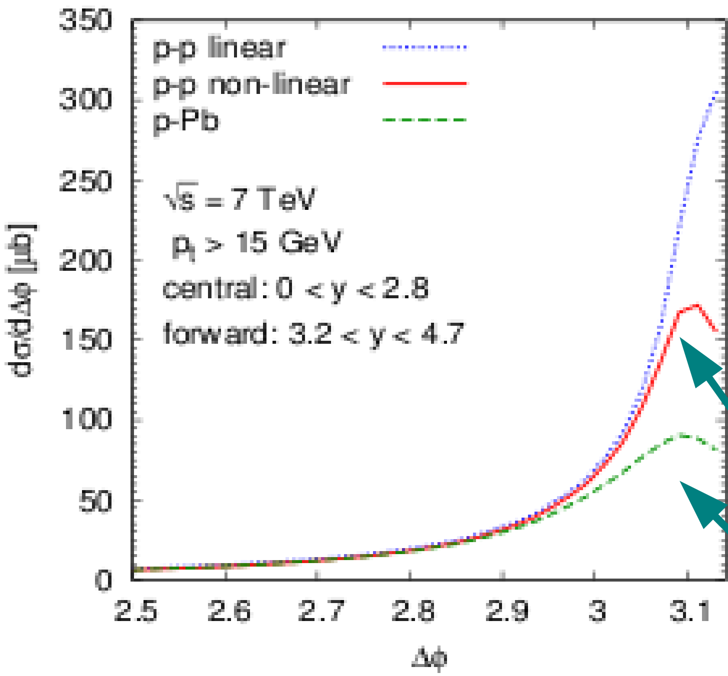
$$\chi^2 = 1.73$$

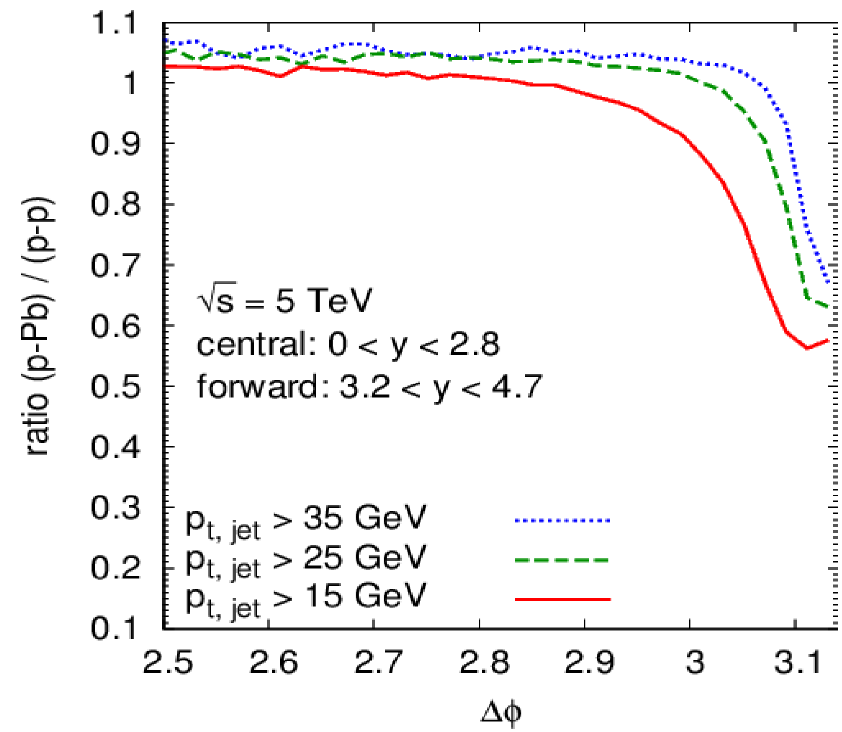
Very good description with BFKL-DGLAP in range $Q^2 > 4.5$ GeV^2

$$\chi^2 = 1.5$$

Signatures of saturation in p-p and p-Pb di-jet production

Kutak, Sapeta. '12 PRD



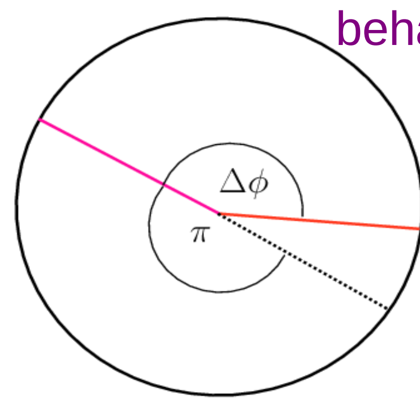


Reflects~ k^2

behavior of gluon at small k^2

Observable suggested to study saturation effects

Leviin, Kharzeev, 05, Marquet ' 07 and BFKL,, CCFM Deak,Jung Kautmann, Kutak '10



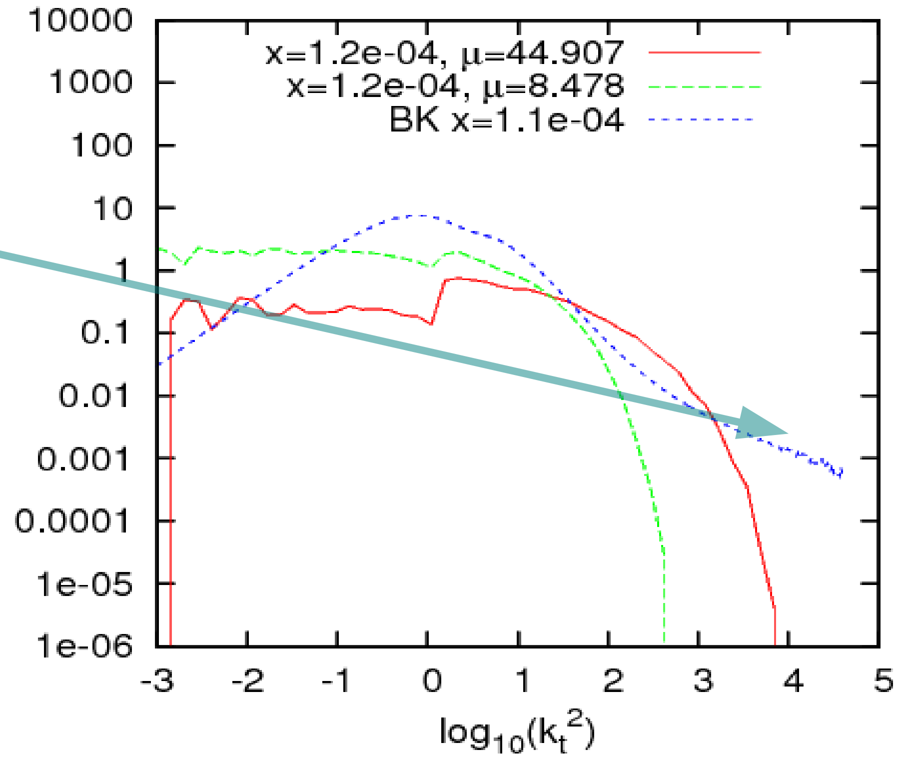
Results for RHIC

Albacete, Marquet '10

Forward physics as the way to constrain gluon both at large and small p_t

- Too flat behaviour of at large k_t
- Lack of saturation in CCFM small k_t

$$\mathcal{F}\left(x,k^2\right) = \frac{N_c}{\alpha_s \pi^2} k^2 \nabla_k^2 \Phi\left(x,k^2\right)$$



Needed a framework which unifies both correct behaviors

Resummed form of the BK

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek '11

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^{\infty} \frac{dl^2}{l^2} \left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}} \right] - \frac{\overline{\alpha}_s}{\pi R^2} \int_{x/x_0}^{1} \frac{dz}{z} \Phi^2(x/z,k^2)$$

Mellin transform and resummation

$$\pi R^2 = 1$$

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 dz \int \frac{d^2\mathbf{q}}{\pi q^2} \theta(q^2-\mu^2) \frac{\Delta_R(z,k,\mu)}{z} \left[\Phi(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2) - q^2 \delta(q^2-k^2)\, \Phi^2(\frac{x}{z}, q^2) \right] \quad (1$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln\frac{1}{z} \ln\frac{k^2}{\mu^2}\right)$$

- The same resumed piece for linear and nonlinear
- Initial distribution also gets multiplied by the Regge form factor
- New scale introduced to equation. One has to check dependence of the solution on it
- Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states

Equation for exclusive states and saturation

Original formulation of BK or BFKL- difficult to address final state problem. One of possible solutions is to combine physics of BK with CCFM

Kutak 2012, JHEP

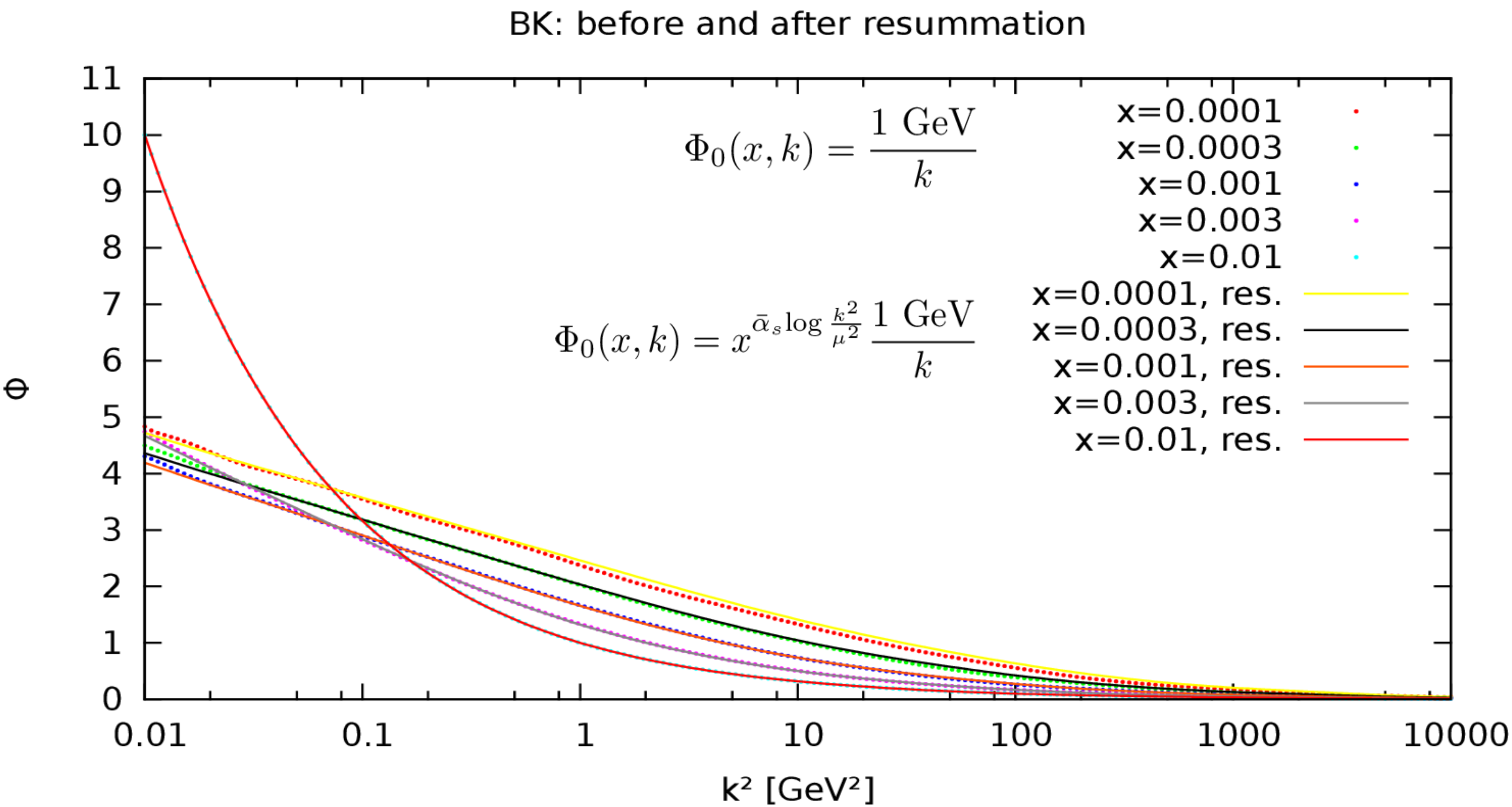
$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\mathbf{q}}{\pi q^2} \theta(p - zq) \Delta_{ns}(z,k,q) \left[\mathcal{E}\left(\frac{x}{z}, k'^2, q\right) - q^2 \delta(q^2 - k^2) \mathcal{E}^2\left(\frac{x}{z}, q^2, q\right) \right]$$

p – linked to some hard scale

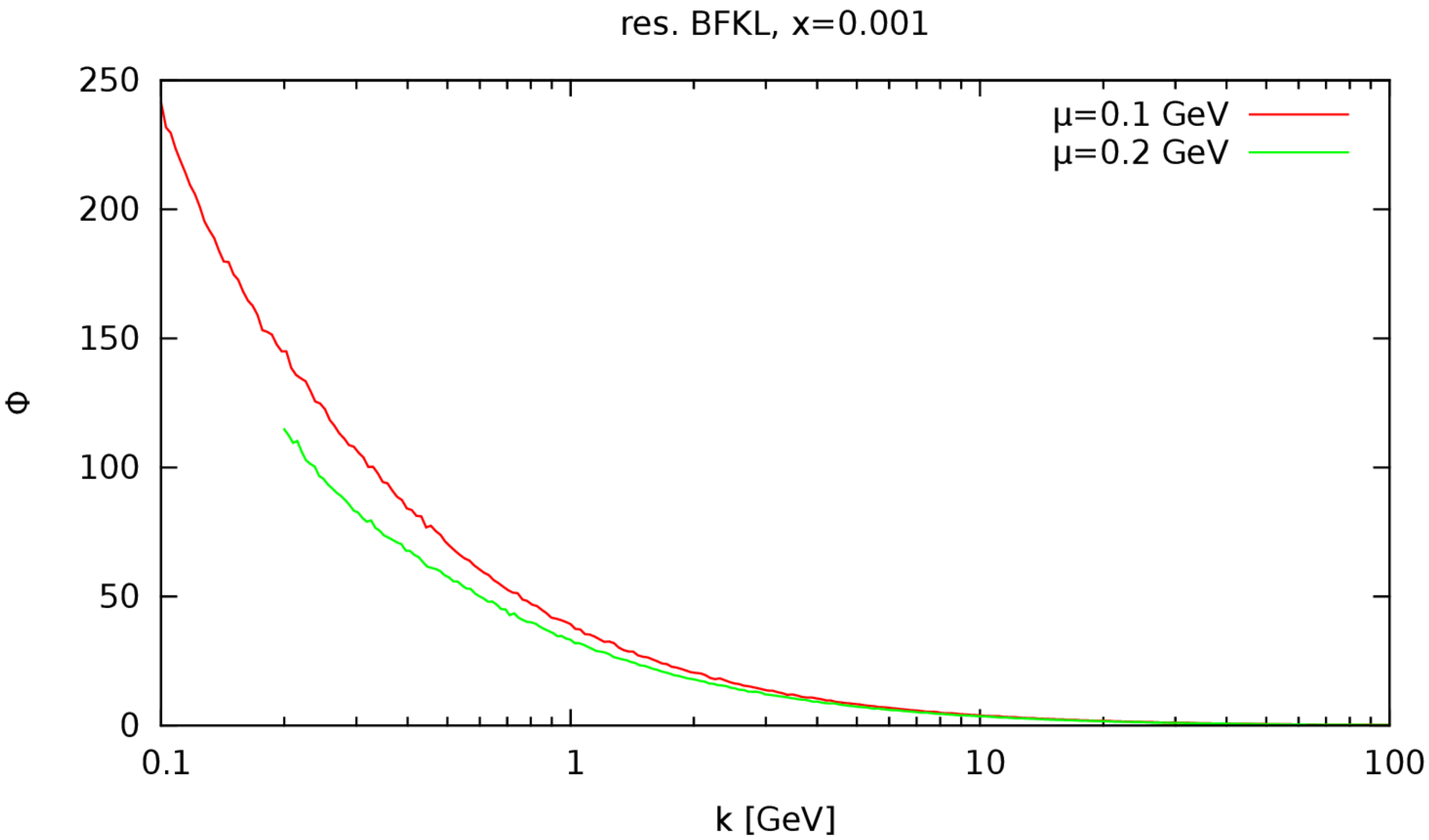
coherence

saturation

Resummation of BK – numerical results

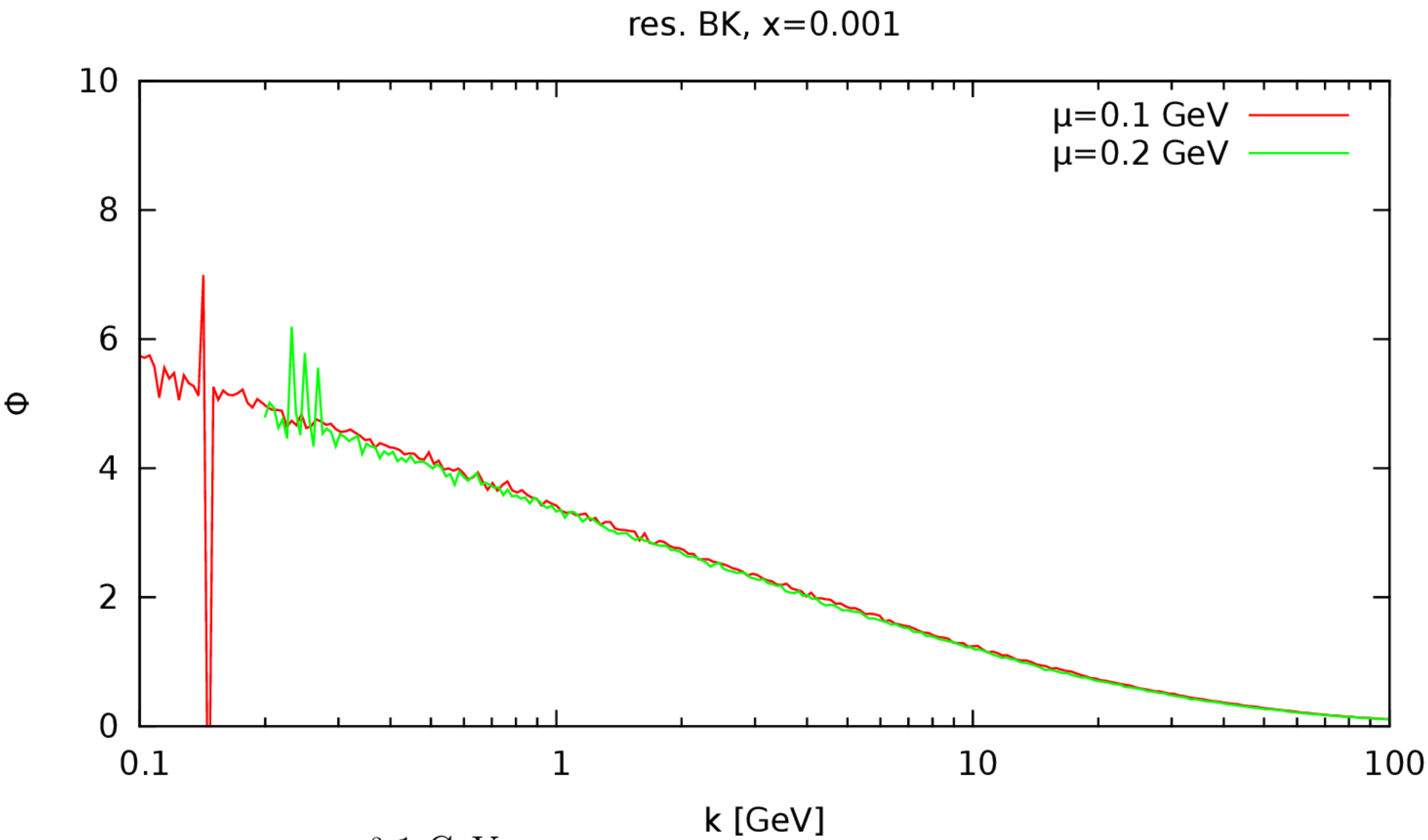


Numerical studies – BFKL



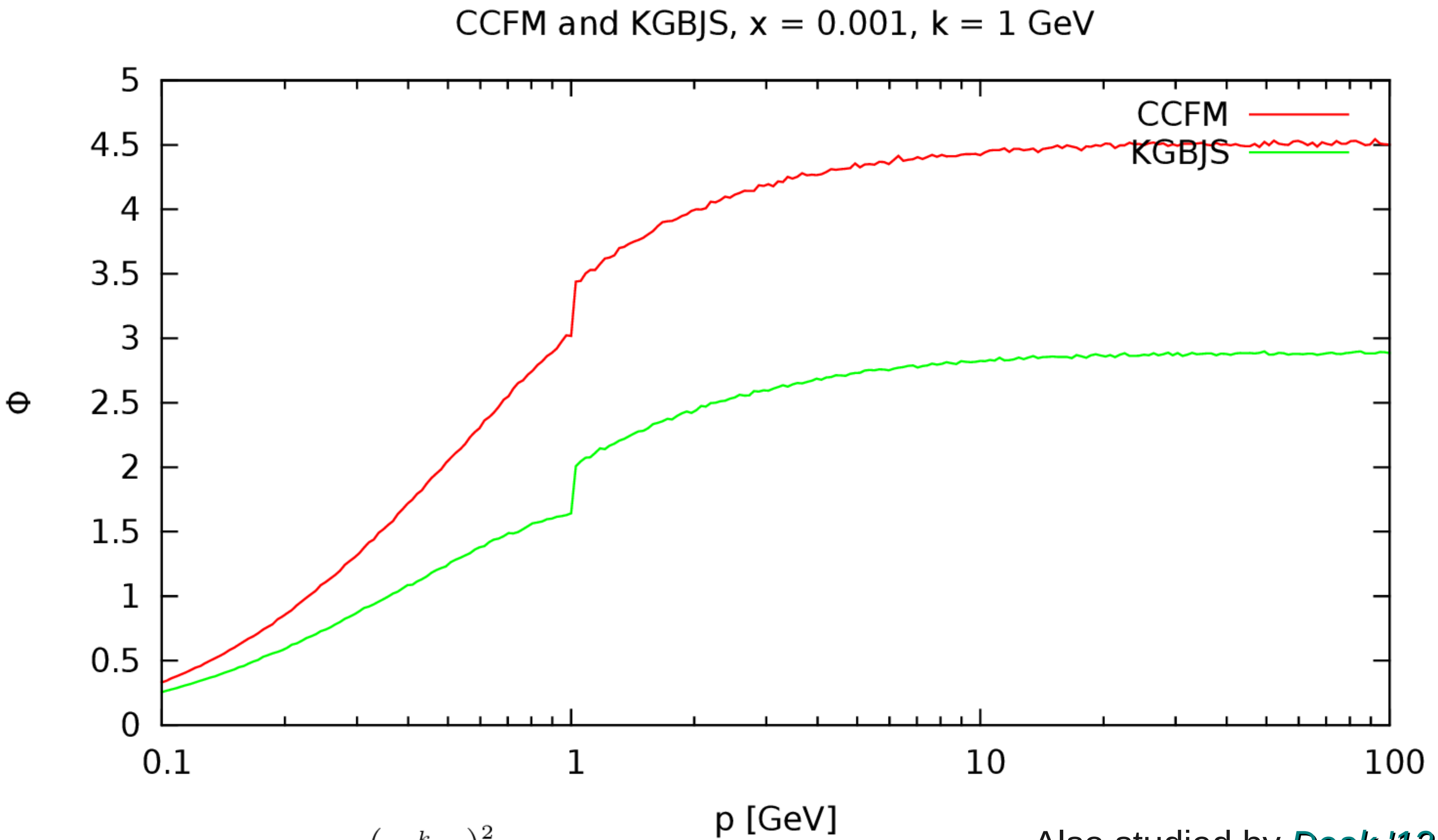
$$\Phi_0(x,k) = x^{\bar{\alpha}_s \log \frac{k^2}{\mu^2}} \frac{1 \text{ GeV}}{k}$$

Numerical studies – BK



$$\Phi_0(x,k) = x^{\bar{\alpha}_s \log \frac{k^2}{\mu^2}} \frac{1\ \mathrm{GeV}}{k}$$

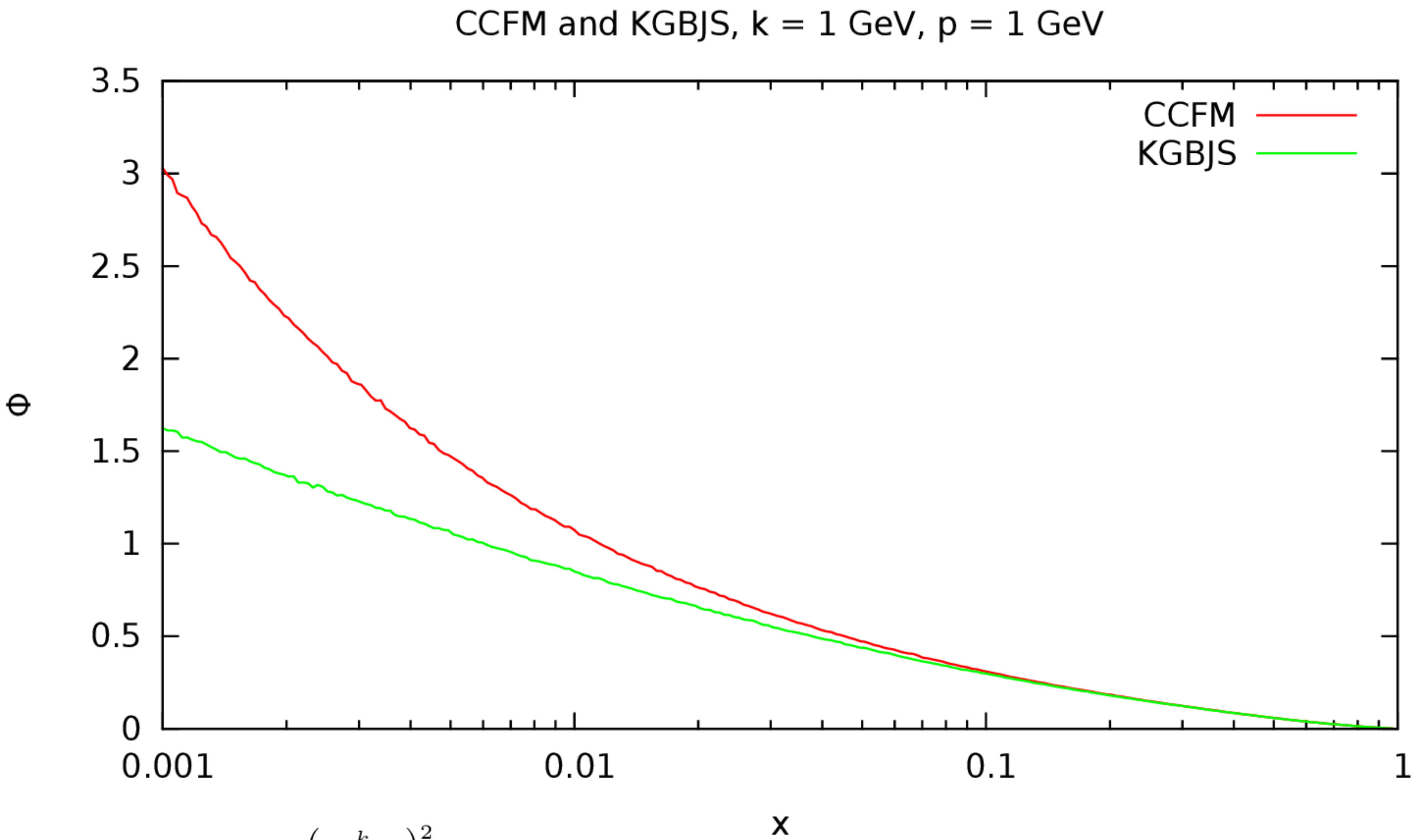
Numerical results – CCFM vs. KGBJS



$$\mathcal{A}_0 = \theta\left(p-k\right) \mathrm{e}^{-\left(\frac{k}{1\ \mathrm{GeV}}\right)^2}$$

Also studied by ***Deak '12***

Numerical results – CCFM vs. KGBJS



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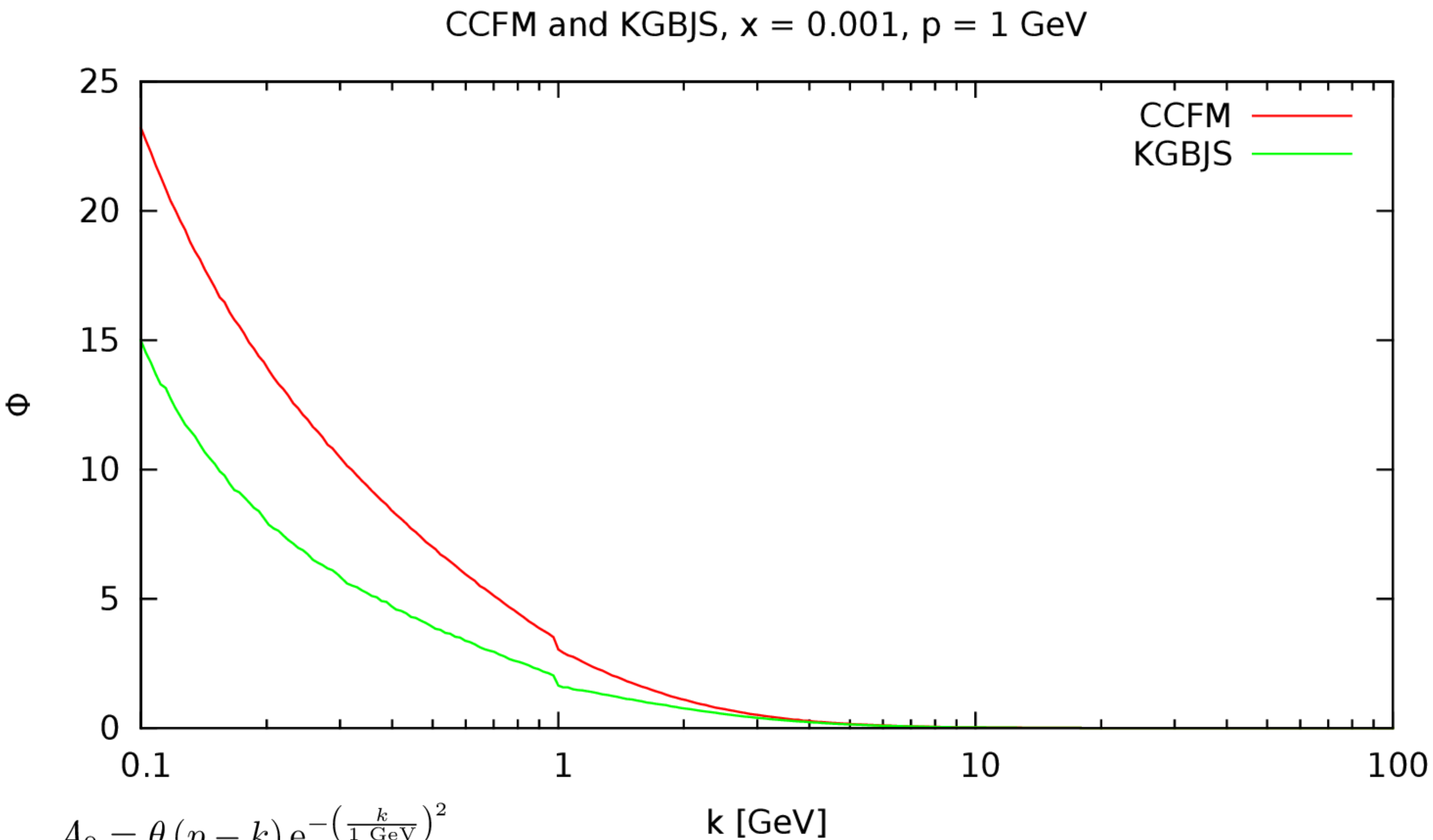
Numerical results – CCFM vs. KGBJS



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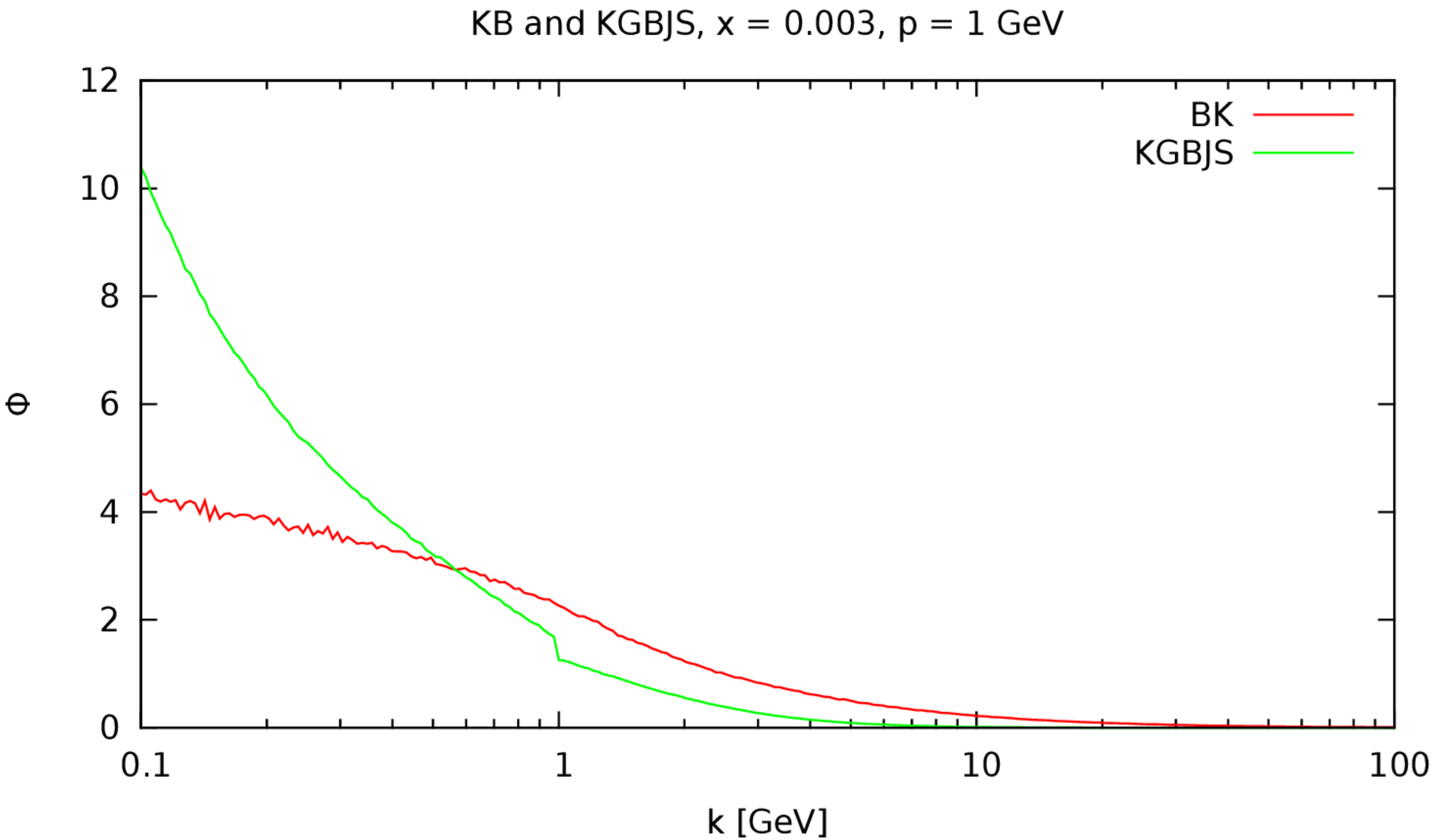
Numerical results – BK vs. KGBJS



$$\mathcal{A}_0 = \theta\left(p-k\right) \mathrm{e}^{-\left(\frac{k}{1\ \mathrm{GeV}}\right)^2}$$

Conclusions and outlook

- Developed framework allowing for saturation and exclusiveness

- The BK and BFKL equations in resummed form have been solved
- The resummation procedure is stable in case of BK.
- The KGBJS equation has been solved and compared to CCFM
- Properties of KGBJS are still to be investigated
- Application to phenomenology of p-Pb and p-p