Theoretical uncertainties in diffractive parton densities

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In collaboration with A.D. Martin and M.G. Ryskin
Diffractive Deep-Inelastic Scattering (DDIS) is characterised by a Large Rapidity Gap (LRG) due to ‘Pomeron’ (vacuum quantum number) exchange.

How do we extract Diffractive Parton Density Functions (DPDFs) from DDIS data?

How ‘wrong’ are the H1 2006 DPDFs due to the oversimplified theory used in their fits?

For the impatient, here’s the answer (right), where:

- “Regge fit” $\sim$ H1 2006 Fit A.
- “pQCD fit” is the subject of this talk.
Diffractive DIS kinematics

\[ \begin{align*}
q^2 & \equiv -Q^2 \\
W^2 & \equiv (q + p)^2 = -Q^2 + 2p \cdot q \\
\Rightarrow \quad x_B & \equiv \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2 + W^2} \text{ (fraction of proton’s momentum carried by struck quark)} \\
t & \equiv (p - p')^2 \approx 0, (p - p') \approx x_P p
\end{align*} \]

\[ \begin{align*}
M_X^2 & \equiv (q + p - p')^2 = -Q^2 + x_P (Q^2 + W^2) \\
\Rightarrow \quad x_P & = \frac{Q^2 + M_X^2}{Q^2 + W^2} \text{ (fraction of proton’s momentum carried by Pomeron)} \\
\beta & \equiv \frac{x_B}{x_P} = \frac{Q^2}{Q^2 + M_X^2} \text{ (fraction of Pomeron’s momentum carried by struck quark)}
\end{align*} \]
Diffractive reduced cross section $\sigma_r^{D(3)}$

- Diffractive cross section (integrated over $t$):

$$\frac{d^3\sigma^D}{dx_P d\beta dQ^2} = \frac{2\pi\alpha_{em}^2}{\beta Q^4} \left[ 1 + (1 - y)^2 \right] \sigma_r^{D(3)}(x_P, \beta, Q^2),$$

where $y = Q^2/(x_B s)$, $s = 4E_eE_p$, and

$$\sigma_r^{D(3)} = F_2^{D(3)} - \frac{y^2}{1 + (1 - y)^2} F_L^{D(3)} \approx F_2^{D(3)}(x_P, \beta, Q^2),$$

for small $y$ or assuming that $F_L^{D(3)} \ll F_2^{D(3)}$

- Measurements of $\sigma_r^{D(3)} \Rightarrow$ diffractive parton density functions (DPDFs)

$$a^D(x_P, z, Q^2) = zq^D(x_P, z, Q^2) \text{ or } zg^D(x_P, z, Q^2),$$

where $\beta \leq z \leq 1$, cf. $x_B \leq x \leq 1$ in DIS.
Leading-twist collinear factorisation in DDIS

\[ F_2^{D(3)} = \sum_{a=q, g} C_{2,a} \otimes a^D + O(1/Q), \]  \tag{1}

where \( C_{2,a} \) are the same coefficient functions as in inclusive DIS and where the DPDFs \( a^D = zq^D \) or \( zg^D \) satisfy DGLAP evolution in \( Q^2 \):

\[ \frac{\partial a^D}{\partial \ln Q^2} = \sum_{a'=q, g} P_{aa'} \otimes a'^D \]  \tag{2}

“The factorisation theorem applies when \( Q \) is made large while \( x_B, x_{IP}, \) and \( t \) are held fixed.” [Collins,'98]

- **Says nothing** about the mechanism for diffraction: information about the diffractive exchange (‘Pomeron’) needs to be parameterised at an input scale \( Q_0 \) and fit to data. Will show later that assuming a ‘QCD Pomeron’ we need to modify both (1) and (2).

- Factorisation should also hold for final states (jets etc.) in DDIS, but is **broken in hadron–hadron collisions**, although hope that same formalism can be applied with extra suppression factor calculable from eikonal models.

- **LO diffractive dijet photoproduction:** resolved photon contribution should be suppressed, but direct photon contribution unsuppressed. Complications at NLO [Klasen–Kramer,'05].
H1 2006 extraction of DPDFs

- Assume Regge factorisation [Ingelman–Schlein,'85]:

\[ a^D(x_P, z, Q^2) = f_P(x_P) a^P(z, Q^2) \]  \hspace{1cm} (3)

- Pomeron flux factor from Regge phenomenology:

\[ f_P(x_P) = \int_{t_{\text{cut}}}^{t_{\text{min}}} dt \ e^{B_P t} x_P^{1-2\alpha_P(t)} \]

\[ (\alpha_P(t) = \alpha_P(0) + \alpha'_P t) \]

“Regge factorisation relates the power of \( x_P \) measured in DDIS to the power of \( s \) measured in hadron–hadron elastic scattering.” [Collins,'98]

- Pomeron PDFs \( a^P(z, Q^2) = z\Sigma^P(z, Q^2) \) or \( zg^P(z, Q^2) \) are DGLAP-evolved from arbitrary inputs at some scale \( Q^2_0 \), with the input parameters fitted to data.

- Fit to H1 FPS data gives \( \alpha_P(t) = 1.11 + 0.06 t \). Fit to H1 LRG data gives \( \alpha_P(0) = 1.12 \) if \( \alpha'_P = 0.06 \), or \( \alpha_P(0) = 1.15 \) if \( \alpha'_P = 0.25 \). So the Pomeron in DDIS is not the universal 'soft Pomeron' [Donnachie–Landshoff,'92] with \( \alpha_P(t) = 1.08 + 0.25 t \). By Collins’ definition, Regge factorisation is broken. H1/ZEUS assume that the \( x_P \) dependence factorises as eq.(3) regardless, with the fitted \( \alpha_P(0) \) independent of \( \beta \) and \( Q^2 \) (also broken, see later).

- Breaking of Regge factorisation with \( \alpha_P(0) > 1.08 \) suggests a significant perturbative QCD (pQCD) contribution to diffractive DIS. In pQCD, Pomeron exchange can be described by two-gluon exchange.
How to reconcile two-gluon exchange with DPDFs?

Two-gluon exchange calculations are the basis for the colour dipole model description of DDIS.

Right: $x_F F_2^{D(3)}$ for $x_F = 0.0042$ as a function of $\beta$

[Golec-Biernat–Wüsthoff,'99].

- dotted lines: $\gamma_T^* \rightarrow q\bar{q}g$,
- dashed lines: $\gamma_T^* \rightarrow q\bar{q}$,
- dot-dashed lines: $\gamma_L^* \rightarrow q\bar{q}$,

important at low, medium, and high $\beta$ respectively.

$\gamma_T^* \rightarrow q\bar{q}g$ and $\gamma_T^* \rightarrow q\bar{q}$ are partly higher-twist, $\gamma_L^* \rightarrow q\bar{q}$ is purely higher-twist, but H1/ZEUS DPDFs only include leading-twist contributions.
## Comparison of two approaches

<table>
<thead>
<tr>
<th>‘Regge factorisation’ approach</th>
<th>Two-gluon exch. (e.g. dipole model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• $\mathcal{P}$ is purely non-perturbative, i.e. a Regge pole.</td>
<td>• $\mathcal{P}$ is purely perturbative, i.e. a gluon ladder.</td>
</tr>
<tr>
<td>• $Q^2$ dependence given by DGLAP.</td>
<td>• $Q^2$ dependence predicted.</td>
</tr>
<tr>
<td>• Need to fit $\beta$ dependence.</td>
<td>• $\beta$ dependence predicted.</td>
</tr>
<tr>
<td>• $x_\mathcal{P}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.</td>
<td>• $x_\mathcal{P}$ dependence given by square of skewed gluon distribution (or dipole cross section).</td>
</tr>
<tr>
<td>• Only leading-twist.</td>
<td>• Goes beyond leading-twist.</td>
</tr>
<tr>
<td>• Full DGLAP evolution in Pomeron structure function.</td>
<td>• Only $q\bar{q}$ and $qg$ final states as products of photon dissociation.</td>
</tr>
<tr>
<td>• Extract universal DPDFs.</td>
<td>• No concept of DPDFs.</td>
</tr>
<tr>
<td>• $x_\mathcal{P}$ dependence factorises.</td>
<td>• $x_\mathcal{P}$ dependence doesn’t factorise.</td>
</tr>
<tr>
<td>• Only applies to inclusive DDIS.</td>
<td>• Also explains exclusive processes.</td>
</tr>
</tbody>
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- In reality, **both** non-perturbative and perturbative Pomeron contributions to inclusive DDIS. Want to combine advantages of both approaches while eliminating the limitations. Improve two-gluon exchange calculations by introducing DGLAP evolution in ‘Pomeron structure function’ allowing universal DPDFs to be extracted.
Combination of two approaches

- Inclusive DDIS consists of **both** non-perturbative and perturbative Pomeron contributions.

<table>
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<th>Non-perturbative ( P ) contribution</th>
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The QCD Pomeron is a parton ladder

- Generalise $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ to arbitrary number of parton emissions [Ryskin,'90; Levin–Wüsthoff,'94].
- Work in Leading Logarithmic Approximation (LLA) ⇒ virtualities of $t$-channel partons are strongly ordered: $\mu_0^2 \ll \ldots \ll \mu^2 \ll \ldots \ll Q^2$, i.e. QCD Pomeron is a DGLAP ladder rather than a BFKL ladder.

**New feature:** integral over scale $\mu^2$ (starting scale for DGLAP evolution of Pomeron PDFs).

\[
F_{2}^{D(3)} = \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \ f_{\mathbb{P}}(x_{\mathbb{P}}; \mu^2) \ F_{2}^{\mathbb{P}}(\beta, Q^2; \mu^2)
\]

\[
f_{\mathbb{P}}(x_{\mathbb{P}}; \mu^2) = \frac{1}{x_{\mathbb{P}}B_D} \ \left[ R_g \ \frac{\alpha_s(\mu^2)}{\mu} \ x_{\mathbb{P}}g(x_{\mathbb{P}}, \mu^2) \right]^2
\]

\[
F_{2}^{\mathbb{P}}(\beta, Q^2; \mu^2) = \sum_{a=q,g} C_{2,a} \otimes a_{\mathbb{P}}
\]

$B_D$ from $t$-integration, $R_g$ from skewedness [Shuvaev et al.,'99]

- Pomeron PDFs $a_{\mathbb{P}}(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale $\mu^2$ up to $Q^2$.
- For $\mu^2 < \mu_0^2 \sim 1 \text{ GeV}^2$, replace lower parton ladder with usual Regge pole contribution. Take $\alpha_{\mathbb{P}}(0) \sim 1.08$ (or fit) and fit Pomeron PDFs DGLAP-evolved from an input scale $\mu_0^2$. 

$\gamma^*$

\[Q^2\]

$\mu_0^2, z$

$x_{\mathbb{P}}$

\[LRG\]

$p$

$\otimes$

$C_{2,a}$

$\sum_{a=q,g}$

$\alpha_{\mathbb{P}}(0)$

$1.08$

$t$-integration

$R_g$

skewedness

[Shuvaev et al.,'99]
Gluonic and sea-quark Pomeron

At low scales, sea-quark density of the proton dominates over gluon density at small $x \Rightarrow$ need to account for sea-quark density in perturbative Pomeron flux factor.

Pomeron structure function $F_2^\gamma(x, Q^2; \mu^2)$ calculated from quark singlet $\Sigma^\gamma(z, Q^2; \mu^2)$ and gluon $g^\gamma(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale $\mu^2$ up to $Q^2$.

Input Pomeron PDFs $\Sigma^\gamma(z, \mu^2; \mu^2)$ and $g^\gamma(z, \mu^2; \mu^2)$ to DGLAP evolution are Pomeron-to-parton splitting functions.
LO Pomeron-to-parton splitting functions


- **Notation:** ‘\( P = G \)’ means gluonic Pomeron, ‘\( P = S \)’ means sea-quark Pomeron, ‘\( P = GS \)’ means interference between these.

\[
\begin{align*}
  z\Sigma^{P=G}(z, \mu^2; \mu^2) &= P_{q,P=G}(z) = z^3 (1 - z), \\
  zg^{P=G}(z, \mu^2; \mu^2) &= P_{g,P=G}(z) = \frac{9}{16} (1 + z)^2 (1 - z)^2, \\
  z\Sigma^{P=S}(z, \mu^2; \mu^2) &= P_{q,P=S}(z) = \frac{4}{81} z (1 - z), \\
  zg^{P=S}(z, \mu^2; \mu^2) &= P_{g,P=S}(z) = \frac{1}{9} (1 - z)^2, \\
  z\Sigma^{P=GS}(z, \mu^2; \mu^2) &= P_{q,P=GS}(z) = \frac{2}{9} z^2 (1 - z), \\
  zg^{P=GS}(z, \mu^2; \mu^2) &= P_{g,P=GS}(z) = \frac{1}{4} (1 + 2z) (1 - z)^2
\end{align*}
\]

Evolve these input Pomeron PDFs from \( \mu^2 \) up to \( Q^2 \) using NLO DGLAP evolution.
Contribution to $F_2^{D(3)}$ as a function of $\mu^2$

$$F_2^{D(3)} = \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} f_P(x_P; \mu^2) F_2^p(\beta, Q^2; \mu^2)$$

$$f_P(x_P; \mu^2) = \frac{1}{x_P B_D} \left[ R_g \frac{\alpha_S(\mu^2)}{\mu} x_P g(x_P, \mu^2) \right]^2$$

- Naïvely, $f_P(x_P; \mu^2) \sim 1/\mu^2$, so contributions from large $\mu^2$ are strongly suppressed.
- But $x_P g(x_P, \mu^2) \sim (\mu^2)^\gamma$, where $\gamma$ is the anomalous dimension. In BFKL limit $\gamma \approx 0.5$, so $f_P(x_P; \mu^2) \sim$ constant.
- HERA domain is in an intermediate region: $\gamma$ is not small, but is less than 0.5.
- Upper plot: $\mu^2 x_P f_P(x_P; \mu^2)$ is not flat for small $x_P$. Lower plot: integrand as a function of $\mu^2$ (using MRST2004F3 NLO PDFs) ⇒ large contribution from large $\mu^2$.
- Recall that fits using ‘Regge factorisation’ include contributions from $\mu^2 \leq Q_0^2$ in the input distributions, but neglect all contributions from $\mu^2 > Q_0^2$. 

![Graph showing the contribution to $F_2^{D(3)}$ as a function of $\mu^2$.](image-url)
Inhomogeneous evolution of DPDFs

\[
F_2^{D(3)} = \sum_{a=q,g} C_{2,a} \otimes a^D,
\]

where \(a^D(x_P, z, Q^2) = \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \, f_P(x_P; \mu^2) \, a^P(z, Q^2; \mu^2)\)

\[
\frac{\partial a^D}{\partial \ln Q^2} = \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \, f_P(x_P; \mu^2) \frac{\partial a^P}{\partial \ln Q^2} + f_P(x_P; \mu^2) \, a^P(z, Q^2; \mu^2) \bigg|_{\mu^2 = Q^2}
\]

\[
= \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \, f_P(x_P; \mu^2) \sum_{a' = q,g} P_{aa'} \otimes a'^D + f_P(x_P; Q^2) \, a^P(z, Q^2; Q^2)
\]

\[
= \sum_{a' = q,g} P_{aa'} \otimes a'^D + \underbrace{f_P(x_P; Q^2) \, P_{aP}(z)}_{\text{DGLAP term}} + \underbrace{f_P(x_P; Q^2) \, a^P(z, Q^2; Q^2)}_{\text{Extra inhomogeneous term}}
\]

Inhomogeneous evolution of DPDFs is not a new idea:

"We introduce a diffractive dissociation structure function and show that it obeys the DGLAP evolution equation, but, with an additional inhomogeneous term." [Levin–Wüsthoff,'94]
Pomeron structure is analogous to photon structure

### Photon structure function

\[
F_2^\gamma(x_B, Q^2) = \sum_{a=q,g} C_{2,a} \otimes a^\gamma + C_{2,\gamma}
\]

where

\[
\frac{\partial a^\gamma(x, Q^2)}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^\gamma + P_{a\gamma}(x)
\]

### Diffractive structure function

\[
F_2^{D(3)}(x_P, \beta, Q^2) = \sum_{a=q,g} C_{2,a} \otimes a^D + C_{2,P}
\]

where

\[
\frac{\partial a^D(x_P, z, Q^2)}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^D + P_{a\gamma}(z) f_p(x_P; Q^2)
\]
Dijets in diffractive photoproduction

<table>
<thead>
<tr>
<th>Resolved photon ($x_\gamma &lt; 1$)</th>
<th>Direct photon ($x_\gamma = 1$)</th>
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</thead>
<tbody>
<tr>
<td>Resolved Pomeron ($z_P &lt; 1$)</td>
<td></td>
</tr>
<tr>
<td>Direct Pomeron ($z_P = 1$)</td>
<td></td>
</tr>
</tbody>
</table>

- Direct Pomeron contributions ($z_P = 1$) are neglected in ‘Regge factorisation’ analyses.
Need for NLO calculations

- NLO analysis of DDIS data is not yet possible.
- Need $C_{2,P}$ and $P_{aP}$ at NLO. Should be calculable with usual methods, e.g. LO diagrams are:

\[
\begin{align*}
\gamma^* & \quad q \quad q - k \\
& \quad k \\
& \quad l_\perp \\
\end{align*}
\]

\[
\begin{align*}
\gamma^* & \quad q \quad q - k \\
& \quad q - k \\
& \quad k + l_\perp \\
& \quad l_\perp \\
\end{align*}
\]

\[
\begin{align*}
\gamma^* & \quad q \quad q - k \\
& \quad q - k \\
& \quad k - l_\perp \\
& \quad l_\perp \\
\end{align*}
\]

**Dimensional regularisation:** work in $4 - 2\varepsilon$ dimensions, collinear singularity appears as $1/\varepsilon$ pole multiplied by $P_{qP}$, subtract in e.g. $\overline{MS}$ factorisation scheme to leave finite remainder $C_{2,P}$.

- Here, take NLO $C_{2,a}$ and $P_{aa'}$ ($a, a' = q, g$), but LO $C_{2,P}$ and $P_{aP}$.
  - Work in Fixed Flavour Number Scheme (no charm DPDF), with charm production via NLO $\gamma^* g^P \rightarrow c\bar{c}$ [Riemersma et al.,'95] and LO $\gamma^* P \rightarrow c\bar{c}$ [Levin–Martin–Ryskin–Teubner,’97].
  - For light quarks, include LO $\gamma_L^* P \rightarrow q\bar{q}$ (higher-twist), with LO $\gamma_T^* P \rightarrow q\bar{q}$ contribution given by $C_{T,P} = F_{T,q\bar{q}}^{D(3)} - F_{T,q\bar{q}}^{D(3)}\mu^2 \ll Q^2$. This subtraction defines a choice of factorisation scheme.
Analysis of H1 LRG data (hep-ex/0606004)

- Take input quark singlet and gluon densities at $Q_0^2 = 2$ GeV$^2$ in the form:
  
  $$ z \Sigma^D(x_P, z, Q_0^2) = f_P(x_P) A_q z^{Bq}(1 - z)^{C_q}, $$
  
  $$ zg^D(x_P, z, Q_0^2) = f_P(x_P) A_g z^{Bg}(1 - z)^{C_g}. $$

- Take $f_P(x_P)$ as in the H1 2006 fit with $\alpha_P(0)$, $A_a$, $B_a$, and $C_a$ ($a = q, g$) as free parameters.

- Treatment of secondary Reggeon as in H1 2006 fit, i.e. using pion PDFs, but using GRV NLO instead of Owens LO. (N.B. No good reason that the $R$ PDFs should be same as pion PDFs.)

- Fit H1 LRG data binned at fixed $x_P$ values with cut $M_X \geq 2$ GeV. Will study effect of cut $Q^2 \geq Q_{\text{min}}^2$ on fitted data.

- Statistical and systematic experimental errors added in quadrature. (Caveat: underestimates numerical values of $\chi^2$, but central DPDFs obtained should be very close to those obtained treating correlated systematic errors separately.)

- Two types of fits:
  - "Regge" = 'Regge factorisation' approach (i.e. no $C_{2,P}$ or $P_{aP}$) $\simeq$ H1 2006 Fit A.
  - "pQCD" = 'perturbative QCD' approach with LO $C_{2,P}$ and $P_{aP}$.

- Use MRST2004F3 NLO PDFs with $\Lambda_{\text{QCD}}^{(n_f=3)} = 407$ MeV.
Stability with respect to $Q_{\text{min}}^2$ variation


<table>
<thead>
<tr>
<th>$Q_{\text{min}}^2$ (GeV$^2$)</th>
<th>Number of data points</th>
<th>$\chi^2(Q^2 \geq 3.5 \text{ GeV}^2)$</th>
<th>$\chi^2(Q^2 \geq 5 \text{ GeV}^2)$</th>
<th>$\chi^2(Q^2 \geq 6.5 \text{ GeV}^2)$</th>
<th>$\chi^2(Q^2 \geq 8.5 \text{ GeV}^2)$</th>
<th>$\chi^2(Q^2 \geq 12 \text{ GeV}^2)$</th>
<th>$\chi^2(Q^2 \geq 15 \text{ GeV}^2)$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>3.5</td>
<td>5.0</td>
<td>6.5</td>
<td>8.5</td>
<td>12</td>
<td>15</td>
</tr>
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<td>15</td>
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<tr>
<td>$\chi^2(Q^2 \geq 3.5 \text{ GeV}^2)$</td>
<td>272</td>
<td>264</td>
<td></td>
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<tr>
<td>$\chi^2(Q^2 \geq 5 \text{ GeV}^2)$</td>
<td>233 222</td>
<td>227 223</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2(Q^2 \geq 6.5 \text{ GeV}^2)$</td>
<td>208 186 174</td>
<td>208 201 186</td>
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<td></td>
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</tr>
<tr>
<td>$\chi^2(Q^2 \geq 8.5 \text{ GeV}^2)$</td>
<td>178 155 144 142</td>
<td>182 172 153 150</td>
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<tr>
<td>$\chi^2(Q^2 \geq 12 \text{ GeV}^2)$</td>
<td>156 136 124 123 122</td>
<td>162 153 135 132 131</td>
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</tr>
<tr>
<td>$\chi^2(Q^2 \geq 15 \text{ GeV}^2)$</td>
<td>133 111 100 98 97 96</td>
<td>138 128 109 104 102 101</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stability measure $\Delta_i^{i+1}$</td>
<td>0.41 0.48 0.08 0.04 0.04</td>
<td>0.15 0.60 0.13 0.04 0.04</td>
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Both Regge and pQCD fits stable for $Q_{\text{min}}^2 \gtrsim 6.5 \text{ GeV}^2$. To compare directly with H1 2006 fits, take $Q_{\text{min}}^2 = 8.5 \text{ GeV}^2$ for default fits (conservative choice).
DPDF and $\alpha_P(0)$ dependence on $Q^2_{\text{min}}$

$Q^2 = 8.5 \text{ GeV}^2$

- Regge fit, $Q^2_{\text{min}} = 3.5 \text{ GeV}^2$
- Regge fit, $Q^2_{\text{min}} = 5.0 \text{ GeV}^2$
- Regge fit, $Q^2_{\text{min}} = 6.5 \text{ GeV}^2$
- Regge fit, $Q^2_{\text{min}} = 8.5 \text{ GeV}^2$

Again, stability for $Q^2_{\text{min}} \gtrsim 6.5 \text{ GeV}^2$.

Required $\alpha_P(0)$ at $Q^2_0 = 2 \text{ GeV}^2$ for pQCD fits is lower than for Regge fits.
dependence of H1 LRG data

- Fit $\sigma_r^{D(3)} \propto f_P(x_P)$ in each $Q^2$ bin containing four or more data points with $x_P \leq 0.01$, $y \leq 0.45$ and $M_X \geq 2$ GeV.
- For $\beta = 0.40$ and $\beta = 0.65$, clear rise with $Q^2$ of effective $\alpha_P(0) \Rightarrow x_P$-factorisation broken.
- Inhomogeneous term depends on $x_P$ and therefore $x_P$-factorisation is broken when evolving upwards from $Q^2_0$ to $Q^2$ (but seems small effect).
Evolution of diffractive gluon distribution

- Extra inhomogeneous term in evolution equation means gluon from pQCD fit needs to be smaller at input scale.
Quark singlet distribution at input scale is larger at low $z$ in pQCD fit and smaller at large $z$. 
dependence of $F_{2}^{D(3)}$

$x_{IP} = 0.003, Q^2 = 8.5 \text{ GeV}^2$

- Total
- Resolved Pomeron contrib.
- Direct Pomeron contrib.
- Secondary Reggeon contrib.

Direct Pomeron contribution only important for $\beta \gtrsim 0.9$. 
\( \beta \) dependence of \( \partial F_2^{D(3)} / \partial \ln Q^2 \)

At LO, \( \frac{\partial F_2^{D(3)}}{\partial \ln Q^2} = \sum_q e_q^2 \left( \sum_{a'=q,g} P_{qa'} \otimes a'^D + P_{a^P} f_{a^P} \right) + (\text{Direct } P) + R. \)

- Peak due to threshold for \( \gamma^* P \rightarrow c\bar{c} \) at \( \beta = Q^2/(Q^2 + 4m_c^2) \).
- Additional contributions to scaling violations apart from DGLAP contribution, important for \( \beta \gtrsim 0.3 \).
Predictions for diffractive charm production

- **Direct Pomeron contribution**, i.e. $\gamma^* P \rightarrow c\bar{c}$ ($z_P = 1$), is significant at moderate/high $\beta$.
- These charm data points are included in the ZEUS LPS fit [ZEUS: Eur. Phys. J. C 38 (2004) 43], but only the $\gamma^* g_P \rightarrow c\bar{c}$ contribution was included and not the $\gamma^* P \rightarrow c\bar{c}$ contribution. Therefore, diffractive gluon from ZEUS LPS fit needed to be artificially large to fit the charm data.
- H1 also neglect the $\gamma^* P \rightarrow c\bar{c}$ contribution (see talk by R. Wolf).
“From the theoretical point of view one also should take into account that the presently available Monte Carlo models are assuming an illegitimate Regge factorisation, in which hard scale dependencies on $x_P$ and $\beta$ as found in theoretical QCD analyses, and which characterise the final state, are neglected. For instance, one treats the charm production as entirely due to the familiar photon–gluon fusion, neglecting the direct charm–anticharm excitation which some theorists claim to be substantial. In this approximation, in order to reproduce the diffractive charm signal one needs a hard glue in the Pomeron fits. Therefore the conclusions drawn from these Monte Carlo studies as to the physical picture underlying the diffractive final states should be handled with care.”

No progress in theory used by H1/ZEUS in 8 years?
Direct Pomeron contribution to dijet production

- Direct Pomeron contribution \( (z_P = 1) \) [EPJC 44 (2005) 69] calculated with ZEUS (prel.) kinematic cuts (see talk by A. Bonato): 31% of data in largest \( \beta \) bin.

- Alternative calculations for exclusive dijets by Braun and Ivanov [PRD 72 (2005) 034016].

- H1 combined fit is to dijet data with \( z_P < 0.9 \) integrated over \( \beta \). Therefore, can neglect direct Pomeron contribution and include only the resolved Pomeron contribution using NLOJET++.

- Aside: inconsistency in heavy quark treatment. H1 2006 fit is done in FFNS with massive heavy quark contributions, but jet coefficient functions used in programs like NLOJET++ and DISENT\(^a\) are computed for massless partons.

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\(^a\)Note that DISENT is known to have a small bug at the 1–2% level [Z. Trócsányi, hep-ph/0512004].
DPDFs with $Q^2_{\text{min}} = 8.5 \text{ GeV}^2$ compared to H1 DPDFs

**Q^2 = 25 \text{ GeV}^2**

- **Quark singlet**
  - Regge fit
  - pQCD fit
  - H1 2002 (prel.)

- **Gluon**
  - $\chi^2_{z^g}(x_p = 0.003, z, Q^2)$

**Q^2 = 90 \text{ GeV}^2**

- **Quark singlet**
  - Regge fit
  - pQCD fit
  - H1 2002 (prel.)

- **Gluon**
  - $\chi^2_{z^g}(x_p = 0.003, z, Q^2)$

- Regge fit $\approx$ H1 2006 Fit A. pQCD fit closer to H1 combined fit than H1 2006 Fit A **without** including jet data $\Rightarrow$ will describe dijet data better than H1 2006 Fit A.

- H1 combined fit determines gluon **directly** from dijet data, whereas fits only to inclusive DDIS data determine gluon only **indirectly** so more sensitive to details of evolution, i.e. better test of theory used. Including dijet data in the fit is **not** necessarily a good thing if the theory is unreliable.

- H1 $\chi^2$ for 190 inclusive DDIS points is 158 (H1 Fit A), 164 (H1 Fit B), 169 (H1 combined fit), so some tension between inclusive DDIS and jet data which is alleviated by inclusion of inhomogeneous term in evolution equation.
Further corrections to DPDF evolution

- NNLO parton-to-parton splitting functions (known).
- NLO Pomeron-to-parton splitting functions (unknown).
- Absorptive corrections. Schematically,

\[ \frac{\partial g^D}{\partial \ln Q^2} = P_{gg} \otimes g^D + P_{gP} \otimes g^2 - 4P_{gP} \otimes gg^D + \ldots \]

Possible that further corrections will stabilise the results of the fit with respect to the $Q^2_{\text{min}}$ cut.
Conclusions

- Collinear factorisation holds, but we need to account for the direct Pomeron coupling:

\[ F_2^{D(3)} = \sum_{a=q,g} C_{2,a} \otimes a^D + C_{2,P} \]

\[ \frac{\partial a^D}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^D + P_{aP}(z) f_P(x_P; Q^2) \]

Direct coupling and inhomogeneous evolution analogous to the photon case. Direct Pomeron contribution should also be included when calculating jet or heavy quark production.

- New analyses from H1 are a dramatic improvement on previous attempts, but still do not include the direct Pomeron contributions.

- Evidence of instability in the fits for \( Q_{\text{min}}^2 \lesssim 6.5 \text{ GeV}^2 \): further theoretical corrections such as NLO \( P_{aP} \) or absorptive corrections may help.

- Claims about factorisation breaking based on previous diffractive PDFs will need to be re-examined. Need to have good understanding of \( \gamma^* p \) (HERA) before extending, in turn, to \( \gamma p \) (HERA), \( p\bar{p} \) (Tevatron) and \( pp \) (LHC). Recent H1 and ZEUS data are a large step towards this goal.
Appendix: Non-linear evolution of inclusive PDFs

\[
\frac{\partial a(x, Q^2)}{\partial \ln Q^2} = \sum_{a' = q, g} P_{aa'} \otimes a' - \int x^1 dx P_{aP}(x/x_P) f_P(x_P; Q^2).
\]

- Interesting application of DDIS formalism to calculate shadowing corrections to inclusive DIS via Abramovsky–Gribov–Kancheli (AGK) cutting rules.
- Inhomogeneous evolution of DPDFs ⇒ non-linear evolution of inclusive PDFs.
- Fit HERA \( F_2 \) data similar to MRST2001 NLO fit. Small-\( x \) gluon enhanced at low scales.