Multi-Parton Interactions at the LHC 2012

December 3 – 7, 2012

CERN, Geneva

A Model Construction of Self-similarity based Double Parton Distribution Functions for proton-proton collision at LHC

Akbari Jahan

Department of Physics, Gauhati University, Guwahati – 781 014, Assam, India.

Abstract

We construct a model for double parton distribution functions (dPDFs) based on the notion of self-similarity, pursued earlier for small x physics at HERA. The most general form of dPDFs contains total thirteen parameters to be fitted from data of proton-proton collision at LHC. It is shown that the constructed dPDF does not factorize into two single PDFs contrary to common expectations, but does satisfy the condition that at the kinematic boundary $x_1 + x_2 = 1$ (where x_1 and x_2 are the longitudinal fractional momenta of two partons), the dPDF vanishes.

Introduction

- Parton Distribution Functions (PDFs) are the most important quantities in Quantum Chromodynamics (QCD) to study the structure function of proton in deep inelastic scattering (DIS).

R G Roberts, *The Structure of the Proton*, Cambridge University Press, Cambridge,120 (1990)

- At LHC, on the other hand, double Parton Distribution Functions (dPDFs) are equally important in the study of double parton scattering cross-sections.

P Bartalini *et al*, Proceedings of the Multi-Parton Interactions at the LHC 2010, DESY, Ed. A Kulesza and Z Nagy (2010); arXiv:1111.0469[hep-ph]

- We outline the model of PDFs based on self-similarity, an inherent property of fractals.

B B Mandelbrot, *Fractal Geometry of Nature*, W H Freeman, New York (1982)

- Relevance of these ideas in the contemporary physics of DIS was first noted by Dremin and Levtchenko in the early nineties where it was shown that the saturation of hadron structure function at small x may proceed further if the highly packed regions of proton have fractal structures.

I M Dremin and B B Levtchenko, *Phys. Lett.* B292, 155 (1992)

- However, these ideas received wider attention in 2002 when Lastovicka of DESY proposed a relevant formalism and a functional form of the structure function $F_2(x,Q^2)$ at small x based on self-similarity.

T Lastovicka, *Euro. Phys. J.* C24, 529 (2002); arXiv:hep-ph/0203260

- In recent years, we have applied the model to

  - deep inelastic scattering ,

    D K Choudhury and Rupjyoti Gogoi, *Indian J. Phys.* 80, 659 (2006); arXiv:hep-ph/0503047

  - longitudinal structure function and

    Akbari Jahan and D K Choudhury, *Indian. J. Phys.* 85, 587 (2011); arXiv:1101.0069[hep-ph]

  - momentum fractions of quarks and gluons in the proton.

    Akbari Jahan and D K Choudhury, Proceedings of the 3rd International Workshop on Multiple Partonic Interactions at the LHC 2011, DESY, Ed. S Platzer and M Diehl, p145 (2011)

    Akbari Jahan and D K Choudhury, *Mod. Phys. Lett.* A27, 1250193 (2012)

❖ Self-similarity based Transverse Momentum Dependent Parton Density Function (TMD PDF) with one hard scale

The formalism is based on the imposition of self-similarity constraints to the dimensionless quark density $f_i(x,k_t^2)$ and relate it to the integrated density $q_i(x,Q^2)$. In other words, using magnification factors $1/x$ and $(1+k_t^2/Q_0^2)$, an unintegrated quark density (TMD) is given as:

$$\log f_i\left(x,k_t^2\right) = D_1 \log\frac{1}{x}\log\left(1+\frac{k_t^2}{Q_0^2}\right) + D_2\log\frac{1}{x} + D_3\log\left(1+\frac{k_t^2}{Q_0^2}\right) + D_0^i - \log M^2 \qquad (1)$$

where i denotes a quark flavor. Here, D_2 and D_3 are the fractal parameters; D_1 is the dimensional correlation relating the two magnification factors; while $D_0{}^i$ is the normalization constant. M^2 is introduced to make PDF $q_i(x,Q^2)$, as defined in the following Eq (2), dimensionless.

$$q_i\left(x,Q^2\right)=\int_0^{Q^2} dk_t^2\ f_i\left(x,k_t^2\right) \tag{2}$$

$$q_i\left(x,Q^2\right)=e^{D_0^i}\ f\left(x,Q^2\right) \tag{3}$$

$$f\left(x,Q^2\right)=\frac{Q_0^2}{M^2}\frac{x^{-D_2}}{1+D_3+D_1\log\left(\frac{1}{x}\right)}\left(\left(\frac{1}{x}\right)^{D_1\log\left(1+\frac{Q^2}{Q_0^2}\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{D_3+1}-1\right) \tag{4}$$

$$F_2\left(x,Q^2\right)=x\sum_i e_i^2\left(q_i\left(x,Q^2\right)+\overline{q_i}\left(x,Q^2\right)\right) \tag{5}$$

$$F_2\left(x,Q^2\right)=e^{D_0}\ x\ f\left(x,Q^2\right) \tag{6}$$

From HERA data, Eq (4) was fitted with

$$D_0 = 0.339 \pm 0.145$$

$$D_1 = 0.073 \pm 0.001$$

$$D_2 = 1.013 \pm 0.01$$

$$D_3 = -1.287 \pm 0.01$$

$$Q_0^2 = 0.062 \pm 0.01\ GeV^2 \qquad (7)$$

H1: C Adloff *et al*, *Eur. Phys. J.* C21, 33 (2001); arXiv:hep-ex/0012053

ZEUS: J Breitweg *et al*, *Phys. Lett.* B487, 53 (2000); arXiv:hep-ex/0005018

in the kinematical region

$$6.2\,.\,10^{-7} \le x \le 0.01 \quad \text{and} \quad 0.045 \le Q^2 \le 120\ \text{GeV}^2 \qquad (8)$$

We set $M^2 = 1$ GeV^2.

❖ Self-similarity based small x TMD PDF extrapolated to large x

The original model was tested for a limited range of small x as noted in Eq (8). It did not take into account the large x behavior of the PDF or structure function

$$\underset{x\to 1}{Lt}\, F_2\left(x, Q^2\right) = 0 \qquad (9)$$

which is not unexpected.

R G Roberts, *The Structure of the Proton*, Cambridge University Press, Cambridge, 120 (1990)

F J Yudurain, *Theory of Quark and Gluon Interactions*, Springer Verlag, Berlin, p129 (1992)

C Pascaud and F Zomer, *preprint* DESY, 96-266 (1996)

The idea of self-similarity is based on the fact that at small x, the behavior of quark density is driven by gluon emissions and splittings such that the parton distribution function at small x and those at still smaller x look similar (upto some magnification factor).

Motivation (for large x)

There is no physical reason for self-similarity at large x and no phenomenological justification till date. But it is not unreasonable to assume that the self-similarity does not terminate abruptly at $x \approx 0.01$, but smoothly vanishes at $x = 1$, the valence quark limit of proton with no trace of self-similarity at all.

We suggest a simple interpolating model of TMD PDF / PDF which approaches the self-similar one at $x \to 0$ (Eq (1)), and still satisfy Eq (9) at large x ($x \to 1$).

A plausible way of achieving it in a parameter free way is to make a formal replacement of $1/x$ factor to $(1/x - 1)$ in Eq (1).

$$\log \widetilde{f}_i\left(x,k_t^2\right)=D_1\log\left(\frac{1}{x}-1\right)\log\left(1+\frac{k_t^2}{Q_0^2}\right)+D_2\log\left(\frac{1}{x}-1\right)+D_3\log\left(1+\frac{k_t^2}{Q_0^2}\right)+D_0^i-\log M^2 \qquad (10)$$

$$\widetilde{q}_i\left(x,Q^2\right)=e^{D_0^i}\ \widetilde{f}\left(x,Q^2\right) \qquad (11)$$

$$F_2\left(x,Q^2\right)=e^{D_0}\ x\,\widetilde{f}\left(x,Q^2\right) \qquad (12)$$

$$\widetilde{f}\left(x,Q^2\right)=\frac{Q_0^2}{M^2}\frac{x^{-D_2}\left(1-x\right)^{D_2}}{1+D_3+D_1\log\left(\frac{1}{x}\right)+D_1\log\left(1-x\right)}\left(\left(1-x\right)^{D_1\log\left(1+\frac{Q^2}{Q_0^2}\right)}x^{-D_1\log\left(1+\frac{Q^2}{Q_0^2}\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{D_3+1}-1\right) \qquad (13)$$

❖ Self-similar TMD dPDF and dPDF at small x_1, x_2

Following the original model, the TMD dPDF for partons of flavors i and j will have the following basic magnification factors:

$$M_1 = \frac{1}{x_1}$$

$$M_2 = \frac{1}{x_2}$$

$$M_3 = \left(1 + \frac{k_t^{2\,(1)}}{k_0^2}\right)$$

$$M_4 = \left(1 + \frac{k_t^{2\,(2)}}{k_0^2}\right) \qquad (14)$$

The TMD dPDF for a pair of partons of flavors *i* and *j* is then given as:

$$\log f_{ij}\left(x_1, x_2, k_t^{2\,(1)}, k_t^{2\,(2)}\right) = D_1 \log M_1 \log M_2 + D_2 \log M_1 \log M_4 + D_3 \log M_2 \log M_3$$

$$+ D_4 \log M_3 \log M_4 + D_5 \log M_1 \log M_2 \log M_3 + D_6 \log M_1 \log M_2 \log M_4$$

$$+ D_7 \log M_1 \log M_3 \log M_4 + D_8 \log M_2 \log M_3 \log M_4$$

$$+ D_9 \log M_1 \log M_2 \log M_3 \log M_4 + D_0^{ij} - \log M^4 \qquad (15)$$

Eq (15) has total 11 parameters, viz. 9 flavor independent fractal parameters (D_1, …,D_9), one flavor dependent normalization constant D_0^{ij} and a mass scale M^4. The additional term log M^4 is added in Eq (15) with dimension $(\text{Mass})^4$ so that the dPDF defined as

$$D_{ij}\left(x_1,x_2,Q^{2(1)},Q^{2(2)}\right)=\int_0^{Q^{2(1)}} dk_t^{2(1)} \int_0^{Q^{2(2)}} dk_t^{2(2)}\ f_{ij}\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right) \qquad (16)$$

is dimensionless.

Using the definition of dPDF (Eq (16)), one has

$$D_{ij}\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right)=\frac{e^{D_0^{ij}}}{M^4}\left(\frac{1}{x_1}\right)^{D_1 \log\frac{1}{x_2}}.I\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right) \qquad (17)$$

where $I\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right)$ is the double integration over $k_t^{2(1)}$ and $k_t^{2(2)}$.

That is,

$$I\left(x_1,x_2,k_t^{2\,(1)},k_t^{2\,(2)}\right)=\int_0^{Q^{2\,(1)}}dk_t^{2\,(1)}\int_0^{Q^{2\,(2)}}dk_t^{2\,(2)}\left(\frac{k_t^{2\,(1)}+k_0^2}{k_0^2}\right)^{\left\{D_3\log\frac{1}{x_2}+D_4\log\left(\frac{k_t^{2\,(2)}+k_0^2}{k_0^2}\right)+D_5\log\frac{1}{x_1}\log\frac{1}{x_2}+D_7\log\frac{1}{x_1}\log\left(\frac{k_t^{2\,(2)}+k_0^2}{k_0^2}\right)\right\}}.$$

$$\left(\frac{k_t^{2\,(2)}+k_0^2}{k_0^2}\right)^{D_2\log\frac{1}{x_1}+D_6\log\frac{1}{x_1}\log\frac{1}{x_2}+D_8\log\frac{1}{x_2}\log\left(\frac{k_t^{2\,(1)}+k_0^2}{k_0^2}\right)+D_9\log\frac{1}{x_1}\log\frac{1}{x_2}\log\left(\frac{k_t^{2\,(1)}+k_0^2}{k_0^2}\right)} \qquad (18)$$

Eq (17) is our main result for self-similar dPDF at small x_1 and x_2. It contains total 12 parameters, viz. 9 fractal parameters (D_1, …,D_9), one normalization constant (D_0^{ij}), one mass scale M^4 and one transverse mass cut off k_0^2.

We note that the integration (Eq (18)) is not factorisable in x_1 and x_2.

Thus the usual factorisability assumption that a dPDF can be considered as a product of two single PDFs

$$D_{ij}\left(x_1, x_2, Q^{2\,(1)}, Q^{2\,(2)}\right) \equiv D_i\left(x_1, Q^{2\,(1)}\right).\, D_j\left(x_2, Q^{2\,(2)}\right) \tag{19}$$

J Bartels, Proceedings of the 3rd International Workshop on Multiple Partonic Interactions at the LHC 2011, DESY, Ed. S Platzer and M Diehl, p151 (2011)

does not hold good in the present self-similarity based dPDF.

❖TMD dPDF and dPDF at the boundary $x_1+x_2 = 1$

$$\underset{x_1\to 1}{Lt}\, D_i\left(x_1, Q^{2\,(1)}\right) = 0 \tag{20}$$

$$\underset{x_2\to 1}{Lt}\, D_j\left(x_2, Q^{2\,(2)}\right) = 0 \tag{21}$$

The corresponding boundary condition of dPDF is

$$\underset{x_1+x_2\to 1}{Lt}\, D_{ij}\left(x_1, x_2, Q^{2\,(1)}, Q^{2\,(2)}\right) = 0 \tag{22}$$

P Bartalini *et al*, Proceedings of the Multi-Parton Interactions at the LHC 2010, DESY, Ed. A Kulesza and Z Nagy (2010); arXiv:1111.0469[hep-ph]

which does not conform to Eq (20) and Eq (21).

Thus the simple assumption of factorisability of dPDF into two PDF fails at the kinematic boundary $x_1 + x_2 = 1$.

Unlike TMD PDF, there is no simple parameter free prescription for incorporating the kinematic boundary condition (Eq (22)) so that it coincides with the original definition (Eq (15)) for x_1, $x_2 \to 0$. A simple way of incorporating such effect is to introduce an additional factor

$M_5 = \left(\frac{1}{x_1 + x_2} - 1\right)$,which for $x_2 = 0$ and $x_1 \to 0$ ($x_2 = 0$ and $x_1 \to 0$) approaches

M_1 (M_2) of Eq (14).

$$\log \widetilde{f}_{ij}\left(x_1, x_2, k_t^{2\,(1)}, k_t^{2\,(2)}\right) = \log f_{ij}\left(x_1, x_2, k_t^{2\,(1)}, k_t^{2\,(2)}\right) + D_{10} \log M_5 \qquad (23)$$

$$\widetilde{f}_{ij}\left(x_1, x_2, k_t^{2\,(1)}, k_t^{2\,(2)}\right) = f_{ij}\left(x_1, x_2, k_t^{2\,(1)}, k_t^{2\,(2)}\right) \cdot \left(\frac{1}{x_1 + x_2} - 1\right)^{D_{10}} \qquad (24)$$

Eq (24) is the expression for the TMD dPDF compatible with the boundary condition (Eq (22)). At small x_1 and x_2 it has self-similar basis.

The corresponding PDF expression is

$$\widetilde{D}_{ij}\left(x_1,x_2,Q^{2(1)},Q^{2(2)}\right)=\frac{e^{D_0^{ij}}}{M^4}\left(\frac{1}{x_1}\right)^{D_1\log\frac{1}{x_2}}.\left(\frac{1-\left(x_1+x_2\right)}{\left(x_1+x_2\right)}\right)^{D_{10}}.\widetilde{I}\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right) \qquad (25)$$

where $\widetilde{I}\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right)$ is the double integration over the transverse momenta $k_t^{2(1)}$ and $k_t^{2(2)}$.

i.e.

$$\widetilde{I}\left(x_1,x_2,k_t^{2(1)},k_t^{2(2)}\right)=\int_0^{Q^{2(1)}}dk_t^{2(1)}\int_0^{Q^{2(2)}}dk_t^{2(2)}\left(\frac{k_t^{2(1)}+k_0^2}{k_0^2}\right)^{\left\{D_3\log\frac{1}{x_2}+D_4\log\left(\frac{k_t^{2(2)}+k_0^2}{k_0^2}\right)+D_5\log\frac{1}{x_1}\log\frac{1}{x_2}+D_7\log\frac{1}{x_1}\log\left(\frac{k_t^{2(2)}+k_0^2}{k_0^2}\right)\right\}}.$$

$$\left(\frac{k_t^{2(2)}+k_0^2}{k_0^2}\right)^{D_2\log\frac{1}{x_1}+D_6\log\frac{1}{x_1}\log\frac{1}{x_2}+D_8\log\frac{1}{x_2}\log\left(\frac{k_t^{2(1)}+k_0^2}{k_0^2}\right)+D_9\log\frac{1}{x_1}\log\frac{1}{x_2}\log\left(\frac{k_t^{2(1)}+k_0^2}{k_0^2}\right)} \qquad (26)$$

Eq (25) of the final expression for dPDF in the approach contains total thirteen parameters to be determined from LHC data. A smooth extrapolation is possible only if D_{10} itself has x_1, x_2 dependence such that $\underset{x_1,x_2\to 0}{Lt} D_{10}\left(x_1,x_2\right)=0$.

Given the paucity of experimental data regarding dPDF, we discuss only some rough qualitative features of the model graphically in the next slide.

For qualitative feature of the model, we plot the PDF $q_i(x,Q^2)$ vs x for representative values of Q^2 = 10, 50 and 100 GeV^2 of HERA range using Eq (3) and Eq (11) respectively. It shows the qualitative difference between the original model and the smooth extrapolation (Eq (12)) for large x.

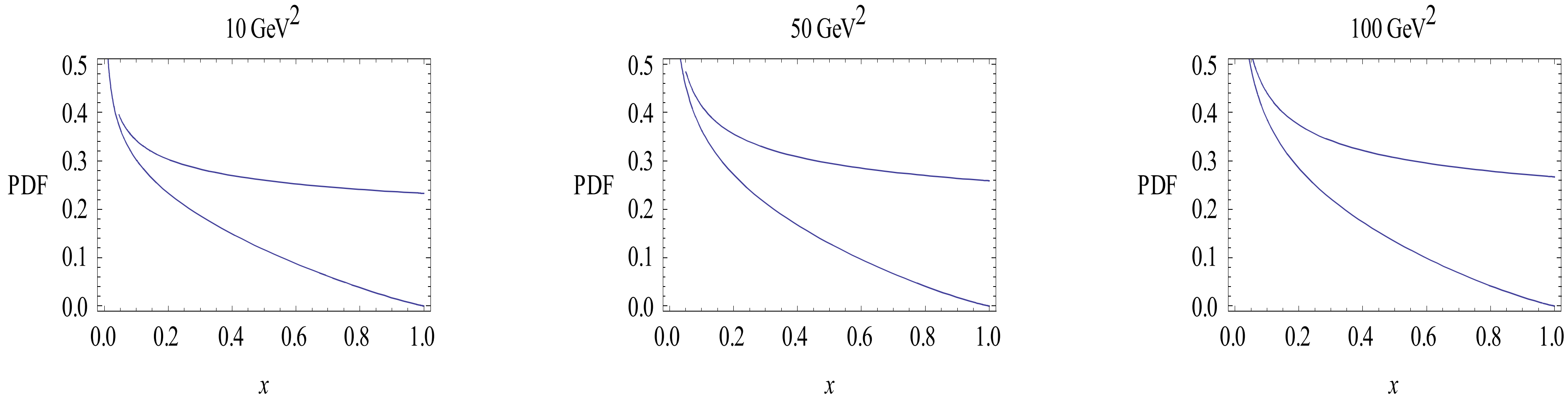

Figure: PDF $q_i(x,Q^2)$ vs x for representative values of Q^2 = 10, 50 and 100 GeV^2 of HERA range using Eq (3) and Eq (11) respectively.

Conclusions

- We have first generalized the model of self-similar PDF through an extrapolated continuous form which conforms to the original model at small x and the desired boundary condition at large x.

- We have then extended the self-similar formalism of PDF to dPDF with small x_1, x_2.

- We find that the constructed dPDF does not factorise into two single PDFs contrary to common expectation.

- We investigate if a smooth continuous form of dPDF can be suggested which has both the expected self-similar behaviour at small x_1 and x_2 while vanishes at the kinematic boundary $x_1 + x_2 = 1$. We achieve it only by introducing an additional factor $D_{10} \log\left(\frac{1}{x_1 + x_2} - 1\right)$ in the defining TMD dPDF.

Thanks