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Partition Evolution, Matching at LO and NLO level

SHOWER ALGORITHM

A NEW PARTON
Very popular
the performance usually is very good.
Very crude approximation but with the tuning

Important tool for detector simulation

uncontrolled.
unphysical parameters are rather

Not a predictive model. The scale and

GCD inspired model

multiplicity event in particle collisions

The parton shower is a tool to model high

INTRODUCTION
Introduction

- Adding higher order correction
- Matching to NLO calculation
- Matching to Born matrix elements
- Covariance, ...
- Kinematics, soft gluon, Lorentz invariance
- We have to rethink and improve the shower
- We need to define a nice formalism
- The shower is strictly pQCD object
\[ \int \mathcal{Z} = 1 \]

Completeness relation:

\[ \prod_{m=1}^{\infty} \left( \frac{Q_{\alpha \beta \gamma \delta}}{p_{\alpha \beta \gamma \delta}} \right) \]

\[ \times \mathcal{G} \left( \frac{u - v}{u} \right)^{\varphi_{\alpha \beta \gamma \delta}} \]

Normalization:

\[ \left( \frac{u - v}{u} \right)^{\varphi_{\alpha \beta \gamma \delta}} \mathcal{G} \left( \frac{u - v}{u} \right)^{\varphi_{\alpha \beta \gamma \delta}} = \left( \frac{u - v}{u} \right)^{\varphi_{\alpha \beta \gamma \delta}} \mathcal{G} \left( \frac{u - v}{u} \right)^{\varphi_{\alpha \beta \gamma \delta}} \]

Basis vector in the configuration space:

An m-parton configuration is

Configuration Space
\[
\left\{ \tau_{e', e''} \right\} \left\{ \tau_{e', e''} \right\} \prod_{i=1}^{m} \int_{a}^{b} \int_{c}^{d} \int_{e}^{f} \equiv \left[ m_{a, b, c, d} \right] \left[ m_{a, b, c, d} \right] \int
\]

Complete relation:

Vector in the configuration space:

\[
\{ m_{a, b, c, d} \} \equiv \left[ m_{a, b, c, d} \right]
\]

An m-parton configuration is

Configuration Space
\[ |\{\{f, d\}, \{f, d\}_m\}| \int \mathcal{Z} = 1 \]

Completeness relation:

\[ \cdot |\{\{f, d\}, \{f, d\}_m\}| \int \mathcal{Z} = |I| \]

The unit vector is

\[ \cdot |\{\{f, d\}, \{f, d\}_m\}| \int \mathcal{Z} = |\mathcal{F}| \]

A general state (e.g., jet function) is

An m-parton configuration is

**Configuration Space**
Cross section in the configuration space

To define the phase space integral we have an operator

Phase Space Integral
We use an evolution variable e.g.:

\[ \log \frac{Q^2}{\hat{p}_1 \cdot \hat{p}_2} = t \in [0, \infty] \]
We use an evolution variable $e^t$:

$[0, \infty] \ni t \mapsto p_{t} \in \mathcal{A}$

By a linear operator $A(t)$, the evolution is given as:

$A(t) = \Omega(t) \Lambda(t) A(t)$

PARTON SHOWER EVOLUTION
Group decomposition

by a linear operator

The evolution is given

We use an evolution variable e.g.:
\[ 1 = (0,t)\mathcal{A}|(0,t)\mathcal{A} 1 \]

\[ (t, \mathcal{T}_{3}, t_{1}) \bigcap (t, \mathcal{T}_{2}, t_{1}) \bigcap (t, \mathcal{T}_{1}, t_{1}) \bigcap (t_{3}, t_{1}) \bigcap (t, \mathcal{T}_{1}) \]

\[ ((0,t)\mathcal{A}|(0,t)\mathcal{A} 1 = (0,t)\mathcal{A}|(0,t)\mathcal{A} 1 \]

Normalization preserves the group decomposition by a linear operator. The evolution is given as:

\[ \log \frac{P_{1} - \mu_{2}}{\sigma} \]
\[ I = (0,t) N (t | t) \cup (0,t) N (t | t) \]

normalization preserves the

\[ \int_{t_3}^{t_1} \left( \mathcal{H}(t_3, t_1) N(t_3, t_1) \cup \mathcal{H}(t_2, t_1) N(t_2, t_1) \right) + \]

no-splitting part

splitting part

[0, \infty] \subseteq t = \frac{1}{\ln \Theta} \int \frac{d\phi}{\theta^2}

We use an evolution variable e.g.:
\[
\begin{align*}
&\left( d, f, c \right) \triangledown \left( t_1, t_2, t_3 \right) \\
&\int_{t_1}^{t_2} d t_2 \left( H(t_2) \mathcal{H} (d) \right) (d, f, c) \triangledown \left( t_1, t_2, t_3 \right) \\
&\left( d, f, c \right) \triangledown = I
\end{align*}
\]

From the normalization

\[
\begin{align*}
&I = (m, p, f, c) \left| \left( t_1, t_2, t_3 \right) \right.
\end{align*}
\]

Sudakov Factor

\[
\begin{align*}
&\left( m, p, f, c \right) \triangledown = (m, p, f, c) \left| \left( t_1, t, t_2, t_3 \right) \right.
\end{align*}
\]

unchanged

The operator \( N(t) \) leaves the basis states

NO-SPILLING OPERATOR
\[
\left( \langle \{a', p', m \} | \langle \tau | (t) H | \tau \rangle \right) \int_{t_2}^{t_1} - \right) \exp \left( \int_{t_2}^{t_1} \right) \nabla = \exp \left( \int_{t_2}^{t_1} \right) \nabla
\]

From the normalization, \( I = (t, t) \cap (t, t) = 1 \)

Sudakov factor
\[
\langle \{a', p', m \} | \langle \tau | (t) | \tau \rangle \rangle \nabla = \langle \{a', p', m \} | (t, t) \rangle \nabla
\]

unchanged

The operator \( N(t, t) \) leaves the basis states unchanged.

No-Splitting Operator
we have only 2 splittings

Since we are interested only at LL and NLL level

\(\{\{a, b, m+n\}\} \leftarrow \{d, f, \{a, b, m\}\}\) transitions that the splitting operator describes all the possible

Splitting Operator
\[
\left( \mathcal{D}/(\Omega_{\mathcal{H}}, \mathcal{L}) \otimes \mathcal{L} \right) \otimes t \right) \frac{\nabla \phi}{\phi p} \int_{\mathcal{D}} \int_{\mathcal{L}} \int_{\mathcal{H}} \int_{\mathcal{L}} \sum_{l=1}^{m} \sum_{k=1}^{l} \sum_{j=1}^{m} \left( \sum_{i=1}^{l} \frac{\nabla \phi}{\phi p} \right) = \left( \mathcal{D}/(\Omega_{\mathcal{H}}, \mathcal{L}) \otimes \mathcal{L} \right) \otimes t \right) \frac{\nabla \phi}{\phi p} \int_{\mathcal{D}} \int_{\mathcal{L}} \int_{\mathcal{H}} \int_{\mathcal{L}} \sum_{l=1}^{m} \sum_{k=1}^{l} \sum_{j=1}^{m} \left( \sum_{i=1}^{l} \frac{\nabla \phi}{\phi p} \right)
\]
\[
\begin{align*}
\text{otherwise } & \begin{cases} 0 \text{ if } l \text{ and } k \text{ are color connected} \\ 1 \end{cases} = C_{l,k}
\end{align*}
\]

\[
\left( \left[ \sum_{m' \neq m} \left\{ c, \frac{f, d}{p} \right\} \right] \right)^{+1} \left\{ c, \frac{f, d}{p} \right\} \times
\]

\[
\left( \sum_{z} \frac{z}{\phi p} \int z p \int \frac{h}{\hat{p}} \int \frac{1}{z} \right) \left( \left[ \sum_{m' \neq m} \left\{ c, \frac{f, d}{p} \right\} \right] \right)^{+1} \left\{ c, \frac{f, d}{p} \right\}
\]

\[
= \left( \left[ \sum_{m' \neq m} \left\{ c, \frac{f, d}{p} \right\} \right] \right)^{+1} \left\{ c, \frac{f, d}{p} \right\}
\]

\text{Splitting Operator}
Sudakov parametrization of the new momenta:
The phase space is exact after the splitting:

\[
\left( \{ \alpha, \beta \} \right| \left( \{ f, g \} \right)_{(w)} \right)_{(1+w)} = \left( \{ c, f \} \right| \left( \{ f, g \} \right)_{(w)} \right)_{(1+w)}
\]
\[
\left[ (z + 1) - \frac{(\hat{h} - 1)z - 1}{2} \right] \frac{\partial}{\partial \mathcal{O}} = (g, h, z) \mathcal{S}_{\mathcal{I}}
\]

spectator, \( g \leftarrow h + g \)

\[E.g.: \text{Final state splitting with final state} \]

\[
\left( \frac{\text{I}[g, \mathcal{S}_{\mathcal{I}}]}{(n, h, z, \hat{p}, d)} \right) \left( \text{I} + 1 \right) \mathcal{Y} \left( g, \text{I} + 1 \right), \mathcal{Y}\{g, f, d\} \right) \times
\]

\[
\frac{\text{S}_{\mathcal{I}}}{(t - \hat{e}) \mathcal{S}_{\mathcal{I}}} \mathcal{O}_{\mathcal{I}} \times
\]

\[
\left( \frac{\text{I}[g, \mathcal{S}_{\mathcal{I}}]}{(n, h, z, \hat{p}, d)} \right) \left( \text{I} + 1 \right) \log + \text{I} \right) \frac{\mu_{\mathcal{S}}}{\phi_{\mathcal{S}}} \int z p \int \frac{h}{\hat{p}} \int \sum_{m=1}^{\mathcal{Y}} \sum_{n=1}^{\mathcal{Y}} \left( \mathcal{Y} \{g, f, d\} \right) (t) \mathcal{Y} \left( g, \text{I} + 1 \right), \mathcal{Y}\{g, f, d\} \right)
\]

Splitting Operator
Splitting Operator

\[
\left( \psi_{f,d}^\prime \right) \left| \left( \left( T, h, z \right)^T, \psi_1 \right) \right| + \sum_{i=1}^{m} \left( \left( T, h, z \right)^T, \psi_i \right) \psi_i \psi_{f,d}^\prime
\]

\[
\frac{1}{\sqrt{2}} \int \left( \sum_{i=1}^{m} \psi_i \psi_{f,d}^\prime \right) = \left( \psi_{f,d}^\prime \left| \left( T, h, z \right)^T \right. \right) \psi_{f,d}^\prime
\]
\[ (\mathcal{M}_{\mathcal{C}/1}\{\{z - 1\}\mathfrak{d}\mathfrak{d}, \mathfrak{d}\mathfrak{d}, \mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\mathfrak{d}\math
The shower cross section is

\[ (\mathcal{P}(t_1, t_2)) \cup \mathcal{P}(t_1) = [\mathcal{P}]^0 \]

Starting hard scale

Infrared cutoff scale

Hadronization

The shower cross section is

\[ pp \rightarrow 2 \text{ particles} \]

\text{e.g.:} \quad pp \rightarrow jets, the simplest configurations are

The evolution starts from the simplest configuration,
Summary: Shower Evolution

... Adding higher order contributions: 1 ← 3, 2 ← 4...

Color contributions.

Better soft gluon treatment: Including subleading...

Freedom to improve this algorithm:

Adding finite terms to the splitting kernels.

What about the freedom?

No external parameters.

Better soft gluon treatment.

Lorentz covariant and invariant.

Kinematics: Exact phase space in every steps.

Improvements describing the parton shower.

We defined a nice operator formalism for
ParTion ShowEr

Matrix Elements To

Matching Born Level

Outline:

- Definition of the scheme
- Connection to the shrinking method
- CKKW method

(CKKW method)
\begin{align*}
\langle \mathcal{H} | I^{(\mu t)} \downarrow H^{(1-\mu t)} \downarrow H \cdots (\mathcal{H}^{(\mu t)} \downarrow H | \mathcal{W} ) = \\
\mathcal{W} | I^{(\mathcal{H}^{(\mu t)} \downarrow H \cdots (1-\mu t) H^{(\mu t)} H | \mathcal{H}}
\end{align*}

For multiple emission:

\textit{For partitions } \mathcal{H}^{(\mu t)} \downarrow H \textit{ always decreases it.}

\textit{Since } \mathcal{H}^{(\mu t)} \downarrow H \textit{ always increases the number of}

\begin{align*}
\langle \mathcal{H} | I^{(\mu t)} H | \mathcal{W} ) = (\mathcal{W} | I^{(\mu t)} H | \mathcal{H}
\end{align*}

\textit{Let us define the operator } \mathcal{H}^{(\mu t)} \textit{ according to}

\textbf{Adjoint Splitting Operator}
Let us define the operator $\mathcal{H}^{\dagger}(t)$ according to

**Adjoint Splitting Operator**
Otherwise

\[
\begin{align*}
(\forall \{ c, f, d \} \in \mathcal{M} \cap (\mathcal{F}, \mathcal{H})) & \left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right) \\
\left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right) & \approx \left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right)
\end{align*}
\]

If \( w \) is known

\[
\begin{align*}
\left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right) & \approx \left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right)
\end{align*}
\]

then the following is a good approximation:

\[
\begin{align*}
\left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right) & \approx \left( \prod_{\exists \mathbf{m} \in \mathcal{F}} \prod_{\mathbf{m} \in \mathcal{F}} \mathbf{m} \right)
\end{align*}
\]
\[
\left( \begin{array}{c}
\{ a, q \}^2 \{ f, d \} \langle \{ f, d \} | \sum_{a} \sum_{q} \sum_{f} \sum_{d} p \rangle \int \sum_{a} \sum_{f} \sum_{d} \langle \{ a, q \}^2 \{ f, d \} | \sum_{a} \sum_{f} \sum_{d} p \rangle \\
\end{array} \right)
\]

Matrix element reweighting operator:

\[
\left( \begin{array}{c}
\{ a, q \}^2 \{ f, d \} \langle \{ f, d \} | \sum_{a} \sum_{q} \sum_{f} \sum_{d} p \rangle \int \sum_{a} \sum_{f} \sum_{d} \langle \{ a, q \}^2 \{ f, d \} | \sum_{a} \sum_{f} \sum_{d} p \rangle \\
\end{array} \right)
\]

\[
\left( \begin{array}{c}
\{ a, q \}^2 \{ f, d \} \langle \{ f, d \} | \sum_{a} \sum_{q} \sum_{f} \sum_{d} p \rangle \int \sum_{a} \sum_{f} \sum_{d} \langle \{ a, q \}^2 \{ f, d \} | \sum_{a} \sum_{f} \sum_{d} p \rangle \\
\end{array} \right)
\]

then the following is a good approximation:

APPROX. MATRIX ELEMENT
Standard shower

Adding and subtracting the same terms we have

Assuming we know $\mathcal{M}$

It is better to use the $Z$-partition matrix element in the

Expanding the first step of the shower cross section:

Matching at Born level
Standard shower

Adding and subtracting the same terms, we have

\[
(\mathcal{M}(t_3, t_2) N(t_3, t_2, t_1) \mathcal{H}) \left( \mathcal{M}(t_3, t_2, t_1) \mathcal{H} \right) + (\mathcal{M}(t_3, t_2, t_1) N(t_3, t_2, t_1) \mathcal{H}) = (\mathcal{M}(t_3) N) \left( (\mathcal{M}(t_3) N) \mathcal{H} \right)
\]

Expanding the first step of the shower cross section:

**MATHING AT BORN LEVEL**
Matching at Born level

Assuming we know $M_3$, $M_4$, \ldots, $M_n$, the matched shower cross section is

$$\sigma(t_\perp, t_t, t_\parallel) \propto \frac{1}{t_t} \left( \sum_{i=1}^{n} \int \frac{d^3p}{2\pi^2} \int \frac{d^3p'}{2\pi^2} \int \frac{d^3p''}{2\pi^2} \right)$$
\[
\forall + \left( \sum_{q \in \mathcal{Q}} \prod \{ \mathcal{H}(\tau, t, \mathcal{N}) \right) \times \\
\int_{t}^{\infty} \int_{t_1}^{t} \cdots \int_{t_{m-1}}^{t} \int_{t_i}^{t} \int_{t_{i-1}}^{t} \int_{t_{i-2}}^{t} \cdots \int_{t_2}^{t} \int_{t_1}^{t} \prod_{q \in \mathcal{Q}} \mathcal{W} \right) \\
\left( \sum_{q \in \mathcal{Q}} \prod \{ \mathcal{H}(\tau, t, \mathcal{N}) \right) \times \\
\int_{t}^{\infty} \int_{t_1}^{t} \cdots \int_{t_{m-1}}^{t} \int_{t_i}^{t} \int_{t_{i-1}}^{t} \int_{t_{i-2}}^{t} \cdots \int_{t_2}^{t} \int_{t_1}^{t} \prod_{q \in \mathcal{Q}} \mathcal{W} \right)
\]

After some algebraic manipulation:

MATCHING AT BORN LEVEL
The CKKW method uses a simplified Sudakov

rewriting operator based on the \( k^\perp \) jet algorithm.

\[
\left( \sum_{\text{all } \mathcal{W}} \text{CKKW}(\mathcal{W}) \right) \mathcal{W}_{\text{CKKW}}(\mathcal{W}) = \left( \sum_{\text{all } \mathcal{W}} \right) \mathcal{W}_{\text{CKKW}}(\mathcal{W})
\]

\[
\left( |\mathcal{O}| \right) \left( |\mathcal{O}| \right) \left( |\mathcal{O}| \right) \left( |\mathcal{O}| \right)
\]

Group decomposition property:
Defining the matching scale \( t^\perp \) and using the

(Catani-Krauss-Kuhn-Meller method)

**SLICING METHOD**
MATCHING PARTON SHOWER TO NLO COMPUTATION
Real - Dipoles

\[ (s^0 \mathcal{O} + \left[ V^0 p - A^0 p \right] )^{1+N} \int + \left( (1) M \frac{\nu z}{\sigma \chi} + \mathcal{E} + 1 \right) \int^N_{\nu} d \mathcal{O} = (\nabla \mathcal{O} | \mathcal{N} \mathcal{F}) \]

"Quasi virtual" Born term

\[ \left( \frac{2N}{1} \right) \frac{(s^0 \mathcal{O})}{(1) \mathcal{E} \frac{\nu z}{\sigma \chi} + (0) \mathcal{E} = \mathcal{E} \quad \text{approx. from} \quad \frac{\nu z}{N} \quad \text{from}\]

"Error term" in \( s \), then we have

\[ \left( 1+N \mathcal{O} \right) \left( (1+2 N \mathcal{F}) \nabla M (1+2 \mathcal{F} \mathcal{O}) N \mathcal{F} \right) \int^2_{\mathcal{F}} + \]

\[ \left( N \mathcal{O} \right) \left( 2 \mathcal{F} \right) \nabla M \mathcal{F} N \mathcal{F} \int^2_{\mathcal{F}} = (\nabla \mathcal{O} | \mathcal{N} \mathcal{F}) \]

Element improved cross section

Let us calculate the N-jet cross section. The matrix

PARTON SHOWERS AT NLO
The NLO parton shower for an N-jet cross section is

\[ (N^\omega | (\mathcal{N}^T, \mathcal{N}^F) \mathcal{M}(N^T, \mathcal{N}^F) \mathcal{N} | \mathcal{N}^H) \mathcal{N}^{\mathcal{F}} \int_{\mathcal{F}} = ((\mathcal{F}^{\mathcal{F}}) \mathcal{O} | \mathcal{N}^H) \]

\[ \mathcal{W} \otimes (\mathcal{X} + (\mathcal{X}^H, \mathcal{X}^F) \mathcal{D}) \sim \]

\[ \mathcal{W} \otimes \mathcal{I} + \text{loop}^{-1} \mathcal{W} \sim \]
Matched to the "NLO matrix elements"

Matched to the LO matrix elements

It is possible to add massive fermions in the same way.

Clear way to add higher order to the shower

Infrared cutoff parameter

No phase space cut parameters at all, only the

formalism, improved soft gluon

Exact kinematics, Lorentz invariant and covariant

parton shower

We defined a new formalism for describing the

SUMMARY